Computer Algebra Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.5-Secant/120-4.5.1.4-d-tan-^n-a+b-sec-^m

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [365]. This is test number [120].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.73 (364)	0.27 (1)
Mathematica	94.52 (345)	5.48 (20)
Maple	90.68 (331)	9.32 (34)
Fricas	71.23 (260)	28.77 (105)
Giac	60.27 (220)	39.73 (145)
Maxima	51.78 (189)	48.22 (176)
Mupad	49.59 (181)	50.41 (184)
Sympy	10.96 (40)	89.04 (325)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

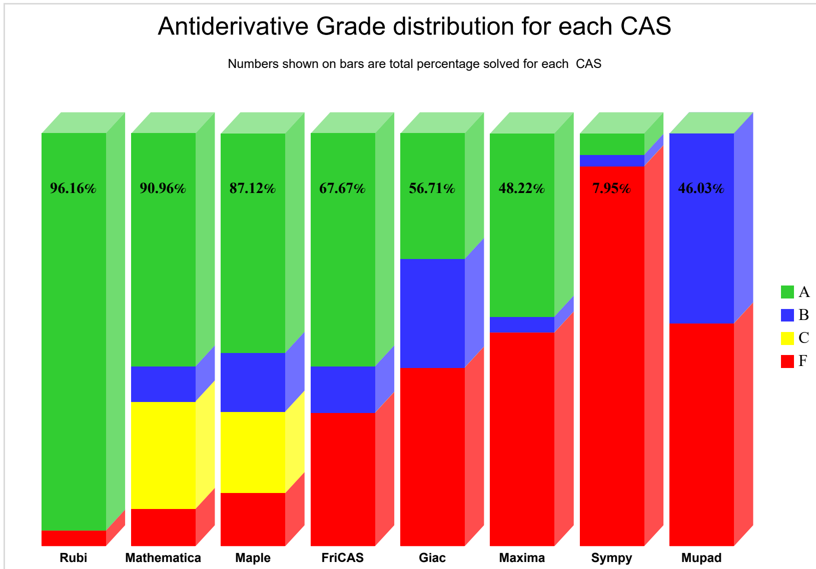
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

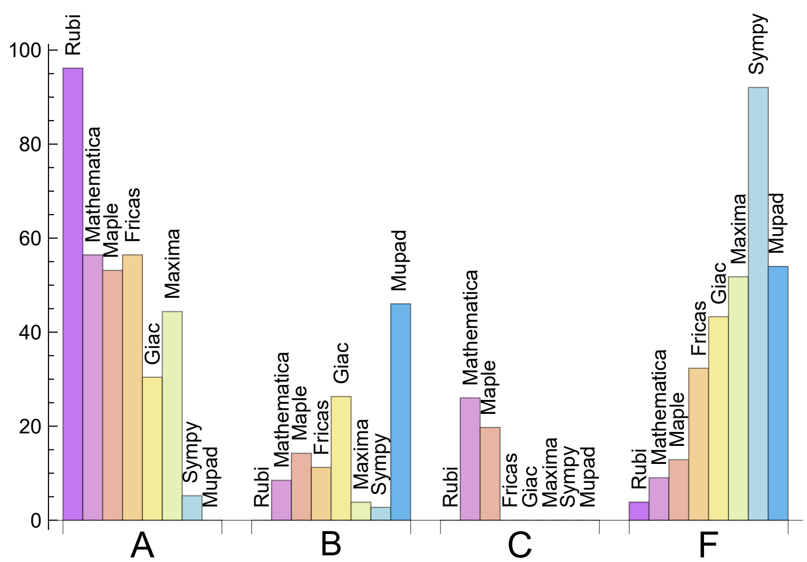
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.164	0.000	0.000	3.836
Mathematica	56.438	8.493	26.027	9.041
Fricas	56.438	11.233	0.000	32.329
Maple	53.151	14.247	19.726	12.877
Maxima	44.384	3.836	0.000	51.781
Giac	30.411	26.301	0.000	43.288
Sympy	5.205	2.740	0.000	92.055
Mupad	0.000	46.027	0.000	53.973

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	20	100.00	0.00	0.00
Maple	34	100.00	0.00	0.00
Fricas	105	41.90	58.10	0.00
Giac	145	97.93	0.00	2.07
Maxima	176	67.05	13.64	19.32
Mupad	184	0.00	100.00	0.00
Sympy	325	86.46	13.54	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.23
Maxima	0.58
Fricas	0.71
Giac	1.54
Mathematica	4.02
Maple	5.54
Sympy	7.51
Mupad	15.50

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	109.20	1.15	94.00	1.00
Sympy	137.72	1.95	97.00	1.36
Rubi	169.94	1.01	129.50	1.00
Giac	224.35	2.01	164.00	1.62
Mathematica	229.78	1.45	125.00	0.97
Fricas	336.38	2.45	178.00	1.60
Maple	366.87	1.72	143.00	1.19
Mupad	418.38	2.89	118.00	1.45

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

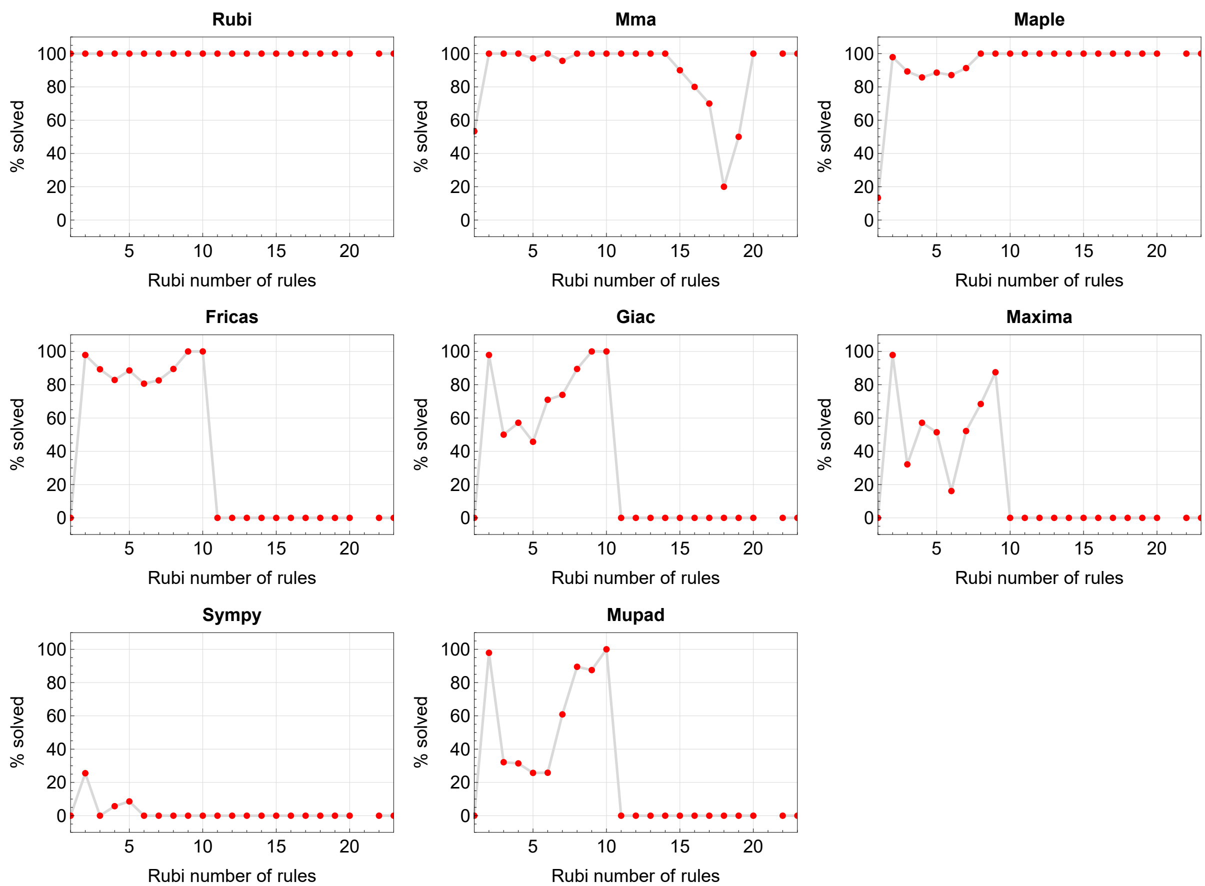


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

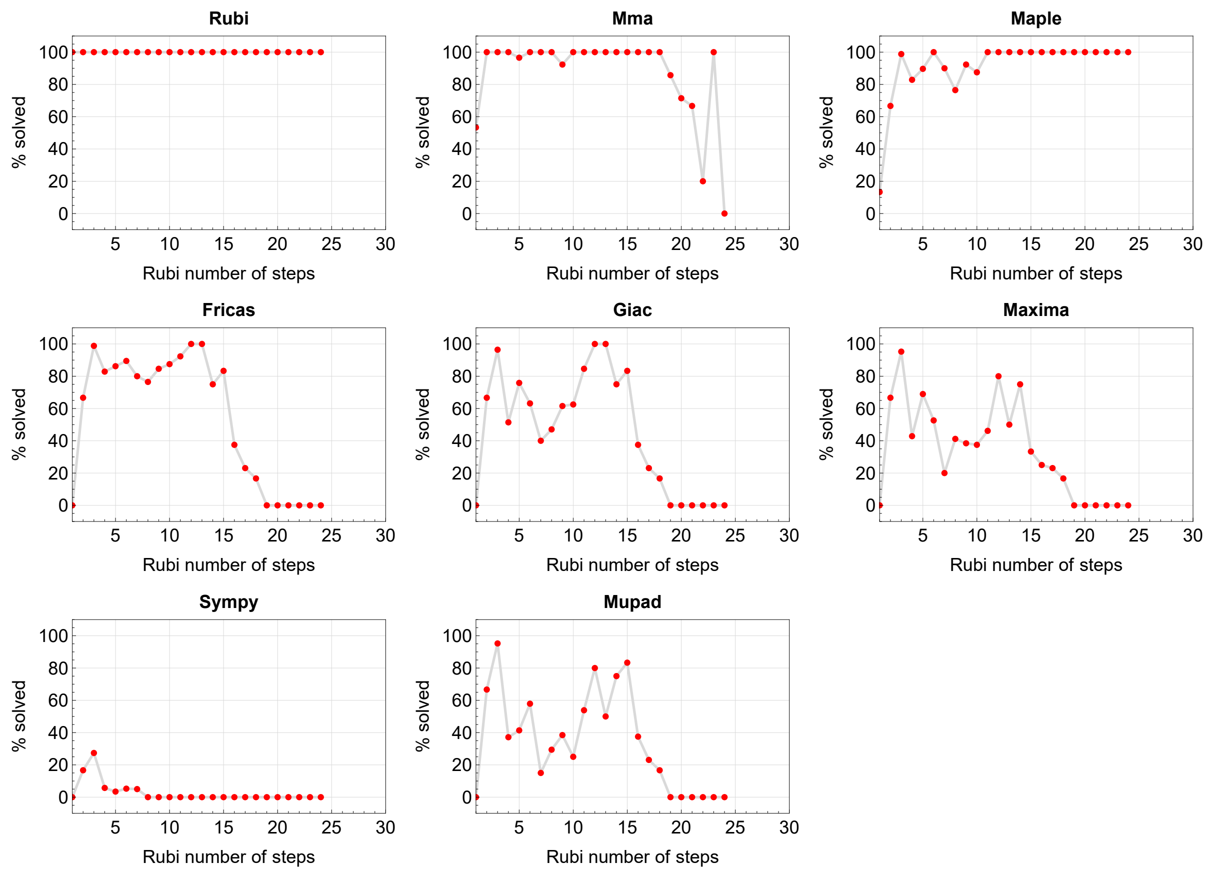


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

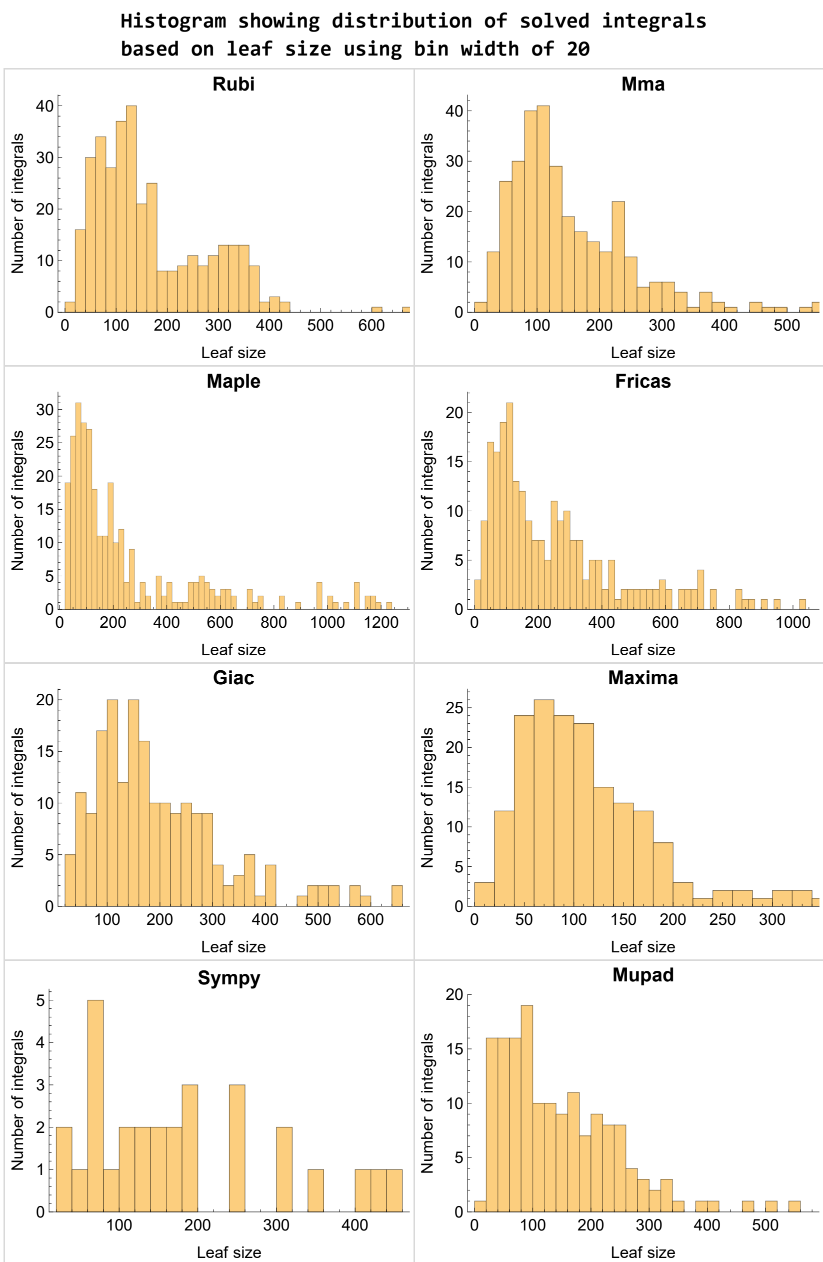


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

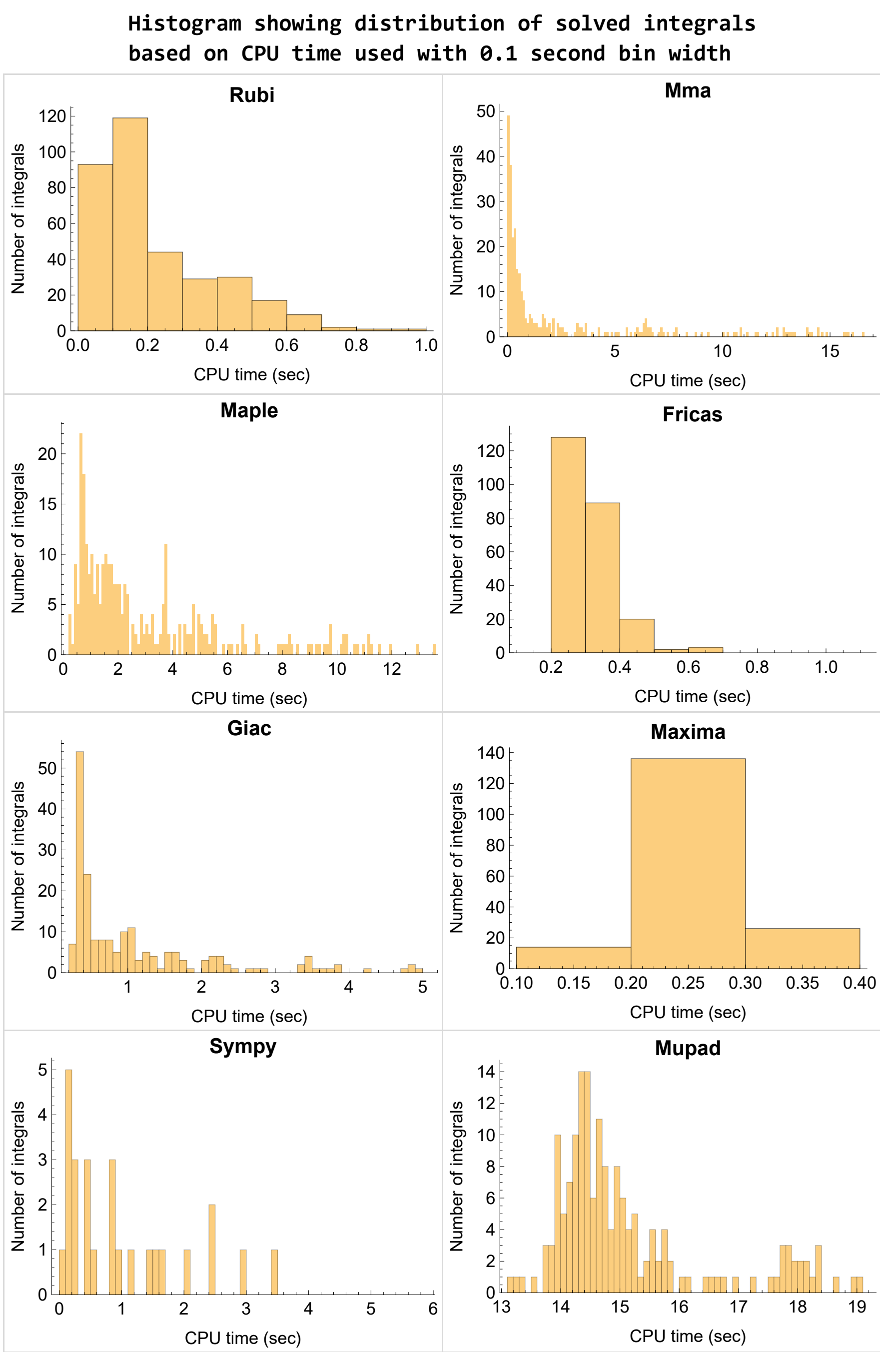


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

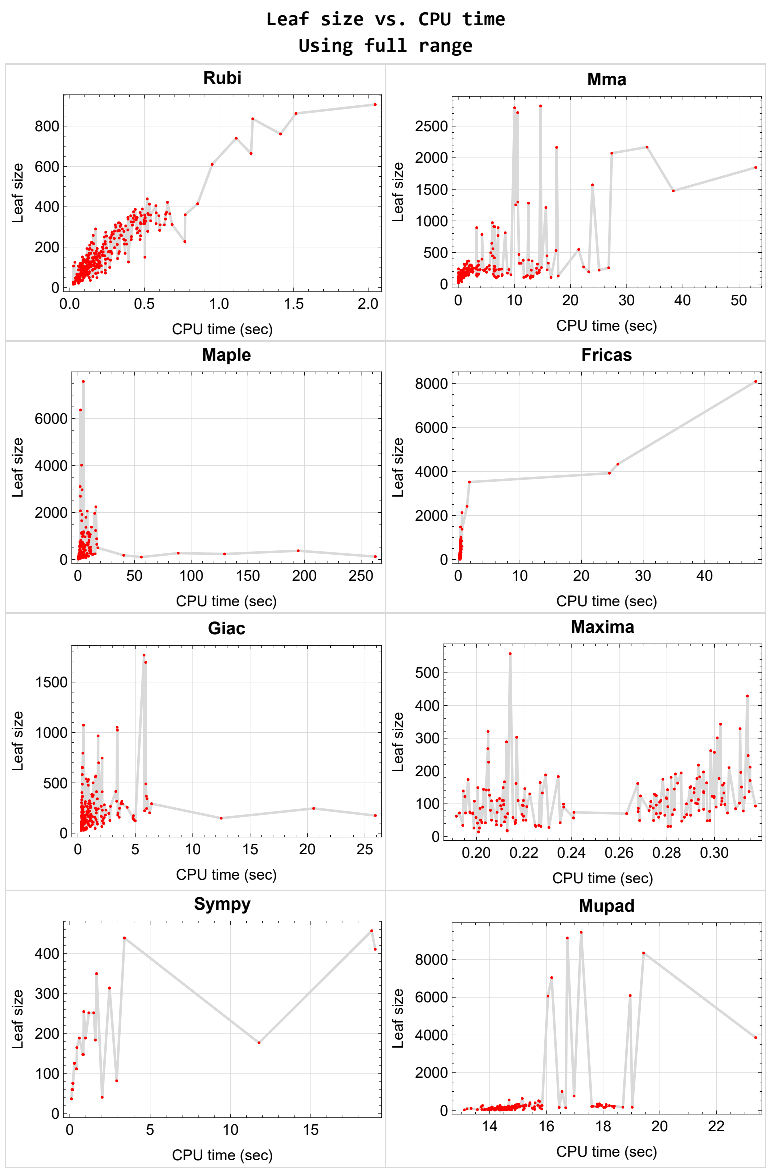


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{348, 349, 350, 351, 352, 358, 359, 360, 361, 362, 363, 364, 365\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{107, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 133, 134, 143, 145, 156, 157, 169, 177, 178, 179, 189, 190, 192, 193, 201, 204, 205, 207, 226, 227, 229, 230, 231, 232, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 312, 313, 314, 315, 316, 317, 332, 340, 341, 347\}$

Maple $\{104, 111, 118, 119, 120, 121, 122, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 155, 167, 189, 192, 201, 204, 207, 243, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 322, 329, 330, 338, 339, 342\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
```

```
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

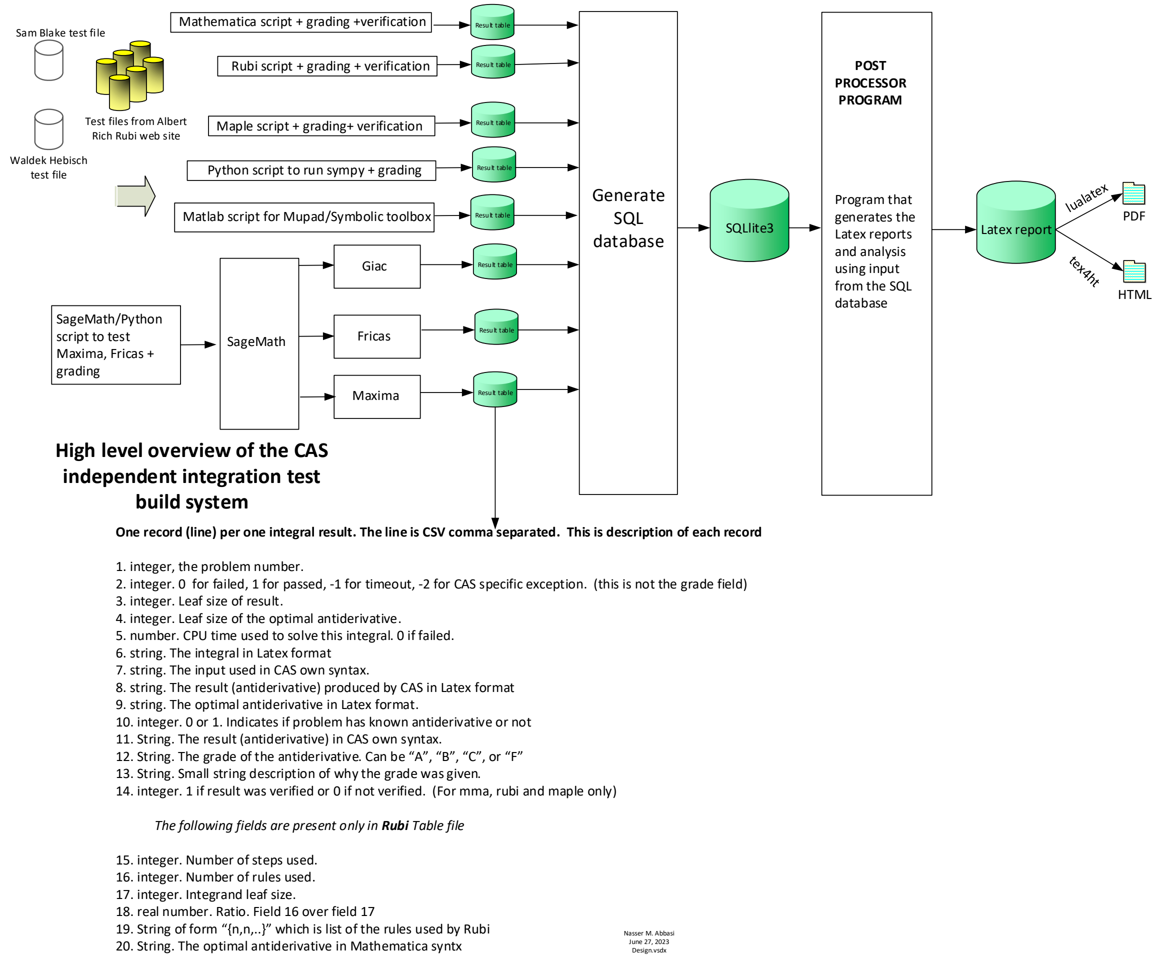
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 353, 354, 355, 356, 357 }

B grade { }

C grade { }

F normal fail { 347 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 56, 57, 58, 59, 60, 61, 62, 63, 68, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 101, 103, 135, 136, 137, 138, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173, 177, 179, 180, 181, 182, 183, 184, 190, 191, 192, 193, 194, 195, 196, 202, 203, 204, 205, 206, 207, 209, 210, 211, 219, 220, 221, 222, 223, 224, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 309, 310, 311, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 340, 341, 343, 344, 345, 346, 353, 354, 355, 356, 357 }

B grade { 6, 50, 64, 65, 66, 67, 69, 70, 79, 80, 81, 84, 97, 98, 99, 100, 102, 201, 226, 227, 229, 230, 231, 232, 294, 295, 298, 307, 308, 342, 347 }

C grade { 14, 15, 16, 17, 18, 32, 33, 34, 35, 36, 51, 52, 53, 54, 55, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 133, 134, 139, 140, 143, 152, 169, 174, 175, 176, 178, 185, 186, 187, 188, 189, 197, 198, 199, 200, 213, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 267, 268, 269, 270, 283, 284, 285, 312, 313, 314, 315, 316, 317, 335, 336, 337, 338, 339 }

F normal fail { 127, 128, 129, 130, 132, 208, 212, 214, 215, 216, 217, 218, 225, 228, 250, 251, 252, 253, 254, 255 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 28, 29, 30, 31, 37, 38, 39, 40, 41, 42, 56, 57, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 99, 100, 101, 102, 103, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 165, 166, 167, 168, 171, 172, 173, 176, 177, 178, 179, 181, 182, 183, 184, 185, 189, 191, 194, 195, 196, 197, 201, 203, 206, 234, 236, 237, 239, 240, 241, 242, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 320, 324, 328, 333, 337 }

B grade { 104, 106, 143, 163, 164, 169, 170, 174, 175, 180, 186, 187, 188, 190, 192, 193, 198, 199, 200, 202, 204, 205, 207, 294, 295, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 341, 342, 343 }

C grade { 14, 15, 16, 17, 25, 26, 27, 32, 33, 34, 35, 36, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 79, 80, 81, 82, 95, 96, 97, 98, 107, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 233, 235, 238, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 282 }

F normal fail { 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 153, 154, 155, 157, 158, 159, 160, 161, 162, 165, 166, 167, 169, 170, 171, 172, 176, 177, 178, 179, 180, 181, 182, 183, 184, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 308, 310, 318, 319, 320, 326, 327, 335, 336 }

B grade { 9, 13, 17, 18, 28, 46, 150, 151, 152, 156, 163, 164, 168, 173, 174, 175, 185, 186, 187, 188, 196, 197, 198, 199, 200, 266, 293, 298, 305, 306, 307, 309, 311, 321, 322, 328, 329, 330, 337, 338, 339 }

C grade { }

F normal fail { 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 323, 324, 325, 331, 332, 333, 334, 340, 341, 342, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 343 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 100, 101, 102, 103, 135, 136, 137, 147, 148, 149, 159, 160, 161, 171, 172, 173, 183, 184, 185, 197, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 299, 300, 301, 302, 303, 304, 305, 318, 319, 320, 326, 327, 328, 335, 336, 337 }

B grade { 64, 65, 66, 67, 79, 80, 81, 96, 97, 98, 99, 195, 196, 306 }

C grade { }

F normal fail { 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 131, 133, 134, 138, 139, 141, 142, 143, 144, 145, 150, 151, 155, 156, 162, 163, 167, 168, 169, 174, 175, 176, 177, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 191, 192, 193, 198, 199, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 244, 245, 246, 247, 248, 249, 250, 252, 254, 312, 313, 314, 315, 316, 317, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 338, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 127, 128, 130, 132, 140, 146, 152, 153, 154, 157, 158, 164, 165, 166, 170, 194, 200, 206, 243, 251, 253, 255, 339, 340 }

F(-2) exception fail { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 294, 295, 296, 297, 298, 307, 308, 309, 310, 311 }

Giac

A grade { 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 171, 172, 173, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 260, 267, 276, 279, 282, 309, 310, 311 }

B grade { 1, 2, 3, 4, 5, 6, 7, 13, 20, 21, 22, 23, 25, 38, 39, 40, 41, 42, 43, 44, 57, 58, 59, 66, 67, 71, 72, 73, 81, 87, 88, 89, 178, 179, 189, 190, 201, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 318, 319, 320, 321, 322, 326, 327, 328, 330, 336, 337 }

C grade { }

F normal fail { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 323, 324, 325, 331, 332, 333, 334, 335, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

F(-1) timedout fail { }

F(-2) exception fail { 329, 338, 339 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 137, 149, 161, 173, 185, 197, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 320, 328, 337 }

C grade { }

F normal fail { }

F(-1) timedout fail { 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 19, 21, 23, 40, 41, 256, 257, 258, 259, 271, 272, 273, 274, 275 }

B grade { 20, 22, 37, 38, 39, 60, 75, 90, 91, 290 }

C grade { }

F normal fail { 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 42, 43, 44, 47, 48, 49, 50, 51, 52, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 160, 161, 167, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 276, 277, 278, 279, 280, 281, 282, 283, 284, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 18, 28, 35, 36, 45, 46, 53, 54, 55, 111, 117, 118, 119, 120, 127, 128, 129, 130, 152, 157, 158, 159, 162, 163, 164, 165, 166, 168, 169, 170, 207, 229, 233, 238, 239, 248, 249, 252, 253, 254, 255, 285, 361, 362 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	134	100	116	123	184	293	259
N.S.	1	1.00	0.89	0.66	0.77	0.81	1.22	1.94	1.72
time (sec)	N/A	0.084	0.570	1.458	0.210	0.291	1.592	6.418	17.949

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	106	82	94	101	148	247	204
N.S.	1	1.00	0.90	0.69	0.80	0.86	1.25	2.09	1.73
time (sec)	N/A	0.069	0.507	1.030	0.220	0.297	0.844	3.630	17.600

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	82	60	72	79	112	201	151
N.S.	1	1.00	0.94	0.69	0.83	0.91	1.29	2.31	1.74
time (sec)	N/A	0.059	0.392	0.862	0.197	0.279	0.414	1.604	18.381

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	44	50	57	76	155	96
N.S.	1	1.00	0.96	0.77	0.88	1.00	1.33	2.72	1.68
time (sec)	N/A	0.050	0.141	0.898	0.202	0.281	0.204	0.637	15.219

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	34	37	106	40
N.S.	1	1.00	1.00	0.80	1.04	1.36	1.48	4.24	1.60
time (sec)	N/A	0.024	0.014	0.292	0.199	0.298	0.098	0.324	14.700

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	38	33	14	16	0	58	34
N.S.	1	1.00	2.38	2.06	0.88	1.00	0.00	3.62	2.12
time (sec)	N/A	0.023	0.018	0.444	0.201	0.278	0.000	0.279	14.579

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	114	75	42	69	0	103	46
N.S.	1	1.00	2.00	1.32	0.74	1.21	0.00	1.81	0.81
time (sec)	N/A	0.052	0.521	0.498	0.235	0.279	0.000	0.314	13.950

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	175	111	86	150	0	149	88
N.S.	1	1.00	1.84	1.17	0.91	1.58	0.00	1.57	0.93
time (sec)	N/A	0.074	0.162	0.697	0.202	0.272	0.000	0.361	13.980

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	165	151	126	241	0	197	118
N.S.	1	1.00	1.24	1.14	0.95	1.81	0.00	1.48	0.89
time (sec)	N/A	0.110	0.441	0.831	0.206	0.285	0.000	0.386	14.184

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	241	180	164	156	0	174	242
N.S.	1	1.00	1.87	1.40	1.27	1.21	0.00	1.35	1.88
time (sec)	N/A	0.156	0.052	1.674	0.297	0.319	0.000	4.832	15.177

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	179	143	134	134	0	146	188
N.S.	1	1.00	1.75	1.40	1.31	1.31	0.00	1.43	1.84
time (sec)	N/A	0.115	0.052	1.460	0.291	0.296	0.000	2.217	14.919

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	117	104	102	112	0	118	134
N.S.	1	1.00	1.60	1.42	1.40	1.53	0.00	1.62	1.84
time (sec)	N/A	0.075	0.035	1.537	0.311	0.284	0.000	0.940	14.871

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	60	67	65	87	0	88	80
N.S.	1	1.00	1.33	1.49	1.44	1.93	0.00	1.96	1.78
time (sec)	N/A	0.039	0.023	0.698	0.290	0.285	0.000	0.504	14.215

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	43	26	31	33	0	26	19
N.S.	1	1.00	1.65	1.00	1.19	1.27	0.00	1.00	0.73
time (sec)	N/A	0.027	0.017	0.443	0.282	0.253	0.000	0.299	14.177

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	62	62	59	72	0	56	53
N.S.	1	1.00	1.13	1.13	1.07	1.31	0.00	1.02	0.96
time (sec)	N/A	0.060	0.029	0.912	0.287	0.293	0.000	0.328	14.331

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	118	79	139	0	83	156
N.S.	1	1.00	0.94	1.40	0.94	1.65	0.00	0.99	1.86
time (sec)	N/A	0.093	0.035	1.069	0.293	0.261	0.000	0.347	14.441

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	92	150	100	210	0	113	204
N.S.	1	1.00	0.83	1.35	0.90	1.89	0.00	1.02	1.84
time (sec)	N/A	0.132	0.040	1.409	0.293	0.276	0.000	0.393	14.787

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	111	205	119	279	0	140	252
N.S.	1	1.00	0.79	1.46	0.85	1.99	0.00	1.00	1.80
time (sec)	N/A	0.182	0.049	1.777	0.300	0.293	0.000	0.429	15.269

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	193	114	149	156	314	342	308
N.S.	1	1.00	1.01	0.59	0.78	0.81	1.64	1.78	1.60
time (sec)	N/A	0.115	1.193	2.273	0.211	0.291	2.473	6.023	18.108

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	159	84	110	117	252	292	249
N.S.	1	1.00	1.20	0.64	0.83	0.89	1.91	2.21	1.89
time (sec)	N/A	0.090	0.743	1.724	0.209	0.299	1.187	3.875	18.220

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	125	74	97	104	189	242	192
N.S.	1	1.00	1.04	0.62	0.81	0.87	1.58	2.02	1.60
time (sec)	N/A	0.079	0.326	1.364	0.211	0.277	0.595	1.775	17.792

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	83	46	58	65	126	192	133
N.S.	1	1.00	1.28	0.71	0.89	1.00	1.94	2.95	2.05
time (sec)	N/A	0.065	0.137	1.078	0.200	0.281	0.297	0.772	16.683

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	34	43	52	60	142	76
N.S.	1	1.00	1.06	0.71	0.90	1.08	1.25	2.96	1.58
time (sec)	N/A	0.039	0.082	0.440	0.216	0.279	0.137	0.403	14.193

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	29	49	31	35	0	64	36
N.S.	1	1.00	0.83	1.40	0.89	1.00	0.00	1.83	1.03
time (sec)	N/A	0.042	0.032	0.497	0.225	0.295	0.000	0.299	14.059

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	56	69	34	48	0	111	50
N.S.	1	1.00	1.40	1.72	0.85	1.20	0.00	2.78	1.25
time (sec)	N/A	0.061	0.060	0.750	0.226	0.271	0.000	0.345	13.861

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	86	114	72	122	0	138	62
N.S.	1	1.00	1.01	1.34	0.85	1.44	0.00	1.62	0.73
time (sec)	N/A	0.075	0.216	1.019	0.199	0.282	0.000	0.388	14.630

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	114	171	109	191	0	186	113
N.S.	1	1.00	0.90	1.35	0.86	1.50	0.00	1.46	0.89
time (sec)	N/A	0.101	0.170	1.595	0.206	0.295	0.000	0.413	13.990

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	146	213	165	322	0	238	149
N.S.	1	1.00	0.86	1.26	0.98	1.91	0.00	1.41	0.88
time (sec)	N/A	0.129	0.272	2.887	0.227	0.291	0.000	0.442	14.456

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	218	168	151	165	0	180	234
N.S.	1	1.00	1.35	1.04	0.94	1.02	0.00	1.12	1.45
time (sec)	N/A	0.219	0.702	2.176	0.311	0.296	0.000	2.398	15.388

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	150	130	119	139	0	148	174
N.S.	1	1.00	1.26	1.09	1.00	1.17	0.00	1.24	1.46
time (sec)	N/A	0.167	0.341	1.630	0.313	0.300	0.000	1.090	14.769

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	81	92	83	111	0	99	101
N.S.	1	1.00	1.12	1.28	1.15	1.54	0.00	1.38	1.40
time (sec)	N/A	0.128	0.219	0.797	0.295	0.297	0.000	0.623	13.347

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	46	30	48	42	0	31	24
N.S.	1	1.00	1.31	0.86	1.37	1.20	0.00	0.89	0.69
time (sec)	N/A	0.086	0.035	1.106	0.297	0.279	0.000	0.324	13.106

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	87	53	77	81	0	50	39
N.S.	1	1.00	1.26	0.77	1.12	1.17	0.00	0.72	0.57
time (sec)	N/A	0.127	0.351	1.319	0.298	0.280	0.000	0.352	13.936

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	105	100	97	118	0	80	78
N.S.	1	1.00	0.98	0.93	0.91	1.10	0.00	0.75	0.73
time (sec)	N/A	0.160	0.336	1.625	0.303	0.288	0.000	0.386	13.984

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	121	143	117	173	0	112	182
N.S.	1	1.00	0.87	1.03	0.84	1.24	0.00	0.81	1.31
time (sec)	N/A	0.177	0.581	2.565	0.303	0.292	0.000	0.414	14.325

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	141	188	137	274	0	145	230
N.S.	1	1.00	0.79	1.05	0.77	1.53	0.00	0.81	1.28
time (sec)	N/A	0.211	1.766	4.477	0.295	0.285	0.000	0.484	15.173

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	214	124	162	169	439	367	337
N.S.	1	1.00	1.02	0.59	0.77	0.80	2.09	1.75	1.60
time (sec)	N/A	0.134	0.783	3.026	0.217	0.313	3.405	5.959	17.795

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	167	86	110	117	350	317	278
N.S.	1	1.00	1.22	0.63	0.80	0.85	2.55	2.31	2.03
time (sec)	N/A	0.098	0.464	2.221	0.211	0.304	1.667	3.829	17.913

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	140	84	110	117	255	267	221
N.S.	1	1.00	1.01	0.61	0.80	0.85	1.85	1.93	1.60
time (sec)	N/A	0.092	0.294	1.659	0.211	0.288	0.871	2.031	17.653

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	92	66	84	91	165	217	162
N.S.	1	1.00	0.93	0.67	0.85	0.92	1.67	2.19	1.64
time (sec)	N/A	0.084	0.235	1.475	0.211	0.302	0.442	0.929	18.074

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	44	58	65	76	167	105
N.S.	1	1.00	0.97	0.67	0.88	0.98	1.15	2.53	1.59
time (sec)	N/A	0.048	0.115	0.694	0.215	0.274	0.193	0.434	14.589

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	36	79	43	61	0	145	86
N.S.	1	1.00	0.75	1.65	0.90	1.27	0.00	3.02	1.79
time (sec)	N/A	0.052	0.064	0.681	0.202	0.289	0.000	0.353	13.956

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	46	69	34	50	0	109	48
N.S.	1	1.00	1.15	1.72	0.85	1.25	0.00	2.72	1.20
time (sec)	N/A	0.067	0.103	1.410	0.210	0.289	0.000	0.350	14.326

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	72	94	59	82	0	138	78
N.S.	1	1.00	1.18	1.54	0.97	1.34	0.00	2.26	1.28
time (sec)	N/A	0.080	0.132	1.575	0.212	0.283	0.000	0.406	14.257

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	102	136	96	178	0	165	94
N.S.	1	1.00	0.95	1.27	0.90	1.66	0.00	1.54	0.88
time (sec)	N/A	0.091	0.503	2.846	0.219	0.303	0.000	0.427	14.407

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	130	193	142	272	0	213	130
N.S.	1	1.00	0.87	1.30	0.95	1.83	0.00	1.43	0.87
time (sec)	N/A	0.124	0.271	5.226	0.205	0.289	0.000	0.484	14.025

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	292	228	262	178	0	196	263
N.S.	1	1.00	1.23	0.96	1.11	0.75	0.00	0.83	1.11
time (sec)	N/A	0.412	1.189	2.927	0.298	0.317	0.000	2.690	15.036

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	200	194	210	152	0	164	203
N.S.	1	1.00	1.18	1.15	1.24	0.90	0.00	0.97	1.20
time (sec)	N/A	0.282	0.457	2.261	0.306	0.285	0.000	1.278	14.832

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	107	139	147	113	0	132	146
N.S.	1	1.00	1.09	1.42	1.50	1.15	0.00	1.35	1.49
time (sec)	N/A	0.196	0.374	1.000	0.291	0.302	0.000	0.738	14.309

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	109	71	85	84	0	66	35
N.S.	1	1.00	2.22	1.45	1.73	1.71	0.00	1.35	0.71
time (sec)	N/A	0.130	0.626	1.302	0.309	0.289	0.000	0.368	13.978

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	85	53	90	82	0	50	39
N.S.	1	1.00	1.23	0.77	1.30	1.19	0.00	0.72	0.57
time (sec)	N/A	0.174	0.260	1.729	0.301	0.279	0.000	0.350	13.568

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	105	76	122	118	0	66	62
N.S.	1	1.00	0.98	0.71	1.14	1.10	0.00	0.62	0.58
time (sec)	N/A	0.211	0.473	2.892	0.300	0.271	0.000	0.397	14.239

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	123	121	152	160	0	96	91
N.S.	1	1.00	0.87	0.86	1.08	1.13	0.00	0.68	0.65
time (sec)	N/A	0.250	0.758	4.797	0.290	0.269	0.000	0.439	14.505

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	141	166	182	235	0	128	206
N.S.	1	1.00	0.79	0.93	1.02	1.31	0.00	0.72	1.15
time (sec)	N/A	0.271	1.793	7.813	0.294	0.282	0.000	0.500	15.082

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	159	209	212	314	0	161	254
N.S.	1	1.00	0.75	0.98	1.00	1.47	0.00	0.76	1.19
time (sec)	N/A	0.318	3.675	12.938	0.315	0.308	0.000	0.571	15.849

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	137	86	90	95	0	245	208
N.S.	1	1.00	1.01	0.64	0.67	0.70	0.00	1.81	1.54
time (sec)	N/A	0.105	0.414	1.239	0.237	0.293	0.000	5.983	18.113

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	103	62	70	75	0	201	153
N.S.	1	1.00	1.06	0.64	0.72	0.77	0.00	2.07	1.58
time (sec)	N/A	0.085	0.194	1.113	0.213	0.307	0.000	3.487	18.022

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	48	50	55	0	157	99
N.S.	1	1.00	0.98	0.73	0.76	0.83	0.00	2.38	1.50
time (sec)	N/A	0.078	0.159	1.018	0.220	0.280	0.000	1.488	14.395

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	21	24	28	33	0	111	44
N.S.	1	1.00	0.75	0.86	1.00	1.18	0.00	3.96	1.57
time (sec)	N/A	0.064	0.054	0.617	0.230	0.274	0.000	0.598	13.941

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	27	17	19	41	31	21
N.S.	1	1.00	1.12	1.59	1.00	1.12	2.41	1.82	1.24
time (sec)	N/A	0.033	0.015	0.241	0.213	0.289	2.017	0.322	14.494

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	67	43	47	60	0	86	49
N.S.	1	1.00	1.10	0.70	0.77	0.98	0.00	1.41	0.80
time (sec)	N/A	0.069	0.088	0.522	0.208	0.279	0.000	0.292	14.418

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	97	67	91	139	0	157	76
N.S.	1	1.00	0.94	0.65	0.88	1.35	0.00	1.52	0.74
time (sec)	N/A	0.101	0.421	0.687	0.221	0.282	0.000	0.338	15.834

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	135	91	130	217	0	211	132
N.S.	1	1.00	0.93	0.63	0.90	1.50	0.00	1.46	0.91
time (sec)	N/A	0.126	0.382	0.649	0.222	0.283	0.000	0.356	14.283

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	301	194	329	127	0	149	193
N.S.	1	1.00	2.87	1.85	3.13	1.21	0.00	1.42	1.84
time (sec)	N/A	0.168	2.327	1.193	0.311	0.289	0.000	4.799	15.578

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	893	151	247	107	0	123	139
N.S.	1	1.00	11.45	1.94	3.17	1.37	0.00	1.58	1.78
time (sec)	N/A	0.132	7.103	0.954	0.314	0.277	0.000	2.134	15.212

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	241	104	163	86	0	96	83
N.S.	1	1.00	4.92	2.12	3.33	1.76	0.00	1.96	1.69
time (sec)	N/A	0.092	1.200	0.662	0.304	0.274	0.000	0.886	14.402

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	60	49	78	35	0	50	25
N.S.	1	1.00	2.86	2.33	3.71	1.67	0.00	2.38	1.19
time (sec)	N/A	0.059	0.372	0.530	0.312	0.271	0.000	0.455	13.829

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	100	59	93	64	0	66	65
N.S.	1	1.00	1.64	0.97	1.52	1.05	0.00	1.08	1.07
time (sec)	N/A	0.115	1.021	0.616	0.317	0.286	0.000	0.326	14.217

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	254	85	137	134	0	98	158
N.S.	1	1.00	2.89	0.97	1.56	1.52	0.00	1.11	1.80
time (sec)	N/A	0.156	1.074	0.665	0.315	0.282	0.000	0.351	14.415

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	359	111	177	198	0	127	206
N.S.	1	1.00	3.07	0.95	1.51	1.69	0.00	1.09	1.76
time (sec)	N/A	0.193	1.632	0.733	0.302	0.279	0.000	0.393	14.903

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	125	78	80	85	0	223	193
N.S.	1	1.00	1.04	0.65	0.67	0.71	0.00	1.86	1.61
time (sec)	N/A	0.086	0.378	1.679	0.206	0.297	0.000	5.808	17.773

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	83	46	50	55	0	180	135
N.S.	1	1.00	1.28	0.71	0.77	0.85	0.00	2.77	2.08
time (sec)	N/A	0.074	0.170	1.334	0.221	0.287	0.000	3.471	15.771

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	38	40	45	0	136	77
N.S.	1	1.00	1.06	0.79	0.83	0.94	0.00	2.83	1.60
time (sec)	N/A	0.059	0.090	0.847	0.216	0.287	0.000	1.580	14.186

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	30	29	31	31	0	33	22
N.S.	1	1.00	0.91	0.88	0.94	0.94	0.00	1.00	0.67
time (sec)	N/A	0.056	0.047	0.692	0.227	0.303	0.000	0.647	14.354

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	56	37	35	43	177	57	35
N.S.	1	1.00	1.56	1.03	0.97	1.19	4.92	1.58	0.97
time (sec)	N/A	0.045	0.106	0.244	0.225	0.286	11.780	0.367	14.186

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	83	55	74	106	0	117	62
N.S.	1	1.00	1.02	0.68	0.91	1.31	0.00	1.44	0.77
time (sec)	N/A	0.077	0.141	0.590	0.241	0.300	0.000	0.319	14.246

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	121	79	110	162	0	186	89
N.S.	1	1.00	0.98	0.64	0.89	1.32	0.00	1.51	0.72
time (sec)	N/A	0.108	0.262	0.704	0.220	0.310	0.000	0.369	14.182

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	154	103	167	283	0	236	151
N.S.	1	1.00	0.93	0.62	1.01	1.72	0.00	1.43	0.92
time (sec)	N/A	0.130	0.610	0.717	0.212	0.297	0.000	0.420	14.320

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	495	161	301	117	0	136	179
N.S.	1	1.00	4.16	1.35	2.53	0.98	0.00	1.14	1.50
time (sec)	N/A	0.239	5.790	1.423	0.301	0.287	0.000	4.858	15.047

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	767	106	196	97	0	99	111
N.S.	1	1.00	10.65	1.47	2.72	1.35	0.00	1.38	1.54
time (sec)	N/A	0.189	7.081	1.057	0.311	0.296	0.000	2.220	14.478

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	177	71	123	66	0	79	61
N.S.	1	1.00	5.21	2.09	3.62	1.94	0.00	2.32	1.79
time (sec)	N/A	0.080	1.050	0.717	0.299	0.272	0.000	0.949	14.212

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	35	42	30	49	42	0	29	22
N.S.	1	1.06	1.27	0.91	1.48	1.27	0.00	0.88	0.67
time (sec)	N/A	0.138	0.020	0.623	0.298	0.283	0.000	0.508	13.988

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	149	72	113	106	0	84	78
N.S.	1	1.00	1.39	0.67	1.06	0.99	0.00	0.79	0.73
time (sec)	N/A	0.208	1.483	0.658	0.293	0.274	0.000	0.337	14.336

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	314	98	157	154	0	114	182
N.S.	1	1.00	2.26	0.71	1.13	1.11	0.00	0.82	1.31
time (sec)	N/A	0.239	1.365	0.619	0.303	0.279	0.000	0.398	14.935

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	276	124	197	250	0	144	230
N.S.	1	1.00	1.54	0.69	1.10	1.40	0.00	0.80	1.28
time (sec)	N/A	0.276	6.498	0.670	0.295	0.292	0.000	0.417	15.781

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	140	86	90	95	0	246	225
N.S.	1	1.00	1.02	0.63	0.66	0.69	0.00	1.80	1.64
time (sec)	N/A	0.098	0.455	4.210	0.203	0.286	0.000	20.535	18.396

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	93	66	70	75	0	202	167
N.S.	1	1.00	0.94	0.67	0.71	0.76	0.00	2.04	1.69
time (sec)	N/A	0.081	0.293	2.391	0.198	0.287	0.000	6.252	18.699

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	64	46	50	55	0	158	109
N.S.	1	1.00	0.98	0.71	0.77	0.85	0.00	2.43	1.68
time (sec)	N/A	0.071	0.140	1.622	0.206	0.275	0.000	3.598	15.504

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	36	37	45	53	0	112	72
N.S.	1	1.00	0.78	0.80	0.98	1.15	0.00	2.43	1.57
time (sec)	N/A	0.065	0.140	1.035	0.212	0.292	0.000	1.666	14.651

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	33	30	36	42	457	56	36
N.S.	1	1.00	0.94	0.86	1.03	1.20	13.06	1.60	1.03
time (sec)	N/A	0.073	0.051	0.807	0.208	0.293	18.795	0.736	14.508

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	79	49	60	76	411	87	48
N.S.	1	1.00	1.41	0.88	1.07	1.36	7.34	1.55	0.86
time (sec)	N/A	0.059	0.113	0.322	0.212	0.296	19.012	0.438	14.629

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	97	67	98	151	0	143	75
N.S.	1	1.00	0.96	0.66	0.97	1.50	0.00	1.42	0.74
time (sec)	N/A	0.085	0.230	0.578	0.211	0.308	0.000	0.354	14.260

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	139	91	146	240	0	212	102
N.S.	1	1.00	0.97	0.64	1.02	1.68	0.00	1.48	0.71
time (sec)	N/A	0.120	0.486	0.749	0.220	0.286	0.000	0.409	14.658

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	169	115	188	317	0	261	170
N.S.	1	1.00	0.91	0.62	1.02	1.71	0.00	1.41	0.92
time (sec)	N/A	0.150	0.891	0.743	0.229	0.291	0.000	0.490	14.537

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	362	227	429	147	0	175	265
N.S.	1	1.00	1.53	0.96	1.81	0.62	0.00	0.74	1.12
time (sec)	N/A	0.471	3.588	5.024	0.314	0.306	0.000	25.908	15.698

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	303	195	343	127	0	149	208
N.S.	1	1.00	1.79	1.15	2.03	0.75	0.00	0.88	1.23
time (sec)	N/A	0.371	3.629	3.214	0.303	0.276	0.000	12.476	15.036

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	230	138	257	97	0	123	148
N.S.	1	1.00	2.32	1.39	2.60	0.98	0.00	1.24	1.49
time (sec)	N/A	0.268	2.075	2.007	0.300	0.304	0.000	4.996	14.976

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	241	105	171	87	0	97	92
N.S.	1	1.00	3.65	1.59	2.59	1.32	0.00	1.47	1.39
time (sec)	N/A	0.122	1.727	1.282	0.315	0.300	0.000	2.249	14.727

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	48	117	61	98	83	0	63	37
N.S.	1	1.04	2.54	1.33	2.13	1.80	0.00	1.37	0.80
time (sec)	N/A	0.182	0.853	0.768	0.304	0.300	0.000	1.073	14.480

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	71	125	45	72	80	0	50	35
N.S.	1	1.18	2.08	0.75	1.20	1.33	0.00	0.83	0.58
time (sec)	N/A	0.239	0.324	0.680	0.305	0.269	0.000	0.608	13.810

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	252	85	133	142	0	99	91
N.S.	1	1.00	1.76	0.59	0.93	0.99	0.00	0.69	0.64
time (sec)	N/A	0.320	1.638	0.637	0.304	0.275	0.000	0.396	14.088

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	366	111	177	216	0	131	205
N.S.	1	1.00	2.07	0.63	1.00	1.22	0.00	0.74	1.16
time (sec)	N/A	0.332	1.885	0.640	0.293	0.283	0.000	0.475	14.279

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	394	137	218	282	0	160	254
N.S.	1	1.00	1.83	0.64	1.01	1.31	0.00	0.74	1.18
time (sec)	N/A	0.381	4.220	0.901	0.293	0.291	0.000	0.533	14.919

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	310	310	186	580	0	0	0	0	0
N.S.	1	1.00	0.60	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.422	2.575	6.000	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	214	372	0	0	0	0	0
N.S.	1	1.00	0.76	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	2.153	5.588	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	182	505	0	0	0	0	0
N.S.	1	1.00	0.67	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	1.381	5.040	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	220	141	0	0	0	0	0
N.S.	1	1.00	0.90	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.870	4.422	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	196	511	0	0	0	0	0
N.S.	1	1.00	0.64	1.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.375	2.632	4.696	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	200	458	0	0	0	0	0
N.S.	1	1.00	0.71	1.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.327	1.638	4.784	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	254	546	0	0	0	0	0
N.S.	1	1.00	0.73	1.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.444	2.748	4.590	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	366	366	117	605	0	0	0	0	0
N.S.	1	1.00	0.32	1.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.523	12.893	6.596	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	257	397	0	0	0	0	0
N.S.	1	1.00	0.77	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.498	26.787	6.591	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	106	530	0	0	0	0	0
N.S.	1	1.00	0.34	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.418	11.676	5.431	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	220	269	0	0	0	0	0
N.S.	1	1.00	0.79	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.363	13.012	5.459	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	238	536	0	0	0	0	0
N.S.	1	1.00	0.77	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.633	7.763	4.951	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	224	483	0	0	0	0	0
N.S.	1	1.00	0.71	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.475	15.823	5.554	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	2820	571	0	0	0	0	0
N.S.	1	1.00	7.62	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.579	14.671	4.913	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	330	330	332	1026	0	0	0	0	0
N.S.	1	1.00	1.01	3.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.542	16.028	4.637	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	326	326	129	1166	0	0	0	0	0
N.S.	1	1.00	0.40	3.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.499	17.800	3.858	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	295	295	271	976	0	0	0	0	0
N.S.	1	1.00	0.92	3.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	22.333	4.027	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	285	285	105	1100	0	0	0	0	0
N.S.	1	1.00	0.37	3.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	16.519	3.637	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	257	257	1211	205	0	0	0	0	0
N.S.	1	1.00	4.71	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.346	15.615	3.749	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	2715	256	0	0	0	0	0
N.S.	1	1.00	8.62	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.495	10.581	2.362	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	1253	611	0	0	0	0	0
N.S.	1	1.00	4.32	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	10.239	3.750	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	359	359	180	1116	0	0	0	0	0
N.S.	1	1.00	0.50	3.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.542	13.994	2.988	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	328	328	1299	744	0	0	0	0	0
N.S.	1	1.00	3.96	2.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.508	10.617	3.870	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	372	372	0	1181	0	0	0	0	0
N.S.	1	1.00	0.00	3.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.649	0.000	5.192	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	339	339	0	1007	0	0	0	0	0
N.S.	1	1.00	0.00	2.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.641	0.000	4.508	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	312	312	0	1141	0	0	0	0	0
N.S.	1	1.00	0.00	3.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.686	0.000	4.916	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	281	281	0	961	0	0	0	0	0
N.S.	1	1.00	0.00	3.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	0.000	6.340	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	310	310	812	261	0	0	0	0	0
N.S.	1	1.00	2.62	0.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.605	8.364	4.490	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	316	316	0	638	0	0	0	0	0
N.S.	1	1.00	0.00	2.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.578	0.000	5.271	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	363	363	2792	705	0	0	0	0	0
N.S.	1	1.00	7.69	1.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.643	10.039	4.229	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	365	365	1281	633	0	0	0	0	0
N.S.	1	1.00	3.51	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.671	12.508	5.598	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	102	101	145	299	0	0	0
N.S.	1	1.00	0.69	0.69	0.99	2.03	0.00	0.00	0.00
time (sec)	N/A	0.151	0.625	5.028	0.283	0.402	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	80	81	107	259	0	0	0
N.S.	1	1.00	0.81	0.82	1.08	2.62	0.00	0.00	0.00
time (sec)	N/A	0.110	0.275	4.745	0.273	0.332	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	60	42	67	184	0	0	47
N.S.	1	1.00	1.18	0.82	1.31	3.61	0.00	0.00	0.92
time (sec)	N/A	0.069	0.101	0.934	0.288	0.312	0.000	0.000	14.576

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	72	92	0	242	0	0	0
N.S.	1	1.00	0.99	1.26	0.00	3.32	0.00	0.00	0.00
time (sec)	N/A	0.108	0.078	2.374	0.000	0.332	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	87	137	0	426	0	0	0
N.S.	1	1.00	0.66	1.05	0.00	3.25	0.00	0.00	0.00
time (sec)	N/A	0.163	0.551	2.620	0.000	0.378	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	102	173	0	529	0	0	0
N.S.	1	1.00	0.53	0.90	0.00	2.74	0.00	0.00	0.00
time (sec)	N/A	0.223	0.585	3.038	0.000	0.373	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	134	233	0	371	0	0	0
N.S.	1	1.00	0.60	1.05	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.145	7.811	5.352	0.000	0.342	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	110	201	0	331	0	0	0
N.S.	1	1.00	0.69	1.26	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.113	6.658	5.176	0.000	0.342	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	226	186	0	283	0	0	0
N.S.	1	1.00	2.35	1.94	0.00	2.95	0.00	0.00	0.00
time (sec)	N/A	0.093	3.484	4.731	0.000	0.297	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	124	161	0	422	0	0	0
N.S.	1	1.00	1.14	1.48	0.00	3.87	0.00	0.00	0.00
time (sec)	N/A	0.122	3.344	2.809	0.000	0.390	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	232	196	0	547	0	0	0
N.S.	1	1.00	1.18	1.00	0.00	2.79	0.00	0.00	0.00
time (sec)	N/A	0.246	7.207	2.781	0.000	0.408	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	210	226	0	646	0	0	0
N.S.	1	1.00	0.75	0.81	0.00	2.31	0.00	0.00	0.00
time (sec)	N/A	0.322	4.657	2.653	0.000	0.418	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	112	112	162	334	0	0	0
N.S.	1	1.00	0.66	0.66	0.96	1.98	0.00	0.00	0.00
time (sec)	N/A	0.166	0.430	4.931	0.268	0.362	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	92	92	124	290	0	0	0
N.S.	1	1.00	0.76	0.76	1.02	2.40	0.00	0.00	0.00
time (sec)	N/A	0.127	0.217	4.260	0.269	0.346	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	70	57	86	238	0	0	67
N.S.	1	1.00	0.96	0.78	1.18	3.26	0.00	0.00	0.92
time (sec)	N/A	0.076	0.122	0.749	0.268	0.355	0.000	0.000	14.334

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	72	91	0	243	0	0	0
N.S.	1	1.00	0.99	1.25	0.00	3.33	0.00	0.00	0.00
time (sec)	N/A	0.097	0.082	1.528	0.000	0.332	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	100	138	0	378	0	0	0
N.S.	1	1.00	0.92	1.27	0.00	3.47	0.00	0.00	0.00
time (sec)	N/A	0.138	0.313	1.939	0.000	0.329	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	104	174	0	589	0	0	0
N.S.	1	1.00	0.61	1.02	0.00	3.44	0.00	0.00	0.00
time (sec)	N/A	0.201	0.721	1.963	0.000	0.422	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	147	250	0	415	0	0	0
N.S.	1	1.00	0.57	0.97	0.00	1.61	0.00	0.00	0.00
time (sec)	N/A	0.157	9.384	4.029	0.000	0.358	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	123	218	0	371	0	0	0
N.S.	1	1.00	0.63	1.12	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.136	7.148	3.724	0.000	0.310	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	128	128	97	187	0	321	0	0	0
N.S.	1	1.00	0.76	1.46	0.00	2.51	0.00	0.00	0.00
time (sec)	N/A	0.122	6.787	3.482	0.000	0.326	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	102	85	0	264	0	0	0
N.S.	1	1.00	1.59	1.33	0.00	4.12	0.00	0.00	0.00
time (sec)	N/A	0.104	0.381	1.939	0.000	0.344	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	226	197	0	531	0	0	0
N.S.	1	1.00	1.57	1.37	0.00	3.69	0.00	0.00	0.00
time (sec)	N/A	0.178	6.687	1.739	0.000	0.404	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	235	227	0	708	0	0	0
N.S.	1	1.00	1.04	1.00	0.00	3.13	0.00	0.00	0.00
time (sec)	N/A	0.274	7.457	1.846	0.000	0.405	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	156	124	181	386	0	0	0
N.S.	1	1.00	0.81	0.64	0.94	2.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.547	262.509	0.282	0.375	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	102	104	143	334	0	0	0
N.S.	1	1.00	0.70	0.72	0.99	2.30	0.00	0.00	0.00
time (sec)	N/A	0.160	0.435	55.967	0.280	0.368	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	82	74	105	282	0	0	92
N.S.	1	1.00	0.85	0.76	1.08	2.91	0.00	0.00	0.95
time (sec)	N/A	0.086	0.160	7.177	0.277	0.336	0.000	0.000	13.961

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	116	0	300	0	0	0
N.S.	1	1.00	0.87	1.22	0.00	3.16	0.00	0.00	0.00
time (sec)	N/A	0.126	0.077	5.423	0.000	0.349	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	96	240	0	398	0	0	0
N.S.	1	1.00	0.91	2.26	0.00	3.75	0.00	0.00	0.00
time (sec)	N/A	0.135	0.334	15.210	0.000	0.339	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	109	272	0	503	0	0	0
N.S.	1	1.00	0.74	1.85	0.00	3.42	0.00	0.00	0.00
time (sec)	N/A	0.171	0.693	88.517	0.000	0.331	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	173	268	0	477	0	0	0
N.S.	1	1.00	0.60	0.92	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.174	8.758	1.253	0.000	0.359	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	149	236	0	425	0	0	0
N.S.	1	1.00	0.67	1.05	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	0.147	7.813	129.408	0.000	0.366	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	160	160	125	233	0	374	0	0	0
N.S.	1	1.00	0.78	1.46	0.00	2.34	0.00	0.00	0.00
time (sec)	N/A	0.128	6.417	13.545	0.000	0.344	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	124	89	0	270	0	0	0
N.S.	1	1.00	1.88	1.35	0.00	4.09	0.00	0.00	0.00
time (sec)	N/A	0.093	0.584	6.621	0.000	0.352	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	81	180	0	355	0	0	0
N.S.	1	1.00	0.84	1.88	0.00	3.70	0.00	0.00	0.00
time (sec)	N/A	0.096	0.196	40.391	0.000	0.343	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	203	372	0	650	0	0	0
N.S.	1	1.00	1.15	2.11	0.00	3.69	0.00	0.00	0.00
time (sec)	N/A	0.224	6.396	194.437	0.000	0.400	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	88	94	129	285	0	167	0
N.S.	1	1.00	0.70	0.75	1.02	2.26	0.00	1.33	0.00
time (sec)	N/A	0.137	0.155	3.783	0.275	0.359	0.000	2.037	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	74	91	241	0	126	0
N.S.	1	1.00	0.85	0.95	1.17	3.09	0.00	1.62	0.00
time (sec)	N/A	0.103	0.075	3.142	0.280	0.374	0.000	1.087	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	44	26	49	137	0	48	27
N.S.	1	1.00	1.42	0.84	1.58	4.42	0.00	1.55	0.87
time (sec)	N/A	0.050	0.033	0.693	0.274	0.318	0.000	0.784	15.183

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	57	229	0	384	0	111	0
N.S.	1	1.00	0.62	2.49	0.00	4.17	0.00	1.21	0.00
time (sec)	N/A	0.116	0.062	1.802	0.000	0.347	0.000	0.752	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	90	272	0	546	0	174	0
N.S.	1	1.00	0.59	1.79	0.00	3.59	0.00	1.14	0.00
time (sec)	N/A	0.167	0.160	1.752	0.000	0.353	0.000	0.843	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	102	308	0	705	0	241	0
N.S.	1	1.00	0.48	1.44	0.00	3.29	0.00	1.13	0.00
time (sec)	N/A	0.226	0.257	1.577	0.000	0.394	0.000	0.890	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	167	220	0	355	0	266	0
N.S.	1	1.00	0.88	1.16	0.00	1.88	0.00	1.41	0.00
time (sec)	N/A	0.130	14.085	3.719	0.000	0.304	0.000	2.839	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	238	188	0	311	0	228	0
N.S.	1	1.00	1.90	1.50	0.00	2.49	0.00	1.82	0.00
time (sec)	N/A	0.113	2.465	3.689	0.000	0.292	0.000	1.601	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	119	95	0	235	0	187	0
N.S.	1	1.00	1.89	1.51	0.00	3.73	0.00	2.97	0.00
time (sec)	N/A	0.087	0.586	3.159	0.000	0.272	0.000	1.142	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	187	336	0	503	0	269	0
N.S.	1	1.00	1.13	2.04	0.00	3.05	0.00	1.63	0.00
time (sec)	N/A	0.177	5.046	2.013	0.000	0.343	0.000	1.095	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	229	372	0	666	0	385	0
N.S.	1	1.00	0.91	1.48	0.00	2.65	0.00	1.53	0.00
time (sec)	N/A	0.277	1.656	2.104	0.000	0.360	0.000	1.206	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	263	408	0	823	0	500	0
N.S.	1	1.00	0.79	1.22	0.00	2.46	0.00	1.49	0.00
time (sec)	N/A	0.445	3.251	1.949	0.000	0.391	0.000	1.322	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	79	82	110	261	0	154	0
N.S.	1	1.00	0.79	0.82	1.10	2.61	0.00	1.54	0.00
time (sec)	N/A	0.181	0.108	3.593	0.290	0.312	0.000	2.290	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	56	65	71	191	0	85	0
N.S.	1	1.00	1.04	1.20	1.31	3.54	0.00	1.57	0.00
time (sec)	N/A	0.155	0.052	3.292	0.277	0.305	0.000	1.252	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	45	70	244	0	87	50
N.S.	1	1.00	0.70	0.83	1.30	4.52	0.00	1.61	0.93
time (sec)	N/A	0.092	0.029	0.872	0.276	0.312	0.000	0.927	14.466

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	60	304	0	485	0	164	0
N.S.	1	1.00	0.50	2.53	0.00	4.04	0.00	1.37	0.00
time (sec)	N/A	0.198	0.048	1.579	0.000	0.360	0.000	0.922	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	90	400	0	592	0	253	0
N.S.	1	1.00	0.51	2.27	0.00	3.36	0.00	1.44	0.00
time (sec)	N/A	0.268	0.128	1.915	0.000	0.376	0.000	1.092	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	99	436	0	837	0	322	0
N.S.	1	1.00	0.42	1.83	0.00	3.52	0.00	1.35	0.00
time (sec)	N/A	0.362	0.207	1.817	0.000	0.396	0.000	1.010	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	157	157	248	233	0	343	0	296	0
N.S.	1	1.00	1.58	1.48	0.00	2.18	0.00	1.89	0.00
time (sec)	N/A	0.191	1.928	3.740	0.000	0.299	0.000	3.748	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	162	174	0	295	0	230	0
N.S.	1	1.00	1.71	1.83	0.00	3.11	0.00	2.42	0.00
time (sec)	N/A	0.119	3.316	3.756	0.000	0.291	0.000	2.444	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	132	136	0	295	0	72	0
N.S.	1	1.00	1.55	1.60	0.00	3.47	0.00	0.85	0.00
time (sec)	N/A	0.113	0.780	1.821	0.000	0.294	0.000	1.141	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	215	215	210	497	0	603	0	143	0
N.S.	1	1.00	0.98	2.31	0.00	2.80	0.00	0.67	0.00
time (sec)	N/A	0.238	6.510	2.004	0.000	0.344	0.000	1.091	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	240	541	0	712	0	261	0
N.S.	1	1.00	0.79	1.79	0.00	2.35	0.00	0.86	0.00
time (sec)	N/A	0.338	2.524	1.870	0.000	0.367	0.000	1.089	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	264	577	0	955	0	379	0
N.S.	1	1.00	0.68	1.49	0.00	2.47	0.00	0.98	0.00
time (sec)	N/A	0.439	5.129	1.587	0.000	0.439	0.000	1.333	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	72	163	241	0	126	0
N.S.	1	1.00	0.88	0.92	2.09	3.09	0.00	1.62	0.00
time (sec)	N/A	0.114	0.232	3.747	0.285	0.310	0.000	2.788	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	87	125	245	0	90	0
N.S.	1	1.00	0.93	1.61	2.31	4.54	0.00	1.67	0.00
time (sec)	N/A	0.099	0.056	1.784	0.276	0.308	0.000	1.706	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	40	62	87	321	0	116	69
N.S.	1	1.00	0.51	0.79	1.12	4.12	0.00	1.49	0.88
time (sec)	N/A	0.075	0.084	0.876	0.274	0.304	0.000	1.227	14.759

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	60	378	0	573	0	206	0
N.S.	1	1.00	0.42	2.62	0.00	3.98	0.00	1.43	0.00
time (sec)	N/A	0.155	0.143	1.732	0.000	0.356	0.000	0.928	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	90	528	0	748	0	295	0
N.S.	1	1.00	0.45	2.64	0.00	3.74	0.00	1.48	0.00
time (sec)	N/A	0.219	0.286	1.758	0.000	0.376	0.000	1.081	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	99	564	0	905	0	364	0
N.S.	1	1.00	0.38	2.15	0.00	3.45	0.00	1.39	0.00
time (sec)	N/A	0.282	0.347	1.675	0.000	0.395	0.000	1.273	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	127	127	447	211	0	323	0	257	0
N.S.	1	1.00	3.52	1.66	0.00	2.54	0.00	2.02	0.00
time (sec)	N/A	0.108	6.090	3.700	0.000	0.293	0.000	4.295	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	188	318	0	414	0	66	0
N.S.	1	1.00	1.66	2.81	0.00	3.66	0.00	0.58	0.00
time (sec)	N/A	0.127	0.982	3.727	0.000	0.331	0.000	2.084	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	193	199	0	492	0	47	0
N.S.	1	1.00	1.52	1.57	0.00	3.87	0.00	0.37	0.00
time (sec)	N/A	0.134	2.348	2.095	0.000	0.325	0.000	1.503	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	265	265	232	589	0	691	0	177	0
N.S.	1	1.00	0.88	2.22	0.00	2.61	0.00	0.67	0.00
time (sec)	N/A	0.291	5.565	1.883	0.000	0.379	0.000	1.085	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	287	710	0	868	0	296	0
N.S.	1	1.00	0.81	2.00	0.00	2.45	0.00	0.83	0.00
time (sec)	N/A	0.409	3.438	1.929	0.000	0.392	0.000	1.391	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	439	292	746	0	1023	0	412	0
N.S.	1	1.00	0.67	1.70	0.00	2.33	0.00	0.94	0.00
time (sec)	N/A	0.519	5.529	1.744	0.000	0.441	0.000	1.620	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	177	227	241	331	0	674	0	109	0
N.S.	1	1.28	1.36	1.87	0.00	3.81	0.00	0.62	0.00
time (sec)	N/A	0.237	6.358	3.069	0.000	0.425	0.000	1.336	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	243	243	226	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	2.103	0.000	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	171	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	1.227	0.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	105	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.432	0.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	329	0	0	0	0	0	0
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	10.842	0.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	252	252	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.104	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.121	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	87	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.140	0.315	0.000	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	72	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.115	0.000	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	49	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	0.039	0.000	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	57	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	96	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.147	0.198	0.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	910	0	0	0	0	0	0
N.S.	1	1.00	8.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	6.330	0.000	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	893	0	0	0	0	0	0
N.S.	1	1.00	8.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	3.283	0.000	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	2072	0	0	0	0	0	0
N.S.	1	1.00	18.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	27.339	0.000	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	238	0	0	0	0	0	0
N.S.	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	4.876	0.000	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	229	0	0	0	0	0	0
N.S.	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	9.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	2164	0	0	0	0	0	0
N.S.	1	1.00	19.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	17.523	0.000	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	185	494	0	0	0	0	0
N.S.	1	1.00	0.58	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	14.429	10.215	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	191	501	0	0	0	0	0
N.S.	1	1.00	0.55	1.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.504	1.244	10.760	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	169	141	0	0	0	0	0
N.S.	1	1.00	0.62	0.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.241	1.832	9.347	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	189	507	0	0	0	0	0
N.S.	1	1.00	0.63	1.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.309	1.423	10.382	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	224	382	0	0	0	0	0
N.S.	1	1.00	0.70	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	2.130	10.122	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	93	496	0	0	0	0	0
N.S.	1	1.00	0.26	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.432	12.830	17.606	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	220	524	0	0	0	0	0
N.S.	1	1.00	0.64	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.425	14.443	10.332	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	118	265	0	0	0	0	0
N.S.	1	1.00	0.38	0.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.343	12.012	11.140	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	339	339	221	531	0	0	0	0	0
N.S.	1	1.00	0.65	1.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	25.080	11.511	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	127	405	0	0	0	0	0
N.S.	1	1.00	0.34	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.427	13.324	11.109	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	405	405	316	1116	0	0	0	0	0
N.S.	1	1.00	0.78	2.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.577	14.104	8.227	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	135	611	0	0	0	0	0
N.S.	1	1.00	0.42	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.426	12.308	8.163	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	249	252	0	0	0	0	0
N.S.	1	1.00	0.72	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.451	13.937	8.317	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	112	207	0	0	0	0	0
N.S.	1	1.00	0.39	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	13.295	7.973	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	325	325	194	1100	0	0	0	0	0
N.S.	1	1.00	0.60	3.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	23.234	9.772	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	335	335	130	976	0	0	0	0	0
N.S.	1	1.00	0.39	2.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.442	12.942	9.795	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	371	371	261	1166	0	0	0	0	0
N.S.	1	1.00	0.70	3.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.472	14.815	9.232	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	413	413	0	705	0	0	0	0	0
N.S.	1	1.00	0.00	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.533	0.000	8.546	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	359	359	0	638	0	0	0	0	0
N.S.	1	1.00	0.00	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.502	0.000	9.009	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	355	355	0	262	0	0	0	0	0
N.S.	1	1.00	0.00	0.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.597	0.000	8.095	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	321	321	0	961	0	0	0	0	0
N.S.	1	1.00	0.00	2.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	0.000	10.284	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	357	357	0	1141	0	0	0	0	0
N.S.	1	1.00	0.00	3.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.479	0.000	8.912	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	389	389	0	1007	0	0	0	0	0
N.S.	1	1.00	0.00	2.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.486	0.000	9.694	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	106	90	94	101	148	317	221
N.S.	1	1.00	0.95	0.81	0.85	0.91	1.33	2.86	1.99
time (sec)	N/A	0.132	0.448	1.407	0.210	0.265	0.812	3.379	18.314

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	82	67	72	79	112	248	162
N.S.	1	1.00	0.98	0.80	0.86	0.94	1.33	2.95	1.93
time (sec)	N/A	0.107	0.203	1.097	0.193	0.285	0.404	1.531	19.026

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	47	50	57	76	179	102
N.S.	1	1.00	1.00	0.85	0.91	1.04	1.38	3.25	1.85
time (sec)	N/A	0.063	0.143	0.845	0.217	0.289	0.191	0.617	15.538

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	26	34	37	107	40
N.S.	1	1.00	1.00	0.92	1.04	1.36	1.48	4.28	1.60
time (sec)	N/A	0.032	0.010	0.412	0.201	0.269	0.102	0.318	14.897

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	64	33	34	38	0	61	51
N.S.	1	1.00	1.49	0.77	0.79	0.88	0.00	1.42	1.19
time (sec)	N/A	0.067	0.018	0.471	0.194	0.261	0.000	0.276	14.476

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	114	75	62	99	0	170	86
N.S.	1	1.00	1.58	1.04	0.86	1.38	0.00	2.36	1.19
time (sec)	N/A	0.121	0.793	0.548	0.220	0.285	0.000	0.295	14.468

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	175	111	99	168	0	266	128
N.S.	1	1.00	1.72	1.09	0.97	1.65	0.00	2.61	1.25
time (sec)	N/A	0.141	0.103	0.716	0.237	0.270	0.000	0.337	14.492

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	216	151	133	237	0	358	170
N.S.	1	1.00	1.66	1.16	1.02	1.82	0.00	2.75	1.31
time (sec)	N/A	0.197	0.490	0.907	0.228	0.271	0.000	0.362	14.093

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	179	143	134	134	0	228	331
N.S.	1	1.00	1.75	1.40	1.31	1.31	0.00	2.24	3.25
time (sec)	N/A	0.127	0.035	1.913	0.295	0.278	0.000	2.127	15.023

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	117	104	102	112	0	172	267
N.S.	1	1.00	1.60	1.42	1.40	1.53	0.00	2.36	3.66
time (sec)	N/A	0.077	0.023	1.505	0.292	0.278	0.000	0.928	15.238

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	60	67	65	87	0	115	96
N.S.	1	1.00	1.33	1.49	1.44	1.93	0.00	2.56	2.13
time (sec)	N/A	0.042	0.007	0.939	0.279	0.285	0.000	0.476	14.610

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	43	35	31	33	0	52	48
N.S.	1	1.00	1.65	1.35	1.19	1.27	0.00	2.00	1.85
time (sec)	N/A	0.031	0.013	0.460	0.280	0.266	0.000	0.302	14.341

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	62	86	59	87	0	112	90
N.S.	1	1.00	1.13	1.56	1.07	1.58	0.00	2.04	1.64
time (sec)	N/A	0.068	0.017	1.232	0.277	0.260	0.000	0.333	14.615

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	129	79	130	0	170	132
N.S.	1	1.00	0.94	1.54	0.94	1.55	0.00	2.02	1.57
time (sec)	N/A	0.112	0.024	1.215	0.273	0.267	0.000	0.335	14.831

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	92	162	100	179	0	225	174
N.S.	1	1.00	0.83	1.46	0.90	1.61	0.00	2.03	1.57
time (sec)	N/A	0.153	0.028	1.694	0.288	0.283	0.000	0.370	14.660

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	217	173	133	174	181	314	489	344
N.S.	1	1.17	0.94	0.72	0.94	0.98	1.70	2.64	1.86
time (sec)	N/A	0.159	1.166	6.163	0.196	0.309	2.475	5.918	17.871

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	169	138	113	139	146	252	415	280
N.S.	1	1.13	0.93	0.76	0.93	0.98	1.69	2.79	1.88
time (sec)	N/A	0.141	0.675	3.736	0.194	0.291	1.493	3.306	17.893

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	131	105	93	108	115	189	341	215
N.S.	1	1.14	0.91	0.81	0.94	1.00	1.64	2.97	1.87
time (sec)	N/A	0.110	0.326	2.283	0.199	0.292	0.983	1.693	17.834

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	74	75	75	82	126	267	151
N.S.	1	1.00	0.85	0.86	0.86	0.94	1.45	3.07	1.74
time (sec)	N/A	0.081	0.385	1.513	0.197	0.304	0.275	0.742	16.451

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	42	40	42	51	60	191	81
N.S.	1	1.00	0.89	0.85	0.89	1.09	1.28	4.06	1.72
time (sec)	N/A	0.039	0.046	0.720	0.204	0.269	0.183	0.357	13.203

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	48	62	68	0	101	96
N.S.	1	1.00	0.89	0.79	1.02	1.11	0.00	1.66	1.57
time (sec)	N/A	0.117	0.147	0.692	0.192	0.277	0.000	0.302	14.380

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	89	92	72	113	0	209	98
N.S.	1	1.00	0.97	1.00	0.78	1.23	0.00	2.27	1.07
time (sec)	N/A	0.167	0.561	0.753	0.195	0.282	0.000	0.330	14.267

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	139	136	122	203	0	360	164
N.S.	1	1.00	1.10	1.08	0.97	1.61	0.00	2.86	1.30
time (sec)	N/A	0.202	3.247	1.136	0.195	0.280	0.000	0.367	14.336

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	132	167	150	184	0	282	403
N.S.	1	1.00	0.84	1.06	0.96	1.17	0.00	1.80	2.57
time (sec)	N/A	0.234	1.259	3.656	0.290	0.278	0.000	2.302	15.420

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	107	129	118	151	0	220	332
N.S.	1	1.00	0.92	1.11	1.02	1.30	0.00	1.90	2.86
time (sec)	N/A	0.185	0.618	2.356	0.280	0.294	0.000	1.045	15.413

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	91	82	115	0	158	227
N.S.	1	1.00	0.94	1.30	1.17	1.64	0.00	2.26	3.24
time (sec)	N/A	0.129	0.224	1.267	0.283	0.275	0.000	0.571	14.663

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	39	47	47	44	0	80	58
N.S.	1	1.00	0.81	0.98	0.98	0.92	0.00	1.67	1.21
time (sec)	N/A	0.094	0.622	1.126	0.286	0.269	0.000	0.295	14.139

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	70	111	76	102	0	176	118
N.S.	1	1.00	0.82	1.31	0.89	1.20	0.00	2.07	1.39
time (sec)	N/A	0.153	0.262	1.317	0.268	0.285	0.000	0.319	14.001

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	84	154	96	152	0	273	191
N.S.	1	1.00	0.69	1.26	0.79	1.25	0.00	2.24	1.57
time (sec)	N/A	0.164	0.375	2.064	0.276	0.272	0.000	0.356	13.700

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	94	187	116	206	0	366	258
N.S.	1	1.00	0.61	1.22	0.76	1.35	0.00	2.39	1.69
time (sec)	N/A	0.186	0.399	3.218	0.280	0.274	0.000	0.397	13.750

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	228	273	268	293	0	1768	631
N.S.	1	1.00	0.91	1.09	1.07	1.17	0.00	7.07	2.52
time (sec)	N/A	0.231	3.910	2.182	0.205	0.344	0.000	5.760	15.150

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	156	184	183	205	0	1052	395
N.S.	1	1.00	0.92	1.08	1.08	1.21	0.00	6.19	2.32
time (sec)	N/A	0.164	6.228	1.683	0.234	0.342	0.000	3.414	15.602

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	98	115	110	129	0	560	227
N.S.	1	1.00	0.91	1.06	1.02	1.19	0.00	5.19	2.10
time (sec)	N/A	0.114	0.362	1.239	0.218	0.321	0.000	1.527	15.085

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	52	57	57	69	0	289	115
N.S.	1	1.00	0.88	0.97	0.97	1.17	0.00	4.90	1.95
time (sec)	N/A	0.086	0.143	0.722	0.241	0.277	0.000	0.614	14.914

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	19	33	19	19	82	114	71
N.S.	1	1.00	0.54	0.94	0.54	0.54	2.34	3.26	2.03
time (sec)	N/A	0.040	0.109	0.279	0.213	0.266	2.924	0.348	14.699

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	82	75	68	75	0	257	93
N.S.	1	1.00	0.87	0.80	0.72	0.80	0.00	2.73	0.99
time (sec)	N/A	0.120	0.150	0.617	0.219	0.286	0.000	0.311	14.753

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	151	126	144	263	0	403	174
N.S.	1	1.00	0.96	0.80	0.92	1.68	0.00	2.57	1.11
time (sec)	N/A	0.221	0.981	0.937	0.203	0.349	0.000	0.351	14.726

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	220	193	289	576	0	649	290
N.S.	1	1.00	0.94	0.82	1.24	2.46	0.00	2.77	1.24
time (sec)	N/A	0.372	4.503	1.216	0.213	0.428	0.000	0.392	15.273

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	271	907	411	0	603	0	746	9148
N.S.	1	1.37	4.58	2.08	0.00	3.05	0.00	3.77	46.20
time (sec)	N/A	0.454	6.529	1.540	0.000	0.612	0.000	2.107	16.738

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	287	234	0	444	0	476	6062
N.S.	1	1.00	2.28	1.86	0.00	3.52	0.00	3.78	48.11
time (sec)	N/A	0.393	1.911	0.880	0.000	0.408	0.000	0.902	16.056

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	115	108	0	253	0	140	121
N.S.	1	1.00	1.51	1.42	0.00	3.33	0.00	1.84	1.59
time (sec)	N/A	0.215	0.329	0.808	0.000	0.321	0.000	0.486	14.406

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	135	147	115	0	362	0	582	1002
N.S.	1	1.27	1.39	1.08	0.00	3.42	0.00	5.49	9.45
time (sec)	N/A	0.273	0.608	0.755	0.000	0.302	0.000	0.384	16.551

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	256	416	184	0	742	0	1073	3859
N.S.	1	1.45	2.35	1.04	0.00	4.19	0.00	6.06	21.80
time (sec)	N/A	0.475	6.439	0.905	0.000	0.315	0.000	0.483	23.381

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	230	287	321	423	0	1696	760
N.S.	1	1.00	0.90	1.13	1.26	1.66	0.00	6.65	2.98
time (sec)	N/A	0.246	0.915	3.602	0.205	0.365	0.000	5.906	16.976

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	163	200	227	312	0	1023	505
N.S.	1	1.00	0.91	1.12	1.27	1.74	0.00	5.72	2.82
time (sec)	N/A	0.178	1.510	2.539	0.205	0.331	0.000	3.435	15.729

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	107	127	149	219	0	568	286
N.S.	1	1.00	0.88	1.05	1.23	1.81	0.00	4.69	2.36
time (sec)	N/A	0.130	0.338	1.268	0.201	0.319	0.000	1.557	14.901

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	62	72	74	102	0	313	124
N.S.	1	1.00	0.84	0.97	1.00	1.38	0.00	4.23	1.68
time (sec)	N/A	0.094	0.183	0.836	0.209	0.295	0.000	0.649	14.307

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	49	41	46	0	238	257
N.S.	1	1.00	1.00	0.91	0.76	0.85	0.00	4.41	4.76
time (sec)	N/A	0.053	0.030	0.496	0.200	0.289	0.000	0.368	14.368

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	136	114	142	234	0	303	160
N.S.	1	1.00	0.99	0.83	1.03	1.70	0.00	2.20	1.16
time (sec)	N/A	0.185	0.431	1.048	0.204	0.345	0.000	0.321	14.617

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	187	164	303	693	0	656	313
N.S.	1	1.00	0.95	0.83	1.54	3.52	0.00	3.33	1.59
time (sec)	N/A	0.300	1.770	1.468	0.217	0.411	0.000	0.379	14.957

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	283	230	558	1378	0	795	471
N.S.	1	1.00	1.02	0.83	2.01	4.96	0.00	2.86	1.69
time (sec)	N/A	0.522	6.451	2.361	0.214	0.618	0.000	0.434	15.760

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	283	559	365	0	843	0	411	9452
N.S.	1	1.42	2.80	1.82	0.00	4.22	0.00	2.06	47.26
time (sec)	N/A	0.602	6.180	2.076	0.000	0.560	0.000	2.113	17.227

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	327	219	0	584	0	294	7044
N.S.	1	1.00	2.18	1.46	0.00	3.89	0.00	1.96	46.96
time (sec)	N/A	0.503	1.534	1.152	0.000	0.343	0.000	0.936	16.187

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	115	0	383	0	144	551
N.S.	1	1.00	0.94	1.35	0.00	4.51	0.00	1.69	6.48
time (sec)	N/A	0.238	0.311	0.789	0.000	0.301	0.000	0.535	14.685

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	209	184	0	705	0	332	6093
N.S.	1	1.00	0.92	0.81	0.00	3.11	0.00	1.46	26.84
time (sec)	N/A	0.771	1.625	1.440	0.000	0.324	0.000	0.335	18.955

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	303	260	0	1481	0	487	8348
N.S.	1	1.00	0.84	0.72	0.00	4.11	0.00	1.35	23.19
time (sec)	N/A	0.773	2.111	1.815	0.000	0.358	0.000	0.383	19.430

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	761	761	1846	2967	0	0	0	0	0
N.S.	1	1.00	2.43	3.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.412	52.967	3.689	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	740	740	548	1651	0	0	0	0	0
N.S.	1	1.00	0.74	2.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.115	21.452	3.363	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	415	415	646	820	0	0	0	0	0
N.S.	1	1.00	1.56	1.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.857	5.978	2.538	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	422	422	1475	1919	0	0	0	0	0
N.S.	1	1.00	3.50	4.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.655	38.297	3.578	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	863	863	1571	4016	0	0	0	0	0
N.S.	1	1.00	1.82	4.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.516	23.898	3.274	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	836	836	2169	7581	0	0	0	0	0
N.S.	1	1.00	2.59	9.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.226	33.622	4.790	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	152	883	191	425	0	966	0
N.S.	1	1.00	0.90	5.22	1.13	2.51	0.00	5.72	0.00
time (sec)	N/A	0.194	1.908	16.530	0.284	0.438	0.000	1.758	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	87	465	108	311	0	539	0
N.S.	1	1.00	0.87	4.65	1.08	3.11	0.00	5.39	0.00
time (sec)	N/A	0.131	0.700	10.670	0.303	0.441	0.000	0.705	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	42	67	192	0	185	47
N.S.	1	1.00	0.96	0.82	1.31	3.76	0.00	3.63	0.92
time (sec)	N/A	0.057	0.136	0.659	0.282	0.427	0.000	0.405	13.724

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	100	543	0	2132	0	293	0
N.S.	1	1.00	0.94	5.12	0.00	20.11	0.00	2.76	0.00
time (sec)	N/A	0.192	0.234	2.245	0.000	0.604	0.000	0.630	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	215	215	212	2072	0	3523	0	514	0
N.S.	1	1.00	0.99	9.64	0.00	16.39	0.00	2.39	0.00
time (sec)	N/A	0.366	2.452	2.199	0.000	1.810	0.000	0.837	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	376	1380	0	0	0	0	0
N.S.	1	1.00	1.09	4.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.450	11.573	11.952	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	151	182	0	0	0	0	0
N.S.	1	1.00	1.21	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.133	6.573	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	154	546	0	0	0	0	0
N.S.	1	1.00	0.63	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	3.607	5.809	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	131	646	175	381	0	699	0
N.S.	1	1.00	0.89	4.36	1.18	2.57	0.00	4.72	0.00
time (sec)	N/A	0.179	0.871	16.651	0.278	0.426	0.000	1.807	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	66	312	92	273	0	265	0
N.S.	1	1.00	0.84	3.95	1.16	3.46	0.00	3.35	0.00
time (sec)	N/A	0.114	0.412	11.293	0.273	0.406	0.000	0.819	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	49	145	0	102	27
N.S.	1	1.00	1.00	0.84	1.58	4.68	0.00	3.29	0.87
time (sec)	N/A	0.056	0.091	0.742	0.269	0.392	0.000	0.493	14.771

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	106	106	100	658	0	2420	0	0	0
N.S.	1	1.00	0.94	6.21	0.00	22.83	0.00	0.00	0.00
time (sec)	N/A	0.181	0.292	2.292	0.000	1.423	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	260	260	280	3107	0	4336	0	533	0
N.S.	1	1.00	1.08	11.95	0.00	16.68	0.00	2.05	0.00
time (sec)	N/A	0.342	2.382	2.011	0.000	25.895	0.000	0.920	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	610	446	2243	0	0	0	0	0
N.S.	1	1.51	1.10	5.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.954	15.799	15.874	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	382	1069	0	0	0	0	0
N.S.	1	1.00	1.23	3.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	12.550	10.915	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	138	147	0	0	0	0	0
N.S.	1	1.00	1.30	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.124	5.436	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	366	1374	0	0	0	0	0
N.S.	1	1.00	1.01	3.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.528	13.148	7.097	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	117	1234	194	467	0	0	0
N.S.	1	1.00	0.79	8.34	1.31	3.16	0.00	0.00	0.00
time (sec)	N/A	0.202	0.560	15.602	0.286	0.503	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	66	731	110	317	0	258	0
N.S.	1	1.00	0.75	8.31	1.25	3.60	0.00	2.93	0.00
time (sec)	N/A	0.155	0.258	9.564	0.277	0.443	0.000	1.105	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	43	45	70	260	0	214	50
N.S.	1	1.00	0.80	0.83	1.30	4.81	0.00	3.96	0.93
time (sec)	N/A	0.063	0.104	0.789	0.263	0.421	0.000	0.778	15.587

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	142	212	2692	0	3924	0	0	0
N.S.	1	1.00	1.49	18.96	0.00	27.63	0.00	0.00	0.00
time (sec)	N/A	0.256	0.672	2.217	0.000	24.540	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	236	316	315	6367	0	8098	0	0	0
N.S.	1	1.34	1.33	26.98	0.00	34.31	0.00	0.00	0.00
time (sec)	N/A	0.501	1.092	2.335	0.000	48.259	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	530	907	531	1965	0	0	0	0	0
N.S.	1	1.71	1.00	3.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.047	17.412	14.836	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	329	828	0	0	0	0	0
N.S.	1	1.00	0.96	2.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.505	11.177	9.790	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	347	347	972	1799	0	0	0	0	0
N.S.	1	1.00	2.80	5.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	6.080	7.080	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	664	469	2062	0	0	0	0	0
N.S.	1	1.48	1.04	4.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.215	10.812	8.203	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	245	238	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.331	3.161	0.000	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	178	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	1.013	0.000	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	106	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	0.584	0.000	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	266	0	786	0	0	0	0	0	0
N.S.	1	0.00	2.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	4.223	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.08
time (sec)	N/A	0.083	29.433	1.182	1.427	0.387	69.961	1.874	18.998

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.08
time (sec)	N/A	0.074	0.888	1.102	1.331	0.378	1.412	1.040	14.985

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.08
time (sec)	N/A	0.073	15.729	1.148	1.334	0.385	0.719	0.836	20.955

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	53	24	25	27
N.S.	1	1.00	1.08	0.92	1.00	2.12	0.96	1.00	1.08
time (sec)	N/A	0.083	19.967	1.130	1.255	0.538	4.317	1.425	28.130

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	27
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.17
time (sec)	N/A	0.052	7.759	1.869	1.886	0.464	33.982	0.829	15.760

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	177	137	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.259	1.374	0.000	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	80	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	0.562	0.000	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	132	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.386	0.000	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	186	0	0	0	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	0.709	0.000	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.19
time (sec)	N/A	0.050	13.790	1.375	19.761	0.300	29.679	1.547	17.989

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.19
time (sec)	N/A	0.392	12.262	0.983	5.804	0.274	3.984	0.710	15.068

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.19
time (sec)	N/A	0.049	8.653	1.199	5.851	0.317	19.226	0.568	18.059

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	0	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.00	1.10	1.19
time (sec)	N/A	0.053	13.642	1.167	18.680	0.606	0.000	0.679	28.690

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	0	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.00	1.00	1.09
time (sec)	N/A	0.060	7.605	1.146	1.918	0.355	0.000	0.429	19.280

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.09
time (sec)	N/A	0.057	10.142	1.515	1.747	0.350	6.022	0.583	14.741

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.09
time (sec)	N/A	0.056	23.219	1.334	1.934	0.354	2.489	0.752	18.949

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	25
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.09
time (sec)	N/A	0.062	10.638	1.118	1.969	0.439	52.603	0.565	20.456

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [316] had the largest ratio of [.920000000000000040]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	19	0.105
2	A	3	2	1.00	19	0.105
3	A	3	2	1.00	19	0.105
4	A	3	2	1.00	19	0.105
5	A	3	2	1.00	17	0.118
6	A	2	2	1.00	17	0.118
7	A	3	2	1.00	19	0.105
8	A	3	2	1.00	19	0.105
9	A	3	2	1.00	19	0.105
10	A	6	2	1.00	19	0.105
11	A	5	2	1.00	19	0.105
12	A	4	2	1.00	19	0.105
13	A	3	2	1.00	19	0.105
14	A	2	2	1.00	19	0.105
15	A	3	2	1.00	19	0.105
16	A	4	2	1.00	19	0.105
17	A	5	2	1.00	19	0.105
18	A	6	2	1.00	19	0.105
19	A	3	2	1.00	21	0.095
20	A	3	2	1.00	21	0.095
21	A	3	2	1.00	21	0.095
22	A	3	2	1.00	21	0.095
23	A	3	2	1.00	19	0.105
24	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	2	1.00	21	0.095
26	A	3	2	1.00	21	0.095
27	A	3	2	1.00	21	0.095
28	A	3	2	1.00	21	0.095
29	A	12	7	1.00	21	0.333
30	A	10	7	1.00	21	0.333
31	A	8	7	1.00	21	0.333
32	A	8	5	1.00	21	0.238
33	A	9	6	1.00	21	0.286
34	A	11	7	1.00	21	0.333
35	A	12	7	1.00	21	0.333
36	A	13	7	1.00	21	0.333
37	A	3	2	1.00	21	0.095
38	A	3	2	1.00	21	0.095
39	A	3	2	1.00	21	0.095
40	A	3	2	1.00	21	0.095
41	A	3	2	1.00	19	0.105
42	A	3	2	1.00	19	0.105
43	A	3	2	1.00	21	0.095
44	A	3	2	1.00	21	0.095
45	A	3	2	1.00	21	0.095
46	A	3	2	1.00	21	0.095
47	A	17	8	1.00	21	0.381
48	A	14	8	1.00	21	0.381
49	A	11	8	1.00	21	0.381
50	A	11	8	1.00	21	0.381
51	A	11	6	1.00	21	0.286
52	A	14	8	1.00	21	0.381
53	A	15	8	1.00	21	0.381
54	A	16	8	1.00	21	0.381
55	A	17	8	1.00	21	0.381
56	A	3	2	1.00	21	0.095
57	A	3	2	1.00	21	0.095
58	A	3	2	1.00	21	0.095
59	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	2	2	1.00	19	0.105
61	A	3	2	1.00	19	0.105
62	A	3	2	1.00	21	0.095
63	A	3	2	1.00	21	0.095
64	A	6	3	1.00	21	0.143
65	A	5	3	1.00	21	0.143
66	A	4	3	1.00	21	0.143
67	A	3	2	1.00	21	0.095
68	A	4	3	1.00	21	0.143
69	A	5	3	1.00	21	0.143
70	A	6	3	1.00	21	0.143
71	A	3	2	1.00	21	0.095
72	A	3	2	1.00	21	0.095
73	A	3	2	1.00	21	0.095
74	A	3	2	1.00	21	0.095
75	A	3	2	1.00	19	0.105
76	A	3	2	1.00	19	0.105
77	A	3	2	1.00	21	0.095
78	A	3	2	1.00	21	0.095
79	A	11	8	1.00	21	0.381
80	A	9	8	1.00	21	0.381
81	A	5	5	1.00	21	0.238
82	A	9	6	1.06	21	0.286
83	A	12	8	1.00	21	0.381
84	A	13	8	1.00	21	0.381
85	A	14	8	1.00	21	0.381
86	A	3	2	1.00	21	0.095
87	A	3	2	1.00	21	0.095
88	A	3	2	1.00	21	0.095
89	A	3	2	1.00	21	0.095
90	A	3	2	1.00	21	0.095
91	A	3	2	1.00	19	0.105
92	A	3	2	1.00	19	0.105
93	A	3	2	1.00	21	0.095
94	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	18	9	1.00	21	0.429
96	A	15	9	1.00	21	0.429
97	A	12	9	1.00	21	0.429
98	A	6	6	1.00	21	0.286
99	A	12	9	1.04	21	0.429
100	A	12	7	1.18	21	0.333
101	A	16	9	1.00	21	0.429
102	A	17	9	1.00	21	0.429
103	A	18	9	1.00	21	0.429
104	A	17	14	1.00	23	0.609
105	A	16	13	1.00	23	0.565
106	A	16	13	1.00	23	0.565
107	A	15	12	1.00	23	0.522
108	A	17	14	1.00	23	0.609
109	A	16	13	1.00	23	0.565
110	A	18	14	1.00	23	0.609
111	A	21	17	1.00	25	0.680
112	A	20	16	1.00	25	0.640
113	A	19	15	1.00	25	0.600
114	A	18	14	1.00	25	0.560
115	A	20	16	1.00	25	0.640
116	A	20	16	1.00	25	0.640
117	A	22	17	1.00	25	0.680
118	A	18	14	1.00	25	0.560
119	A	18	15	1.00	25	0.600
120	A	17	14	1.00	25	0.560
121	A	17	14	1.00	25	0.560
122	A	16	13	1.00	25	0.520
123	A	18	15	1.00	25	0.600
124	A	17	14	1.00	25	0.560
125	A	19	15	1.00	25	0.600
126	A	18	14	1.00	25	0.560
127	A	22	18	1.00	25	0.720
128	A	21	17	1.00	25	0.680
129	A	20	16	1.00	25	0.640

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	19	15	1.00	25	0.600
131	A	21	17	1.00	25	0.680
132	A	21	17	1.00	25	0.680
133	A	23	18	1.00	25	0.720
134	A	23	17	1.00	25	0.680
135	A	8	5	1.00	23	0.217
136	A	6	5	1.00	23	0.217
137	A	4	4	1.00	21	0.190
138	A	6	4	1.00	21	0.190
139	A	8	6	1.00	23	0.261
140	A	10	7	1.00	23	0.304
141	A	4	3	1.00	23	0.130
142	A	4	3	1.00	23	0.130
143	A	4	4	1.00	23	0.174
144	A	5	4	1.00	23	0.174
145	A	7	5	1.00	23	0.217
146	A	9	6	1.00	23	0.261
147	A	9	5	1.00	23	0.217
148	A	7	5	1.00	23	0.217
149	A	5	4	1.00	21	0.190
150	A	6	4	1.00	21	0.190
151	A	7	5	1.00	23	0.217
152	A	9	7	1.00	23	0.304
153	A	4	3	1.00	23	0.130
154	A	4	3	1.00	23	0.130
155	A	4	3	1.00	23	0.130
156	A	3	3	1.00	23	0.130
157	A	6	5	1.00	23	0.217
158	A	8	5	1.00	23	0.217
159	A	10	5	1.00	23	0.217
160	A	8	5	1.00	23	0.217
161	A	6	4	1.00	21	0.190
162	A	7	5	1.00	21	0.238
163	A	7	5	1.00	23	0.217
164	A	8	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	4	3	1.00	23	0.130
166	A	4	3	1.00	23	0.130
167	A	4	3	1.00	23	0.130
168	A	3	3	1.00	23	0.130
169	A	4	3	1.00	23	0.130
170	A	7	5	1.00	23	0.217
171	A	7	5	1.00	23	0.217
172	A	5	5	1.00	23	0.217
173	A	3	3	1.00	21	0.143
174	A	7	5	1.00	21	0.238
175	A	9	6	1.00	23	0.261
176	A	11	7	1.00	23	0.304
177	A	4	3	1.00	23	0.130
178	A	5	4	1.00	23	0.174
179	A	3	3	1.00	23	0.130
180	A	6	5	1.00	23	0.217
181	A	8	6	1.00	23	0.261
182	A	10	6	1.00	23	0.261
183	A	6	5	1.00	23	0.217
184	A	4	4	1.00	23	0.174
185	A	4	4	1.00	21	0.190
186	A	8	6	1.00	21	0.286
187	A	10	6	1.00	23	0.261
188	A	12	7	1.00	23	0.304
189	A	5	4	1.00	23	0.174
190	A	4	3	1.00	23	0.130
191	A	4	3	1.00	23	0.130
192	A	7	6	1.00	23	0.261
193	A	9	6	1.00	23	0.261
194	A	11	6	1.00	23	0.261
195	A	5	4	1.00	23	0.174
196	A	4	4	1.00	23	0.174
197	A	5	4	1.00	21	0.190
198	A	9	6	1.00	21	0.286
199	A	11	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	13	7	1.00	23	0.304
201	A	4	3	1.00	23	0.130
202	A	5	4	1.00	23	0.174
203	A	5	4	1.00	23	0.174
204	A	8	6	1.00	23	0.261
205	A	10	6	1.00	23	0.261
206	A	12	6	1.00	23	0.261
207	A	7	5	1.28	23	0.217
208	A	1	1	1.00	23	0.043
209	A	8	6	1.00	23	0.261
210	A	7	6	1.00	23	0.261
211	A	4	4	1.00	21	0.190
212	A	5	5	1.00	23	0.217
213	A	8	7	1.00	23	0.304
214	A	9	7	1.00	23	0.304
215	A	1	1	1.00	25	0.040
216	A	1	1	1.00	25	0.040
217	A	1	1	1.00	25	0.040
218	A	1	1	1.00	25	0.040
219	A	4	3	1.00	21	0.143
220	A	4	3	1.00	21	0.143
221	A	3	3	1.00	21	0.143
222	A	2	2	1.00	19	0.105
223	A	4	4	1.00	19	0.210
224	A	5	5	1.00	21	0.238
225	A	1	1	1.00	21	0.048
226	A	1	1	1.00	21	0.048
227	A	1	1	1.00	21	0.048
228	A	1	1	1.00	21	0.048
229	A	1	1	1.00	23	0.043
230	A	1	1	1.00	23	0.043
231	A	1	1	1.00	23	0.043
232	A	1	1	1.00	23	0.043
233	A	17	14	1.00	23	0.609
234	A	18	15	1.00	23	0.652

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	16	13	1.00	23	0.565
236	A	17	14	1.00	23	0.609
237	A	17	14	1.00	23	0.609
238	A	21	17	1.00	25	0.680
239	A	21	17	1.00	25	0.680
240	A	19	15	1.00	25	0.600
241	A	20	16	1.00	25	0.640
242	A	21	17	1.00	25	0.680
243	A	20	16	1.00	25	0.640
244	A	18	15	1.00	25	0.600
245	A	19	16	1.00	25	0.640
246	A	17	14	1.00	25	0.560
247	A	18	15	1.00	25	0.600
248	A	18	15	1.00	25	0.600
249	A	19	16	1.00	25	0.640
250	A	24	19	1.00	25	0.760
251	A	22	18	1.00	25	0.720
252	A	22	18	1.00	25	0.720
253	A	20	16	1.00	25	0.640
254	A	21	17	1.00	25	0.680
255	A	22	18	1.00	25	0.720
256	A	7	5	1.00	19	0.263
257	A	6	5	1.00	19	0.263
258	A	5	5	1.00	19	0.263
259	A	4	4	1.00	17	0.235
260	A	5	4	1.00	17	0.235
261	A	6	5	1.00	19	0.263
262	A	7	5	1.00	19	0.263
263	A	8	5	1.00	19	0.263
264	A	5	2	1.00	19	0.105
265	A	4	2	1.00	19	0.105
266	A	3	2	1.00	19	0.105
267	A	2	2	1.00	19	0.105
268	A	3	2	1.00	19	0.105
269	A	4	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	5	2	1.00	19	0.105
271	A	3	2	1.17	21	0.095
272	A	3	2	1.13	21	0.095
273	A	3	2	1.14	21	0.095
274	A	3	2	1.00	21	0.095
275	A	3	2	1.00	19	0.105
276	A	3	2	1.00	19	0.105
277	A	4	3	1.00	21	0.143
278	A	5	4	1.00	21	0.190
279	A	12	7	1.00	21	0.333
280	A	10	7	1.00	21	0.333
281	A	8	7	1.00	21	0.333
282	A	8	5	1.00	21	0.238
283	A	9	6	1.00	21	0.286
284	A	11	7	1.00	21	0.333
285	A	12	7	1.00	21	0.333
286	A	3	2	1.00	21	0.095
287	A	3	2	1.00	21	0.095
288	A	3	2	1.00	21	0.095
289	A	3	2	1.00	21	0.095
290	A	4	4	1.00	19	0.210
291	A	3	2	1.00	19	0.105
292	A	3	2	1.00	21	0.095
293	A	3	2	1.00	21	0.095
294	A	15	8	1.37	21	0.381
295	A	6	6	1.00	21	0.286
296	A	7	7	1.00	21	0.333
297	A	9	8	1.27	21	0.381
298	A	15	8	1.45	21	0.381
299	A	3	2	1.00	21	0.095
300	A	3	2	1.00	21	0.095
301	A	3	2	1.00	21	0.095
302	A	3	2	1.00	21	0.095
303	A	3	2	1.00	19	0.105
304	A	3	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	3	2	1.00	21	0.095
306	A	3	2	1.00	21	0.095
307	A	16	10	1.42	21	0.476
308	A	6	6	1.00	21	0.286
309	A	6	6	1.00	21	0.286
310	A	11	7	1.00	21	0.333
311	A	15	8	1.00	21	0.381
312	A	38	22	1.00	25	0.880
313	A	35	19	1.00	25	0.760
314	A	21	16	1.00	25	0.640
315	A	19	14	1.00	25	0.560
316	A	39	23	1.00	25	0.920
317	A	36	20	1.00	25	0.800
318	A	5	4	1.00	23	0.174
319	A	5	4	1.00	23	0.174
320	A	4	4	1.00	21	0.190
321	A	7	5	1.00	21	0.238
322	A	13	9	1.00	23	0.391
323	A	7	7	1.00	23	0.304
324	A	1	1	1.00	14	0.071
325	A	5	4	1.00	23	0.174
326	A	5	4	1.00	23	0.174
327	A	5	4	1.00	23	0.174
328	A	3	3	1.00	21	0.143
329	A	7	5	1.00	21	0.238
330	A	11	6	1.00	23	0.261
331	A	11	8	1.51	23	0.348
332	A	6	6	1.00	23	0.261
333	A	1	1	1.00	14	0.071
334	A	9	8	1.00	23	0.348
335	A	5	4	1.00	23	0.174
336	A	5	4	1.00	23	0.174
337	A	4	4	1.00	21	0.190
338	A	7	4	1.00	21	0.190
339	A	11	5	1.34	23	0.217

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	17	11	1.71	23	0.478
341	A	7	7	1.00	23	0.304
342	A	6	6	1.00	14	0.429
343	A	14	11	1.48	23	0.478
344	A	8	6	1.00	23	0.261
345	A	7	6	1.00	23	0.261
346	A	4	4	1.00	21	0.190
347	F	0	0	N/A	0.000	N/A
348	N/A	0	0	1.00	25	0.000
349	N/A	0	0	1.00	25	0.000
350	N/A	0	0	1.00	25	0.000
351	N/A	0	0	1.00	25	0.000
352	N/A	0	0	1.00	23	0.000
353	A	5	4	1.00	21	0.190
354	A	4	4	1.00	21	0.190
355	A	2	2	1.00	19	0.105
356	A	8	5	1.00	19	0.263
357	A	10	5	1.00	21	0.238
358	N/A	0	0	1.00	21	0.000
359	N/A	0	0	1.00	21	0.000
360	N/A	0	0	1.00	21	0.000
361	N/A	0	0	1.00	21	0.000
362	N/A	0	0	1.00	23	0.000
363	N/A	0	0	1.00	23	0.000
364	N/A	0	0	1.00	23	0.000
365	N/A	0	0	1.00	23	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+a\sec(c+dx))\tan^9(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 151

$$\int (a+a\sec(c+dx))\tan^9(c+dx)\,dx = -\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}-\frac{2a\sec^2(c+dx)}{d}-\frac{4a\sec^3(c+dx)}{3d}+\frac{3a\sec^4(c+dx)}{2d}+\frac{6a\sec^5(c+dx)}{5d}-\frac{2a\sec^6(c+dx)}{3d}-\frac{4a\sec^7(c+dx)}{7d}+\frac{a\sec^8(c+dx)}{8d}+\frac{a\sec^9(c+dx)}{9d}$$

```
[Out] -a*ln(cos(d*x+c))/d+a*sec(d*x+c)/d-2*a*sec(d*x+c)^2/d-4/3*a*sec(d*x+c)^3/d+
3/2*a*sec(d*x+c)^4/d+6/5*a*sec(d*x+c)^5/d-2/3*a*sec(d*x+c)^6/d-4/7*a*sec(d*
x+c)^7/d+1/8*a*sec(d*x+c)^8/d+1/9*a*sec(d*x+c)^9/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used

$= \{3964, 90\}$

$$\int (a+a\sec(c+dx))\tan^9(c+dx)\,dx = \frac{a\sec^9(c+dx)}{9d}+\frac{a\sec^8(c+dx)}{8d}-\frac{4a\sec^7(c+dx)}{7d}-\frac{2a\sec^6(c+dx)}{3d}+\frac{6a\sec^5(c+dx)}{5d}+\frac{3a\sec^4(c+dx)}{2d}-\frac{4a\sec^3(c+dx)}{3d}-\frac{2a\sec^2(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^9,x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d - (2*a*Sec[c + d*x]^2)/d -
(4*a*Sec[c + d*x]^3)/(3*d) + (3*a*Sec[c + d*x]^4)/(2*d) + (6*a*Sec[c + d*x]
^5)/(5*d) - (2*a*Sec[c + d*x]^6)/(3*d) - (4*a*Sec[c + d*x]^7)/(7*d) + (a*Se
c[c + d*x]^8)/(8*d) + (a*Sec[c + d*x]^9)/(9*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^5}{x^{10}}\,dx, x, \cos(c+dx)\right)}{a^8d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a^9}{x^{10}}+\frac{a^9}{x^9}-\frac{4a^9}{x^8}-\frac{4a^9}{x^7}+\frac{6a^9}{x^6}+\frac{6a^9}{x^5}-\frac{4a^9}{x^4}-\frac{4a^9}{x^3}+\frac{a^9}{x^2}+\frac{a^9}{x}\right)dx, x, \cos(c+dx)\right)}{a^8d}$$

$$= -\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}-\frac{2a\sec^2(c+dx)}{d}-\frac{4a\sec^3(c+dx)}{3d}+\frac{3a\sec^4(c+dx)}{2d}+\frac{6a\sec^5(c+dx)}{5d}-\frac{2a\sec^6(c+dx)}{3d}-\frac{4a\sec^7(c+dx)}{7d}+\frac{a\sec^8(c+dx)}{8d}+\frac{a\sec^9(c+dx)}{9d}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89

$$\int (a+a\sec(c+dx))\tan^9(c+dx)\,dx$$
$$=\frac{a\sec(c+dx)}{d}-\frac{4a\sec^3(c+dx)}{3d}+\frac{6a\sec^5(c+dx)}{5d}-\frac{4a\sec^7(c+dx)}{7d}+\frac{a\sec^9(c+dx)}{9d}$$
$$-\frac{a(24\log(\cos(c+dx))+12\tan^2(c+dx)-6\tan^4(c+dx)+4\tan^6(c+dx)-3\tan^8(c+dx))}{24d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^9,x]
```

```
[Out] (a*Sec[c + d*x])/d - (4*a*Sec[c + d*x]^3)/(3*d) + (6*a*Sec[c + d*x]^5)/(5*d
) - (4*a*Sec[c + d*x]^7)/(7*d) + (a*Sec[c + d*x]^9)/(9*d) - (a*(24*Log[Cos[
c + d*x]] + 12*Tan[c + d*x]^2 - 6*Tan[c + d*x]^4 + 4*Tan[c + d*x]^6 - 3*Tan
[c + d*x]^8))/(24*d)
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{a\bigg(\frac{\sec(dx+c)^9}{9}+\frac{\sec(dx+c)^8}{8}-\frac{4\sec(dx+c)^7}{7}-\frac{2\sec(dx+c)^6}{3}+\frac{6\sec(dx+c)^5}{5}+\frac{3\sec(dx+c)^4}{2}-\frac{4\sec(dx+c)^3}{3}-2\sec(dx+c)^2+\sec(dx+}{d}$
default	$\frac{a\bigg(\frac{\sec(dx+c)^9}{9}+\frac{\sec(dx+c)^8}{8}-\frac{4\sec(dx+c)^7}{7}-\frac{2\sec(dx+c)^6}{3}+\frac{6\sec(dx+c)^5}{5}+\frac{3\sec(dx+c)^4}{2}-\frac{4\sec(dx+c)^3}{3}-2\sec(dx+c)^2+\sec(dx+}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^8}{8}-\frac{\tan(dx+c)^6}{6}+\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a\bigg(\frac{\sec(dx+c)^9}{9}-\frac{4\sec(dx+c)^7}{7}+\frac{6\sec(dx+c)^5}{5}}{d}$
risch	$iax+\frac{2iac}{d}+\frac{2a\big(315\,e^{17i(dx+c)}-1260\,e^{16i(dx+c)}+840\,e^{15i(dx+c)}-5040\,e^{14i(dx+c)}+4788\,e^{13i(dx+c)}-14280\,e^{12i(dx+c)}}{}$

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] a/d*(1/9*sec(d*x+c)^9+1/8*sec(d*x+c)^8-4/7*sec(d*x+c)^7-2/3*sec(d*x+c)^6+6/
5*sec(d*x+c)^5+3/2*sec(d*x+c)^4-4/3*sec(d*x+c)^3-2*sec(d*x+c)^2+sec(d*x+c)+
ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int (a+a\sec(c+dx))\tan^9(c+dx)\,dx =$$
$$-\frac{2520\,a\cos\left(dx+c\right)^{9}\log\left(-\cos\left(dx+c\right)\right)-2520\,a\cos\left(dx+c\right)^{8}+5040\,a\cos\left(dx+c\right)^{7}+3360\,a\cos\left(dx-\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(2520*a*cos(d*x + c)^9*log(-cos(d*x + c)) - 2520*a*cos(d*x + c)^8 +
 5040*a*cos(d*x + c)^7 + 3360*a*cos(d*x + c)^6 - 3780*a*cos(d*x + c)^5 - 30
24*a*cos(d*x + c)^4 + 1680*a*cos(d*x + c)^3 + 1440*a*cos(d*x + c)^2 - 315*a
*cos(d*x + c) - 280*a)/(d*cos(d*x + c)^9)
```

Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.22

$$\int (a+a\sec(c+dx))\tan^9(c+dx)\,dx$$
$$=\begin{cases}\frac{a\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^8\left(c+dx\right)\sec\left(c+dx\right)}{9d}+\frac{a\tan^8\left(c+dx\right)}{8d}-\frac{8a\tan^6\left(c+dx\right)\sec\left(c+dx\right)}{63d}-\frac{a\tan^6\left(c+dx\right)}{6d}+\frac{16a\tan^4\left(c+dx\right)\mathrm{s}}{105d}\\ x(a\sec\left(c\right)+a)\tan^9\left(c\right)\end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**8*sec(c + d*x
)/(9*d) + a*tan(c + d*x)**8/(8*d) - 8*a*tan(c + d*x)**6*sec(c + d*x)/(63*d)
 - a*tan(c + d*x)**6/(6*d) + 16*a*tan(c + d*x)**4*sec(c + d*x)/(105*d) + a*
tan(c + d*x)**4/(4*d) - 64*a*tan(c + d*x)**2*sec(c + d*x)/(315*d) - a*tan(c
 + d*x)**2/(2*d) + 128*a*sec(c + d*x)/(315*d), Ne(d, 0)), (x*(a*sec(c) + a)
*tan(c)**9, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.77

$$\int (a+a\sec(c+dx))\tan^9(c+dx)\,dx = -\frac{2520\,a\log\left(\cos\left(dx+c\right)\right) - \frac{2520\,a\cos(dx+c)^8-5040\,a\cos(dx+c)^7-3360\,a\cos(dx+c)^6+3780\,a\cos(dx+c)^5+3024\,a\cos(dx+c)^4-1680\,a}{\cos(dx+c)^9}}{2520\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(2520*a*log(cos(d*x + c)) - (2520*a*cos(d*x + c)^8 - 5040*a*cos(d*x
 + c)^7 - 3360*a*cos(d*x + c)^6 + 3780*a*cos(d*x + c)^5 + 3024*a*cos(d*x +
c)^4 - 1680*a*cos(d*x + c)^3 - 1440*a*cos(d*x + c)^2 + 315*a*cos(d*x + c) +
 280*a)/cos(d*x + c)^9)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(137) = 274.

Time = 6.42 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.94

$$\int (a+a\sec(c+dx))\tan^9(c+dx)\,dx = \frac{2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{9177\,a+\frac{87633\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{375732\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{}}{}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2520*
a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (9177*a + 87633*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 375732*a*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 953988*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1594
782*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1336734*a*(cos(d*x + c) -
 1)^5/(cos(d*x + c) + 1)^5 + 781956*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) +
1)^6 + 302004*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 69201*a*(cos(d*
x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 7129*a*(cos(d*x + c) - 1)^9/(cos(d*x +
 c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^9)/d
```

Mupad [B] (verification not implemented)

Time = 17.95 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.72

$$\int (a + a\sec(c+dx))\tan^9(c+dx)\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

$$-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-18\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+\frac{218\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}-174\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{1382\,a\tan(}{5}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-84\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+126\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\right.}$$

```
[In] int(tan(c + d*x)^9*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - ((256*a)/315 - (326*a*tan(c/2 + (d*x)
/2)^2)/35 + (1654*a*tan(c/2 + (d*x)/2)^4)/35 - (2114*a*tan(c/2 + (d*x)/2)^6
)/15 + (1382*a*tan(c/2 + (d*x)/2)^8)/5 - 174*a*tan(c/2 + (d*x)/2)^10 + (218
*a*tan(c/2 + (d*x)/2)^12)/3 - 18*a*tan(c/2 + (d*x)/2)^14 + 2*a*tan(c/2 + (d
*x)/2)^16)/(d*(9*tan(c/2 + (d*x)/2)^2 - 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/
2 + (d*x)/2)^6 - 126*tan(c/2 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 - 84*
tan(c/2 + (d*x)/2)^12 + 36*tan(c/2 + (d*x)/2)^14 - 9*tan(c/2 + (d*x)/2)^16
+ tan(c/2 + (d*x)/2)^18 - 1))
```

3.2 $\int (a+a\sec(c+dx))\tan^7(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int (a+a\sec(c+dx))\tan^7(c+dx)\,dx=\frac{a\log(\cos(c+dx))}{d}-\frac{a\sec(c+dx)}{d}+\frac{3a\sec^2(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{d}-\frac{3a\sec^4(c+dx)}{4d}-\frac{3a\sec^5(c+dx)}{5d}+\frac{a\sec^6(c+dx)}{6d}+\frac{a\sec^7(c+dx)}{7d}$$

```
[Out] a*ln(cos(d*x+c))/d-a*sec(d*x+c)/d+3/2*a*sec(d*x+c)^2/d+a*sec(d*x+c)^3/d-3/4
*a*sec(d*x+c)^4/d-3/5*a*sec(d*x+c)^5/d+1/6*a*sec(d*x+c)^6/d+1/7*a*sec(d*x+c
)^7/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 90}

$$\int (a+a\sec(c+dx))\tan^7(c+dx)\,dx=\frac{a\sec^7(c+dx)}{7d}+\frac{a\sec^6(c+dx)}{6d}-\frac{3a\sec^5(c+dx)}{5d}-\frac{3a\sec^4(c+dx)}{4d}+\frac{a\sec^3(c+dx)}{d}+\frac{3a\sec^2(c+dx)}{2d}-\frac{a\sec(c+dx)}{d}+\frac{a\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (a*Sec[c + d*x])/d + (3*a*Sec[c + d*x]^2)/(2*d) +
 (a*Sec[c + d*x]^3)/d - (3*a*Sec[c + d*x]^4)/(4*d) - (3*a*Sec[c + d*x]^5)/(
5*d) + (a*Sec[c + d*x]^6)/(6*d) + (a*Sec[c + d*x]^7)/(7*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^4}{x^8}\,dx, x, \cos(c+dx)\right)}{a^6 d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}+\frac{a^7}{x^7}-\frac{3a^7}{x^6}-\frac{3a^7}{x^5}+\frac{3a^7}{x^4}+\frac{3a^7}{x^3}-\frac{a^7}{x^2}-\frac{a^7}{x}\right)dx, x, \cos(c+dx)\right)}{a^6 d}\\
&= \frac{a\log(\cos(c+dx))}{d}-\frac{a\sec(c+dx)}{d}+\frac{3a\sec^2(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{d}\\
&\quad-\frac{3a\sec^4(c+dx)}{4d}-\frac{3a\sec^5(c+dx)}{5d}+\frac{a\sec^6(c+dx)}{6d}+\frac{a\sec^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.90

$$\begin{aligned}
&\int (a+a\sec(c+dx))\tan^7(c+dx)\,dx\\
&= -\frac{a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{d}-\frac{3a\sec^5(c+dx)}{5d}+\frac{a\sec^7(c+dx)}{7d}\\
&\quad+\frac{a(12\log(\cos(c+dx))+6\tan^2(c+dx)-3\tan^4(c+dx)+2\tan^6(c+dx))}{12d}
\end{aligned}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/d - (3*a*Sec[c + d*x]^5)/(5*d) +
 (a*Sec[c + d*x]^7)/(7*d) + (a*(12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 - 3
*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)^7}{7}+\frac{\sec(dx+c)^6}{6}-\frac{3\sec(dx+c)^5}{5}-\frac{3\sec(dx+c)^4}{4}+\sec(dx+c)^3+\frac{3\sec(dx+c)^2}{2}-\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)^7}{7}+\frac{\sec(dx+c)^6}{6}-\frac{3\sec(dx+c)^5}{5}-\frac{3\sec(dx+c)^4}{4}+\sec(dx+c)^3+\frac{3\sec(dx+c)^2}{2}-\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^6}{6}-\frac{\tan(dx+c)^4}{4}+\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a\left(\frac{\sec(dx+c)^7}{7}-\frac{3\sec(dx+c)^5}{5}+\sec(dx+c)^3-\sec(dx+c)\right)}{d}$
risch	$-iax-\frac{2iac}{d}-\frac{2a\left(105\,e^{13i(dx+c)}-315\,e^{12i(dx+c)}+210\,e^{11i(dx+c)}-945\,e^{10i(dx+c)}+903\,e^{9i(dx+c)}-1820\,e^{8i(dx+c)}+63}{105d\left(e^{2i(}\right.}$

[In] int((a+a*sec(d*x+c))*tan(d*x+c)^7,x,method=_RETURNVERBOSE)

[Out] a/d*(1/7*sec(d*x+c)^7+1/6*sec(d*x+c)^6-3/5*sec(d*x+c)^5-3/4*sec(d*x+c)^4+se
c(d*x+c)^3+3/2*sec(d*x+c)^2-sec(d*x+c)-ln(sec(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int (a+a\sec(c+dx))\tan^7(c+dx)\,dx$$
$$=\frac{420\,a\cos(dx+c)^7\log(-\cos(dx+c))-420\,a\cos(dx+c)^6+630\,a\cos(dx+c)^5+420\,a\cos(dx+c)^4-3}{420\,d\cos(dx+c)^7}$$

[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="fricas")

[Out] 1/420*(420*a*cos(d*x + c)^7*log(-cos(d*x + c)) - 420*a*cos(d*x + c)^6 + 630
*a*cos(d*x + c)^5 + 420*a*cos(d*x + c)^4 - 315*a*cos(d*x + c)^3 - 252*a*cos
(d*x + c)^2 + 70*a*cos(d*x + c) + 60*a)/(d*cos(d*x + c)^7)

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.25

$$\int (a+a\sec(c+dx))\tan^7(c+dx)\,dx$$
$$=\begin{cases} -\frac{a\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a\tan^6(c+dx)\sec(c+dx)}{7d}+\frac{a\tan^6(c+dx)}{6d}-\frac{6a\tan^4(c+dx)\sec(c+dx)}{35d}-\frac{a\tan^4(c+dx)}{4d}+\frac{8a\tan^2(c+dx)}{35d} \\ x(a\sec(c)+a)\tan^7(c) \end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**6*sec(c + d*
x)/(7*d) + a*tan(c + d*x)**6/(6*d) - 6*a*tan(c + d*x)**4*sec(c + d*x)/(35*d
) - a*tan(c + d*x)**4/(4*d) + 8*a*tan(c + d*x)**2*sec(c + d*x)/(35*d) + a*t
an(c + d*x)**2/(2*d) - 16*a*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c) +
a)*tan(c)**7, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int (a+a\sec(c+dx))\tan^7(c+dx)\,dx$$
$$=\frac{420\,a\log(\cos(dx+c))-\frac{420\,a\cos(dx+c)^6-630\,a\cos(dx+c)^5-420\,a\cos(dx+c)^4+315\,a\cos(dx+c)^3+252\,a\cos(dx+c)^2-70\,a\cos(dx+c)}{\cos(dx+c)^7}}{420\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(420*a*log(cos(d*x + c)) - (420*a*cos(d*x + c)^6 - 630*a*cos(d*x + c)
^5 - 420*a*cos(d*x + c)^4 + 315*a*cos(d*x + c)^3 + 252*a*cos(d*x + c)^2 - 7
0*a*cos(d*x + c) - 60*a)/cos(d*x + c)^7)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(108) = 216$.

Time = 3.63 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.09

$$\int (a+a\sec(c+dx))\tan^7(c+dx)\,dx =$$
$$-\frac{420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{1473\,a+\frac{11151\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{36813\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{}}{420\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1473*a + 11151*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 36813*a*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2 + 69475*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a*(cos(d*x + c) - 1)^5/(c
os(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 108
9*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 17.60 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.73

$$\int (a + a\sec(c + dx))\tan^7(c + dx)\,dx$$
$$= \frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + \frac{128\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3} - \frac{224\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3} + \frac{166\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{5} - }{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} - 7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} + 21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} + 35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} - 21\tan\right.}$$
$$- \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\right)}{d}$$

```
[In] int(tan(c + d*x)^7*(a + a/cos(c + d*x)),x)
```

```
[Out] ((32*a)/35 - (42*a*tan(c/2 + (d*x)/2)^2)/5 + (166*a*tan(c/2 + (d*x)/2)^4)/5
 - (224*a*tan(c/2 + (d*x)/2)^6)/3 + (128*a*tan(c/2 + (d*x)/2)^8)/3 - 14*a*t
an(c/2 + (d*x)/2)^10 + 2*a*tan(c/2 + (d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/2)^
2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/
2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)
/2)^14 - 1)) - (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.3 $\int (a+a\sec(c+dx))\tan^5(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int (a+a\sec(c+dx))\tan^5(c+dx)\,dx=-\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\sec^2(c+dx)}{d}-\frac{2a\sec^3(c+dx)}{3d}+\frac{a\sec^4(c+dx)}{4d}+\frac{a\sec^5(c+dx)}{5d}$$

```
[Out] -a*ln(cos(d*x+c))/d+a*sec(d*x+c)/d-a*sec(d*x+c)^2/d-2/3*a*sec(d*x+c)^3/d+1/
4*a*sec(d*x+c)^4/d+1/5*a*sec(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 90}

$$\int (a+a\sec(c+dx))\tan^5(c+dx)\,dx=\frac{a\sec^5(c+dx)}{5d}+\frac{a\sec^4(c+dx)}{4d}-\frac{2a\sec^3(c+dx)}{3d}-\frac{a\sec^2(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d - (a*Sec[c + d*x]^2)/d - (2
*a*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x]^4)/(4*d) + (a*Sec[c + d*x]^5)/(5
*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)^3}{x^6}\,dx, x, \cos(c+dx)\right)}{a^4d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{a^5}{x^5}-\frac{2a^5}{x^4}-\frac{2a^5}{x^3}+\frac{a^5}{x^2}+\frac{a^5}{x}\right)\,dx, x, \cos(c+dx)\right)}{a^4d}\\
&= -\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\sec^2(c+dx)}{d}\\
&\quad -\frac{2a\sec^3(c+dx)}{3d}+\frac{a\sec^4(c+dx)}{4d}+\frac{a\sec^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\begin{aligned}
&\int (a+a\sec(c+dx))\tan^5(c+dx)\,dx\\
&= \frac{a\sec(c+dx)}{d}-\frac{2a\sec^3(c+dx)}{3d}+\frac{a\sec^5(c+dx)}{5d}\\
&\quad -\frac{a(4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx))}{4d}
\end{aligned}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (a*Sec[c + d*x])/d - (2*a*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x]^5)/(5*d)
- (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)^5}{5}+\frac{\sec(dx+c)^4}{4}-\frac{2\sec(dx+c)^3}{3}-\sec(dx+c)^2+\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)^5}{5}+\frac{\sec(dx+c)^4}{4}-\frac{2\sec(dx+c)^3}{3}-\sec(dx+c)^2+\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a\left(\frac{\sec(dx+c)^5}{5}-\frac{2\sec(dx+c)^3}{3}+\sec(dx+c)\right)}{d}$
risch	$iax+\frac{2iac}{d}+\frac{2a\left(15\,\mathrm{e}^{9i(dx+c)}-30\,\mathrm{e}^{8i(dx+c)}+20\,\mathrm{e}^{7i(dx+c)}-60\,\mathrm{e}^{6i(dx+c)}+58\,\mathrm{e}^{5i(dx+c)}-60\,\mathrm{e}^{4i(dx+c)}+20\,\mathrm{e}^{3i(dx+c)}-3\right.}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] a/d*(1/5*sec(d*x+c)^5+1/4*sec(d*x+c)^4-2/3*sec(d*x+c)^3-sec(d*x+c)^2+sec(d*
x+c)+ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int (a+a\sec(c+dx))\tan^5(c+dx)\,dx =$$
$$-\frac{60\,a\cos\left(dx+c\right)^5\log\left(-\cos\left(dx+c\right)\right)-60\,a\cos\left(dx+c\right)^4+60\,a\cos\left(dx+c\right)^3+40\,a\cos\left(dx+c\right)^2-15}{60\,d\cos\left(dx+c\right)^5}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*cos(d*x + c)^5*log(-cos(d*x + c)) - 60*a*cos(d*x + c)^4 + 60*a*
cos(d*x + c)^3 + 40*a*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 12*a)/(d*cos(d*x
 + c)^5)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.29

$$\int (a + a\sec(c+dx))\tan^5(c+dx)\,dx$$
$$= \begin{cases} \frac{a\log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a\tan^4(c+dx)\sec(c+dx)}{5d} + \frac{a\tan^4(c+dx)}{4d} - \frac{4a\tan^2(c+dx)\sec(c+dx)}{15d} - \frac{a\tan^2(c+dx)}{2d} + \frac{8a\sec(c+dx)}{15d} & \text{fo} \\ x(a\sec(c)+a)\tan^5(c) & \text{ot} \end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**4*sec(c + d*x
)/(5*d) + a*tan(c + d*x)**4/(4*d) - 4*a*tan(c + d*x)**2*sec(c + d*x)/(15*d)
 - a*tan(c + d*x)**2/(2*d) + 8*a*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a*sec(
c) + a)*tan(c)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int (a + a\sec(c+dx))\tan^5(c+dx)\,dx$$
$$= -\frac{60\,a\log\left(\cos\left(dx+c\right)\right) - \frac{60\,a\cos(dx+c)^4 - 60\,a\cos(dx+c)^3 - 40\,a\cos(dx+c)^2 + 15\,a\cos(dx+c) + 12\,a}{\cos(dx+c)^5}}{60\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a*log(cos(d*x + c)) - (60*a*cos(d*x + c)^4 - 60*a*cos(d*x + c)^3
- 40*a*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 12*a)/cos(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(81) = 162$.

Time = 1.60 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.31

$$\int (a + a\sec(c+dx))\tan^5(c+dx)\,dx$$
$$= \frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{201\,a + \frac{1125\,a(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{2610\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{1970\,a(\cos}{(\cos(d}}{\left(\frac{\cos(dx+c)-}{\cos(dx+c)+}\right.}}{60\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (201*a + 1125*a*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 2610*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*a*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a*(cos(d*x + c) - 1)^5/(cos(d*x + c)
+ 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 18.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.74

$$\int (a+a\sec(c+dx))\tan^5(c+dx)\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d} - \frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{62\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{22\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{16\,a}{15}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - ((16*a)/15 - (22*a*tan(c/2 + (d*x)/2)
^2)/3 + (62*a*tan(c/2 + (d*x)/2)^4)/3 - 10*a*tan(c/2 + (d*x)/2)^6 + 2*a*tan
(c/2 + (d*x)/2)^8)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 1
0*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1
))
```

3.4 $\int (a+a\sec(c+dx))\tan^3(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int (a+a\sec(c+dx))\tan^3(c+dx)\,dx=\frac{a\log(\cos(c+dx))}{d}-\frac{a\sec(c+dx)}{d}+\frac{a\sec^2(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}$$

```
[Out] a*ln(cos(d*x+c))/d-a*sec(d*x+c)/d+1/2*a*sec(d*x+c)^2/d+1/3*a*sec(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 76}

$$\int (a+a\sec(c+dx))\tan^3(c+dx)\,dx=\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec^2(c+dx)}{2d}-\frac{a\sec(c+dx)}{d}+\frac{a\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^2)/(2*d) + (
a*Sec[c + d*x]^3)/(3*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
```

```
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)(a+ax)^2}{x^4}\,dx, x, \cos(c+dx)\right)}{a^2 d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{a^3}{x^3}-\frac{a^3}{x^2}-\frac{a^3}{x}\right)\,dx, x, \cos(c+dx)\right)}{a^2 d}\\
&= \frac{a\log(\cos(c+dx))}{d}-\frac{a\sec(c+dx)}{d}+\frac{a\sec^2(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int (a+a\sec(c+dx))\tan^3(c+dx)\,dx = -\frac{a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a(2\log(\cos(c+dx))+\tan^2(c+dx))}{2d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^3,x]
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/(3*d) + (a*(2*Log[Cos[c + d*x]]
+ Tan[c + d*x]^2))/(2*d)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)^3}{3}+\frac{\sec(dx+c)^2}{2}-\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$	44
default	$\frac{a\left(\frac{\sec(dx+c)^3}{3}+\frac{\sec(dx+c)^2}{2}-\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$	44
parts	$\frac{a\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a\left(\frac{\sec(dx+c)^3}{3}-\sec(dx+c)\right)}{d}$	55
risch	$-iax-\frac{2iac}{d}-\frac{2a\left(3\,\mathrm{e}^{5i(dx+c)}-3\,\mathrm{e}^{4i(dx+c)}+2\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	107

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
[Out] a/d*(1/3*sec(d*x+c)^3+1/2*sec(d*x+c)^2-sec(d*x+c)-ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (a+a\sec(c+dx))\tan^3(c+dx)\,dx = \frac{6\,a\cos\left(dx+c\right)^3\log\left(-\cos\left(dx+c\right)\right)-6\,a\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+2\,a}{6\,d\cos\left(dx+c\right)^3}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
[Out] 1/6*(6*a*cos(d*x + c)^3*log(-cos(d*x + c)) - 6*a*cos(d*x + c)^2 + 3*a*cos(d
*x + c) + 2*a)/(d*cos(d*x + c)^3)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int (a+a\sec(c+dx))\tan^3(c+dx)\,dx = \begin{cases} -\frac{a\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{3d}+\frac{a\tan^2\left(c+dx\right)}{2d}-\frac{2a\sec\left(c+dx\right)}{3d} & \text{for}\ d\neq 0 \\ x(a\sec\left(c\right)+a)\tan^3\left(c\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**3,x)
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**2*sec(c + d*
x)/(3*d) + a*tan(c + d*x)**2/(2*d) - 2*a*sec(c + d*x)/(3*d), Ne(d, 0)), (x*
(a*sec(c) + a)*tan(c)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int (a + a\sec(c + dx))\tan^3(c + dx)\,dx = \frac{6\,a\log\left(\cos\left(dx + c\right)\right) - \frac{6\,a\cos(dx+c)^2 - 3\,a\cos(dx+c) - 2\,a}{\cos(dx+c)^3}}{6\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a*log(cos(d*x + c)) - (6*a*cos(d*x + c)^2 - 3*a*cos(d*x + c) - 2*a)/
cos(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(53) = 106.

Time = 0.64 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.72

$$\int (a + a\sec(c + dx))\tan^3(c + dx)\,dx =$$

$$-\frac{6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{19\,a + \frac{69\,a(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{45\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{11\,a(\cos(dx+}{(\cos(dx+c)}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (19*a + 69*a*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 11*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 15.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

$$\int (a + a\sec(c + dx))\tan^3(c + dx)\,dx$$

$$= \frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + \frac{4\,a}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1\right)} - \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

```
[Out] ((4*a)/3 - 6*a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(c
/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (2*
a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.5 $\int (a+a\sec(c+dx))\tan(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int (a+a\sec(c+dx))\tan(c+dx)\,dx=-\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}$$

```
[Out] -a*ln(cos(d*x+c))/d+a*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3964, 45}

$$\int (a+a\sec(c+dx))\tan(c+dx)\,dx=\frac{a\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{a+ax}{x^2}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{a}{x}\right)\,dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+a\sec(c+dx))\tan(c+dx)\,dx = -\frac{a\log(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x],x]
[Out] -((a*Log[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{a(\sec(dx+c)+\ln(\sec(dx+c)))}{d}$	20
default	$\frac{a(\sec(dx+c)+\ln(\sec(dx+c)))}{d}$	20
parts	$\frac{a\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{a\sec(dx+c)}{d}$	30
risch	$iax+\frac{2iac}{d}+\frac{2a\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	61

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*a*(sec(d*x+c)+ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int (a + a\sec(c + dx))\tan(c + dx)\,dx = -\frac{a\cos(dx + c)\log(-\cos(dx + c)) - a}{d\cos(dx + c)}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="fricas")
[Out] -(a*cos(d*x + c)*log(-cos(d*x + c)) - a)/(d*cos(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int (a + a\sec(c + dx))\tan(c + dx)\,dx = \begin{cases} \frac{a\log(\tan^2(c+dx)+1)}{2d} + \frac{a\sec(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sec(c) + a)\tan(c) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x)
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*sec(c + d*x)/d, Ne(d, 0)),
(x*(a*sec(c) + a)*tan(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int (a + a\sec(c + dx))\tan(c + dx)\,dx = -\frac{a\log(\cos(dx + c)) - \frac{a}{\cos(dx+c)}}{d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="maxima")
[Out] -(a*log(cos(d*x + c)) - a/cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(25) = 50$.

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.24

$$\int (a + a\sec(c + dx))\tan(c + dx)\,dx = \frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{3\,a + \frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

```
[Out] (a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3*a + a*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 14.70 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int (a + a\sec(c + dx))\tan(c + dx)\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2\right)}{d} - \frac{2\,a}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 - 1\right)}$$

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.6 $\int \cot(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx=\frac{a\log(1-\cos(c+dx))}{d}$$

```
[Out] a*ln(1-cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3964, 31}

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx=\frac{a\log(1-\cos(c+dx))}{d}$$

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Log[1 - Cos[c + d*x]])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
```

[n]

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a^2\text{Subst}\left(\int \frac{1}{a-ax}\,dx, x, \cos(c+dx)\right)}{d}\\
&= \frac{a\log(1-\cos(c+dx))}{d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 38 vs. $2(16) = 32$.

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$
\int \cot(c+dx)(a+a\sec(c+dx))\,dx = a\left(\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{2\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}\right)
$$

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x]),x]
[Out] a*((2*Log[Cos[(c + d*x)/2]])/d + (2*Log[Tan[c/2 + (d*x)/2]])/d)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

method	result	size
derivativedivides	$\frac{a\ln(-\cot(dx+c)+\csc(dx+c))+a\ln(\sin(dx+c))}{d}$	33
default	$\frac{a\ln(-\cot(dx+c)+\csc(dx+c))+a\ln(\sin(dx+c))}{d}$	33
risch	$-iax-\frac{2iac}{d}+\frac{2a\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d}$	33

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*ln(-cot(d*x+c)+csc(d*x+c))+a*ln(sin(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{d}$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] a*log(-1/2*cos(d*x + c) + 1/2)/d
```

Sympy [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot\left(c+dx\right)\sec\left(c+dx\right)dx+\int \cot\left(c+dx\right)dx\right)$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(cot(c + d*x)*sec(c + d*x), x) + Integral(cot(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\cos\left(dx+c\right)-1\right)}{d}$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] a*log(cos(d*x + c) - 1)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(16) = 32.

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.62

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{d}$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] (a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a*log(abs(-(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \cot(c+dx)(a+a\sec(c+dx))\,dx=\frac{a\left(2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\right)}{d}$$

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x)),x)
[Out] (a*(2*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^2 + 1)))/d
```

3.7 $\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx=-\frac{a}{2d(1-\cos(c+dx))}-\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(1+\cos(c+dx))}{4d}$$

```
[Out] -1/2*a/d/(1-cos(d*x+c))-3/4*a*ln(1-cos(d*x+c))/d-1/4*a*ln(1+cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 90}

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx=-\frac{a}{2d(1-\cos(c+dx))}-\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx)+1)}{4d}$$

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*a/(d*(1 - Cos[c + d*x])) - (3*a*Log[1 - Cos[c + d*x]])/(4*d) - (a*Log[
1 + Cos[c + d*x]])/(4*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^4 \text{Subst}\left(\int \frac{x^2}{(a-ax)^2(a+ax)}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{a^4 \text{Subst}\left(\int \left(\frac{1}{2a^3(-1+x)^2} + \frac{3}{4a^3(-1+x)} + \frac{1}{4a^3(1+x)}\right)dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{a}{2d(1-\cos(c+dx))} - \frac{3a\log(1-\cos(c+dx))}{4d} - \frac{a\log(1+\cos(c+dx))}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.00

$$\begin{aligned}
&\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx \\
&= -\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} \\
&\quad - \frac{a(\cot^2(c+dx) + 2\log(\cos(c+dx)) + 2\log(\tan(c+dx)))}{2d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}
\end{aligned}$$

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x]),x]
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d + (a*Log[Cos[(c + d*x)/2]])/(2*d) - (a*Log[Si
n[(c + d*x)/2]])/(2*d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[T
an[c + d*x]]))/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+a\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)}{d}$	75
default	$\frac{a\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+a\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)}{d}$	75
risch	$iax+\frac{2iac}{d}+\frac{a\,e^{i(dx+c)}}{d\left(e^{i(dx+c)}-1\right)^2}-\frac{3a\ln\left(e^{i(dx+c)}-1\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}$	78

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(-cot(d*x+c)+cs
c(d*x+c)))+a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{(a\cos(dx+c)-a)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3\,(a\cos(dx+c)-a)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\,a}{4\,(d\cos(dx+c)-d)}$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/4*((a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x + c)
- a)*log(-1/2*cos(d*x + c) + 1/2) - 2*a)/(d*cos(d*x + c) - d)
```

Sympy [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^3(c+dx)\sec(c+dx)\,dx+\int \cot^3(c+dx)\,dx\right)$$

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(cot(c + d*x)**3*sec(c + d*x), x) + Integral(cot(c + d*x)**3, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\log\left(\cos\left(dx+c\right)+1\right)+3\,a\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\,a}{\cos(dx+c)-1}}{4\,d}$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(a*log(cos(d*x + c) + 1) + 3*a*log(cos(d*x + c) - 1) - 2*a/(cos(d*x +
c) - 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(49) = 98$.

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.81

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{3\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-4\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{4\,d}$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(3*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a*log(abs(-
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + 3*a*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \cot^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\left(\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{4\,d}$$

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(6*log(tan(c/2 + (d*x)/2)) - 4*log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
+ (d*x)/2)^2))/(4*d)
```

3.8 $\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 95

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a}{8d(1-\cos(c+dx))^2}+\frac{3a}{4d(1-\cos(c+dx))} + \frac{a}{8d(1+\cos(c+dx))}+\frac{11a\log(1-\cos(c+dx))}{16d} + \frac{5a\log(1+\cos(c+dx))}{16d}$$

```
[Out] -1/8*a/d/(1-cos(d*x+c))^2+3/4*a/d/(1-cos(d*x+c))+1/8*a/d/(1+cos(d*x+c))+11/
16*a*ln(1-cos(d*x+c))/d+5/16*a*ln(1+cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 90}

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx = \frac{3a}{4d(1-\cos(c+dx))}+\frac{a}{8d(\cos(c+dx)+1)} - \frac{a}{8d(1-\cos(c+dx))^2}+\frac{11a\log(1-\cos(c+dx))}{16d} + \frac{5a\log(\cos(c+dx)+1)}{16d}$$

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*a/(d*(1 - Cos[c + d*x])^2) + (3*a)/(4*d*(1 - Cos[c + d*x])) + a/(8*d*(
1 + Cos[c + d*x])) + (11*a*Log[1 - Cos[c + d*x]])/(16*d) + (5*a*Log[1 + Cos
[c + d*x]])/(16*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^6\text{Subst}\left(\int \frac{x^4}{(a-ax)^3(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^6\text{Subst}\left(\int \left(-\frac{1}{4a^5(-1+x)^3}-\frac{3}{4a^5(-1+x)^2}-\frac{11}{16a^5(-1+x)}+\frac{1}{8a^5(1+x)^2}-\frac{5}{16a^5(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a}{8d(1-\cos(c+dx))^2}+\frac{3a}{4d(1-\cos(c+dx))}+\frac{a}{8d(1+\cos(c+dx))}\\
&\quad+\frac{11a\log(1-\cos(c+dx))}{16d}+\frac{5a\log(1+\cos(c+dx))}{16d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.84

$$\begin{aligned}
\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx &= \frac{a\cot^2(c+dx)}{2d}-\frac{a\cot^4(c+dx)}{4d}\\
&\quad+\frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}\\
&\quad-\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{a\log(\cos(c+dx))}{d}\\
&\quad+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{a\log(\tan(c+dx))}{d}\\
&\quad-\frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}
\end{aligned}$$

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

[Out] (a*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^4)/(4*d) + (5*a*Csc[(c + d*x)/2]
^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (3*a*Log[Cos[(c + d*x)/2]])/(8
*d) + (a*Log[Cos[c + d*x]])/d + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) + (a*Log[
Tan[c + d*x]])/d - (5*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)
/(64*d)

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+a\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\ln\right.}{d}$
default	$\frac{a\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+a\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\ln\right.}{d}$
risch	$-iax-\frac{2iac}{d}-\frac{a\left(5\,e^{5i(dx+c)}+6\,e^{4i(dx+c)}-14\,e^{3i(dx+c)}+6\,e^{2i(dx+c)}+5\,e^{i(dx+c)}\right)}{4d\left(e^{i(dx+c)}-1\right)^4\left(e^{i(dx+c)}+1\right)^2}+\frac{5a\ln\left(e^{i(dx+c)}+1\right)}{8d}+\frac{11a\ln(e}{}$

[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-1/4/sin(d*x+c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*co
s(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(-cot(d*x+c)+csc(d*x+c)))+a*(-1/4*cot(d*x+c
)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c))))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.58

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{10\,a\cos\left(dx+c\right)^2+6\,a\cos\left(dx+c\right)-5\left(a\cos\left(dx+c\right)^3-a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)+a\right)\log\left(\frac{1}{2}\cos\right.}{16\left(d\cos\left(dx+c\right)^3-d\cos\right.}$$

[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/16*(10*a*cos(d*x + c)^2 + 6*a*cos(d*x + c) - 5*(a*cos(d*x + c)^3 - a*cos
(d*x + c)^2 - a*cos(d*x + c) + a)*log(1/2*cos(d*x + c) + 1/2) - 11*(a*cos(d
*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-1/2*cos(d*x + c) +
1/2) - 12*a)/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)

Sympy [F]

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^5(c+dx)\sec(c+dx)\,dx + \int \cot^5(c+dx)\,dx\right)$$

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(cot(c + d*x)**5*sec(c + d*x), x) + Integral(cot(c + d*x)**5, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx = \frac{5\,a\log(\cos(dx+c)+1) + 11\,a\log(\cos(dx+c)-1) - \frac{2\left(5\,a\cos(dx+c)^2+3\,a\cos(dx+c)-6\,a\right)}{\cos(dx+c)^3-\cos(dx+c)^2-\cos(dx+c)+1}}{16\,d}$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/16*(5*a*log(cos(d*x + c) + 1) + 11*a*log(cos(d*x + c) - 1) - 2*(5*a*cos(d
*x + c)^2 + 3*a*cos(d*x + c) - 6*a)/(cos(d*x + c)^3 - cos(d*x + c)^2 - cos(
d*x + c) + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.57

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx = \frac{22\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 32\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{\left(a+\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{33\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2} -}{32\,d}$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] 1/32*(22*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + 10*a*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 33*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \cot^5(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{16\,d} - \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a}{4}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}\right)}{8\,d} + \frac{11\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,d}$$

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x)),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(16*d) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (co
t(c/2 + (d*x)/2)^4*(a/4 - (5*a*tan(c/2 + (d*x)/2)^2)/2))/(8*d) + (11*a*log(
tan(c/2 + (d*x)/2)))/(8*d)
```

3.9 $\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\begin{aligned}\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx= & -\frac{a}{24d(1-\cos(c+dx))^3}+\frac{9a}{32d(1-\cos(c+dx))^2}\\ & -\frac{15a}{16d(1-\cos(c+dx))}+\frac{a}{32d(1+\cos(c+dx))^2}\\ & -\frac{a}{4d(1+\cos(c+dx))}-\frac{21a\log(1-\cos(c+dx))}{32d}\\ & -\frac{11a\log(1+\cos(c+dx))}{32d}\end{aligned}$$

```
[Out] -1/24*a/d/(1-cos(d*x+c))^3+9/32*a/d/(1-cos(d*x+c))^2-15/16*a/d/(1-cos(d*x+c
))+1/32*a/d/(1+cos(d*x+c))^2-1/4*a/d/(1+cos(d*x+c))-21/32*a*ln(1-cos(d*x+c)
)/d-11/32*a*ln(1+cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 90}

$$\begin{aligned}\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx= & -\frac{15a}{16d(1-\cos(c+dx))}-\frac{a}{4d(\cos(c+dx)+1)}\\ & +\frac{9a}{32d(1-\cos(c+dx))^2}+\frac{a}{32d(\cos(c+dx)+1)^2}\\ & -\frac{a}{24d(1-\cos(c+dx))^3}-\frac{21a\log(1-\cos(c+dx))}{32d}\\ & -\frac{11a\log(\cos(c+dx)+1)}{32d}\end{aligned}$$

```
[In] Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/24*a/(d*(1 - Cos[c + d*x])^3) + (9*a)/(32*d*(1 - Cos[c + d*x])^2) - (15*
a)/(16*d*(1 - Cos[c + d*x])) + a/(32*d*(1 + Cos[c + d*x])^2) - a/(4*d*(1 +
Cos[c + d*x])) - (21*a*Log[1 - Cos[c + d*x]])/(32*d) - (11*a*Log[1 + Cos[c
+ d*x]])/(32*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\text{integral} = -\frac{a^8\text{Subst}\left(\int \frac{x^6}{(a-ax)^4(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^8\text{Subst}\left(\int \left(\frac{1}{8a^7(-1+x)^4} + \frac{9}{16a^7(-1+x)^3} + \frac{15}{16a^7(-1+x)^2} + \frac{21}{32a^7(-1+x)} + \frac{1}{16a^7(1+x)^3} - \frac{1}{4a^7(1+x)^2} + \frac{11}{32a^7(1+x}\right.\right.}{d}$$

$$= -\frac{a}{24d(1-\cos(c+dx))^3} + \frac{9a}{32d(1-\cos(c+dx))^2} - \frac{15a}{16d(1-\cos(c+dx))} + \frac{a}{32d(1+\cos(c+dx))^2} - \frac{a}{4d(1+\cos(c+dx))} - \frac{21a\log(1-\cos(c+dx))}{32d} - \frac{11a\log(1+\cos(c+dx))}{32d}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.24

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{a\left(192\cot^2(c+dx)-96\cot^4(c+dx)+64\cot^6(c+dx)+66\csc^2\left(\frac{1}{2}(c+dx)\right)-12\csc^4\left(\frac{1}{2}(c+dx)\right)+\mathrm{c}\right.}{}$$

[In] Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x]),x]

[Out] -1/384*(a*(192*Cot[c + d*x]^2 - 96*Cot[c + d*x]^4 + 64*Cot[c + d*x]^6 + 66*
Csc[(c + d*x)/2]^2 - 12*Csc[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6 - 120*Log[C
os[(c + d*x)/2]] + 384*Log[Cos[c + d*x]] + 120*Log[Sin[(c + d*x)/2]] + 384*
Log[Tan[c + d*x]] - 66*Sec[(c + d*x)/2]^2 + 12*Sec[(c + d*x)/2]^4 - Sec[(c
+ d*x)/2]^6))/d

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^7}{6\sin(dx+c)^6}+\frac{\cos(dx+c)^7}{24\sin(dx+c)^4}-\frac{\cos(dx+c)^7}{16\sin(dx+c)^2}-\frac{\cos(dx+c)^5}{16}-\frac{5\cos(dx+c)^3}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right)+\mathrm{c}}{d}$
default	$\frac{a\left(-\frac{\cos(dx+c)^7}{6\sin(dx+c)^6}+\frac{\cos(dx+c)^7}{24\sin(dx+c)^4}-\frac{\cos(dx+c)^7}{16\sin(dx+c)^2}-\frac{\cos(dx+c)^5}{16}-\frac{5\cos(dx+c)^3}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right)+\mathrm{c}}{d}$
risch	$iax+\frac{2iac}{d}+\frac{a\left(33\,\mathrm{e}^{9i(dx+c)}+78\,\mathrm{e}^{8i(dx+c)}-184\,\mathrm{e}^{7i(dx+c)}+2\,\mathrm{e}^{6i(dx+c)}+270\,\mathrm{e}^{5i(dx+c)}+2\,\mathrm{e}^{4i(dx+c)}-184\,\mathrm{e}^{3i(dx+c)}+7\right.}{24d\left(\mathrm{e}^{i(dx+c)}-1\right)^6\left(\mathrm{e}^{i(dx+c)}+1\right)^4}$

[In] int(cot(d*x+c)^7*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/24/sin(d*x+c)^4*cos(d*x+c)^7-1/16/
sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^5-5/48*cos(d*x+c)^3-5/16*cos(d*x+
c)-5/16*ln(-cot(d*x+c)+csc(d*x+c)))+a*(-1/6*cot(d*x+c)^6+1/4*cot(d*x+c)^4-1
/2*cot(d*x+c)^2-ln(sin(d*x+c))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(113) = 226.

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.81

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{66\,a\cos\left(dx+c\right)^4+78\,a\cos\left(dx+c\right)^3-158\,a\cos\left(dx+c\right)^2-58\,a\cos\left(dx+c\right)-33\,\left(a\cos\left(dx+c\right)^5-a\cos\right.}{}$$

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] 1/96*(66*a*cos(d*x + c)^4 + 78*a*cos(d*x + c)^3 - 158*a*cos(d*x + c)^2 - 58
*a*cos(d*x + c) - 33*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c
)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2)
- 63*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*
x + c)^2 + a*cos(d*x + c) - a)*log(-1/2*cos(d*x + c) + 1/2) + 88*a)/(d*cos(
d*x + c)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d
*cos(d*x + c) - d)
```

Sympy [F]

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^7\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cot^7\left(c+dx\right)dx\right)$$

```
[In] integrate(cot(d*x+c)**7*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(cot(c + d*x)**7*sec(c + d*x), x) + Integral(cot(c + d*x)**7, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx =$$
$$-\frac{33\,a\log\left(\cos\left(dx+c\right)+1\right)+63\,a\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(33\,a\cos(dx+c)^4+39\,a\cos(dx+c)^3-79\,a\cos(dx+c)^2-29\,a\cos\right.}{\cos(dx+c)^5-\cos(dx+c)^4-2\,\cos(dx+c)^3+2\,\cos(dx+c)^2+\cos}}{96\,d}$$

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/96*(33*a*log(cos(d*x + c) + 1) + 63*a*log(cos(d*x + c) - 1) - 2*(33*a*co
s(d*x + c)^4 + 39*a*cos(d*x + c)^3 - 79*a*cos(d*x + c)^2 - 29*a*cos(d*x + c
) + 44*a)/(cos(d*x + c)^5 - cos(d*x + c)^4 - 2*cos(d*x + c)^3 + 2*cos(d*x +
 c)^2 + cos(d*x + c) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.48

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{252\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(2\,a+\frac{21\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{462\,a(\cos(dx\ldots}{(\cos(dx+\ldots}\right.}{(\cos(dx+c)-1)^3}}{384\,d}$$

[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] -1/384*(252*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 384*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (2*a + 21*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 132*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 462*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(cos(d*x + c) - 1)^3 - 42*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/d

Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

$$\int \cot^7(c+dx)(a+a\sec(c+dx))\,dx$$

$$=\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}+\frac{a}{6}\right)}{32\,d}$$
$$-\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{128\,d}-\frac{21\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{16\,d}$$

[In] int(cot(c + d*x)^7*(a + a/cos(c + d*x)),x)

[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (cot(c/2 + (d*x)/2)^6*(a/6 - (7*a*tan(c/2 + (d*x)/2)^2)/4 + 11*a*tan(c/2 + (d*x)/2)^4))/(32*d) - (7*a*tan(c/2 + (d*x)/2)^2)/(64*d) + (a*tan(c/2 + (d*x)/2)^4)/(128*d) - (21*a*log(tan(c/2 + (d*x)/2)))/(16*d)

3.10 $\int (a + a\sec(c + dx))\tan^8(c + dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 129

$$\int (a + a\sec(c + dx))\tan^8(c + dx)\,dx = ax + \frac{35a\,\mathrm{arctanh}(\sin(c + dx))}{128d} - \frac{(128a + 35a\sec(c + dx))\tan(c + dx)}{128d} + \frac{(64a + 35a\sec(c + dx))\tan^3(c + dx)}{192d} - \frac{(48a + 35a\sec(c + dx))\tan^5(c + dx)}{240d} + \frac{(8a + 7a\sec(c + dx))\tan^7(c + dx)}{56d}$$

```
[Out] a*x+35/128*a*arctanh(sin(d*x+c))/d-1/128*(128*a+35*a*sec(d*x+c))*tan(d*x+c)
/d+1/192*(64*a+35*a*sec(d*x+c))*tan(d*x+c)^3/d-1/240*(48*a+35*a*sec(d*x+c))
*tan(d*x+c)^5/d+1/56*(8*a+7*a*sec(d*x+c))*tan(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used

$= \{3966, 3855\}$

$$\int (a+a\sec(c+dx))\tan^8(c+dx)\,dx = \frac{35a\text{arctanh}(\sin(c+dx))}{128d} + \frac{\tan^7(c+dx)(7a\sec(c+dx)+8a)}{56d} - \frac{\tan^5(c+dx)(35a\sec(c+dx)+48a)}{240d} + \frac{\tan^3(c+dx)(35a\sec(c+dx)+64a)}{192d} - \frac{\tan(c+dx)(35a\sec(c+dx)+128a)}{128d} + ax$$

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^8,x]
```

```
[Out] a*x + (35*a*ArcTanh[Sin[c + d*x]])/(128*d) - ((128*a + 35*a*Sec[c + d*x])*T
an[c + d*x])/(128*d) + ((64*a + 35*a*Sec[c + d*x])*Tan[c + d*x]^3)/(192*d)
- ((48*a + 35*a*Sec[c + d*x])*Tan[c + d*x]^5)/(240*d) + ((8*a + 7*a*Sec[c +
 d*x])*Tan[c + d*x]^7)/(56*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(8a+7a\sec(c+dx))\tan^7(c+dx)}{56d} - \frac{1}{8}\int (8a+7a\sec(c+dx))\tan^6(c+dx)\,dx \\
&= -\frac{(48a+35a\sec(c+dx))\tan^5(c+dx)}{240d} + \frac{(8a+7a\sec(c+dx))\tan^7(c+dx)}{56d} \\
&\quad + \frac{1}{48}\int (48a+35a\sec(c+dx))\tan^4(c+dx)\,dx \\
&= \frac{(64a+35a\sec(c+dx))\tan^3(c+dx)}{192d} - \frac{(48a+35a\sec(c+dx))\tan^5(c+dx)}{240d} \\
&\quad + \frac{(8a+7a\sec(c+dx))\tan^7(c+dx)}{56d} - \frac{1}{192}\int (192a+105a\sec(c+dx))\tan^2(c \\
&\quad + dx)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(128a+35a\sec(c+dx))\tan(c+dx)}{128d}+\frac{(64a+35a\sec(c+dx))\tan^3(c+dx)}{192d} \\
&\quad -\frac{(48a+35a\sec(c+dx))\tan^5(c+dx)}{240d}+\frac{(8a+7a\sec(c+dx))\tan^7(c+dx)}{56d} \\
&\quad +\frac{1}{384}\int(384a+105a\sec(c+dx))\,dx \\
&= ax-\frac{(128a+35a\sec(c+dx))\tan(c+dx)}{128d} \\
&\quad +\frac{(64a+35a\sec(c+dx))\tan^3(c+dx)}{192d}-\frac{(48a+35a\sec(c+dx))\tan^5(c+dx)}{240d} \\
&\quad +\frac{(8a+7a\sec(c+dx))\tan^7(c+dx)}{56d}+\frac{1}{128}(35a)\int\sec(c+dx)\,dx \\
&= ax+\frac{35a\text{arctanh}(\sin(c+dx))}{128d}-\frac{(128a+35a\sec(c+dx))\tan(c+dx)}{128d} \\
&\quad +\frac{(64a+35a\sec(c+dx))\tan^3(c+dx)}{192d} \\
&\quad -\frac{(48a+35a\sec(c+dx))\tan^5(c+dx)}{240d}+\frac{(8a+7a\sec(c+dx))\tan^7(c+dx)}{56d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.87

$$
\begin{aligned}
\int(a+a\sec(c+dx))\tan^8(c+dx)\,dx &= \frac{a\arctan(\tan(c+dx))}{d}+\frac{35a\text{arctanh}(\sin(c+dx))}{128d} \\
&\quad -\frac{a\tan(c+dx)}{d}+\frac{35a\sec(c+dx)\tan(c+dx)}{128d} \\
&\quad +\frac{35a\sec^3(c+dx)\tan(c+dx)}{192d} \\
&\quad +\frac{7a\sec^5(c+dx)\tan(c+dx)}{48d} \\
&\quad -\frac{7a\sec^7(c+dx)\tan(c+dx)}{8d}+\frac{a\tan^3(c+dx)}{3d} \\
&\quad +\frac{7a\sec^5(c+dx)\tan^3(c+dx)}{3d} \\
&\quad -\frac{a\tan^5(c+dx)}{5d}-\frac{7a\sec^3(c+dx)\tan^5(c+dx)}{3d} \\
&\quad +\frac{a\tan^7(c+dx)}{7d}+\frac{a\sec(c+dx)\tan^7(c+dx)}{d}
\end{aligned}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^8,x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d + (35*a*ArcTanh[Sin[c + d*x]])/(128*d) - (a*Tan[
c + d*x])/d + (35*a*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (35*a*Sec[c + d*x]
```

```
^3*Tan[c + d*x])/(192*d) + (7*a*Sec[c + d*x]^5*Tan[c + d*x])/(48*d) - (7*a*
Sec[c + d*x]^7*Tan[c + d*x])/(8*d) + (a*Tan[c + d*x]^3)/(3*d) + (7*a*Sec[c
+ d*x]^5*Tan[c + d*x]^3)/(3*d) - (a*Tan[c + d*x]^5)/(5*d) - (7*a*Sec[c + d*
x]^3*Tan[c + d*x]^5)/(3*d) + (a*Tan[c + d*x]^7)/(7*d) + (a*Sec[c + d*x]*Tan
[c + d*x]^7)/d
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{a\left(\frac{\sin(dx+c)^9}{8\cos(dx+c)^8}-\frac{\sin(dx+c)^9}{48\cos(dx+c)^6}+\frac{\sin(dx+c)^9}{64\cos(dx+c)^4}-\frac{5\sin(dx+c)^9}{128\cos(dx+c)^2}-\frac{5\sin(dx+c)^7}{128}-\frac{7\sin(dx+c)^5}{128}-\frac{35\sin(dx+c)^3}{384}-\frac{35\sin(dx+c)}{128}+\right.}{d}$
default	$\frac{a\left(\frac{\sin(dx+c)^9}{8\cos(dx+c)^8}-\frac{\sin(dx+c)^9}{48\cos(dx+c)^6}+\frac{\sin(dx+c)^9}{64\cos(dx+c)^4}-\frac{5\sin(dx+c)^9}{128\cos(dx+c)^2}-\frac{5\sin(dx+c)^7}{128}-\frac{7\sin(dx+c)^5}{128}-\frac{35\sin(dx+c)^3}{384}-\frac{35\sin(dx+c)}{128}+\right.}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^7}{7}-\frac{\tan(dx+c)^5}{5}+\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{a\left(\frac{\sin(dx+c)^9}{8\cos(dx+c)^8}-\frac{\sin(dx+c)^9}{48\cos(dx+c)^6}+\frac{\sin(dx}{64\cos(d}\right.}{}$
risch	$ax+\frac{ia(9765\,e^{15i(dx+c)}-53760\,e^{14i(dx+c)}+3185\,e^{13i(dx+c)}-215040\,e^{12i(dx+c)}+62965\,e^{11i(dx+c)}-555520\,e^{10i(dx+c)}}{}$

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/8*sin(d*x+c)^9/cos(d*x+c)^8-1/48*sin(d*x+c)^9/cos(d*x+c)^6+1/64*s
in(d*x+c)^9/cos(d*x+c)^4-5/128*sin(d*x+c)^9/cos(d*x+c)^2-5/128*sin(d*x+c)^7
-7/128*sin(d*x+c)^5-35/384*sin(d*x+c)^3-35/128*sin(d*x+c)+35/128*ln(sec(d*x
+c)+tan(d*x+c)))+a*(1/7*tan(d*x+c)^7-1/5*tan(d*x+c)^5+1/3*tan(d*x+c)^3-tan(
d*x+c)+d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.21

$$\int (a+a\sec(c+dx))\tan^8(c+dx)\,dx$$
$$=\frac{26880\,adx\cos{(dx+c)}^8+3675\,a\cos{(dx+c)}^8\log{(\sin{(dx+c)}+1)}-3675\,a\cos{(dx+c)}^8\log{(-\sin{(dx-}}}{}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/26880*(26880*a*d*x*cos(d*x + c)^8 + 3675*a*cos(d*x + c)^8*log(sin(d*x + c
) + 1) - 3675*a*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(22528*a*cos(d*x
+ c)^7 + 9765*a*cos(d*x + c)^6 - 15616*a*cos(d*x + c)^5 - 11410*a*cos(d*x +
 c)^4 + 8448*a*cos(d*x + c)^3 + 7000*a*cos(d*x + c)^2 - 1920*a*cos(d*x + c)
 - 1680*a)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F]

$$\int (a+a\sec(c+dx))\tan^8(c+dx)\,dx = a\left(\int \tan^8(c+dx)\sec(c+dx)\,dx + \int \tan^8(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**8,x)
[Out] a*(Integral(tan(c + d*x)**8*sec(c + d*x), x) + Integral(tan(c + d*x)**8, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.27

$$\int (a+a\sec(c+dx))\tan^8(c+dx)\,dx$$
$$= \frac{256\left(15\tan(dx+c)^7-21\tan(dx+c)^5+35\tan(dx+c)^3+105\,dx+105\,c-105\tan(dx+c)\right)a+35\,a\left(\right.}{2}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="maxima")
[Out] 1/26880*(256*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 1
05*d*x + 105*c - 105*tan(d*x + c))*a + 35*a*(2*(279*sin(d*x + c)^7 - 511*si
n(d*x + c)^5 + 385*sin(d*x + c)^3 - 105*sin(d*x + c))/(sin(d*x + c)^8 - 4*s
in(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) + 105*log(sin(d*x
+ c) + 1) - 105*log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 4.83 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.35

$$\int (a+a\sec(c+dx))\tan^8(c+dx)\,dx$$
$$= \frac{13440\,(dx+c)a+3675\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3675\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(9765\,a\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 1/13440*(13440*(d*x + c)*a + 3675*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 36
75*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9765*a*tan(1/2*d*x + 1/2*c)^15
 - 83825*a*tan(1/2*d*x + 1/2*c)^13 + 321013*a*tan(1/2*d*x + 1/2*c)^11 - 724
649*a*tan(1/2*d*x + 1/2*c)^9 + 1078359*a*tan(1/2*d*x + 1/2*c)^7 - 508683*a*
tan(1/2*d*x + 1/2*c)^5 + 140175*a*tan(1/2*d*x + 1/2*c)^3 - 17115*a*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 15.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.88

$$\int (a + a\sec(c + dx))\tan^8(c + dx)\,dx = a\,x - \frac{-\frac{93\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64} + \frac{2395\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192} - \frac{45859\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{960} + \frac{724649\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{6720} - \frac{359453\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2240} + \cdots}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{16} - 8\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{14} + 28\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{12} - 56\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{10} + 70\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{8} - 56 \cdots\right.} + \frac{35\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{64\,d}$$

```
[In] int(tan(c + d*x)^8*(a + a/cos(c + d*x)),x)
```

```
[Out] a*x - ((163*a*tan(c/2 + (d*x)/2))/64 - (1335*a*tan(c/2 + (d*x)/2)^3)/64 + (
24223*a*tan(c/2 + (d*x)/2)^5)/320 - (359453*a*tan(c/2 + (d*x)/2)^7)/2240 +
(724649*a*tan(c/2 + (d*x)/2)^9)/6720 - (45859*a*tan(c/2 + (d*x)/2)^11)/960
+ (2395*a*tan(c/2 + (d*x)/2)^13)/192 - (93*a*tan(c/2 + (d*x)/2)^15)/64)/(d*
(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/2 + (d*x)/2)^6
 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/
2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1)) + (35*a*atanh
(tan(c/2 + (d*x)/2)))/(64*d)
```

3.11 $\int (a+a\sec(c+dx))\tan^6(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int (a+a\sec(c+dx))\tan^6(c+dx)\,dx = -ax-\frac{5a\text{arctanh}(\sin(c+dx))}{16d}+\frac{(16a+5a\sec(c+dx))\tan(c+dx)}{16d}-\frac{(8a+5a\sec(c+dx))\tan^3(c+dx)}{24d}+\frac{(6a+5a\sec(c+dx))\tan^5(c+dx)}{30d}$$

```
[Out] -a*x-5/16*a*arctanh(sin(d*x+c))/d+1/16*(16*a+5*a*sec(d*x+c))*tan(d*x+c)/d-1
/24*(8*a+5*a*sec(d*x+c))*tan(d*x+c)^3/d+1/30*(6*a+5*a*sec(d*x+c))*tan(d*x+c
)^5/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3966, 3855}

$$\int (a+a\sec(c+dx))\tan^6(c+dx)\,dx = -\frac{5a\text{arctanh}(\sin(c+dx))}{16d}+\frac{\tan^5(c+dx)(5a\sec(c+dx)+6a)}{30d}-\frac{\tan^3(c+dx)(5a\sec(c+dx)+8a)}{24d}+\frac{\tan(c+dx)(5a\sec(c+dx)+16a)}{16d}-ax$$

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -(a*x) - (5*a*ArcTanh[Sin[c + d*x]])/(16*d) + ((16*a + 5*a*Sec[c + d*x])*Ta
n[c + d*x])/(16*d) - ((8*a + 5*a*Sec[c + d*x])*Tan[c + d*x]^3)/(24*d) + ((6
*a + 5*a*Sec[c + d*x])*Tan[c + d*x]^5)/(30*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(6a+5a\sec(c+dx))\tan^5(c+dx)}{30d}-\frac{1}{6}\int(6a+5a\sec(c+dx))\tan^4(c+dx)\,dx\\
&= -\frac{(8a+5a\sec(c+dx))\tan^3(c+dx)}{24d}+\frac{(6a+5a\sec(c+dx))\tan^5(c+dx)}{30d}\\
&\quad+\frac{1}{24}\int(24a+15a\sec(c+dx))\tan^2(c+dx)\,dx\\
&= \frac{(16a+5a\sec(c+dx))\tan(c+dx)}{16d}-\frac{(8a+5a\sec(c+dx))\tan^3(c+dx)}{24d}\\
&\quad+\frac{(6a+5a\sec(c+dx))\tan^5(c+dx)}{30d}-\frac{1}{48}\int(48a+15a\sec(c+dx))\,dx\\
&= -ax+\frac{(16a+5a\sec(c+dx))\tan(c+dx)}{16d}-\frac{(8a+5a\sec(c+dx))\tan^3(c+dx)}{24d}\\
&\quad+\frac{(6a+5a\sec(c+dx))\tan^5(c+dx)}{30d}-\frac{1}{16}(5a)\int\sec(c+dx)\,dx\\
&= -ax-\frac{5a\operatorname{arctanh}(\sin(c+dx))}{16d}+\frac{(16a+5a\sec(c+dx))\tan(c+dx)}{16d}\\
&\quad-\frac{(8a+5a\sec(c+dx))\tan^3(c+dx)}{24d}+\frac{(6a+5a\sec(c+dx))\tan^5(c+dx)}{30d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.75

$$
\begin{aligned}
\int (a+a\sec(c+dx))\tan^6(c+dx)\,dx = & -\frac{a\arctan(\tan(c+dx))}{d}-\frac{5a\text{arctanh}(\sin(c+dx))}{16d} \\
& +\frac{a\tan(c+dx)}{d}-\frac{5a\sec(c+dx)\tan(c+dx)}{16d} \\
& -\frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} \\
& +\frac{5a\sec^5(c+dx)\tan(c+dx)}{6d}-\frac{a\tan^3(c+dx)}{3d} \\
& -\frac{5a\sec^3(c+dx)\tan^3(c+dx)}{3d} \\
& +\frac{a\tan^5(c+dx)}{5d}+\frac{a\sec(c+dx)\tan^5(c+dx)}{d}
\end{aligned}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) - (5*a*ArcTanh[Sin[c + d*x]])/(16*d) + (a*Tan
[c + d*x])/d - (5*a*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (5*a*Sec[c + d*x]^3
*Tan[c + d*x])/(24*d) + (5*a*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) - (a*Tan[c
+ d*x]^3)/(3*d) - (5*a*Sec[c + d*x]^3*Tan[c + d*x]^3)/(3*d) + (a*Tan[c + d*
x]^5)/(5*d) + (a*Sec[c + d*x]*Tan[c + d*x]^5)/d
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{a\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(dx+c)^3}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+a\left(\frac{\tan}{}\right.}{d}$
default	$\frac{a\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(dx+c)^3}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+a\left(\frac{\tan}{}\right.}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{a\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+}{16}\right.}{}$
risch	$-ax-\frac{ia\left(165\,\mathrm{e}^{11i(dx+c)}-720\,\mathrm{e}^{10i(dx+c)}-25\,\mathrm{e}^{9i(dx+c)}-2160\,\mathrm{e}^{8i(dx+c)}+450\,\mathrm{e}^{7i(dx+c)}-3680\,\mathrm{e}^{6i(dx+c)}-450\,\mathrm{e}^{5i(dx+c)}-\right.}{120d\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}$

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/6*sin(d*x+c)^7/cos(d*x+c)^6-1/24*sin(d*x+c)^7/cos(d*x+c)^4+1/16*s
in(d*x+c)^7/cos(d*x+c)^2+1/16*sin(d*x+c)^5+5/48*sin(d*x+c)^3+5/16*sin(d*x+c
)-5/16*ln(sec(d*x+c)+tan(d*x+c)))+a*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(
d*x+c)-d*x-c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.31

$$\int (a + a\sec(c + dx))\tan^6(c + dx)\,dx =$$
$$-\frac{480\,adx\cos(dx+c)^6 + 75\,a\cos(dx+c)^6\log(\sin(dx+c)+1) - 75\,a\cos(dx+c)^6\log(-\sin(dx+c)$$

[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")

[Out] -1/480*(480*a*d*x*cos(d*x + c)^6 + 75*a*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 75*a*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*a*cos(d*x + c)^5 + 165*a*cos(d*x + c)^4 - 176*a*cos(d*x + c)^3 - 130*a*cos(d*x + c)^2 + 48*a*c os(d*x + c) + 40*a)*sin(d*x + c))/(d*cos(d*x + c)^6)

Sympy [F]

$$\int (a + a\sec(c + dx))\tan^6(c + dx)\,dx = a\left(\int \tan^6(c + dx)\sec(c + dx)\,dx + \int \tan^6(c + dx)\,dx\right)$$

[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**6,x)

[Out] a*(Integral(tan(c + d*x)**6*sec(c + d*x), x) + Integral(tan(c + d*x)**6, x))

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.31

$$\int (a + a\sec(c + dx))\tan^6(c + dx)\,dx$$
$$= \frac{32\left(3\tan(dx+c)^5 - 5\tan(dx+c)^3 - 15\,dx - 15\,c + 15\tan(dx+c)\right)a - 5\,a\left(\frac{2\left(33\sin(dx+c)^5 - 40\sin(dx+c)^3}{\sin(dx+c)^6 - 3\sin(dx+c)^4 + 3\,s}\right.}{480\,d}$$

[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")

[Out] 1/480*(32*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a - 5*a*(2*(33*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 15*sin(d*x + c)) /(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 15*log(sin(d* x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d

Giac [A] (verification not implemented)

none

Time = 2.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.43

$$\int (a + a\sec(c+dx))\tan^6(c+dx)\,dx =$$

$$-\frac{240\,(dx+c)a + 75\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 75\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(165\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{1}\right.}{}}{240\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(240*(d*x + c)*a + 75*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 75*a*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(165*a*tan(1/2*d*x + 1/2*c)^11 - 1095*
a*tan(1/2*d*x + 1/2*c)^9 + 3138*a*tan(1/2*d*x + 1/2*c)^7 - 5118*a*tan(1/2*d
*x + 1/2*c)^5 + 1945*a*tan(1/2*d*x + 1/2*c)^3 - 315*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.84

$$\int (a + a\sec(c+dx))\tan^6(c+dx)\,dx$$

$$= \frac{-\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} + \frac{73\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{8} - \frac{523\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20} + \frac{853\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20} - \frac{389\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24} + \frac{21\,a\tan\left(\frac{c}{2}\right.}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + 15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} - 20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} + 15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4} - 6\tan\left(\frac{c}{2}\right.\right.}$$

$$-\,a\,x - \frac{5\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x)),x)
```

```
[Out] ((21*a*tan(c/2 + (d*x)/2))/8 - (389*a*tan(c/2 + (d*x)/2)^3)/24 + (853*a*tan
(c/2 + (d*x)/2)^5)/20 - (523*a*tan(c/2 + (d*x)/2)^7)/20 + (73*a*tan(c/2 + (
d*x)/2)^9)/8 - (11*a*tan(c/2 + (d*x)/2)^11)/8)/(d*(15*tan(c/2 + (d*x)/2)^4
- 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^
8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - a*x - (5*a*atan
h(tan(c/2 + (d*x)/2)))/(8*d)
```

3.12 $\int (a+a\sec(c+dx))\tan^4(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (a+a\sec(c+dx))\tan^4(c+dx)\,dx = ax+\frac{3a\text{arctanh}(\sin(c+dx))}{8d} - \frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d} + \frac{(4a+3a\sec(c+dx))\tan^3(c+dx)}{12d}$$

```
[Out] a*x+3/8*a*arctanh(sin(d*x+c))/d-1/8*(8*a+3*a*sec(d*x+c))*tan(d*x+c)/d+1/12*
(4*a+3*a*sec(d*x+c))*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3966, 3855}

$$\int (a+a\sec(c+dx))\tan^4(c+dx)\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d} + \frac{\tan^3(c+dx)(3a\sec(c+dx)+4a)}{12d} - \frac{\tan(c+dx)(3a\sec(c+dx)+8a)}{8d} + ax$$

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x + (3*a*ArcTanh[Sin[c + d*x]])/(8*d) - ((8*a + 3*a*Sec[c + d*x])*Tan[c +
 d*x])/(8*d) + ((4*a + 3*a*Sec[c + d*x])*Tan[c + d*x]^3)/(12*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(4a+3a\sec(c+dx))\tan^3(c+dx)}{12d} - \frac{1}{4}\int (4a+3a\sec(c+dx))\tan^2(c+dx)\,dx \\
&= -\frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d} \\
&\quad + \frac{(4a+3a\sec(c+dx))\tan^3(c+dx)}{12d} + \frac{1}{8}\int (8a+3a\sec(c+dx))\,dx \\
&= ax - \frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d} \\
&\quad + \frac{(4a+3a\sec(c+dx))\tan^3(c+dx)}{12d} + \frac{1}{8}(3a)\int \sec(c+dx)\,dx \\
&= ax + \frac{3a\text{arctanh}(\sin(c+dx))}{8d} - \frac{(8a+3a\sec(c+dx))\tan(c+dx)}{8d} \\
&\quad + \frac{(4a+3a\sec(c+dx))\tan^3(c+dx)}{12d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$
\begin{aligned}
\int (a+a\sec(c+dx))\tan^4(c+dx)\,dx &= \frac{a\arctan(\tan(c+dx))}{d} + \frac{3a\text{arctanh}(\sin(c+dx))}{8d} \\
&\quad - \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} \\
&\quad - \frac{3a\sec^3(c+dx)\tan(c+dx)}{4d} \\
&\quad + \frac{a\tan^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan^3(c+dx)}{d}
\end{aligned}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

[Out] (a*ArcTan[Tan[c + d*x]])/d + (3*a*ArcTanh[Sin[c + d*x]])/(8*d) - (a*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*x]^3)/(3*d) + (a*Sec[c + d*x]*Tan[c + d*x]^3)/d

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{a\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+dx+c\right.}{d}$
default	$\frac{a\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+dx+c\right.}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{a\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(d\right.}{d}$
risch	$ax+\frac{ia\left(15\,\mathrm{e}^{7i(dx+c)}-48\,\mathrm{e}^{6i(dx+c)}-9\,\mathrm{e}^{5i(dx+c)}-96\,\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{3i(dx+c)}-80\,\mathrm{e}^{2i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}-32\right)}{12d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}-\frac{3a\ln(\mathrm{e}}{}$

[In] int((a+a*sec(d*x+c))*tan(d*x+c)^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.53

$$\int (a+a\sec(c+dx))\tan^4(c+dx)\,dx$$
$$=\frac{48\,adx\cos(dx+c)^4+9\,a\cos(dx+c)^4\log(\sin(dx+c)+1)-9\,a\cos(dx+c)^4\log(-\sin(dx+c)+1)-}{48\,d\cos(dx+c)^4}$$

[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")

[Out] 1/48*(48*a*d*x*cos(d*x + c)^4 + 9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*a*cos(d*x + c)^3 + 15*a*cos(d*x + c)^2 - 8*a*cos(d*x + c) - 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)

Sympy [F]

$$\int (a + a\sec(c + dx))\tan^4(c + dx)\,dx = a\left(\int \tan^4(c + dx)\sec(c + dx)\,dx + \int \tan^4(c + dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**4,x)
[Out] a*(Integral(tan(c + d*x)**4*sec(c + d*x), x) + Integral(tan(c + d*x)**4, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int (a + a\sec(c + dx))\tan^4(c + dx)\,dx = \frac{16\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a + 3\,a\left(\frac{2\left(5\,\sin(dx+c)^3 - 3\,\sin(dx+c)\right)}{\sin(dx+c)^4 - 2\,\sin(dx+c)^2 + 1} + 3\,\log(\sin(dx+c) + 1)\right.}{48\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a + 3*a*(2*(5*sin(
d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*lo
g(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.94 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.62

$$\int (a + a\sec(c + dx))\tan^4(c + dx)\,dx = \frac{24\,(dx+c)a + 9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^7 - 71\,a\tan\right.}{}}{24\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
[Out] 1/24*(24*(d*x + c)*a + 9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs
(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 71*a*tan(1/2
*d*x + 1/2*c)^5 + 137*a*tan(1/2*d*x + 1/2*c)^3 - 33*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 14.87 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.84

$$\int (a+a\sec(c+dx))\tan^4(c+dx)\,dx$$
$$= a\,x - \frac{-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{71\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}-\frac{137\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$
$$+\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

```
[Out] a*x - ((11*a*tan(c/2 + (d*x)/2))/4 - (137*a*tan(c/2 + (d*x)/2)^3)/12 + (71*
a*tan(c/2 + (d*x)/2)^5)/12 - (5*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(6*tan(c/2 +
(d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d
*x)/2)^8 + 1)) + (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.13 $\int (a+a\sec(c+dx))\tan^2(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int (a+a\sec(c+dx))\tan^2(c+dx)\,dx=-ax-\frac{a\text{arctanh}(\sin(c+dx))}{2d}+\frac{(2a+a\sec(c+dx))\tan(c+dx)}{2d}$$

```
[Out] -a*x-1/2*a*arctanh(sin(d*x+c))/d+1/2*(2*a+a*sec(d*x+c))*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3966, 3855}

$$\int (a+a\sec(c+dx))\tan^2(c+dx)\,dx=-\frac{a\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)(a\sec(c+dx)+2a)}{2d}-ax$$

```
[In] Int[(a + a*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) - (a*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a + a*Sec[c + d*x])*Tan[c +
d*x])/(2*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(2a + a\sec(c+dx))\tan(c+dx)}{2d} - \frac{1}{2}\int (2a + a\sec(c+dx))\,dx\\ &= -ax + \frac{(2a + a\sec(c+dx))\tan(c+dx)}{2d} - \frac{1}{2}a\int \sec(c+dx)\,dx\\ &= -ax - \frac{a\text{arctanh}(\sin(c+dx))}{2d} + \frac{(2a + a\sec(c+dx))\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int (a + a\sec(c+dx))\tan^2(c+dx)\,dx = -\frac{a\arctan(\tan(c+dx))}{d} - \frac{a\text{arctanh}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*Tan[c + d*x]^2,x]
[Out] -((a*ArcTan[Tan[c + d*x]])/d) - (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c
+ d*x])/d + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.49

method	result	size
derivativedivides	$\frac{a\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a(\tan(dx+c)-dx-c)}{d}$	67
default	$\frac{a\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a(\tan(dx+c)-dx-c)}{d}$	67
parts	$\frac{a(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{a\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	71
risch	$-ax-\frac{ia\left(e^{3i(dx+c)}-2\,e^{2i(dx+c)}-e^{i(dx+c)}-2\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$	97

```
[In] int((a+a*sec(d*x+c))*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(
d*x+c)))+a*(tan(d*x+c)-d*x-c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(41) = 82$.

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.93

$$\int (a + a\sec(c + dx))\tan^2(c + dx)\,dx =$$

$$-\frac{4\,adx\cos{(dx+c)}^2 + a\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)} - a\cos{(dx+c)}^2\log{(-\sin{(dx+c)}+1)} - 2\,(}{4\,d\cos{(dx+c)}^2}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
[Out] -1/4*(4*a*d*x*cos(d*x + c)^2 + a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*c
os(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a*cos(d*x + c) + a)*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a + a\sec(c + dx))\tan^2(c + dx)\,dx = a\left(\int \tan^2{(c+dx)}\sec{(c+dx)}\,dx + \int \tan^2{(c+dx)}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)**2,x)
[Out] a*(Integral(tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)**2, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int (a + a\sec(c + dx))\tan^2(c + dx)\,dx =$$

$$-\frac{4\,(dx+c-\tan{(dx+c)})a + a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} + \log{(\sin{(dx+c)}+1)} - \log{(\sin{(dx+c)}-1)}\right)}{4\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
[Out] -1/4*(4*(d*x + c - tan(d*x + c))*a + a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
 + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(41) = 82$.

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.96

$$\int (a + a\sec(c + dx))\tan^2(c + dx)\, dx = -\frac{2\,(dx+c)a + a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 3\,a\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

```
[In] integrate((a+a*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a + a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.78

$$\int (a + a\sec(c + dx))\tan^2(c + dx)\, dx = \frac{3\,a\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right) - a\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4 - 2\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)} - a\,x - \frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d}$$

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x)),x)
```

```
[Out] (3*a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4
- 2*tan(c/2 + (d*x)/2)^2 + 1)) - a*x - (a*atanh(tan(c/2 + (d*x)/2)))/d
```

3.14 $\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx = -ax - \frac{\cot(c+dx)(a+a\sec(c+dx))}{d}$$

```
[Out] -a*x-cot(d*x+c)*(a+a*sec(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3967, 8}

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\cot(c+dx)(a\sec(c+dx)+a)}{d} - ax$$

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]*(a + a*Sec[c + d*x]))/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
```

```
Q[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cot(c+dx)(a+a\sec(c+dx))}{d} - \int a\,dx \\
&= -ax - \frac{\cot(c+dx)(a+a\sec(c+dx))}{d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$
\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\csc(c+dx)}{d} - \frac{a\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d}
$$

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x]),x]
[Out] -((a*Csc[c + d*x])/d) - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Ta
n[c + d*x]^2])/d
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
risch	$-ax-\frac{2ia}{d\left(e^{i(dx+c)}-1\right)}$	26
derivativedivides	$\frac{-\frac{a}{\sin(dx+c)}+a(-\cot(dx+c)-dx-c)}{d}$	35
default	$\frac{-\frac{a}{\sin(dx+c)}+a(-\cot(dx+c)-dx-c)}{d}$	35

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -a*x-2*I*a/d/(exp(I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx = -\frac{adx\sin(dx+c)+a\cos(dx+c)+a}{d\sin(dx+c)}$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -(a*d*x*sin(d*x + c) + a*cos(d*x + c) + a)/(d*sin(d*x + c))
```

Sympy [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^2(c+dx)\sec(c+dx)\,dx + \int \cot^2(c+dx)\,dx\right)$$

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(cot(c + d*x)**2*sec(c + d*x), x) + Integral(cot(c + d*x)**2, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a+\frac{a}{\sin(dx+c)}}{d}$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -((d*x + c + 1/tan(d*x + c))*a + a/sin(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx = -\frac{(dx+c)a+\frac{a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{d}$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] -((d*x + c)*a + a/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \cot^2(c+dx)(a+a\sec(c+dx))\,dx=-\frac{a\left(\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)+d\,x\right)}{d}$$

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x)),x)
[Out] -(a*(cot(c/2 + (d*x)/2) + d*x))/d
```

3.15 $\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = ax-\frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2a\sec(c+dx))}{3d}$$

```
[Out] a*x-1/3*cot(d*x+c)^3*(a+a*sec(d*x+c))/d+1/3*cot(d*x+c)*(3*a+2*a*sec(d*x+c))
/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3967, 8}

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\cot^3(c+dx)(a\sec(c+dx)+a)}{3d}+\frac{\cot(c+dx)(2a\sec(c+dx)+3a)}{3d}+ax$$

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^3*(a + a*Sec[c + d*x]))/(3*d) + (Cot[c + d*x]*(3*a + 2*
a*Sec[c + d*x]))/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d}+\frac{1}{3}\int\cot^2(c+dx)(-3a-2a\sec(c+dx))\,dx\\
&= -\frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2a\sec(c+dx))}{3d}+\frac{1}{3}\int 3a\,dx\\
&= ax-\frac{\cot^3(c+dx)(a+a\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2a\sec(c+dx))}{3d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\begin{aligned}
&\int\cot^4(c+dx)(a+a\sec(c+dx))\,dx\\
&= \frac{a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}\\
&\quad-\frac{a\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d}
\end{aligned}$$

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x]),x]
[Out] (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) - (a*Cot[c + d*x]^3*Hypergeom
etric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

method	result	size
risch	$ax + \frac{2ia(3\,\mathrm{e}^{3i(dx+c)}-5\,\mathrm{e}^{i(dx+c)}+4)}{3d(\mathrm{e}^{i(dx+c)}-1)^3(\mathrm{e}^{i(dx+c)}+1)}$	62
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{(2+\cos(dx+c)^2)\sin(dx+c)}{3}\right)+a\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)}{d}$	86
default	$\frac{a\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{(2+\cos(dx+c)^2)\sin(dx+c)}{3}\right)+a\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)}{d}$	86

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] a*x+2/3*I*a*(3*exp(3*I*(d*x+c))-5*exp(I*(d*x+c))+4)/d/(exp(I*(d*x+c))-1)^3/
(exp(I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{4\,a\cos{(dx+c)}^2 - a\cos{(dx+c)} + 3\,(adx\cos{(dx+c)} - adx)\sin{(dx+c)} - 2\,a}{3\,(d\cos{(dx+c)} - d)\sin{(dx+c)}}$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] 1/3*(4*a*cos(d*x + c)^2 - a*cos(d*x + c) + 3*(a*d*x*cos(d*x + c) - a*d*x)*s
in(d*x + c) - 2*a)/((d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^4{(c+dx)}\sec{(c+dx)}\,dx + \int \cot^4{(c+dx)}\,dx\right)$$

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(cot(c + d*x)**4*sec(c + d*x), x) + Integral(cot(c + d*x)**4, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+\frac{\left(3\,\sin(dx+c)^2-1\right)a}{\sin(dx+c)^3}}{3\,d}$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + (3*sin(d*x +
 c)^2 - 1)*a/sin(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{12\,(dx+c)a-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{12\,d}$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(d*x + c)*a - 3*a*tan(1/2*d*x + 1/2*c) + (12*a*tan(1/2*d*x + 1/2*c
)^2 - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 14.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \cot^4(c+dx)(a+a\sec(c+dx))\,dx = a\,x-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d}-\frac{\frac{a}{12}-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

```
[Out] a*x - (a*tan(c/2 + (d*x)/2))/(4*d) - (a/12 - a*tan(c/2 + (d*x)/2)^2)/(d*tan
(c/2 + (d*x)/2)^3)
```

3.16 $\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx = -ax-\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4a\sec(c+dx))}{15d}-\frac{\cot(c+dx)(15a+8a\sec(c+dx))}{15d}$$

```
[Out] -a*x-1/5*cot(d*x+c)^5*(a+a*sec(d*x+c))/d+1/15*cot(d*x+c)^3*(5*a+4*a*sec(d*x
+c))/d-1/15*cot(d*x+c)*(15*a+8*a*sec(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3967, 8}

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\cot^5(c+dx)(a\sec(c+dx)+a)}{5d}+\frac{\cot^3(c+dx)(4a\sec(c+dx)+5a)}{15d}-\frac{\cot(c+dx)(8a\sec(c+dx)+15a)}{15d}-ax$$

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]^5*(a + a*Sec[c + d*x]))/(5*d) + (Cot[c + d*x]^3*(5*a
 + 4*a*Sec[c + d*x]))/(15*d) - (Cot[c + d*x]*(15*a + 8*a*Sec[c + d*x]))/(15
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d} + \frac{1}{5}\int \cot^4(c+dx)(-5a-4a\sec(c+dx))\,dx \\
&= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d} + \frac{\cot^3(c+dx)(5a+4a\sec(c+dx))}{15d} \\
&\quad + \frac{1}{15}\int \cot^2(c+dx)(15a+8a\sec(c+dx))\,dx \\
&= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d} + \frac{\cot^3(c+dx)(5a+4a\sec(c+dx))}{15d} \\
&\quad - \frac{\cot(c+dx)(15a+8a\sec(c+dx))}{15d} + \frac{1}{15}\int -15a\,dx \\
&= -ax - \frac{\cot^5(c+dx)(a+a\sec(c+dx))}{5d} \\
&\quad + \frac{\cot^3(c+dx)(5a+4a\sec(c+dx))}{15d} - \frac{\cot(c+dx)(15a+8a\sec(c+dx))}{15d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$
\begin{aligned}
&\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx \\
&= -\frac{a\csc(c+dx)}{d} + \frac{2a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d} \\
&\quad - \frac{a\cot^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5d}
\end{aligned}
$$

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x]),x]
[Out] -((a*Csc[c + d*x])/d) + (2*a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*
d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(
5*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

method	result
risch	$-ax-\frac{2ia\left(15\,\mathrm{e}^{7i(dx+c)}+15\,\mathrm{e}^{6i(dx+c)}-65\,\mathrm{e}^{5i(dx+c)}+25\,\mathrm{e}^{4i(dx+c)}+73\,\mathrm{e}^{3i(dx+c)}-31\,\mathrm{e}^{2i(dx+c)}-31\,\mathrm{e}^{i(dx+c)}+23\right)}{15d\left(\mathrm{e}^{i(dx+c)}-1\right)^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}$
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}\right)+a\left(-\frac{\cot(dx+c)^5}{5}+\frac{\cot(dx+c)^3}{3}-\right.}{d}$
default	$\frac{a\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}\right)+a\left(-\frac{\cot(dx+c)^5}{5}+\frac{\cot(dx+c)^3}{3}-\right.}{d}$

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -a*x-2/15*I*a*(15*exp(7*I*(d*x+c))+15*exp(6*I*(d*x+c))-65*exp(5*I*(d*x+c))+
25*exp(4*I*(d*x+c))+73*exp(3*I*(d*x+c))-31*exp(2*I*(d*x+c))-31*exp(I*(d*x+c
))+23)/d/(exp(I*(d*x+c))-1)^5/(exp(I*(d*x+c))+1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.65

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx = -\frac{23\,a\cos\left(dx+c\right)^4-8\,a\cos\left(dx+c\right)^3-27\,a\cos\left(dx+c\right)^2+7\,a\cos\left(dx+c\right)+15\,\left(adx\cos\left(dx+c\right)^3-ad\right.}{15\,\left(d\cos\left(dx+c\right)^3-d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+d\right)}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(23*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^3 - 27*a*cos(d*x + c)^2 + 7*a
*cos(d*x + c) + 15*(a*d*x*cos(d*x + c)^3 - a*d*x*cos(d*x + c)^2 - a*d*x*cos
(d*x + c) + a*d*x)*sin(d*x + c) + 8*a)/((d*cos(d*x + c)^3 - d*cos(d*x + c)^
2 - d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F]

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^6{(c+dx)}\sec{(c+dx)}\,dx + \int \cot^6{(c+dx)}\,dx\right)$$

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(cot(c + d*x)**6*sec(c + d*x), x) + Integral(cot(c + d*x)**6, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\left(15\,dx + 15\,c + \frac{15\,\tan(dx+c)^4 - 5\,\tan(dx+c)^2 + 3}{\tan(dx+c)^5}\right)a + \frac{\left(15\,\sin(dx+c)^4 - 10\,\sin(dx+c)^2 + 3\right)a}{\sin(dx+c)^5}}{15\,d}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a + (15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a/sin(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx = -\frac{5\,a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 240\,(dx+c)a - 90\,a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + \frac{3\left(80\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{240\,d}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] -1/240*(5*a*tan(1/2*d*x + 1/2*c)^3 + 240*(d*x + c)*a - 90*a*tan(1/2*d*x + 1
/2*c) + 3*(80*a*tan(1/2*d*x + 1/2*c)^4 - 10*a*tan(1/2*d*x + 1/2*c)^2 + a)/t
an(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.86

$$\int \cot^6(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{a\left(3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8+5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8-90\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4\right.}{240\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d}{2}\right.}$$

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(3*cos(c/2 + (d*x)/2)^8 + 5*sin(c/2 + (d*x)/2)^8 - 90*cos(c/2 + (d*x)/2
)^2*sin(c/2 + (d*x)/2)^6 + 240*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4 -
30*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 + 240*cos(c/2 + (d*x)/2)^3*sin
(c/2 + (d*x)/2)^5*(c + d*x)))/(240*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2
)^5)
```

3.17 $\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx = ax-\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d}+\frac{\cot(c+dx)(35a+16a\sec(c+dx))}{35d}-\frac{\cot^3(c+dx)(35a+24a\sec(c+dx))}{105d}$$

```
[Out] a*x-1/7*cot(d*x+c)^7*(a+a*sec(d*x+c))/d+1/35*cot(d*x+c)^5*(7*a+6*a*sec(d*x+
c))/d+1/35*cot(d*x+c)*(35*a+16*a*sec(d*x+c))/d-1/105*cot(d*x+c)^3*(35*a+24*
a*sec(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3967, 8}

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\cot^7(c+dx)(a\sec(c+dx)+a)}{7d}+\frac{\cot^5(c+dx)(6a\sec(c+dx)+7a)}{35d}-\frac{\cot^3(c+dx)(24a\sec(c+dx)+35a)}{105d}+\frac{\cot(c+dx)(16a\sec(c+dx)+35a)}{35d}+ax$$

```
[In] Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^7*(a + a*Sec[c + d*x]))/(7*d) + (Cot[c + d*x]^5*(7*a +
6*a*Sec[c + d*x]))/(35*d) + (Cot[c + d*x]*(35*a + 16*a*Sec[c + d*x]))/(35*d
) - (Cot[c + d*x]^3*(35*a + 24*a*Sec[c + d*x]))/(105*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d}+\frac{1}{7}\int\cot^6(c+dx)(-7a-6a\sec(c+dx))\,dx\\
&= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d}\\
&\quad+\frac{1}{35}\int\cot^4(c+dx)(35a+24a\sec(c+dx))\,dx\\
&= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d}\\
&\quad-\frac{\cot^3(c+dx)(35a+24a\sec(c+dx))}{105d}+\frac{1}{105}\int\cot^2(c+dx)(-105a\\
&\qquad-48a\sec(c+dx))\,dx\\
&= -\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d}\\
&\quad+\frac{\cot(c+dx)(35a+16a\sec(c+dx))}{35d}\\
&\quad-\frac{\cot^3(c+dx)(35a+24a\sec(c+dx))}{105d}+\frac{1}{105}\int 105a\,dx\\
&= ax-\frac{\cot^7(c+dx)(a+a\sec(c+dx))}{7d}+\frac{\cot^5(c+dx)(7a+6a\sec(c+dx))}{35d}\\
&\quad+\frac{\cot(c+dx)(35a+16a\sec(c+dx))}{35d}-\frac{\cot^3(c+dx)(35a+24a\sec(c+dx))}{105d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.83

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^5(c+dx)}{5d}-\frac{a\csc^7(c+dx)}{7d}$$
$$-\frac{a\cot^7(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7d}$$

```
[In] Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*a*Csc[c + d*x]^5)/(5*d) - (a
*Csc[c + d*x]^7)/(7*d) - (a*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2,
 -Tan[c + d*x]^2])/(7*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.35

method	result
risch	$ax+\frac{2ia\left(105\,e^{11i(dx+c)}+210\,e^{10i(dx+c)}-735\,e^{9i(dx+c)}+1638\,e^{7i(dx+c)}-196\,e^{6i(dx+c)}-1882\,e^{5i(dx+c)}+880\,e^{4i(dx+c)}\right.}{105d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)^5}$
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(dx+c)^8}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}\right)}{d}$
default	$\frac{a\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(dx+c)^8}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}\right)}{d}$

```
[In] int(cot(d*x+c)^8*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+2/105*I*a*(105*exp(11*I*(d*x+c))+210*exp(10*I*(d*x+c))-735*exp(9*I*(d*x
+c))+1638*exp(7*I*(d*x+c))-196*exp(6*I*(d*x+c))-1882*exp(5*I*(d*x+c))+880*e
xp(4*I*(d*x+c))+1025*exp(3*I*(d*x+c))-494*exp(2*I*(d*x+c))-247*exp(I*(d*x+c
))+176)/d/(exp(I*(d*x+c))-1)^7/(exp(I*(d*x+c))+1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(103) = 206.

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.89

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx = \frac{176\,a\cos(dx+c)^6-71\,a\cos(dx+c)^5-335\,a\cos(dx+c)^4+125\,a\cos(dx+c)^3+225\,a\cos(dx+c)^2-5}{105\,\left(d\cos(dx+c)^5-d\cos(dx+}$$

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/105*(176*a*cos(d*x + c)^6 - 71*a*cos(d*x + c)^5 - 335*a*cos(d*x + c)^4 +
125*a*cos(d*x + c)^3 + 225*a*cos(d*x + c)^2 - 57*a*cos(d*x + c) + 105*(a*d*
x*cos(d*x + c)^5 - a*d*x*cos(d*x + c)^4 - 2*a*d*x*cos(d*x + c)^3 + 2*a*d*x*
cos(d*x + c)^2 + a*d*x*cos(d*x + c) - a*d*x)*sin(d*x + c) - 48*a)/((d*cos(d
*x + c)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d*
cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F]

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cot^8(c+dx)\sec(c+dx)\,dx+\int \cot^8(c+dx)\,dx\right)$$

```
[In] integrate(cot(d*x+c)**8*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cot(c + d*x)**8*sec(c + d*x), x) + Integral(cot(c + d*x)**8, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx = \frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{\tan(dx+c)^7}\right)a+\frac{3\left(35\,\sin(dx+c)^6-35\,\sin(dx+c)^4+21\,\sin(dx+c)^2-5\right)a}{\sin(dx+c)^7}}{105\,d}$$

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a + 3*(35*sin(d*x + c)^6 - 35*sin(d*x + c)
^4 + 21*sin(d*x + c)^2 - 5)*a/sin(d*x + c)^7)/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6720\,(dx+c)a+3045\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{6720\,a\tan\left(\frac{1}{2}\right.}{}}{6720\,d}$$

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(21*a*tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^3 - 6720*
(d*x + c)*a + 3045*a*tan(1/2*d*x + 1/2*c) - (6720*a*tan(1/2*d*x + 1/2*c)^6
- 1015*a*tan(1/2*d*x + 1/2*c)^4 + 168*a*tan(1/2*d*x + 1/2*c)^2 - 15*a)/tan(
1/2*d*x + 1/2*c)^7)/d
```

Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.84

$$\int \cot^8(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{a\left(15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+21\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-280\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+3045\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}\right.\right.}{}$$

```
[In] int(cot(c + d*x)^8*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(15*cos(c/2 + (d*x)/2)^12 + 21*sin(c/2 + (d*x)/2)^12 - 280*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 3045*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/
2)^8 - 6720*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6 + 1015*cos(c/2 + (d*x
)/2)^8*sin(c/2 + (d*x)/2)^4 - 168*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^
2 - 6720*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7*(c + d*x)))/(6720*d*cos(
c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7)
```

3.18 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 140

$$\begin{aligned}\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx = &-ax-\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d}\\&+\frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d}\\&-\frac{\cot^5(c+dx)(21a+16a\sec(c+dx))}{105d}\\&+\frac{\cot^3(c+dx)(105a+64a\sec(c+dx))}{315d}\\&-\frac{\cot(c+dx)(315a+128a\sec(c+dx))}{315d}\end{aligned}$$

```
[Out] -a*x-1/9*cot(d*x+c)^9*(a+a*sec(d*x+c))/d+1/63*cot(d*x+c)^7*(9*a+8*a*sec(d*x
+c))/d-1/105*cot(d*x+c)^5*(21*a+16*a*sec(d*x+c))/d+1/315*cot(d*x+c)^3*(105*
a+64*a*sec(d*x+c))/d-1/315*cot(d*x+c)*(315*a+128*a*sec(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used

$= \{3967, 8\}$

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx = -\frac{\cot^9(c+dx)(a\sec(c+dx)+a)}{9d} + \frac{\cot^7(c+dx)(8a\sec(c+dx)+9a)}{63d} - \frac{\cot^5(c+dx)(16a\sec(c+dx)+21a)}{105d} + \frac{\cot^3(c+dx)(64a\sec(c+dx)+105a)}{315d} - \frac{\cot(c+dx)(128a\sec(c+dx)+315a)}{315d} - ax$$

```
[In] Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]^9*(a + a*Sec[c + d*x]))/(9*d) + (Cot[c + d*x]^7*(9*a
 + 8*a*Sec[c + d*x]))/(63*d) - (Cot[c + d*x]^5*(21*a + 16*a*Sec[c + d*x]))/
(105*d) + (Cot[c + d*x]^3*(105*a + 64*a*Sec[c + d*x]))/(315*d) - (Cot[c + d
*x]*(315*a + 128*a*Sec[c + d*x]))/(315*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{1}{9}\int \cot^8(c+dx)(-9a-8a\sec(c+dx))\,dx \\
&= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d} \\
&\quad + \frac{1}{63}\int \cot^6(c+dx)(63a+48a\sec(c+dx))\,dx \\
&= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d} \\
&\quad - \frac{\cot^5(c+dx)(21a+16a\sec(c+dx))}{105d} + \frac{1}{315}\int \cot^4(c+dx)(-315a \\
&\quad - 192a\sec(c+dx))\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d} \\
&\quad - \frac{\cot^5(c+dx)(21a+16a\sec(c+dx))}{105d} + \frac{\cot^3(c+dx)(105a+64a\sec(c+dx))}{315d} \\
&\quad + \frac{1}{945}\int \cot^2(c+dx)(945a+384a\sec(c+dx))\,dx \\
&= -\frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d} \\
&\quad - \frac{\cot^5(c+dx)(21a+16a\sec(c+dx))}{105d} + \frac{\cot^3(c+dx)(105a+64a\sec(c+dx))}{315d} \\
&\quad - \frac{\cot(c+dx)(315a+128a\sec(c+dx))}{315d} + \frac{1}{945}\int -945a\,dx \\
&= -ax - \frac{\cot^9(c+dx)(a+a\sec(c+dx))}{9d} + \frac{\cot^7(c+dx)(9a+8a\sec(c+dx))}{63d} \\
&\quad - \frac{\cot^5(c+dx)(21a+16a\sec(c+dx))}{105d} + \frac{\cot^3(c+dx)(105a+64a\sec(c+dx))}{315d} \\
&\quad - \frac{\cot(c+dx)(315a+128a\sec(c+dx))}{315d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

$$
\begin{aligned}
&\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx \\
&= -\frac{a\csc(c+dx)}{d} + \frac{4a\csc^3(c+dx)}{3d} - \frac{6a\csc^5(c+dx)}{5d} + \frac{4a\csc^7(c+dx)}{7d} \\
&\quad - \frac{a\csc^9(c+dx)}{9d} - \frac{a\cot^9(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(c+dx)\right)}{9d}
\end{aligned}
$$

```
[In] Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (4*a*Csc[c + d*x]^3)/(3*d) - (6*a*Csc[c + d*x]^5)/(
5*d) + (4*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d) - (a*Cot[c + d
*x]^9*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[c + d*x]^2])/(9*d)
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{a\left(-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)^9}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^7}-\frac{\cos(dx+c)^{10}}{105\sin(dx+c)^5}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^3}-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8\cos(dx+c)^6}{7}+\frac{48\cos(dx}{35}\right.}{9}\right.}{d}$
default	$\frac{a\left(-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)^9}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^7}-\frac{\cos(dx+c)^{10}}{105\sin(dx+c)^5}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^3}-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8\cos(dx+c)^6}{7}+\frac{48\cos(dx}{35}\right.}{9}\right.}{d}$
risch	$-ax-\frac{2ia(315\,\mathrm{e}^{15i(dx+c)}+945\,\mathrm{e}^{14i(dx+c)}-3045\,\mathrm{e}^{13i(dx+c)}-1155\,\mathrm{e}^{12i(dx+c)}+10143\,\mathrm{e}^{11i(dx+c)}+1869\,\mathrm{e}^{10i(dx+c)}-18}{}$

```
[In] int(cot(d*x+c)^10*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^10+1/63/sin(d*x+c)^7*cos(d*x+c)^10-1/1
05/sin(d*x+c)^5*cos(d*x+c)^10+1/63/sin(d*x+c)^3*cos(d*x+c)^10-1/9/sin(d*x+c
)*cos(d*x+c)^10-1/9*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^
4+64/35*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/9*cot(d*x+c)^9+1/7*cot(d*x+c)^7-1/5
*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(130) = 260$.

Time = 0.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.99

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx = -\frac{563\,a\cos(dx+c)^8-248\,a\cos(dx+c)^7-1498\,a\cos(dx+c)^6+658\,a\cos(dx+c)^5+1610\,a\cos(dx+}{31}$$

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/315*(563*a*cos(d*x + c)^8 - 248*a*cos(d*x + c)^7 - 1498*a*cos(d*x + c)^6
 + 658*a*cos(d*x + c)^5 + 1610*a*cos(d*x + c)^4 - 602*a*cos(d*x + c)^3 - 76
3*a*cos(d*x + c)^2 + 187*a*cos(d*x + c) + 315*(a*d*x*cos(d*x + c)^7 - a*d*x
*cos(d*x + c)^6 - 3*a*d*x*cos(d*x + c)^5 + 3*a*d*x*cos(d*x + c)^4 + 3*a*d*x
*cos(d*x + c)^3 - 3*a*d*x*cos(d*x + c)^2 - a*d*x*cos(d*x + c) + a*d*x)*sin(
d*x + c) + 128*a)/((d*cos(d*x + c)^7 - d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^
5 + 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^3 - 3*d*cos(d*x + c)^2 - d*cos(d*
x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**10*(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.85

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{\left(315\,dx+315\,c+\frac{315\,\tan(dx+c)^8-105\,\tan(dx+c)^6+63\,\tan(dx+c)^4-45\,\tan(dx+c)^2+35}{\tan(dx+c)^9}\right)a+\frac{\left(315\,\sin(dx+c)^8-420\,\sin(dx+c)^6+3\cdots\right.}{\sin(d\cdots}}{315\,d}$$

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*ta
n(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a + (315*sin(d*x + c
)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 + 35)*a/
sin(d*x + c)^9)/d
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx =$$

$$-\frac{45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4830\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80640\,(dx+c)a-40950\,a\,\mathrm{t}\cdots}{80640\,d}$$

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] -1/80640*(45*a*tan(1/2*d*x + 1/2*c)^7 - 630*a*tan(1/2*d*x + 1/2*c)^5 + 4830
*a*tan(1/2*d*x + 1/2*c)^3 + 80640*(d*x + c)*a - 40950*a*tan(1/2*d*x + 1/2*c
) + (80640*a*tan(1/2*d*x + 1/2*c)^8 - 13650*a*tan(1/2*d*x + 1/2*c)^6 + 2898
*a*tan(1/2*d*x + 1/2*c)^4 - 450*a*tan(1/2*d*x + 1/2*c)^2 + 35*a)/tan(1/2*d*
x + 1/2*c)^9)/d
```

Mupad [B] (verification not implemented)

Time = 15.27 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.80

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\left(35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+45\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-630\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+4830\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}\right.\right.}{}$$

```
[In] int(cot(c + d*x)^10*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(35*cos(c/2 + (d*x)/2)^16 + 45*sin(c/2 + (d*x)/2)^16 - 630*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 4830*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/
2)^12 - 40950*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 80640*cos(c/2 +
(d*x)/2)^8*sin(c/2 + (d*x)/2)^8 - 13650*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*
x)/2)^6 + 2898*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 450*cos(c/2 + (
d*x)/2)^14*sin(c/2 + (d*x)/2)^2 + 80640*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x
)/2)^9*(c + d*x)))/(80640*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^9)
```

3.19 $\int (a+a\sec(c+dx))^2\tan^9(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 192

$$\begin{aligned}\int (a+a\sec(c+dx))^2\tan^9(c+dx)\,dx = &-\frac{a^2\log(\cos(c+dx))}{d}\\ &+\frac{2a^2\sec(c+dx)}{d}-\frac{3a^2\sec^2(c+dx)}{2d}\\ &-\frac{8a^2\sec^3(c+dx)}{3d}+\frac{a^2\sec^4(c+dx)}{2d}\\ &+\frac{12a^2\sec^5(c+dx)}{5d}+\frac{a^2\sec^6(c+dx)}{3d}\\ &-\frac{8a^2\sec^7(c+dx)}{7d}-\frac{3a^2\sec^8(c+dx)}{8d}\\ &+\frac{2a^2\sec^9(c+dx)}{9d}+\frac{a^2\sec^{10}(c+dx)}{10d}\end{aligned}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d-3/2*a^2*sec(d*x+c)^2/d-8/3*a^2*sec
(d*x+c)^3/d+1/2*a^2*sec(d*x+c)^4/d+12/5*a^2*sec(d*x+c)^5/d+1/3*a^2*sec(d*x+
c)^6/d-8/7*a^2*sec(d*x+c)^7/d-3/8*a^2*sec(d*x+c)^8/d+2/9*a^2*sec(d*x+c)^9/d
+1/10*a^2*sec(d*x+c)^10/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used

= {3964, 90}

$$\int (a+a\sec(c+dx))^2\tan^9(c+dx)\,dx = \frac{a^2\sec^{10}(c+dx)}{10d}+\frac{2a^2\sec^9(c+dx)}{9d} - \frac{3a^2\sec^8(c+dx)}{8d}-\frac{8a^2\sec^7(c+dx)}{7d} + \frac{a^2\sec^6(c+dx)}{3d}+\frac{12a^2\sec^5(c+dx)}{5d} + \frac{a^2\sec^4(c+dx)}{2d}-\frac{8a^2\sec^3(c+dx)}{3d} - \frac{3a^2\sec^2(c+dx)}{2d}+\frac{2a^2\sec(c+dx)}{d} - \frac{a^2\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d - (3*a^2*Sec[c + d*x]
^2)/(2*d) - (8*a^2*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]^4)/(2*d) + (12
*a^2*Sec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^6)/(3*d) - (8*a^2*Sec[c + d*
x]^7)/(7*d) - (3*a^2*Sec[c + d*x]^8)/(8*d) + (2*a^2*Sec[c + d*x]^9)/(9*d) +
 (a^2*Sec[c + d*x]^10)/(10*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^6}{x^{11}}\,dx, x, \cos(c+dx)\right)}{a^8d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a^{10}}{x^{11}}+\frac{2a^{10}}{x^{10}}-\frac{3a^{10}}{x^9}-\frac{8a^{10}}{x^8}+\frac{2a^{10}}{x^7}+\frac{12a^{10}}{x^6}+\frac{2a^{10}}{x^5}-\frac{8a^{10}}{x^4}-\frac{3a^{10}}{x^3}+\frac{2a^{10}}{x^2}+\frac{a^{10}}{x}\right)dx, x, \cos(c\right.}{a^8d}$$

$$
\begin{aligned}
&= -\frac{a^2 \log(\cos(c+dx))}{d} + \frac{2a^2 \sec(c+dx)}{d} - \frac{3a^2 \sec^2(c+dx)}{2d} \\
&\quad - \frac{8a^2 \sec^3(c+dx)}{3d} + \frac{a^2 \sec^4(c+dx)}{2d} + \frac{12a^2 \sec^5(c+dx)}{5d} + \frac{a^2 \sec^6(c+dx)}{3d} \\
&\quad - \frac{8a^2 \sec^7(c+dx)}{7d} - \frac{3a^2 \sec^8(c+dx)}{8d} + \frac{2a^2 \sec^9(c+dx)}{9d} + \frac{a^2 \sec^{10}(c+dx)}{10d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01

$$
\int (a+a\sec(c+dx))^2 \tan^9(c+dx)\,dx = \frac{a^2(63080\cos(c+dx) + 39600\cos(3(c+dx)) - 1050\cos(2(c+dx))(68+63\log(\cos(c+dx))) - 21(1662 -
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] (a^2*(63080*Cos[c + d*x] + 39600*Cos[3*(c + d*x)] - 1050*Cos[2*(c + d*x)]*(
68 + 63*Log[Cos[c + d*x]]) - 21*(1662 - 1072*Cos[5*(c + d*x)] + 600*Cos[6*(
c + d*x)] - 220*Cos[7*(c + d*x)] + 90*Cos[8*(c + d*x)] - 60*Cos[9*(c + d*x)
] + 1890*Log[Cos[c + d*x]] + 675*Cos[6*(c + d*x)]*Log[Cos[c + d*x]] + 150*C
os[8*(c + d*x)]*Log[Cos[c + d*x]] + 15*Cos[10*(c + d*x)]*Log[Cos[c + d*x]]
+ 40*Cos[4*(c + d*x)]*(37 + 45*Log[Cos[c + d*x]])))*Sec[c + d*x]^10)/(16128
0*d)
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)^{10}}{10}+\frac{2\sec(dx+c)^9}{9}-\frac{3\sec(dx+c)^8}{8}-\frac{8\sec(dx+c)^7}{7}+\frac{\sec(dx+c)^6}{3}+\frac{12\sec(dx+c)^5}{5}+\frac{\sec(dx+c)^4}{2}-\frac{8\sec(dx+c)^3}{3}-\frac{3\sec(dx}{2}\right.}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)^{10}}{10}+\frac{2\sec(dx+c)^9}{9}-\frac{3\sec(dx+c)^8}{8}-\frac{8\sec(dx+c)^7}{7}+\frac{\sec(dx+c)^6}{3}+\frac{12\sec(dx+c)^5}{5}+\frac{\sec(dx+c)^4}{2}-\frac{8\sec(dx+c)^3}{3}-\frac{3\sec(dx}{2}\right.}{d}$
parts	$\frac{a^2\left(\frac{\tan(dx+c)^8}{8}-\frac{\tan(dx+c)^6}{6}+\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a^2\tan(dx+c)^{10}}{10d}+\frac{2a^2\left(\frac{\sec(dx+c)^9}{9}-\frac{4\,\mathrm{se}}{}\right.}{}$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2a^2\left(630\,\mathrm{e}^{19i(dx+c)}-945\,\mathrm{e}^{18i(dx+c)}+2310\,\mathrm{e}^{17i(dx+c)}-6300\,\mathrm{e}^{16i(dx+c)}+11256\,\mathrm{e}^{15i(dx+c)}-15540\,\mathrm{e}^{14i(}\right.}{}$

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(1/10*sec(d*x+c)^10+2/9*sec(d*x+c)^9-3/8*sec(d*x+c)^8-8/7*sec(d*x+c)^
7+1/3*sec(d*x+c)^6+12/5*sec(d*x+c)^5+1/2*sec(d*x+c)^4-8/3*sec(d*x+c)^3-3/2*
```

```
sec(d*x+c)^2+2*sec(d*x+c)+ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.81

$$\int (a+a\sec(c+dx))^2\tan^9(c+dx)\,dx = -\frac{2520\,a^2\cos{(dx+c)}^{10}\log{(-\cos{(dx+c)})}-5040\,a^2\cos{(dx+c)}^9+3780\,a^2\cos{(dx+c)}^8+6720\,a^2\cos$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(2520*a^2*cos(d*x + c)^10*log(-cos(d*x + c)) - 5040*a^2*cos(d*x + c
)^9 + 3780*a^2*cos(d*x + c)^8 + 6720*a^2*cos(d*x + c)^7 - 1260*a^2*cos(d*x
+ c)^6 - 6048*a^2*cos(d*x + c)^5 - 840*a^2*cos(d*x + c)^4 + 2880*a^2*cos(d*
x + c)^3 + 945*a^2*cos(d*x + c)^2 - 560*a^2*cos(d*x + c) - 252*a^2)/(d*cos(
d*x + c)^10)
```

Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.64

$$\int (a+a\sec(c+dx))^2\tan^9(c+dx)\,dx = \begin{cases} \frac{a^2\log{(\tan^2{(c+dx)}+1)}}{2d}+\frac{a^2\tan^8{(c+dx)}\sec^2{(c+dx)}}{10d}+\frac{2a^2\tan^8{(c+dx)}\sec{(c+dx)}}{9d}+\frac{a^2\tan^8{(c+dx)}}{8d}-\frac{a^2\tan^6{(c+dx)}\sec^2{(c+dx)}}{10d}- \\ x(a\sec{(c)}+a)^2\tan^9{(c)} \end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**8*sec(c
 + d*x)**2/(10*d) + 2*a**2*tan(c + d*x)**8*sec(c + d*x)/(9*d) + a**2*tan(c
+ d*x)**8/(8*d) - a**2*tan(c + d*x)**6*sec(c + d*x)**2/(10*d) - 16*a**2*tan
(c + d*x)**6*sec(c + d*x)/(63*d) - a**2*tan(c + d*x)**6/(6*d) + a**2*tan(c
+ d*x)**4*sec(c + d*x)**2/(10*d) + 32*a**2*tan(c + d*x)**4*sec(c + d*x)/(10
5*d) + a**2*tan(c + d*x)**4/(4*d) - a**2*tan(c + d*x)**2*sec(c + d*x)**2/(1
0*d) - 128*a**2*tan(c + d*x)**2*sec(c + d*x)/(315*d) - a**2*tan(c + d*x)**2
/(2*d) + a**2*sec(c + d*x)**2/(10*d) + 256*a**2*sec(c + d*x)/(315*d), Ne(d,
 0)), (x*(a*sec(c) + a)**2*tan(c)**9, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int (a+a\sec(c+dx))^2\tan^9(c+dx)\,dx = -\frac{2520\,a^2\log(\cos(dx+c)) - \frac{5040\,a^2\cos(dx+c)^9-3780\,a^2\cos(dx+c)^8-6720\,a^2\cos(dx+c)^7+1260\,a^2\cos(dx+c)^6+6048\,a^2\cos(dx+c)^5}{\cos(dx+c)^{10}}}{2520\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(2520*a^2*log(cos(d*x + c)) - (5040*a^2*cos(d*x + c)^9 - 3780*a^2*c
os(d*x + c)^8 - 6720*a^2*cos(d*x + c)^7 + 1260*a^2*cos(d*x + c)^6 + 6048*a^
2*cos(d*x + c)^5 + 840*a^2*cos(d*x + c)^4 - 2880*a^2*cos(d*x + c)^3 - 945*a
^2*cos(d*x + c)^2 + 560*a^2*cos(d*x + c) + 252*a^2)/cos(d*x + c)^10)/d
```

Giac [A] (verification not implemented)

none

Time = 6.02 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.78

$$\int (a+a\sec(c+dx))^2\tan^9(c+dx)\,dx = \frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{11477\,a^2+\frac{119810\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{566865\,a^2(\cos(dx}{(\cos(dx+c)+}}{}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 252
0*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11477*a^2 + 1
19810*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 566865*a^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + 1605720*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 3031770*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 2995020*
a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 2171610*a^2*(cos(d*x + c) -
 1)^6/(cos(d*x + c) + 1)^6 + 1114200*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c)
 + 1)^7 + 382545*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 78850*a^2*
(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9 + 7381*a^2*(cos(d*x + c) - 1)^10/
(cos(d*x + c) + 1)^10)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^10)/d
```

Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.60

$$\int (a+a\sec(c+dx))^2\tan^9(c+dx)\,dx=\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$
$$-\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-20\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+\frac{272\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}}{3}-\frac{740\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}+\frac{2252\,a^2\tan\left(\right.}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+45\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-120\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+210\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\right.}$$

```
[In] int(tan(c + d*x)^9*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d - ((1150*a^2*tan(c/2 + (d*x)/2)^2)/63
 - (652*a^2*tan(c/2 + (d*x)/2)^4)/7 + (2000*a^2*tan(c/2 + (d*x)/2)^6)/7 - 5
88*a^2*tan(c/2 + (d*x)/2)^8 + (2252*a^2*tan(c/2 + (d*x)/2)^10)/5 - (740*a^2
*tan(c/2 + (d*x)/2)^12)/3 + (272*a^2*tan(c/2 + (d*x)/2)^14)/3 - 20*a^2*tan(
c/2 + (d*x)/2)^16 + 2*a^2*tan(c/2 + (d*x)/2)^18 - (512*a^2)/315)/(d*(45*tan
(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 - 120*tan(c/2 + (d*x)/2)^6 + 21
0*tan(c/2 + (d*x)/2)^8 - 252*tan(c/2 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)
^12 - 120*tan(c/2 + (d*x)/2)^14 + 45*tan(c/2 + (d*x)/2)^16 - 10*tan(c/2 + (
d*x)/2)^18 + tan(c/2 + (d*x)/2)^20 + 1))
```

3.20 $\int (a+a\sec(c+dx))^2\tan^7(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int (a+a\sec(c+dx))^2\tan^7(c+dx)\,dx=\frac{a^2\log(\cos(c+dx))}{d}-\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{d}+\frac{2a^2\sec^3(c+dx)}{d}-\frac{6a^2\sec^5(c+dx)}{5d}-\frac{a^2\sec^6(c+dx)}{3d}+\frac{2a^2\sec^7(c+dx)}{7d}+\frac{a^2\sec^8(c+dx)}{8d}$$

```
[Out] a^2*ln(cos(d*x+c))/d-2*a^2*sec(d*x+c)/d+a^2*sec(d*x+c)^2/d+2*a^2*sec(d*x+c)
^3/d-6/5*a^2*sec(d*x+c)^5/d-1/3*a^2*sec(d*x+c)^6/d+2/7*a^2*sec(d*x+c)^7/d+1
/8*a^2*sec(d*x+c)^8/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int (a+a\sec(c+dx))^2\tan^7(c+dx)\,dx=\frac{a^2\sec^8(c+dx)}{8d}+\frac{2a^2\sec^7(c+dx)}{7d}-\frac{a^2\sec^6(c+dx)}{3d}-\frac{6a^2\sec^5(c+dx)}{5d}+\frac{2a^2\sec^3(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{d}-\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a^2*Sec[c + d*x])/d + (a^2*Sec[c + d*x]^2)/d
 + (2*a^2*Sec[c + d*x]^3)/d - (6*a^2*Sec[c + d*x]^5)/(5*d) - (a^2*Sec[c + d
*x]^6)/(3*d) + (2*a^2*Sec[c + d*x]^7)/(7*d) + (a^2*Sec[c + d*x]^8)/(8*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^5}{x^9}\,dx, x, \cos(c+dx)\right)}{a^6 d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^8}{x^9}+\frac{2a^8}{x^8}-\frac{2a^8}{x^7}-\frac{6a^8}{x^6}+\frac{6a^8}{x^4}+\frac{2a^8}{x^3}-\frac{2a^8}{x^2}-\frac{a^8}{x}\right)dx, x, \cos(c+dx)\right)}{a^6 d}\\
&= \frac{a^2\log(\cos(c+dx))}{d}-\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{d}+\frac{2a^2\sec^3(c+dx)}{d}\\
&\quad -\frac{6a^2\sec^5(c+dx)}{5d}-\frac{a^2\sec^6(c+dx)}{3d}+\frac{2a^2\sec^7(c+dx)}{7d}+\frac{a^2\sec^8(c+dx)}{8d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20

$$
\int (a+a\sec(c+dx))^2\tan^7(c+dx)\,dx = \frac{a^2(-6156\cos(c+dx)+7(-636\cos(3(c+dx))+20\cos(2(c+dx))(29+42\log(\cos(c+dx)))+5(104-}{}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (a^2*(-6156*Cos[c + d*x] + 7*(-636*Cos[3*(c + d*x)] + 20*Cos[2*(c + d*x)]*(
29 + 42*Log[Cos[c + d*x]]) + 5*(104 - 36*Cos[5*(c + d*x)] + 12*Cos[6*(c + d
*x)] - 12*Cos[7*(c + d*x)] + 105*Log[Cos[c + d*x]] + 24*Cos[6*(c + d*x)]*Lo
g[Cos[c + d*x]] + 3*Cos[8*(c + d*x)]*Log[Cos[c + d*x]] + 12*Cos[4*(c + d*x)
]*(6 + 7*Log[Cos[c + d*x]])))))*Sec[c + d*x]^8)/(13440*d)
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{a^{2}\left(\frac{\sec(dx+c)^{8}}{8}+\frac{2\sec(dx+c)^{7}}{7}-\frac{\sec(dx+c)^{6}}{3}-\frac{6\sec(dx+c)^{5}}{5}+2\sec(dx+c)^{3}+\sec(dx+c)^{2}-2\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^{2}\left(\frac{\sec(dx+c)^{8}}{8}+\frac{2\sec(dx+c)^{7}}{7}-\frac{\sec(dx+c)^{6}}{3}-\frac{6\sec(dx+c)^{5}}{5}+2\sec(dx+c)^{3}+\sec(dx+c)^{2}-2\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a^{2}\left(\frac{\tan(dx+c)^{6}}{6}-\frac{\tan(dx+c)^{4}}{4}+\frac{\tan(dx+c)^{2}}{2}-\frac{\ln\left(1+\tan(dx+c)^{2}\right)}{2}\right)}{d}+\frac{a^{2}\tan(dx+c)^{8}}{8d}+\frac{2a^{2}\left(\frac{\sec(dx+c)^{7}}{7}-\frac{3\sec(dx+c)^{5}}{5}+\sec\right.}{d}$
risch	$-ia^{2}x-\frac{2ia^{2}c}{d}-\frac{4a^{2}\left(105\,e^{15i(dx+c)}-105\,e^{14i(dx+c)}+315\,e^{13i(dx+c)}-630\,e^{12i(dx+c)}+1113\,e^{11i(dx+c)}-1015\,e^{10i(dx}\right.}{}$

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(1/8*sec(d*x+c)^8+2/7*sec(d*x+c)^7-1/3*sec(d*x+c)^6-6/5*sec(d*x+c)^5+
2*sec(d*x+c)^3+sec(d*x+c)^2-2*sec(d*x+c)-ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int (a+a\sec(c+dx))^2\tan^7(c+dx)\,dx$$
$$=\frac{840\,a^{2}\cos\left(dx+c\right)^{8}\log\left(-\cos\left(dx+c\right)\right)-1680\,a^{2}\cos\left(dx+c\right)^{7}+840\,a^{2}\cos\left(dx+c\right)^{6}+1680\,a^{2}\cos\left(dx+c\right.}{840\,d\cos\left(dx+c\right)^{8}}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/840*(840*a^2*cos(d*x + c)^8*log(-cos(d*x + c)) - 1680*a^2*cos(d*x + c)^7
+ 840*a^2*cos(d*x + c)^6 + 1680*a^2*cos(d*x + c)^5 - 1008*a^2*cos(d*x + c)^
3 - 280*a^2*cos(d*x + c)^2 + 240*a^2*cos(d*x + c) + 105*a^2)/(d*cos(d*x + c
)^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(119) = 238$.

Time = 1.19 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.91

$$\int (a + a\sec(c + dx))^2 \tan^7(c + dx)\,dx$$
$$= \begin{cases} -\frac{a^2 \log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^2 \tan^6(c+dx)\sec^2(c+dx)}{8d} + \frac{2a^2 \tan^6(c+dx)\sec(c+dx)}{7d} + \frac{a^2 \tan^6(c+dx)}{6d} - \frac{a^2 \tan^4(c+dx)\sec^2(c+dx)}{8d} \\ x(a\sec(c) + a)^2 \tan^7(c) \end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**6*sec(
c + d*x)**2/(8*d) + 2*a**2*tan(c + d*x)**6*sec(c + d*x)/(7*d) + a**2*tan(c
+ d*x)**6/(6*d) - a**2*tan(c + d*x)**4*sec(c + d*x)**2/(8*d) - 12*a**2*tan(
c + d*x)**4*sec(c + d*x)/(35*d) - a**2*tan(c + d*x)**4/(4*d) + a**2*tan(c +
 d*x)**2*sec(c + d*x)**2/(8*d) + 16*a**2*tan(c + d*x)**2*sec(c + d*x)/(35*d
) + a**2*tan(c + d*x)**2/(2*d) - a**2*sec(c + d*x)**2/(8*d) - 32*a**2*sec(c
 + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c) + a)**2*tan(c)**7, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int (a + a\sec(c + dx))^2 \tan^7(c + dx)\,dx$$
$$= \frac{840\,a^2 \log(\cos(dx + c)) - \frac{1680\,a^2 \cos(dx+c)^7 - 840\,a^2 \cos(dx+c)^6 - 1680\,a^2 \cos(dx+c)^5 + 1008\,a^2 \cos(dx+c)^3 + 280\,a^2 \cos(dx+c)^2 - 240}{\cos(dx+c)^8}}{840\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/840*(840*a^2*log(cos(d*x + c)) - (1680*a^2*cos(d*x + c)^7 - 840*a^2*cos(d
*x + c)^6 - 1680*a^2*cos(d*x + c)^5 + 1008*a^2*cos(d*x + c)^3 + 280*a^2*cos
(d*x + c)^2 - 240*a^2*cos(d*x + c) - 105*a^2)/cos(d*x + c)^8)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(124) = 248$.

Time = 3.88 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.21

$$\int (a + a\sec(c + dx))^2 \tan^7(c + dx)\,dx =$$

$$-\frac{840\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 840\,a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{3819\,a^2 + \frac{32232\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{120372\,a^2(\cos(dx+c}{(\cos(dx+c)+1)}}{}}{}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/840*(840*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 840*
a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3819*a^2 + 3223
2*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 120372*a^2*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 261464*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1
)^3 + 258370*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 175448*a^2*(co
s(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 77364*a^2*(cos(d*x + c) - 1)^6/(co
s(d*x + c) + 1)^6 + 19944*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 2
283*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8)/((cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 18.22 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.89

$$\int (a + a\sec(c + dx))^2 \tan^7(c + dx)\,dx$$

$$= \frac{2\,a^2 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{14} - 16\,a^2 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{12} + \frac{170\,a^2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{3} - \frac{352\,a^2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3} + \frac{2386\,a^2 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{15}}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{16} - 8\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{14} + 28\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{12} - 56\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{10} + 70\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{8} - 56\tan\right.}$$
$$- \frac{2\,a^2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2\right)}{d}$$

```
[In] int(tan(c + d*x)^7*(a + a/cos(c + d*x))^2,x)
```

```
[Out] ((582*a^2*tan(c/2 + (d*x)/2)^2)/35 - (336*a^2*tan(c/2 + (d*x)/2)^4)/5 + (23
86*a^2*tan(c/2 + (d*x)/2)^6)/15 - (352*a^2*tan(c/2 + (d*x)/2)^8)/3 + (170*a
^2*tan(c/2 + (d*x)/2)^10)/3 - 16*a^2*tan(c/2 + (d*x)/2)^12 + 2*a^2*tan(c/2
+ (d*x)/2)^14 - (64*a^2)/35)/(d*(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x
)/2)^2 - 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (
d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 +
 (d*x)/2)^16 + 1)) - (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.21 $\int (a+a\sec(c+dx))^2\tan^5(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 120

$$\int (a+a\sec(c+dx))^2\tan^5(c+dx)\,dx = -\frac{a^2\log(\cos(c+dx))}{d} + \frac{2a^2\sec(c+dx)}{d} - \frac{a^2\sec^2(c+dx)}{2d} - \frac{4a^2\sec^3(c+dx)}{3d} - \frac{a^2\sec^4(c+dx)}{4d} + \frac{2a^2\sec^5(c+dx)}{5d} + \frac{a^2\sec^6(c+dx)}{6d}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d-1/2*a^2*sec(d*x+c)^2/d-4/3*a^2*sec
(d*x+c)^3/d-1/4*a^2*sec(d*x+c)^4/d+2/5*a^2*sec(d*x+c)^5/d+1/6*a^2*sec(d*x+c
)^6/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int (a+a\sec(c+dx))^2\tan^5(c+dx)\,dx = \frac{a^2\sec^6(c+dx)}{6d} + \frac{2a^2\sec^5(c+dx)}{5d} - \frac{a^2\sec^4(c+dx)}{4d} - \frac{4a^2\sec^3(c+dx)}{3d} - \frac{a^2\sec^2(c+dx)}{2d} + \frac{2a^2\sec(c+dx)}{d} - \frac{a^2\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d - (a^2*Sec[c + d*x]^2
)/(2*d) - (4*a^2*Sec[c + d*x]^3)/(3*d) - (a^2*Sec[c + d*x]^4)/(4*d) + (2*a^
2*Sec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^6)/(6*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)^4}{x^7}\,dx, x, \cos(c+dx)\right)}{a^4d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^6}{x^7}+\frac{2a^6}{x^6}-\frac{a^6}{x^5}-\frac{4a^6}{x^4}-\frac{a^6}{x^3}+\frac{2a^6}{x^2}+\frac{a^6}{x}\right)dx, x, \cos(c+dx)\right)}{a^4d}\\
&= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\sec^2(c+dx)}{2d}\\
&\quad -\frac{4a^2\sec^3(c+dx)}{3d}-\frac{a^2\sec^4(c+dx)}{4d}+\frac{2a^2\sec^5(c+dx)}{5d}+\frac{a^2\sec^6(c+dx)}{6d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04

$$
\int (a+a\sec(c+dx))^2\tan^5(c+dx)\,dx
= \frac{a^2(312\cos(c+dx)-5(14-28\cos(3(c+dx))+6\cos(4(c+dx))-12\cos(5(c+dx))+30\log(\cos(c+dx)}{}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (a^2*(312*Cos[c + d*x] - 5*(14 - 28*Cos[3*(c + d*x)] + 6*Cos[4*(c + d*x)] -
 12*Cos[5*(c + d*x)] + 30*Log[Cos[c + d*x]] + 18*Cos[4*(c + d*x)]*Log[Cos[c
 + d*x]] + 3*Cos[6*(c + d*x)]*Log[Cos[c + d*x]] + 9*Cos[2*(c + d*x)]*(4 + 5
*Log[Cos[c + d*x]])))*Sec[c + d*x]^6)/(480*d)
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{a^{2}\left(\frac{\sec(dx+c)^{6}}{6}+\frac{2\sec(dx+c)^{5}}{5}-\frac{\sec(dx+c)^{4}}{4}-\frac{4\sec(dx+c)^{3}}{3}-\frac{\sec(dx+c)^{2}}{2}+2\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^{2}\left(\frac{\sec(dx+c)^{6}}{6}+\frac{2\sec(dx+c)^{5}}{5}-\frac{\sec(dx+c)^{4}}{4}-\frac{4\sec(dx+c)^{3}}{3}-\frac{\sec(dx+c)^{2}}{2}+2\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a^{2}\left(\frac{\tan(dx+c)^{4}}{4}-\frac{\tan(dx+c)^{2}}{2}+\frac{\ln\left(1+\tan(dx+c)^{2}\right)}{2}\right)}{d}+\frac{a^{2}\tan(dx+c)^{6}}{6d}+\frac{2a^{2}\left(\frac{\sec(dx+c)^{5}}{5}-\frac{2\sec(dx+c)^{3}}{3}+\sec(dx+c)\right)}{d}$
risch	$ia^{2}x+\frac{2ia^{2}c}{d}+\frac{2a^{2}\left(30\,\mathrm{e}^{11i(dx+c)}-15\,\mathrm{e}^{10i(dx+c)}+70\,\mathrm{e}^{9i(dx+c)}-90\,\mathrm{e}^{8i(dx+c)}+156\,\mathrm{e}^{7i(dx+c)}-70\,\mathrm{e}^{6i(dx+c)}+156\,\mathrm{e}^{5i(}\right.}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{6}}$

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(1/6*sec(d*x+c)^6+2/5*sec(d*x+c)^5-1/4*sec(d*x+c)^4-4/3*sec(d*x+c)^3-
1/2*sec(d*x+c)^2+2*sec(d*x+c)+ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int (a+a\sec(c+dx))^2\tan^5(c+dx)\,dx =$$
$$-\frac{60\,a^{2}\cos\left(dx+c\right)^{6}\log\left(-\cos\left(dx+c\right)\right)-120\,a^{2}\cos\left(dx+c\right)^{5}+30\,a^{2}\cos\left(dx+c\right)^{4}+80\,a^{2}\cos\left(dx+c\right)^{3}}{60\,d\cos\left(dx+c\right)^{6}}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a^2*cos(d*x + c)^6*log(-cos(d*x + c)) - 120*a^2*cos(d*x + c)^5 +
30*a^2*cos(d*x + c)^4 + 80*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c)^2 - 24*
a^2*cos(d*x + c) - 10*a^2)/(d*cos(d*x + c)^6)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.58

$$\int (a + a\sec(c + dx))^2 \tan^5(c + dx)\,dx$$
$$= \begin{cases} \frac{a^2 \log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^2\tan^4(c+dx)\sec^2(c+dx)}{6d} + \frac{2a^2\tan^4(c+dx)\sec(c+dx)}{5d} + \frac{a^2\tan^4(c+dx)}{4d} - \frac{a^2\tan^2(c+dx)\sec^2(c+dx)}{6d} - \frac{8}{} \\ x(a\sec(c)+a)^2\tan^5(c) \end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**4*sec(c
 + d*x)**2/(6*d) + 2*a**2*tan(c + d*x)**4*sec(c + d*x)/(5*d) + a**2*tan(c +
 d*x)**4/(4*d) - a**2*tan(c + d*x)**2*sec(c + d*x)**2/(6*d) - 8*a**2*tan(c
+ d*x)**2*sec(c + d*x)/(15*d) - a**2*tan(c + d*x)**2/(2*d) + a**2*sec(c + d
*x)**2/(6*d) + 16*a**2*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a*sec(c) + a)**2
*tan(c)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.81

$$\int (a + a\sec(c + dx))^2 \tan^5(c + dx)\,dx =$$
$$-\frac{60\,a^2\log\left(\cos\left(dx+c\right)\right) - \frac{120\,a^2\cos(dx+c)^5 - 30\,a^2\cos(dx+c)^4 - 80\,a^2\cos(dx+c)^3 - 15\,a^2\cos(dx+c)^2 + 24\,a^2\cos(dx+c) + 10\,a^2}{\cos(dx+c)^6}}{60\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a^2*log(cos(d*x + c)) - (120*a^2*cos(d*x + c)^5 - 30*a^2*cos(d*x
+ c)^4 - 80*a^2*cos(d*x + c)^3 - 15*a^2*cos(d*x + c)^2 + 24*a^2*cos(d*x + c
) + 10*a^2)/cos(d*x + c)^6)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(110) = 220.

Time = 1.77 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.02

$$\int (a + a\sec(c + dx))^2 \tan^5(c + dx)\,dx$$
$$= \frac{60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{275\,a^2 + \frac{1770\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{4845\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{478}{}}{}}{60\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (275*a^2 + 1770*a^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4845*a^2*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 4780*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925
*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^2*(cos(d*x + c) - 1
)^5/(cos(d*x + c) + 1)^5 + 147*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^
6)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 17.79 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.60

$$\int (a+a\sec(c+dx))^2\tan^5(c+dx)\,dx = \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d} - \frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 12\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + \frac{92\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3} - 44\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + \frac{74\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + 15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + 15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 6\,\mathrm{ta}\right.}$$

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d - ((74*a^2*tan(c/2 + (d*x)/2)^2)/5 -
44*a^2*tan(c/2 + (d*x)/2)^4 + (92*a^2*tan(c/2 + (d*x)/2)^6)/3 - 12*a^2*tan(
c/2 + (d*x)/2)^8 + 2*a^2*tan(c/2 + (d*x)/2)^10 - (32*a^2)/15)/(d*(15*tan(c/
2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(
c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.22 $\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx=\frac{a^2\log(\cos(c+dx))}{d}-\frac{2a^2\sec(c+dx)}{d}+\frac{2a^2\sec^3(c+dx)}{3d}+\frac{a^2\sec^4(c+dx)}{4d}$$

```
[Out] a^2*ln(cos(d*x+c))/d-2*a^2*sec(d*x+c)/d+2/3*a^2*sec(d*x+c)^3/d+1/4*a^2*sec(
d*x+c)^4/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 76}

$$\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx=\frac{a^2\sec^4(c+dx)}{4d}+\frac{2a^2\sec^3(c+dx)}{3d}-\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a^2*Sec[c + d*x])/d + (2*a^2*Sec[c + d*x]^3)
/(3*d) + (a^2*Sec[c + d*x]^4)/(4*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
```

```
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)(a+ax)^3}{x^5}\,dx, x, \cos(c+dx)\right)}{a^2 d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^4}{x^5}+\frac{2a^4}{x^4}-\frac{2a^4}{x^2}-\frac{a^4}{x}\right)dx, x, \cos(c+dx)\right)}{a^2 d}\\
&= \frac{a^2\log(\cos(c+dx))}{d}-\frac{2a^2\sec(c+dx)}{d}+\frac{2a^2\sec^3(c+dx)}{3d}+\frac{a^2\sec^4(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

$$\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx$$
$$= \frac{a^2(-20\cos(c+dx)+3(2-4\cos(3(c+dx))+3\log(\cos(c+dx))+4\cos(2(c+dx))\log(\cos(c+dx))+\text{c}}{24d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*(-20*Cos[c + d*x] + 3*(2 - 4*Cos[3*(c + d*x)] + 3*Log[Cos[c + d*x]] +
4*Cos[2*(c + d*x)]*Log[Cos[c + d*x]] + Cos[4*(c + d*x)]*Log[Cos[c + d*x]]))
*Sec[c + d*x]^4)/(24*d)
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{a^{2}\left(\frac{\sec(dx+c)^{4}}{4}+\frac{2\sec(dx+c)^{3}}{3}-2\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$	46
default	$\frac{a^{2}\left(\frac{\sec(dx+c)^{4}}{4}+\frac{2\sec(dx+c)^{3}}{3}-2\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$	46
parts	$\frac{a^{2}\left(\frac{\tan(dx+c)^{2}}{2}-\frac{\ln\left(1+\tan(dx+c)^{2}\right)}{2}\right)}{d}+\frac{a^{2}\tan(dx+c)^{4}}{4d}+\frac{2a^{2}\left(\frac{\sec(dx+c)^{3}}{3}-\sec(dx+c)\right)}{d}$	76
risch	$-ia^{2}x-\frac{2ia^{2}c}{d}-\frac{4a^{2}\left(3\,e^{7i(dx+c)}+5\,e^{5i(dx+c)}-3\,e^{4i(dx+c)}+5\,e^{3i(dx+c)}+3\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^{4}}+\frac{a^{2}\ln\left(e^{2i(dx+c)}+1\right)}{d}$	115

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
[Out] a^2/d*(1/4*sec(d*x+c)^4+2/3*sec(d*x+c)^3-2*sec(d*x+c)-ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx = \frac{12\,a^{2}\cos\left(dx+c\right)^{4}\log\left(-\cos\left(dx+c\right)\right)-24\,a^{2}\cos\left(dx+c\right)^{3}+8\,a^{2}\cos\left(dx+c\right)+3\,a^{2}}{12\,d\cos\left(dx+c\right)^{4}}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
[Out] 1/12*(12*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) - 24*a^2*cos(d*x + c)^3 + 8*
a^2*cos(d*x + c) + 3*a^2)/(d*cos(d*x + c)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(58) = 116$.

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.94

$$\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx = \begin{cases} -\frac{a^{2}\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{a^{2}\tan^{2}\left(c+dx\right)\sec^{2}\left(c+dx\right)}{4d}+\frac{2a^{2}\tan^{2}\left(c+dx\right)\sec\left(c+dx\right)}{3d}+\frac{a^{2}\tan^{2}\left(c+dx\right)}{2d}-\frac{a^{2}\sec^{2}\left(c+dx\right)}{4d}-\frac{4a^{2}\sec\left(c+\right.}{3d} \\ x\left(a\sec\left(c\right)+a\right)^{2}\tan^{3}\left(c\right) \end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**2*sec(
c + d*x)**2/(4*d) + 2*a**2*tan(c + d*x)**2*sec(c + d*x)/(3*d) + a**2*tan(c
+ d*x)**2/(2*d) - a**2*sec(c + d*x)**2/(4*d) - 4*a**2*sec(c + d*x)/(3*d), N
e(d, 0)), (x*(a*sec(c) + a)**2*tan(c)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx = \frac{12\,a^2\log\left(\cos\left(dx+c\right)\right) - \frac{24\,a^2\cos(dx+c)^3-8\,a^2\cos(dx+c)-3\,a^2}{\cos(dx+c)^4}}{12\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*a^2*log(cos(d*x + c)) - (24*a^2*cos(d*x + c)^3 - 8*a^2*cos(d*x + c
) - 3*a^2)/cos(d*x + c)^4)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(61) = 122.

Time = 0.77 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.95

$$\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx =$$

$$-\frac{12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{57\,a^2+\frac{252\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{246\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+}{(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+}}{12\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^2
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (57*a^2 + 252*a^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 246*a^2*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 25*a^2*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 16.68 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.05

$$\int (a+a\sec(c+dx))^2\tan^3(c+dx)\,dx$$
$$=\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{38\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{8\,a^2}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$
$$-\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

```
[Out] ((38*a^2*tan(c/2 + (d*x)/2)^2)/3 - 8*a^2*tan(c/2 + (d*x)/2)^4 + 2*a^2*tan(c
/2 + (d*x)/2)^6 - (8*a^2)/3)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)
/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (2*a^2*atanh(
tan(c/2 + (d*x)/2)^2))/d
```

3.23 $\int (a+a\sec (c+dx))^2 \tan (c+dx) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int (a+a\sec (c+dx))^2 \tan (c+dx) \, dx = -\frac{a^2 \log (\cos (c+dx))}{d} + \frac{2 a^2 \sec (c+dx)}{d} + \frac{a^2 \sec ^2(c+dx)}{2 d}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d+1/2*a^2*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 45}

$$\int (a+a\sec (c+dx))^2 \tan (c+dx) \, dx = \frac{a^2 \sec ^2(c+dx)}{2 d} + \frac{2 a^2 \sec (c+dx)}{d} - \frac{a^2 \log (\cos (c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d + (a^2*Sec[c + d*x]^2
)/(2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a+ax)^2}{x^3}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3}+\frac{2a^2}{x^2}+\frac{a^2}{x}\right)\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^2\log(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}+\frac{a^2\sec^2(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int (a+a\sec(c+dx))^2\tan(c+dx)\,dx = -\frac{a^2(-1-4\cos(c+dx)+\log(\cos(c+dx))+\cos(2(c+dx))\log(\cos(c+dx)))\sec^2(c+dx)}{2d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x],x]
[Out] -1/2*(a^2*(-1 - 4*Cos[c + d*x] + Log[Cos[c + d*x]] + Cos[2*(c + d*x)]*Log[C
os[c + d*x]])*Sec[c + d*x]^2)/d
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)^2}{2}+2\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$	34
default	$\frac{a^2\left(\frac{\sec(dx+c)^2}{2}+2\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$	34
parts	$\frac{a^2\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{a^2\sec(dx+c)^2}{2d}+\frac{2a^2\sec(dx+c)}{d}$	51
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2a^2\left(2\,\mathrm{e}^{3i(dx+c)}+\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{i(dx+c)}\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{a^2\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	92

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^2*(1/2*sec(d*x+c)^2+2*sec(d*x+c)+ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int (a+a\sec(c+dx))^2\tan(c+dx)\,dx$$
$$= -\frac{2\,a^2\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-4\,a^2\cos\left(dx+c\right)-a^2}{2\,d\cos\left(dx+c\right)^2}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*cos(d*x + c)^2*log(-cos(d*x + c)) - 4*a^2*cos(d*x + c) - a^2)/(
d*cos(d*x + c)^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int (a+a\sec(c+dx))^2\tan(c+dx)\,dx$$
$$= \begin{cases} \frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\sec^2\left(c+dx\right)}{2d}+\frac{2a^2\sec\left(c+dx\right)}{d} & \text{for } d\neq 0 \\ x(a\sec\left(c\right)+a)^2\tan\left(c\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c),x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*sec(c + d*x)**2/(2*d)
 + 2*a**2*sec(c + d*x)/d, Ne(d, 0)), (x*(a*sec(c) + a)**2*tan(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int (a+a\sec(c+dx))^2\tan(c+dx)\,dx = -\frac{2\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{4\,a^2\cos(dx+c)+a^2}{\cos(dx+c)^2}}{2\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
[Out] -1/2*(2*a^2*log(cos(d*x + c)) - (4*a^2*cos(d*x + c) + a^2)/cos(d*x + c)^2)/
d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(46) = 92.

Time = 0.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.96

$$\int (a+a\sec(c+dx))^2\tan(c+dx)\,dx$$
$$= \frac{2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{11\,a^2+\frac{10\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
[Out] 1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11*a^2 + 10*a^2*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\int (a+a\sec(c+dx))^2\tan(c+dx)\,dx = \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d} - \frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,a^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^2,x)
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a^2*tan(c/2 + (d*x)/2)^2 - 4*a^2
)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.24 $\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 35

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\log(1-\cos(c+dx))}{d} - \frac{a^2\log(\cos(c+dx))}{d}$$

```
[Out] 2*a^2*ln(1-cos(d*x+c))/d-a^2*ln(cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3964, 78}

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2a^2\log(1-\cos(c+dx))}{d} - \frac{a^2\log(\cos(c+dx))}{d}$$

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*Log[1 - Cos[c + d*x]])/d - (a^2*Log[Cos[c + d*x]])/d
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a^2\text{Subst}\left(\int \frac{a+ax}{x(a-ax)}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{a^2\text{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right)\,dx, x, \cos(c+dx)\right)}{d}\\ &= \frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{a^2\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\left(\log(\cos(c+dx))-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^2,x]
[Out] -((a^2*(Log[Cos[c + d*x]] - 4*Log[Sin[(c + d*x)/2]]))/d)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.40

method	result	size
derivativedivides	$\frac{a^2\ln(\tan(dx+c))+2a^2\ln(-\cot(dx+c)+\csc(dx+c))+a^2\ln(\sin(dx+c))}{d}$	49
default	$\frac{a^2\ln(\tan(dx+c))+2a^2\ln(-\cot(dx+c)+\csc(dx+c))+a^2\ln(\sin(dx+c))}{d}$	49
risch	$-ia^2x-\frac{2ia^2c}{d}+\frac{4a^2\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$	59

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*ln(tan(d*x+c))+2*a^2*ln(-cot(d*x+c)+csc(d*x+c))+a^2*ln(sin(d*x+c))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= -\frac{a^2\log\left(-\cos\left(dx+c\right)\right)-2\,a^2\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{d}$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -(a^2*log(-cos(d*x + c)) - 2*a^2*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cot\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cot\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cot\left(c+dx\right)dx\right)$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**2,x)
[Out] a**2*(Integral(2*cot(c + d*x)*sec(c + d*x), x) + Integral(cot(c + d*x)*sec(
c + d*x)**2, x) + Integral(cot(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\,a^2\log\left(\cos\left(dx+c\right)-1\right)-a^2\log\left(\cos\left(dx+c\right)\right)}{d}$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] (2*a^2*log(cos(d*x + c) - 1) - a^2*log(cos(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - a^2\log\left(\left|\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - 1\right|\right)}{d}$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^2*log(abs((cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \cot(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\left(4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right) - \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 1\right)\right)}{d}$$

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*(4*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^4 - 1)))/d
```

3.25 $\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2}{d(1-\cos(c+dx))}-\frac{a^2\log(1-\cos(c+dx))}{d}$$

```
[Out] -a^2/d/(1-cos(d*x+c))-a^2*ln(1-cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 45}

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2}{d(1-\cos(c+dx))}-\frac{a^2\log(1-\cos(c+dx))}{d}$$

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2/(d*(1 - Cos[c + d*x]))) - (a^2*Log[1 - Cos[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
```

```
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^4 \text{Subst}\left(\int \frac{x}{(a-ax)^2}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{a^4 \text{Subst}\left(\int \left(\frac{1}{a^2(-1+x)^2}+\frac{1}{a^2(-1+x)}\right)dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{a^2}{d(1-\cos(c+dx))}-\frac{a^2\log(1-\cos(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\csc^2\left(\frac{1}{2}(c+dx)\right)\left(-1-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\cos(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2d}$$

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
[Out] (a^2*Csc[(c + d*x)/2]^2*(-1 - 2*Log[Sin[(c + d*x)/2]] + 2*Cos[c + d*x]*Log[
Sin[(c + d*x)/2]]))/(2*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

method	result	size
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2a^2\mathrm{e}^{i(dx+c)}}{d\left(\mathrm{e}^{i(dx+c)}-1\right)^2}-\frac{2a^2\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d}$	69
derivativedivides	$\frac{-\frac{a^2}{2\sin(dx+c)^2}+2a^2\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)}{d}$	93
default	$\frac{-\frac{a^2}{2\sin(dx+c)^2}+2a^2\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)+a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)}{d}$	93

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] I*a^2*x+2*I/d*a^2*c+2*a^2*exp(I*(d*x+c))/d/(exp(I*(d*x+c))-1)^2-2/d*a^2*ln(
exp(I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2 - (a^2\cos(dx+c) - a^2)\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)}{d\cos(dx+c) - d}$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] (a^2 - (a^2*cos(d*x + c) - a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x +
c) - d)
```

Sympy [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cot^3(c+dx)\sec(c+dx)\,dx + \int \cot^3(c+dx)\sec^2(c+dx)\,dx + \int \cot^3(c+dx)\,dx\right)$$

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
[Out] a**2*(Integral(2*cot(c + d*x)**3*sec(c + d*x), x) + Integral(cot(c + d*x)**
3*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\log(\cos(dx+c) - 1) - \frac{a^2}{\cos(dx+c)-1}}{d}$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -(a^2*log(cos(d*x + c) - 1) - a^2/(cos(d*x + c) - 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(37) = 74$.

Time = 0.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.78

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{\left(a^2+\frac{2\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{2\,d}$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a^2*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^2 + 2*a^2*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 13.86 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\left(\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{d}$$

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(a^2*(2*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
+ (d*x)/2)^2/2))/d
```

3.26 $\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2}{4d(1-\cos(c+dx))^2}+\frac{5a^2}{4d(1-\cos(c+dx))}+\frac{7a^2\log(1-\cos(c+dx))}{8d}+\frac{a^2\log(1+\cos(c+dx))}{8d}$$

```
[Out] -1/4*a^2/d/(1-cos(d*x+c))^2+5/4*a^2/d/(1-cos(d*x+c))+7/8*a^2*ln(1-cos(d*x+c
))/d+1/8*a^2*ln(1+cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{5a^2}{4d(1-\cos(c+dx))}-\frac{a^2}{4d(1-\cos(c+dx))^2}+\frac{7a^2\log(1-\cos(c+dx))}{8d}+\frac{a^2\log(\cos(c+dx)+1)}{8d}$$

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*a^2/(d*(1 - Cos[c + d*x])^2) + (5*a^2)/(4*d*(1 - Cos[c + d*x])) + (7*a
^2*Log[1 - Cos[c + d*x]])/(8*d) + (a^2*Log[1 + Cos[c + d*x]])/(8*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a^6\text{Subst}\left(\int \frac{x^3}{(a-ax)^3(a+ax)}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^6\text{Subst}\left(\int \left(-\frac{1}{2a^4(-1+x)^3}-\frac{5}{4a^4(-1+x)^2}-\frac{7}{8a^4(-1+x)}-\frac{1}{8a^4(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^2}{4d(1-\cos(c+dx))^2}+\frac{5a^2}{4d(1-\cos(c+dx))}\\
&\quad+\frac{7a^2\log(1-\cos(c+dx))}{8d}+\frac{a^2\log(1+\cos(c+dx))}{8d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$
\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2(1+\cos(c+dx))^2\left(-10\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)-4\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+7\log\left(\sin\left(\frac{1}{2}(\right.\right.\right.\right.}{64d}
$$

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/64*(a^2*(1 + Cos[c + d*x])^2*(-10*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^
4 - 4*(Log[Cos[(c + d*x)/2]] + 7*Log[Sin[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4
)/d
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

method	result
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{a^2\left(5\,\mathrm{e}^{3i(dx+c)}-8\,\mathrm{e}^{2i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}\right)}{2d\left(\mathrm{e}^{i(dx+c)}-1\right)^4}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{4d}+\frac{7a^2\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{4d}$
derivativedivides	$\frac{-\frac{a^2\cos(dx+c)^4}{4\sin(dx+c)^4}+2a^2\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+a^2\left(-\frac{\cot(d}{}\right.}{d}$
default	$\frac{-\frac{a^2\cos(dx+c)^4}{4\sin(dx+c)^4}+2a^2\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)+a^2\left(-\frac{\cot(d}{}\right.}{d}$

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -I*a^2*x-2*I/d*a^2*c-1/2*a^2/d/(exp(I*(d*x+c))-1)^4*(5*exp(3*I*(d*x+c))-8*e
xp(2*I*(d*x+c))+5*exp(I*(d*x+c)))+1/4/d*a^2*ln(exp(I*(d*x+c))+1)+7/4/d*a^2*
ln(exp(I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int\cot^5(c+dx)(a+a\sec(c+dx))^2\,dx=$$
$$-\frac{10\,a^2\cos\left(dx+c\right)-8\,a^2-\left(a^2\cos\left(dx+c\right)^2-2\,a^2\cos\left(dx+c\right)+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-7\,\left(a^2\mathrm{c}\right.}{8\,\left(d\cos\left(dx+c\right)^2-2\,d\cos\left(dx+c\right)+d\right)}$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(10*a^2*cos(d*x + c) - 8*a^2 - (a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c
) + a^2)*log(1/2*cos(d*x + c) + 1/2) - 7*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*
x + c) + a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - 2*d*cos(d*x
 + c) + d)
```

Sympy [F]

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cot^5(c+dx)\sec(c+dx)\,dx + \int \cot^5(c+dx)\sec^2(c+dx)\,dx + \int \cot^5(c+dx)\,dx\right)$$

[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**2,x)

[Out] a**2*(Integral(2*cot(c + d*x)**5*sec(c + d*x), x) + Integral(cot(c + d*x)**5*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**5, x))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\log(\cos(dx+c)+1) + 7a^2\log(\cos(dx+c)-1) - \frac{2(5a^2\cos(dx+c)-4a^2)}{\cos(dx+c)^2-2\cos(dx+c)+1}}{8d}$$

[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] 1/8*(a^2*log(cos(d*x + c) + 1) + 7*a^2*log(cos(d*x + c) - 1) - 2*(5*a^2*cos(d*x + c) - 4*a^2)/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1))/d

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.62

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{14a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 16a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{\left(a^2+\frac{8a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{21a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}}{16d}$$

[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] 1/16*(14*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 16*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^2 + 8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 21*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d

Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.73

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\left(-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{7\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4}-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{d}$$

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
[Out] (a^2*((7*log(tan(c/2 + (d*x)/2)))/4 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c
/2 + (d*x)/2)^2/2 - cot(c/2 + (d*x)/2)^4/16))/d
```

3.27 $\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2}{12d(1-\cos(c+dx))^3}+\frac{a^2}{2d(1-\cos(c+dx))^2}-\frac{23a^2}{16d(1-\cos(c+dx))}-\frac{a^2}{16d(1+\cos(c+dx))}-\frac{13a^2\log(1-\cos(c+dx))}{16d}-\frac{3a^2\log(1+\cos(c+dx))}{16d}$$

```
[Out] -1/12*a^2/d/(1-cos(d*x+c))^3+1/2*a^2/d/(1-cos(d*x+c))^2-23/16*a^2/d/(1-cos(
d*x+c))-1/16*a^2/d/(1+cos(d*x+c))-13/16*a^2*ln(1-cos(d*x+c))/d-3/16*a^2*ln(
1+cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{23a^2}{16d(1-\cos(c+dx))}-\frac{a^2}{16d(\cos(c+dx)+1)}+\frac{a^2}{2d(1-\cos(c+dx))^2}-\frac{a^2}{12d(1-\cos(c+dx))^3}-\frac{13a^2\log(1-\cos(c+dx))}{16d}-\frac{3a^2\log(\cos(c+dx)+1)}{16d}$$

```
[In] Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/12*a^2/(d*(1 - Cos[c + d*x])^3) + a^2/(2*d*(1 - Cos[c + d*x])^2) - (23*a
^2)/(16*d*(1 - Cos[c + d*x])) - a^2/(16*d*(1 + Cos[c + d*x])) - (13*a^2*Log
[1 - Cos[c + d*x]])/(16*d) - (3*a^2*Log[1 + Cos[c + d*x]])/(16*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^8\text{Subst}\left(\int \frac{x^5}{(a-ax)^4(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{a^8\text{Subst}\left(\int \left(\frac{1}{4a^6(-1+x)^4}+\frac{1}{a^6(-1+x)^3}+\frac{23}{16a^6(-1+x)^2}+\frac{13}{16a^6(-1+x)}-\frac{1}{16a^6(1+x)^2}+\frac{3}{16a^6(1+x)}\right)dx, x, \cos
\end{aligned}$$

$$= -\frac{a^2}{12d(1-\cos(c+dx))^3}+\frac{a^2}{2d(1-\cos(c+dx))^2}-\frac{23a^2}{16d(1-\cos(c+dx))} - \frac{a^2}{16d(1+\cos(c+dx))}-\frac{13a^2\log(1-\cos(c+dx))}{16d}-\frac{3a^2\log(1+\cos(c+dx))}{16d}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(69\csc^2\left(\frac{1}{2}(c+dx)\right)-12\csc^4\left(\frac{1}{2}(c+dx)\right)+\csc^6\left(\frac{1}{2}(c+dx)\right)+3\right.}{384d}$$

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/384*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(69*Csc[(c + d*x)/2]^2
- 12*Csc[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6 + 3*(12*Log[Cos[(c + d*x)/2]]
+ 52*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2)))/d
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.35

method	result
risch	$i a^{2} x+\frac{2 i a^{2} c}{d}+\frac{a^{2} \left(33 \,{\mathrm e}^{7 i \left(d x +c \right)}-36 \,{\mathrm e}^{6 i \left(d x +c \right)}-49 \,{\mathrm e}^{5 i \left(d x +c \right)}+136 \,{\mathrm e}^{4 i \left(d x +c \right)}-49 \,{\mathrm e}^{3 i \left(d x +c \right)}-36 \,{\mathrm e}^{2 i \left(d x +c \right)}+33 \,{\mathrm e}^{i \left(d x +c \right)}\right)}{12 d \left({\mathrm e}^{i \left(d x +c \right)}-1\right)^{6} \left({\mathrm e}^{i \left(d x +c \right)}+1\right)^{2}}$
derivativedivides	$\frac{-\frac{a^{2} \cos \left(d x +c \right)^{6}}{6 \sin \left(d x +c \right)^{6}}+2 a^{2} \left(-\frac{\cos \left(d x +c \right)^{7}}{6 \sin \left(d x +c \right)^{6}}+\frac{\cos \left(d x +c \right)^{7}}{24 \sin \left(d x +c \right)^{4}}-\frac{\cos \left(d x +c \right)^{7}}{16 \sin \left(d x +c \right)^{2}}-\frac{\cos \left(d x +c \right)^{5}}{16}-\frac{5 \cos \left(d x +c \right)^{3}}{48}-\frac{5 \cos \left(d x +c \right)}{16}-\frac{5 \ln \left(-\cot \left(d x +\right.\right.}{1}}{d}$
default	$\frac{-\frac{a^{2} \cos \left(d x +c \right)^{6}}{6 \sin \left(d x +c \right)^{6}}+2 a^{2} \left(-\frac{\cos \left(d x +c \right)^{7}}{6 \sin \left(d x +c \right)^{6}}+\frac{\cos \left(d x +c \right)^{7}}{24 \sin \left(d x +c \right)^{4}}-\frac{\cos \left(d x +c \right)^{7}}{16 \sin \left(d x +c \right)^{2}}-\frac{\cos \left(d x +c \right)^{5}}{16}-\frac{5 \cos \left(d x +c \right)^{3}}{48}-\frac{5 \cos \left(d x +c \right)}{16}-\frac{5 \ln \left(-\cot \left(d x +\right.\right.}{1}}{d}$

```
[In] int(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] I*a^2*x+2*I/d*a^2*c+1/12*a^2/d/(exp(I*(d*x+c))-1)^6/(exp(I*(d*x+c))+1)^2*(3
3*exp(7*I*(d*x+c))-36*exp(6*I*(d*x+c))-49*exp(5*I*(d*x+c))+136*exp(4*I*(d*x
+c))-49*exp(3*I*(d*x+c))-36*exp(2*I*(d*x+c))+33*exp(I*(d*x+c)))-13/8/d*a^2*
ln(exp(I*(d*x+c))-1)-3/8/d*a^2*ln(exp(I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.50

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{66\,a^2\cos\left(dx+c\right)^3-36\,a^2\cos\left(dx+c\right)^2-74\,a^2\cos\left(dx+c\right)+52\,a^2-9\,\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right.\right.}{48\,\left(d\cos\left(d\right.\right.}$$

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(66*a^2*cos(d*x + c)^3 - 36*a^2*cos(d*x + c)^2 - 74*a^2*cos(d*x + c) +
 52*a^2 - 9*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)
 - a^2)*log(1/2*cos(d*x + c) + 1/2) - 39*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*
x + c)^3 + 2*a^2*cos(d*x + c) - a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d
*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c) - d)
```

Sympy [F]

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cot^7(c+dx)\sec(c+dx)\,dx + \int \cot^7(c+dx)\sec^2(c+dx)\,dx + \int \cot^7(c+dx)\,dx\right)$$

```
[In] integrate(cot(d*x+c)**7*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cot(c + d*x)**7*sec(c + d*x), x) + Integral(cot(c + d*x)**
7*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**7, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{9\,a^2\log\left(\cos\left(dx+c\right)+1\right)+39\,a^2\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(33\,a^2\cos(dx+c)^3-18\,a^2\cos(dx+c)^2-37\,a^2\cos(dx+c)+26}{\cos(dx+c)^4-2\cos(dx+c)^3+2\cos(dx+c)-1}}{48\,d}$$

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/48*(9*a^2*log(cos(d*x + c) + 1) + 39*a^2*log(cos(d*x + c) - 1) - 2*(33*a
^2*cos(d*x + c)^3 - 18*a^2*cos(d*x + c)^2 - 37*a^2*cos(d*x + c) + 26*a^2)/(
cos(d*x + c)^4 - 2*cos(d*x + c)^3 + 2*cos(d*x + c) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.46

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{78\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-96\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{3\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{\left(a^2+\frac{9\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48\,a^2(\cos(}{(\cos(dx}}{96\,d}$$

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*(78*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 96*a^2*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 3*a^2*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - (a^2 + 9*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 48*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 143*a^2*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(cos(d*x + c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{13\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$
$$-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{a^2}{6}\right)}{16\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{32\,d}$$

```
[In] int(cot(c + d*x)^7*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (13*a^2*log(tan(c/2 + (d*x)/2)))/(8
*d) - (cot(c/2 + (d*x)/2)^6*(8*a^2*tan(c/2 + (d*x)/2)^4 - (3*a^2*tan(c/2 +
(d*x)/2)^2)/2 + a^2/6))/(16*d) - (a^2*tan(c/2 + (d*x)/2)^2)/(32*d)
```

3.28 $\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 169

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2}{32d(1-\cos(c+dx))^4}+\frac{11a^2}{48d(1-\cos(c+dx))^3} - \frac{3a^2}{4d(1-\cos(c+dx))^2}+\frac{51a^2}{32d(1-\cos(c+dx))} - \frac{a^2}{64d(1+\cos(c+dx))^2}+\frac{9a^2}{64d(1+\cos(c+dx))} + \frac{99a^2\log(1-\cos(c+dx))}{128d} + \frac{29a^2\log(1+\cos(c+dx))}{128d}$$

```
[Out] -1/32*a^2/d/(1-cos(d*x+c))^4+11/48*a^2/d/(1-cos(d*x+c))^3-3/4*a^2/d/(1-cos(
d*x+c))^2+51/32*a^2/d/(1-cos(d*x+c))-1/64*a^2/d/(1+cos(d*x+c))^2+9/64*a^2/d
/(1+cos(d*x+c))+99/128*a^2*ln(1-cos(d*x+c))/d+29/128*a^2*ln(1+cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used

$= \{3964, 90\}$

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{51a^2}{32d(1-\cos(c+dx))} + \frac{9a^2}{64d(\cos(c+dx)+1)} - \frac{3a^2}{4d(1-\cos(c+dx))^2} - \frac{a^2}{64d(\cos(c+dx)+1)^2} + \frac{11a^2}{48d(1-\cos(c+dx))^3} - \frac{a^2}{32d(1-\cos(c+dx))^4} + \frac{99a^2\log(1-\cos(c+dx))}{128d} + \frac{29a^2\log(\cos(c+dx)+1)}{128d}$$

```
[In] Int[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/32*a^2/(d*(1 - Cos[c + d*x])^4) + (11*a^2)/(48*d*(1 - Cos[c + d*x])^3) -
 (3*a^2)/(4*d*(1 - Cos[c + d*x])^2) + (51*a^2)/(32*d*(1 - Cos[c + d*x])) -
a^2/(64*d*(1 + Cos[c + d*x])^2) + (9*a^2)/(64*d*(1 + Cos[c + d*x])) + (99*a
^2*Log[1 - Cos[c + d*x]])/(128*d) + (29*a^2*Log[1 + Cos[c + d*x]])/(128*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\text{integral} = -\frac{a^{10}\text{Subst}\left(\int \frac{x^7}{(a-ax)^5(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^{10}\text{Subst}\left(\int \left(-\frac{1}{8a^8(-1+x)^5} - \frac{11}{16a^8(-1+x)^4} - \frac{3}{2a^8(-1+x)^3} - \frac{51}{32a^8(-1+x)^2} - \frac{99}{128a^8(-1+x)} - \frac{1}{32a^8(1+x)^3} + \frac{}{64a}\right.\right.}{d}$$

$$
\begin{aligned}
&= -\frac{a^2}{32d(1-\cos(c+dx))^4} + \frac{11a^2}{48d(1-\cos(c+dx))^3} - \frac{3a^2}{4d(1-\cos(c+dx))^2} \\
&\quad + \frac{51a^2}{32d(1-\cos(c+dx))} - \frac{a^2}{64d(1+\cos(c+dx))^2} + \frac{9a^2}{64d(1+\cos(c+dx))} \\
&\quad + \frac{99a^2\log(1-\cos(c+dx))}{128d} + \frac{29a^2\log(1+\cos(c+dx))}{128d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.86

$$
\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx =
$$

$$
-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-1224\csc^2\left(\frac{1}{2}(c+dx)\right)+288\csc^4\left(\frac{1}{2}(c+dx)\right)-44\csc^6\left(\frac{1}{2}(c+\right.\right.}{}
$$

```
[In] Integrate[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/6144*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-1224*Csc[(c + d*x)/2
]^2 + 288*Csc[(c + d*x)/2]^4 - 44*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^8
 - 6*(116*Log[Cos[(c + d*x)/2]] + 396*Log[Sin[(c + d*x)/2]] + 18*Sec[(c + d
*x)/2]^2 - Sec[(c + d*x)/2]^4)))/d
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-\frac{a^2\cos(dx+c)^8}{8\sin(dx+c)^8}+2a^2\left(-\frac{\cos(dx+c)^9}{8\sin(dx+c)^8}+\frac{\cos(dx+c)^9}{48\sin(dx+c)^6}-\frac{\cos(dx+c)^9}{64\sin(dx+c)^4}+\frac{5\cos(dx+c)^9}{128\sin(dx+c)^2}+\frac{5\cos(dx+c)^7}{128}+\frac{7\cos(dx+c)^5}{128}+\frac{35\cos(d}{38}\right.}{}$
default	$\frac{-\frac{a^2\cos(dx+c)^8}{8\sin(dx+c)^8}+2a^2\left(-\frac{\cos(dx+c)^9}{8\sin(dx+c)^8}+\frac{\cos(dx+c)^9}{48\sin(dx+c)^6}-\frac{\cos(dx+c)^9}{64\sin(dx+c)^4}+\frac{5\cos(dx+c)^9}{128\sin(dx+c)^2}+\frac{5\cos(dx+c)^7}{128}+\frac{7\cos(dx+c)^5}{128}+\frac{35\cos(d}{38}\right.}{}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{a^2\left(279\,e^{11i(dx+c)}-156\,e^{10i(dx+c)}-1141\,e^{9i(dx+c)}+2080\,e^{8i(dx+c)}+670\,e^{7i(dx+c)}-2696\,e^{6i(dx+c}\right.}{96d\left(e^{i(dx+c)}-1\right)^8\left(e^{i(dx+}\right.}$

```
[In] int(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*a^2/sin(d*x+c)^8*cos(d*x+c)^8+2*a^2*(-1/8/sin(d*x+c)^8*cos(d*x+c)
^9+1/48/sin(d*x+c)^6*cos(d*x+c)^9-1/64/sin(d*x+c)^4*cos(d*x+c)^9+5/128/sin(
d*x+c)^2*cos(d*x+c)^9+5/128*cos(d*x+c)^7+7/128*cos(d*x+c)^5+35/384*cos(d*x+
c)^3+35/128*cos(d*x+c)+35/128*ln(-cot(d*x+c)+csc(d*x+c)))+a^2*(-1/8*cot(d*x
+c)^8+1/6*cot(d*x+c)^6-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(145) = 290$.

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.91

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{558\,a^2\cos(dx+c)^5 - 156\,a^2\cos(dx+c)^4 - 1268\,a^2\cos(dx+c)^3 + 676\,a^2\cos(dx+c)^2 + 686\,a^2\cos(dx$$

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/384*(558*a^2*cos(d*x + c)^5 - 156*a^2*cos(d*x + c)^4 - 1268*a^2*cos(d*x
 + c)^3 + 676*a^2*cos(d*x + c)^2 + 686*a^2*cos(d*x + c) - 448*a^2 - 87*(a^2*
cos(d*x + c)^6 - 2*a^2*cos(d*x + c)^5 - a^2*cos(d*x + c)^4 + 4*a^2*cos(d*x
 + c)^3 - a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*log(1/2*cos(d*x + c
) + 1/2) - 297*(a^2*cos(d*x + c)^6 - 2*a^2*cos(d*x + c)^5 - a^2*cos(d*x + c
)^4 + 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)
*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^5 - d*c
os(d*x + c)^4 + 4*d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - 2*d*cos(d*x + c) +
d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**9*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{87\,a^2\log(\cos(dx+c)+1) + 297\,a^2\log(\cos(dx+c)-1) - \frac{2\left(279\,a^2\cos(dx+c)^5 - 78\,a^2\cos(dx+c)^4 - 634\,a^2\cos(dx+c)^3 + 3\right.}{\cos(dx+c)^6 - 2\cos(dx+c)^5 - \cos(dx+c)^4 + 4\cos(dx+}}{384\,d}$$

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/384*(87*a^2*log(cos(d*x + c) + 1) + 297*a^2*log(cos(d*x + c) - 1) - 2*(27
9*a^2*cos(d*x + c)^5 - 78*a^2*cos(d*x + c)^4 - 634*a^2*cos(d*x + c)^3 + 338
*a^2*cos(d*x + c)^2 + 343*a^2*cos(d*x + c) - 224*a^2)/(cos(d*x + c)^6 - 2*c
os(d*x + c)^5 - cos(d*x + c)^4 + 4*cos(d*x + c)^3 - cos(d*x + c)^2 - 2*cos(
d*x + c) + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.41

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{1188\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 1536\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{96\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{6\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{(3\,a^2}{}}{1536\,d}$$

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(1188*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 1536*a
^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 96*a^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 6*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 - (3*a^2 + 32*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 174*a^2*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 768*a^2*(cos(d*x + c) - 1)^3/(cos(d*x
 + c) + 1)^3 + 2475*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)*(cos(d*x
 + c) + 1)^4/(cos(d*x + c) - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 14.46 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \cot^9(c+dx)(a+a\sec(c+dx))^2\,dx &= \frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d} - \frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d} + \frac{99\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,d} \\ &\quad + \frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(32\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - \frac{29\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{4} + \frac{4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3} - \frac{a^2}{8}\right)}{64\,d} \\ &\quad - \frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}\end{aligned}$$

```
[In] int(cot(c + d*x)^9*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(16*d) - (a^2*tan(c/2 + (d*x)/2)^4)/(256*d) + (9
9*a^2*log(tan(c/2 + (d*x)/2)))/(64*d) + (cot(c/2 + (d*x)/2)^8*((4*a^2*tan(c
/2 + (d*x)/2)^2)/3 - (29*a^2*tan(c/2 + (d*x)/2)^4)/4 + 32*a^2*tan(c/2 + (d*
x)/2)^6 - a^2/8))/(64*d) - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.29 $\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx = -a^2x-\frac{5a^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)}{d}+\frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^2\tan^3(c+dx)}{3d}-\frac{5a^2\sec(c+dx)\tan^3(c+dx)}{12d}+\frac{a^2\tan^5(c+dx)}{5d}+\frac{a^2\sec(c+dx)\tan^5(c+dx)}{3d}+\frac{a^2\tan^7(c+dx)}{7d}$$

```
[Out] -a^2*x-5/8*a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+5/8*a^2*sec(d*x+c)*ta
n(d*x+c)/d-1/3*a^2*tan(d*x+c)^3/d-5/12*a^2*sec(d*x+c)*tan(d*x+c)^3/d+1/5*a^
2*tan(d*x+c)^5/d+1/3*a^2*sec(d*x+c)*tan(d*x+c)^5/d+1/7*a^2*tan(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30}

$$\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx = -\frac{5a^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^2\tan^7(c+dx)}{7d}+\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^5(c+dx)\sec(c+dx)}{3d}-\frac{5a^2\tan^3(c+dx)\sec(c+dx)}{12d}+\frac{5a^2\tan(c+dx)\sec(c+dx)}{8d}-a^2x$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] -(a^2*x) - (5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*Tan[c + d*x])/d + (5*
a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Tan[c + d*x]^3)/(3*d) - (5*a^2*
Sec[c + d*x]*Tan[c + d*x]^3)/(12*d) + (a^2*Tan[c + d*x]^5)/(5*d) + (a^2*Sec
[c + d*x]*Tan[c + d*x]^5)/(3*d) + (a^2*Tan[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2 \tan^6(c+dx) + 2a^2 \sec(c+dx) \tan^6(c+dx) + a^2 \sec^2(c+dx) \tan^6(c+dx)\right) \, dx$$

$$= a^2 \int \tan^6(c+dx) \, dx + a^2 \int \sec^2(c+dx) \tan^6(c+dx) \, dx + \left(2a^2\right) \int \sec(c+dx) \tan^6(c+dx) \, dx$$

$$= \frac{a^2 \tan^5(c+dx)}{5d} + \frac{a^2 \sec(c+dx) \tan^5(c+dx)}{3d} - a^2 \int \tan^4(c+dx) \, dx - \frac{1}{3}\left(5a^2\right) \int \sec(c+dx) \tan^4(c+dx) \, dx + \frac{a^2 \text{Subst}\left(\int x^6 \, dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{a^2 \tan^3(c+dx)}{3d} - \frac{5a^2 \sec(c+dx) \tan^3(c+dx)}{12d} + \frac{a^2 \tan^5(c+dx)}{5d} + \frac{a^2 \sec(c+dx) \tan^5(c+dx)}{3d} + \frac{a^2 \tan^7(c+dx)}{7d} + a^2 \int \tan^2(c+dx) \, dx + \frac{1}{4}\left(5a^2\right) \int \sec(c+dx) \tan^2(c+dx) \, dx$$

$$= \frac{a^2 \tan(c+dx)}{d} + \frac{5a^2 \sec(c+dx) \tan(c+dx)}{8d} - \frac{a^2 \tan^3(c+dx)}{3d} - \frac{5a^2 \sec(c+dx) \tan^3(c+dx)}{12d} + \frac{a^2 \tan^5(c+dx)}{5d} + \frac{a^2 \sec(c+dx) \tan^5(c+dx)}{3d} + \frac{a^2 \tan^7(c+dx)}{7d} - \frac{1}{8}\left(5a^2\right) \int \sec(c+dx) \, dx - a^2 \int 1 \, dx$$

$$= -a^2 x - \frac{5a^2 \text{arctanh}(\sin(c+dx))}{8d} + \frac{a^2 \tan(c+dx)}{d} + \frac{5a^2 \sec(c+dx) \tan(c+dx)}{8d} - \frac{a^2 \tan^3(c+dx)}{3d} - \frac{5a^2 \sec(c+dx) \tan^3(c+dx)}{12d} + \frac{a^2 \tan^5(c+dx)}{5d} + \frac{a^2 \sec(c+dx) \tan^5(c+dx)}{3d} + \frac{a^2 \tan^7(c+dx)}{7d}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.35

$$\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx = -\frac{a^2\arctan(\tan(c+dx))}{d} - \frac{5a^2\text{arctanh}(\sin(c+dx))}{8d} + \frac{a^2\tan(c+dx)}{d} - \frac{5a^2\sec(c+dx)\tan(c+dx)}{8d} - \frac{5a^2\sec^3(c+dx)\tan(c+dx)}{12d} + \frac{5a^2\sec^5(c+dx)\tan(c+dx)}{3d} - \frac{a^2\tan^3(c+dx)}{3d} - \frac{10a^2\sec^3(c+dx)\tan^3(c+dx)}{3d} + \frac{a^2\tan^5(c+dx)}{5d} + \frac{2a^2\sec(c+dx)\tan^5(c+dx)}{d} + \frac{a^2\tan^7(c+dx)}{7d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] -((a^2*ArcTan[Tan[c + d*x]])/d) - (5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (a^
2*Tan[c + d*x])/d - (5*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (5*a^2*Sec[c
+ d*x]^3*Tan[c + d*x])/(12*d) + (5*a^2*Sec[c + d*x]^5*Tan[c + d*x])/(3*d) -
 (a^2*Tan[c + d*x]^3)/(3*d) - (10*a^2*Sec[c + d*x]^3*Tan[c + d*x]^3)/(3*d)
+ (a^2*Tan[c + d*x]^5)/(5*d) + (2*a^2*Sec[c + d*x]*Tan[c + d*x]^5)/d + (a^2
*Tan[c + d*x]^7)/(7*d)
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04

method	result
parts	$\frac{a^2\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{a^2\tan(dx+c)^7}{7d}+\frac{2a^2\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\right.}{}$
derivativedivides	$\frac{\frac{a^2\sin(dx+c)^7}{7\cos(dx+c)^7}+2a^2\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(dx+c)^3}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+c)+\tan}{16}\right.}{d}$
default	$\frac{\frac{a^2\sin(dx+c)^7}{7\cos(dx+c)^7}+2a^2\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(dx+c)^3}{48}+\frac{5\sin(dx+c)}{16}-\frac{5\ln(\sec(dx+c)+\tan}{16}\right.}{d}$
risch	$-a^2x-\frac{ia^2(1155\,\mathrm{e}^{13i(dx+c)}-1680\,\mathrm{e}^{12i(dx+c)}+980\,\mathrm{e}^{11i(dx+c)}-10080\,\mathrm{e}^{10i(dx+c)}+2975\,\mathrm{e}^{9i(dx+c)}-16240\,\mathrm{e}^{8i(dx+c)}-24}{420d(\mathrm{e}^{2i(dx+c)}}$

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-arctan(tan(d*x+c)))+1/7
*a^2*tan(d*x+c)^7/d+2*a^2/d*(1/6*sin(d*x+c)^7/cos(d*x+c)^6-1/24*sin(d*x+c)^
7/cos(d*x+c)^4+1/16*sin(d*x+c)^7/cos(d*x+c)^2+1/16*sin(d*x+c)^5+5/48*sin(d*
x+c)^3+5/16*sin(d*x+c)-5/16*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx =$$
$$-\frac{1680\,a^2dx\cos\left(dx+c\right)^7+525\,a^2\cos\left(dx+c\right)^7\log\left(\sin\left(dx+c\right)+1\right)-525\,a^2\cos\left(dx+c\right)^7\log\left(-\sin\left(dx\right.\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/1680*(1680*a^2*d*x*cos(d*x + c)^7 + 525*a^2*cos(d*x + c)^7*log(sin(d*x +
 c) + 1) - 525*a^2*cos(d*x + c)^7*log(-sin(d*x + c) + 1) - 2*(1168*a^2*cos(
d*x + c)^6 + 1155*a^2*cos(d*x + c)^5 - 256*a^2*cos(d*x + c)^4 - 910*a^2*cos
(d*x + c)^3 - 192*a^2*cos(d*x + c)^2 + 280*a^2*cos(d*x + c) + 120*a^2)*sin(
d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx = a^2\left(\int 2\tan^6\left(c+dx\right)\sec\left(c+dx\right)dx + \int \tan^6\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \tan^6\left(c+dx\right)dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**6,x)
```

```
[Out] a**2*(Integral(2*tan(c + d*x)**6*sec(c + d*x), x) + Integral(tan(c + d*x)**
6*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx$$

$$= \frac{240\,a^2\tan\left(dx+c\right)^7+112\,\left(3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)a^2-35\,a^2\Big(}{1680\,d}$$

[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")

[Out] 1/1680*(240*a^2*tan(d*x + c)^7 + 112*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^2 - 35*a^2*(2*(33*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d

Giac [A] (verification not implemented)

none

Time = 2.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.12

$$\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx =$$

$$-\frac{840\,(dx+c)a^2+525\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-525\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(315\,a^2\tan\left(\frac{1}{2}\,d\right.\right.}{}}{}$$

[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")

[Out] -1/840*(840*(d*x + c)*a^2 + 525*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 525*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(315*a^2*tan(1/2*d*x + 1/2*c)^13 - 2660*a^2*tan(1/2*d*x + 1/2*c)^11 + 9863*a^2*tan(1/2*d*x + 1/2*c)^9 - 21216*a^2*tan(1/2*d*x + 1/2*c)^7 + 29673*a^2*tan(1/2*d*x + 1/2*c)^5 - 9660*a^2*tan(1/2*d*x + 1/2*c)^3 + 1365*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d

Mupad [B] (verification not implemented)

Time = 15.39 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.45

$$\int (a+a\sec(c+dx))^2\tan^6(c+dx)\,dx = -a^2\,x-\frac{5\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{4}-\frac{19\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3}+\frac{1409\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{60}-\frac{1768\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{35}+\frac{1413\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}-23\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-21\right.}$$

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^2,x)
```

```
[Out] - a^2*x - (5*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((1413*a^2*tan(c/2 + (d
*x)/2)^5)/20 - 23*a^2*tan(c/2 + (d*x)/2)^3 - (1768*a^2*tan(c/2 + (d*x)/2)^7
)/35 + (1409*a^2*tan(c/2 + (d*x)/2)^9)/60 - (19*a^2*tan(c/2 + (d*x)/2)^11)/
3 + (3*a^2*tan(c/2 + (d*x)/2)^13)/4 + (13*a^2*tan(c/2 + (d*x)/2))/4)/(d*(7*
tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 -
35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^1
2 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.30 $\int (a+a\sec(c+dx))^2\tan^4(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int (a+a\sec(c+dx))^2\tan^4(c+dx)\,dx = a^2x+\frac{3a^2\text{arctanh}(\sin(c+dx))}{4d}-\frac{a^2\tan(c+dx)}{d}-\frac{3a^2\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan^3(c+dx)}{2d}+\frac{a^2\tan^5(c+dx)}{5d}$$

```
[Out] a^2*x+3/4*a^2*arctanh(sin(d*x+c))/d-a^2*tan(d*x+c)/d-3/4*a^2*sec(d*x+c)*tan
(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d+1/2*a^2*sec(d*x+c)*tan(d*x+c)^3/d+1/5*a^2*
tan(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30}

$$\int (a+a\sec(c+dx))^2\tan^4(c+dx)\,dx = \frac{3a^2\text{arctanh}(\sin(c+dx))}{4d}+\frac{a^2\tan^5(c+dx)}{5d}+\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{4d}+a^2x$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] a^2*x + (3*a^2*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*Tan[c + d*x])/d - (3*a^2
*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (a^2*Sec[c
 + d*x]*Tan[c + d*x]^3)/(2*d) + (a^2*Tan[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2 \tan^4(c+dx)+2a^2 \sec(c+dx)\tan^4(c+dx)+a^2\sec^2(c+dx)\tan^4(c+dx)\right)\,dx\\
&= a^2\int \tan^4(c+dx)\,dx+a^2\int \sec^2(c+dx)\tan^4(c+dx)\,dx+\left(2a^2\right)\int \sec(c+dx)\tan^4(c+dx)\,dx\\
&= \frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan^3(c+dx)}{2d}-a^2\int \tan^2(c+dx)\,dx\\
&\quad -\frac{1}{2}\left(3a^2\right)\int \sec(c+dx)\tan^2(c+dx)\,dx+\frac{a^2\text{Subst}\left(\int x^4\,dx,x,\tan(c+dx)\right)}{d}\\
&= -\frac{a^2\tan(c+dx)}{d}-\frac{3a^2\sec(c+dx)\tan(c+dx)}{4d}\\
&\quad +\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan^3(c+dx)}{2d}\\
&\quad +\frac{a^2\tan^5(c+dx)}{5d}+\frac{1}{4}\left(3a^2\right)\int \sec(c+dx)\,dx+a^2\int 1\,dx\\
&= a^2x+\frac{3a^2\text{arctanh}(\sin(c+dx))}{4d}-\frac{a^2\tan(c+dx)}{d}-\frac{3a^2\sec(c+dx)\tan(c+dx)}{4d}\\
&\quad +\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan^3(c+dx)}{2d}+\frac{a^2\tan^5(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$
\begin{aligned}
\int (a+a\sec(c+dx))^2\tan^4(c+dx)\,dx &= \frac{a^2\arctan(\tan(c+dx))}{d}+\frac{3a^2\text{arctanh}(\sin(c+dx))}{4d}\\
&\quad -\frac{a^2\tan(c+dx)}{d}+\frac{3a^2\sec(c+dx)\tan(c+dx)}{4d}\\
&\quad -\frac{3a^2\sec^3(c+dx)\tan(c+dx)}{2d}+\frac{a^2\tan^3(c+dx)}{3d}\\
&\quad +\frac{2a^2\sec(c+dx)\tan^3(c+dx)}{d}+\frac{a^2\tan^5(c+dx)}{5d}
\end{aligned}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (a^2*ArcTan[Tan[c + d*x]])/d + (3*a^2*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*T
an[c + d*x])/d + (3*a^2*Sec[c + d*x]*Tan[c + d*x])/(4*d) - (3*a^2*Sec[c + d
*x]^3*Tan[c + d*x])/(2*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (2*a^2*Sec[c + d*x
]*Tan[c + d*x]^3)/d + (a^2*Tan[c + d*x]^5)/(5*d)
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{a^2 \sin(dx+c)^5}{5\cos(dx+c)^5}+2a^2\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a^2\left(\frac{\tan(dx+c)^3}{3}-\right.}{d}$
default	$\frac{\frac{a^2 \sin(dx+c)^5}{5\cos(dx+c)^5}+2a^2\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a^2\left(\frac{\tan(dx+c)^3}{3}-\right.}{d}$
parts	$\frac{a^2\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{a^2\tan(dx+c)^5}{5d}+\frac{2a^2\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}\right.}{d}$
risch	$a^2x+\frac{ia^2\left(75\,e^{9i(dx+c)}-60\,e^{8i(dx+c)}+30\,e^{7i(dx+c)}-360\,e^{6i(dx+c)}-320\,e^{4i(dx+c)}-30\,e^{3i(dx+c)}-280\,e^{2i(dx+c)}-75\,e^{i(\right.}{30d\left(e^{2i(dx+c)}+1\right)^5}$

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*a^2*sin(d*x+c)^5/cos(d*x+c)^5+2*a^2*(1/4*sin(d*x+c)^5/cos(d*x+c)^4
-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d
*x+c)+tan(d*x+c)))+a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.17

$$\int (a+a\sec(c+dx))^2\tan^4(c+dx)\,dx$$
$$=\frac{120\,a^2dx\cos(dx+c)^5+45\,a^2\cos(dx+c)^5\log(\sin(dx+c)+1)-45\,a^2\cos(dx+c)^5\log(-\sin(dx+c)}{120\,d\,c}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/120*(120*a^2*d*x*cos(d*x + c)^5 + 45*a^2*cos(d*x + c)^5*log(sin(d*x + c)
+ 1) - 45*a^2*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(68*a^2*cos(d*x + c
)^4 + 75*a^2*cos(d*x + c)^3 + 4*a^2*cos(d*x + c)^2 - 30*a^2*cos(d*x + c) -
12*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int (a + a\sec(c + dx))^2 \tan^4(c + dx)\, dx = a^2 \left(\int 2\tan^4(c + dx)\sec(c + dx)\, dx + \int \tan^4(c + dx)\sec^2(c + dx)\, dx + \int \tan^4(c + dx)\, dx \right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**4,x)
[Out] a**2*(Integral(2*tan(c + d*x)**4*sec(c + d*x), x) + Integral(tan(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int (a + a\sec(c + dx))^2 \tan^4(c + dx)\, dx = \frac{24\, a^2 \tan(dx + c)^5 + 40\left(\tan(dx + c)^3 + 3\, dx + 3\, c - 3\, \tan(dx + c)\right)a^2 + 15\, a^2 \left(\frac{2\left(5\, \sin(dx+c)^3 - 3\, \sin(dx+c)\right)}{\sin(dx+c)^4 - 2\, \sin(dx+c)^2 + 1} + \cdots\right.}{120\, d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
[Out] 1/120*(24*a^2*tan(d*x + c)^5 + 40*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x
 + c))*a^2 + 15*a^2*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4
- 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)
))/d
```

Giac [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.24

$$\int (a + a\sec(c + dx))^2 \tan^4(c + dx)\, dx = \frac{60\,(dx + c)a^2 + 45\, a^2 \log\left(\left|\tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right) + 1\right|\right) - 45\, a^2 \log\left(\left|\tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right) - 1\right|\right) + \frac{2\left(15\, a^2 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^9 - \cdots\right.}{\cdots}}{60\, d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*a^2 + 45*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a^2
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 - 11
0*a^2*tan(1/2*d*x + 1/2*c)^7 + 328*a^2*tan(1/2*d*x + 1/2*c)^5 - 530*a^2*tan
(1/2*d*x + 1/2*c)^3 + 105*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.46

$$\int (a+a\sec(c+dx))^2\tan^4(c+dx)\,dx = a^2\,x+\frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d} + \frac{\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2}-\frac{11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{164\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}-\frac{53\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{7\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (3*a^2*atanh(tan(c/2 + (d*x)/2)))/(2*d) + ((164*a^2*tan(c/2 + (d*x)
/2)^5)/15 - (53*a^2*tan(c/2 + (d*x)/2)^3)/3 - (11*a^2*tan(c/2 + (d*x)/2)^7)
/3 + (a^2*tan(c/2 + (d*x)/2)^9)/2 + (7*a^2*tan(c/2 + (d*x)/2))/2)/(d*(5*tan
(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*t
an(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.31 $\int (a+a\sec(c+dx))^2\tan^2(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int (a+a\sec(c+dx))^2\tan^2(c+dx)\,dx=-a^2x-\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}$$

[Out] -a^2*x-a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+a^2*sec(d*x+c)*tan(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30}

$$\int (a+a\sec(c+dx))^2\tan^2(c+dx)\,dx=-\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{d}-a^2x$$

[In] Int[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^2,x]

[Out] -(a^2*x) - (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a^2*Sec[c + d*x]*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2\tan^2(c+dx)+2a^2\sec(c+dx)\tan^2(c+dx)+a^2\sec^2(c+dx)\tan^2(c+dx)\right)\,dx\\
&= a^2\int \tan^2(c+dx)\,dx+a^2\int \sec^2(c+dx)\tan^2(c+dx)\,dx+\left(2a^2\right)\int \sec(c+dx)\tan^2(c+dx)\,dx\\
&= \frac{a^2\tan(c+dx)}{d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{d}-a^2\int 1\,dx\\
&\quad -a^2\int \sec(c+dx)\,dx+\frac{a^2\text{Subst}\left(\int x^2\,dx,x,\tan(c+dx)\right)}{d}
\end{aligned}
$$

$$= -a^2x - \frac{a^2\text{arctanh}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{d} + \frac{a^2\tan^3(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

$$\int (a+a\sec(c+dx))^2\tan^2(c+dx)\,dx = -\frac{a^2\arctan(\tan(c+dx))}{d} - \frac{a^2\text{arctanh}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{d} + \frac{a^2\tan^3(c+dx)}{3d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -((a^2*ArcTan[Tan[c + d*x]])/d) - (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[
c + d*x])/d + (a^2*Sec[c + d*x]*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d
)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

method	result	s
parts	$\frac{a^2(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{a^2\tan(dx+c)^3}{3d}+\frac{2a^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	9
derivativedivides	$\frac{\frac{a^2\sin(dx+c)^3}{3\cos(dx+c)^3}+2a^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2(\tan(dx+c)-dx-c)}{d}$	9
default	$\frac{\frac{a^2\sin(dx+c)^3}{3\cos(dx+c)^3}+2a^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2(\tan(dx+c)-dx-c)}{d}$	9
risch	$-a^2x-\frac{2ia^2\left(3\,e^{5i(dx+c)}-6\,e^{2i(dx+c)}-3\,e^{i(dx+c)}-2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{d}+\frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{d}$	1

```
[In] int((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(tan(d*x+c)-arctan(tan(d*x+c)))+1/3*a^2*tan(d*x+c)^3/d+2*a^2/d*(1/2*s
in(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int (a+a\sec(c+dx))^2\tan^2(c+dx)\,dx = -\frac{6\,a^2dx\cos\left(dx+c\right)^3+3\,a^2\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+\right.}{6\,d\cos\left(dx+c\right)^3}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
[Out] -1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*a^2*cos(d*x + c)^2 +
3*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^2\tan^2(c+dx)\,dx = a^2\left(\int 2\tan^2(c+dx)\sec(c+dx)\,dx + \int \tan^2(c+dx)\sec^2(c+dx)\,dx + \int \tan^2(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*tan(d*x+c)**2,x)
[Out] a**2*(Integral(2*tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int (a+a\sec(c+dx))^2\tan^2(c+dx)\,dx = \frac{2\,a^2\tan\left(dx+c\right)^3-6\left(dx+c-\tan\left(dx+c\right)\right)a^2-3\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx\right.\right.\right.}{6\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
[Out] 1/6*(2*a^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a^2*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int (a + a\sec(c + dx))^2 \tan^2(c + dx)\,dx =$$

$$-\frac{3\,(dx + c)a^2 + 3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{4\left(a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 3\,a^2}{\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}}{3\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)*a^2 + 3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*(a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 13.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int (a + a\sec(c + dx))^2 \tan^2(c + dx)\,dx$$

$$= \frac{\frac{4\,a^2\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3}{3} - 4\,a^2\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)} - \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d} - a^2\,x$$

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

```
[Out] ((4*a^2*tan(c/2 + (d*x)/2)^3)/3 - 4*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 +
 (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (2*a^2*
atanh(tan(c/2 + (d*x)/2)))/d - a^2*x
```

3.32 $\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx = -a^2x - \frac{2a^2\cot(c+dx)}{d} - \frac{2a^2\csc(c+dx)}{d}$$

```
[Out] -a^2*x-2*a^2*cot(d*x+c)/d-2*a^2*csc(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3971, 3554, 8, 2686, 3852}

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2a^2\cot(c+dx)}{d} - \frac{2a^2\csc(c+dx)}{d} + a^2(-x)$$

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (2*a^2*Cot[c + d*x])/d - (2*a^2*Csc[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2\cot^2(c+dx)+2a^2\cot(c+dx)\csc(c+dx)+a^2\csc^2(c+dx)\right)\,dx\\
&= a^2\int \cot^2(c+dx)\,dx+a^2\int \csc^2(c+dx)\,dx+\left(2a^2\right)\int \cot(c+dx)\csc(c+dx)\,dx\\
&= -\frac{a^2\cot(c+dx)}{d}-a^2\int 1\,dx-\frac{a^2\text{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{d}\\
&\quad-\frac{\left(2a^2\right)\text{Subst}\left(\int 1\,dx,x,\csc(c+dx)\right)}{d}\\
&= -a^2x-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\begin{aligned}
&\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx\\
&= -\frac{2a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}
\end{aligned}$$

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
[Out] (-2*a^2*Cot[c/2 + (d*x)/2]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c/2 + (d*x)
/2]^2])/d
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
risch	$-a^2x-\frac{4ia^2}{d\left(e^{i(dx+c)}-1\right)}$	30
derivativedivides	$\frac{-a^2\cot(dx+c)-\frac{2a^2}{\sin(dx+c)}+a^2(-\cot(dx+c)-dx-c)}{d}$	50
default	$\frac{-a^2\cot(dx+c)-\frac{2a^2}{\sin(dx+c)}+a^2(-\cot(dx+c)-dx-c)}{d}$	50

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -a^2*x-4*I*a^2/d/(exp(I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx=-\frac{a^2dx\sin(dx+c)+2\,a^2\cos(dx+c)+2\,a^2}{d\sin(dx+c)}$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -(a^2*d*x*sin(d*x + c) + 2*a^2*cos(d*x + c) + 2*a^2)/(d*sin(d*x + c))
```

Sympy [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx=a^2\left(\int 2\cot^2(c+dx)\sec(c+dx)\,dx+\int \cot^2(c+dx)\sec^2(c+dx)\,dx+\int \cot^2(c+dx)\,dx\right)$$

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
[Out] a**2*(Integral(2*cot(c + d*x)**2*sec(c + d*x), x) + Integral(cot(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2+\frac{2\,a^2}{\sin(dx+c)}+\frac{a^2}{\tan(dx+c)}}{d}$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -((d*x + c + 1/tan(d*x + c))*a^2 + 2*a^2/sin(d*x + c) + a^2/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{(dx+c)a^2+\frac{2\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] -((d*x + c)*a^2 + 2*a^2/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B] (verification not implemented)

Time = 13.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^2\,dx = -a^2\,x-\frac{2\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
[Out] - a^2*x - (2*a^2*cot(c/2 + (d*x)/2))/d
```

3.33 $\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx = a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}$$

```
[Out] a^2*x+a^2*cot(d*x+c)/d-2/3*a^2*cot(d*x+c)^3/d+2*a^2*csc(d*x+c)/d-2/3*a^2*cs
c(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3971, 3554, 8, 2686, 2687, 30}

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}+a^2x$$

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3*d) + (2*a^2*Csc[c
 + d*x])/d - (2*a^2*Csc[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2\cot^4(c+dx)+2a^2\cot^3(c+dx)\csc(c+dx)+a^2\cot^2(c+dx)\csc^2(c+dx)\right)\,dx\\
&= a^2\int \cot^4(c+dx)\,dx+a^2\int \cot^2(c+dx)\csc^2(c+dx)\,dx+\left(2a^2\right)\int \cot^3(c+dx)\csc(c+dx)\,dx\\
&= -\frac{a^2\cot^3(c+dx)}{3d}-a^2\int \cot^2(c+dx)\,dx+\frac{a^2\text{Subst}\left(\int x^2\,dx,x,-\cot(c+dx)\right)}{d}\\
&\quad -\frac{\left(2a^2\right)\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&= \frac{a^2\cot(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}+a^2\int 1\,dx\\
&= a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= -\frac{a^2\cot^3(c+dx)}{3d}+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{3d} - \frac{a^2\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d}$$

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*(a^2*Cot[c + d*x]^3)/d + (2*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c + d*x]^
3)/(3*d) - (a^2*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*
x]^2])/(3*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

method	result
risch	$a^2x+\frac{2ia^2\left(6\,e^{2i(dx+c)}-9\,e^{i(dx+c)}+5\right)}{3d\left(e^{i(dx+c)}-1\right)^3}$
derivativedivides	$\frac{-\frac{a^2\cos(dx+c)^3}{3\sin(dx+c)^3}+2a^2\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)+a^2\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)}{d}$
default	$\frac{-\frac{a^2\cos(dx+c)^3}{3\sin(dx+c)^3}+2a^2\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)+a^2\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)}{d}$

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+2/3*I*a^2*(6*exp(2*I*(d*x+c))-9*exp(I*(d*x+c))+5)/d/(exp(I*(d*x+c))-1
)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{5\,a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)-4\,a^2+3\left(a^2dx\cos\left(dx+c\right)-a^2dx\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] 1/3*(5*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) - 4*a^2 + 3*(a^2*d*x*cos(d*x +
 c) - a^2*d*x)*sin(d*x + c))/((d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cot^4\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cot^4\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cot^4\left(c+dx\right)dx\right)$$

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
[Out] a**2*(Integral(2*cot(c + d*x)**4*sec(c + d*x), x) + Integral(cot(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^2-1}{\tan\left(dx+c\right)^3}\right)a^2+\frac{2\left(3\,\sin\left(dx+c\right)^2-1\right)a^2}{\sin\left(dx+c\right)^3}-\frac{a^2}{\tan\left(dx+c\right)^3}}{3\,d}$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 + 2*(3*sin(d
*x + c)^2 - 1)*a^2/sin(d*x + c)^3 - a^2/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{6\,(dx+c)a^2 + \frac{9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{6\,d}$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*a^2 + (9*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2)/tan(1/2*d*x + 1
/2*c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\,x + \frac{a^2\,\left(9\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}{6\,d}$$

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (a^2*(9*cot(c/2 + (d*x)/2) - cot(c/2 + (d*x)/2)^3))/(6*d)
```

3.34 $\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx = -a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\csc(c+dx)}{d}+\frac{4a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc^5(c+dx)}{5d}$$

```
[Out] -a^2*x-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/d-2/5*a^2*cot(d*x+c)^5/d-2*a^2
*csc(d*x+c)/d+4/3*a^2*csc(d*x+c)^3/d-2/5*a^2*csc(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3971, 3554, 8, 2686, 200, 2687, 30}

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{4a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}-a^2x$$

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (2*a^2*Cot[c
 + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x])/d + (4*a^2*Csc[c + d*x]^3)/(3*d) -
(2*a^2*Csc[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2 \cot^6(c+dx) + 2a^2 \cot^5(c+dx)\csc(c+dx) + a^2 \cot^4(c+dx)\csc^2(c+dx)\right) dx$$

$$= a^2 \int \cot^6(c+dx)\,dx + a^2 \int \cot^4(c+dx)\csc^2(c+dx)\,dx + \left(2a^2\right) \int \cot^5(c+dx)\csc(c+dx)\,dx$$

$$= -\frac{a^2\cot^5(c+dx)}{5d} - a^2\int\cot^4(c+dx)\,dx + \frac{a^2\text{Subst}\left(\int x^4\,dx, x, -\cot(c+dx)\right)}{d} - \frac{(2a^2)\,\text{Subst}\left(\int\left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{d}$$

$$= \frac{a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+dx)}{5d} + a^2\int\cot^2(c+dx)\,dx - \frac{(2a^2)\,\text{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx, x, \csc(c+dx)\right)}{d}$$

$$= -\frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\csc(c+dx)}{d} + \frac{4a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc^5(c+dx)}{5d} - a^2\int 1\,dx$$

$$= -a^2x - \frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\csc(c+dx)}{d} + \frac{4a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc^5(c+dx)}{5d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int\cot^6(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\cot^5(c+dx)}{5d} - \frac{2a^2\csc(c+dx)}{d} + \frac{4a^2\csc^3(c+dx)}{3d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{a^2\cot^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2}, 1, -\frac{3}{2}, -\tan^2(c+dx)\right)}{5d}$$

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/5*(a^2*Cot[c + d*x]^5)/d - (2*a^2*Csc[c + d*x])/d + (4*a^2*Csc[c + d*x]^
3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2*Cot[c + d*x]^5*Hypergeometri
c2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(5*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

method	result
risch	$-a^2x-\frac{4ia^2\left(15\,\mathrm{e}^{5i(dx+c)}-30\,\mathrm{e}^{4i(dx+c)}+10\,\mathrm{e}^{3i(dx+c)}+35\,\mathrm{e}^{2i(dx+c)}-37\,\mathrm{e}^{i(dx+c)}+13\right)}{15d\left(\mathrm{e}^{i(dx+c)}-1\right)^5\left(\mathrm{e}^{i(dx+c)}+1\right)}$
derivativedivides	$\frac{-\frac{a^2\cos(dx+c)^5}{5\sin(dx+c)^5}+2a^2\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}\right)+a^2\left(-\cot\right.}{d}$
default	$\frac{-\frac{a^2\cos(dx+c)^5}{5\sin(dx+c)^5}+2a^2\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}\right)+a^2\left(-\cot\right.}{d}$

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*x-4/15*I*a^2*(15*exp(5*I*(d*x+c))-30*exp(4*I*(d*x+c))+10*exp(3*I*(d*x+
c))+35*exp(2*I*(d*x+c))-37*exp(I*(d*x+c))+13)/d/(exp(I*(d*x+c))-1)^5/(exp(I
*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.10

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{26\,a^2\cos(dx+c)^3-22\,a^2\cos(dx+c)^2-17\,a^2\cos(dx+c)+16\,a^2+15\left(a^2dx\cos(dx+c)^2-2\,a^2dx\,\mathrm{c}\right.}{15\left(d\cos(dx+c)^2-2\,d\cos(dx+c)+d\right)\sin(dx+c)}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(26*a^2*cos(d*x + c)^3 - 22*a^2*cos(d*x + c)^2 - 17*a^2*cos(d*x + c)
+ 16*a^2 + 15*(a^2*d*x*cos(d*x + c)^2 - 2*a^2*d*x*cos(d*x + c) + a^2*d*x)*s
in(d*x + c))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F]

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cot^6(c+dx)\sec(c+dx)\,dx + \int \cot^6(c+dx)\sec^2(c+dx)\,dx + \int \cot^6(c+dx)\,dx\right)$$

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**2,x)
[Out] a**2*(Integral(2*cot(c + d*x)**6*sec(c + d*x), x) + Integral(cot(c + d*x)**
6*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.91

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2+\frac{2\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)a^2}{\sin(dx+c)^5}+\frac{3\,a^2}{\tan(dx+c)^5}}{15\,d}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a^2 + 2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a^2/sin(d*x + c
)^5 + 3*a^2/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{120\,(dx+c)a^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{165\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-25\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{120\,d}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] -1/120*(120*(d*x + c)*a^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + (165*a^2*tan(1/2*
d*x + 1/2*c)^4 - 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2)/tan(1/2*d*x + 1/2*c
)^5)/d
```

Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.73

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d} - \frac{\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{8} - \frac{5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{24} + \frac{a^2}{40}}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5} - a^2\,x$$

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2))/(8*d) - ((11*a^2*tan(c/2 + (d*x)/2)^4)/8 - (5*a^2*
tan(c/2 + (d*x)/2)^2)/24 + a^2/40)/(d*tan(c/2 + (d*x)/2)^5) - a^2*x
```

3.35 $\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx = a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{d}+\frac{6a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^7(c+dx)}{7d}$$

```
[Out] a^2*x+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d+1/5*a^2*cot(d*x+c)^5/d-2/7*a^
2*cot(d*x+c)^7/d+2*a^2*csc(d*x+c)/d-2*a^2*csc(d*x+c)^3/d+6/5*a^2*csc(d*x+c)
^5/d-2/7*a^2*csc(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3971, 3554, 8, 2686, 200, 2687, 30}

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^7(c+dx)}{7d}+\frac{6a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^3(c+dx)}{d}+\frac{2a^2\csc(c+dx)}{d}+a^2x$$

```
[In] Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*
x]^5)/(5*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) + (2*a^2*Csc[c + d*x])/d - (2*a^
2*Csc[c + d*x]^3)/d + (6*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)
/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2\cot^8(c+dx)+2a^2\cot^7(c+dx)\csc(c+dx)+a^2\cot^6(c+dx)\csc^2(c+dx)\right)\,dx\\
&= a^2\int \cot^8(c+dx)\,dx+a^2\int \cot^6(c+dx)\csc^2(c+dx)\,dx+\left(2a^2\right)\int \cot^7(c+dx)\csc(c+dx)\,dx\\
&= -\frac{a^2\cot^7(c+dx)}{7d}-a^2\int \cot^6(c+dx)\,dx+\frac{a^2\text{Subst}\left(\int x^6\,dx,x,-\cot(c+dx)\right)}{d}\\
&\quad-\frac{\left(2a^2\right)\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\csc(c+dx)\right)}{d}\\
&= \frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^7(c+dx)}{7d}+a^2\int \cot^4(c+dx)\,dx\\
&\quad-\frac{\left(2a^2\right)\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&= -\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{2a^2\csc(c+dx)}{d}\\
&\quad-\frac{2a^2\csc^3(c+dx)}{d}+\frac{6a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^7(c+dx)}{7d}-a^2\int \cot^2(c+dx)\,dx\\
&= \frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}\\
&\quad-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{d}\\
&\quad+\frac{6a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^7(c+dx)}{7d}+a^2\int 1\,dx\\
&= a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^7(c+dx)}{7d}\\
&\quad+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{d}+\frac{6a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^7(c+dx)}{7d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.58 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.87

$$
\begin{aligned}
&\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx\\
&= -\frac{a^2\cot^7(c+dx)}{7d}+\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(c+dx)}{d}+\frac{6a^2\csc^5(c+dx)}{5d}\\
&\quad-\frac{2a^2\csc^7(c+dx)}{7d}-\frac{a^2\cot^7(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7d}
\end{aligned}
$$

```
[In] Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/7*(a^2*Cot[c + d*x]^7)/d + (2*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c + d*x]^
3)/d + (6*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(7*d) - (a^2*C
ot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[c + d*x]^2])/(7*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.56 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03

method	result
risch	$a^2x + \frac{2ia^2\left(210\,e^{9i(dx+c)}-315\,e^{8i(dx+c)}-420\,e^{7i(dx+c)}+1470\,e^{6i(dx+c)}-504\,e^{5i(dx+c)}-1204\,e^{4i(dx+c)}+1108\,e^{3i(dx+\cdots}{105d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)^3}$
derivativedivides	$\frac{-\frac{a^2\cos(dx+c)^7}{7\sin(dx+c)^7}+2a^2\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(dx+c)^8}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(d\cdots}{5}\right.}{7}\right.}{d}$
default	$\frac{-\frac{a^2\cos(dx+c)^7}{7\sin(dx+c)^7}+2a^2\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(dx+c)^8}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(d\cdots}{5}\right.}{7}\right.}{d}$

```
[In] int(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+2/105*I*a^2*(210*exp(9*I*(d*x+c))-315*exp(8*I*(d*x+c))-420*exp(7*I*(d
*x+c))+1470*exp(6*I*(d*x+c))-504*exp(5*I*(d*x+c))-1204*exp(4*I*(d*x+c))+110
8*exp(3*I*(d*x+c))+258*exp(2*I*(d*x+c))-554*exp(I*(d*x+c))+191)/d/(exp(I*(d
*x+c))-1)^7/(exp(I*(d*x+c))+1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.24

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{191\,a^2\cos\left(dx+c\right)^5-172\,a^2\cos\left(dx+c\right)^4-253\,a^2\cos\left(dx+c\right)^3+258\,a^2\cos\left(dx+c\right)^2+87\,a^2\cos\left(dx+\cdots\right.}{105\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^3\cdots\right.}$$

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(191*a^2*cos(d*x + c)^5 - 172*a^2*cos(d*x + c)^4 - 253*a^2*cos(d*x +
c)^3 + 258*a^2*cos(d*x + c)^2 + 87*a^2*cos(d*x + c) - 96*a^2 + 105*(a^2*d*x
*cos(d*x + c)^4 - 2*a^2*d*x*cos(d*x + c)^3 + 2*a^2*d*x*cos(d*x + c) - a^2*d
*x)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c
) - d)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**8*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{\tan(dx+c)^7}\right)a^2+\frac{6\left(35\,\sin(dx+c)^6-35\,\sin(dx+c)^4+21\,\sin(dx+c)^2-5\right)a^2}{\sin(dx+c)^7}}{105\,d}$$

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a^2 + 6*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*a^2/sin(d*x + c)^7 - 15*a^2/tan(d*x + c)^7)/d
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.81

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3360\,(dx+c)a^2-735\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4410\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-770\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{3360\,d}$$

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3360*(35*a^2*tan(1/2*d*x + 1/2*c)^3 + 3360*(d*x + c)*a^2 - 735*a^2*tan(1/
2*d*x + 1/2*c) + (4410*a^2*tan(1/2*d*x + 1/2*c)^6 - 770*a^2*tan(1/2*d*x + 1
/2*c)^4 + 147*a^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/
d
```

Mupad [B] (verification not implemented)

Time = 14.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.31

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \frac{a^2\left(35\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-735\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+4410\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\right.\right.}{3360\,d\cos}$$

```
[In] int(cot(c + d*x)^8*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*(35*sin(c/2 + (d*x)/2)^10 - 15*cos(c/2 + (d*x)/2)^10 - 735*cos(c/2 + (
d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 4410*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/
2)^6 - 770*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 147*cos(c/2 + (d*x)/
2)^8*sin(c/2 + (d*x)/2)^2 + 3360*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^7*
(c + d*x)))/(3360*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^7)
```

3.36 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\begin{aligned}\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = &-a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}\\&-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^7(c+dx)}{7d}\\&-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{2a^2\csc(c+dx)}{d}\\&+\frac{8a^2\csc^3(c+dx)}{3d}-\frac{12a^2\csc^5(c+dx)}{5d}\\&+\frac{8a^2\csc^7(c+dx)}{7d}-\frac{2a^2\csc^9(c+dx)}{9d}\end{aligned}$$

```
[Out] -a^2*x-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/d-1/5*a^2*cot(d*x+c)^5/d+1/7*a
^2*cot(d*x+c)^7/d-2/9*a^2*cot(d*x+c)^9/d-2*a^2*csc(d*x+c)/d+8/3*a^2*csc(d*x
+c)^3/d-12/5*a^2*csc(d*x+c)^5/d+8/7*a^2*csc(d*x+c)^7/d-2/9*a^2*csc(d*x+c)^9
/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

= {3971, 3554, 8, 2686, 200, 2687, 30}

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2a^2\cot^9(c+dx)}{9d}+\frac{a^2\cot^7(c+dx)}{7d} -\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d} -\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc^9(c+dx)}{9d} +\frac{8a^2\csc^7(c+dx)}{7d}-\frac{12a^2\csc^5(c+dx)}{5d} +\frac{8a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}-a^2x$$

```
[In] Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c +
 d*x]^5)/(5*d) + (a^2*Cot[c + d*x]^7)/(7*d) - (2*a^2*Cot[c + d*x]^9)/(9*d)
- (2*a^2*Csc[c + d*x])/d + (8*a^2*Csc[c + d*x]^3)/(3*d) - (12*a^2*Csc[c + d
*x]^5)/(5*d) + (8*a^2*Csc[c + d*x]^7)/(7*d) - (2*a^2*Csc[c + d*x]^9)/(9*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2\cot^{10}(c+dx)+2a^2\cot^9(c+dx)\csc(c+dx)+a^2\cot^8(c+dx)\csc^2(c+dx)\right)\,dx\\
&= a^2\int \cot^{10}(c+dx)\,dx+a^2\int \cot^8(c+dx)\csc^2(c+dx)\,dx+\left(2a^2\right)\int \cot^9(c+dx)\csc(c+dx)\,dx\\
&= -\frac{a^2\cot^9(c+dx)}{9d}-a^2\int \cot^8(c+dx)\,dx+\frac{a^2\text{Subst}\left(\int x^8\,dx,x,-\cot(c+dx)\right)}{d}\\
&\quad-\frac{\left(2a^2\right)\text{Subst}\left(\int \left(-1+x^2\right)^4\,dx,x,\csc(c+dx)\right)}{d}\\
&= \frac{a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^9(c+dx)}{9d}+a^2\int \cot^6(c+dx)\,dx\\
&\quad-\frac{\left(2a^2\right)\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&= -\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{2a^2\csc(c+dx)}{d}+\frac{8a^2\csc^3(c+dx)}{3d}\\
&\quad-\frac{12a^2\csc^5(c+dx)}{5d}+\frac{8a^2\csc^7(c+dx)}{7d}-\frac{2a^2\csc^9(c+dx)}{9d}-a^2\int \cot^4(c+dx)\,dx\\
&= \frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^9(c+dx)}{9d}\\
&\quad-\frac{2a^2\csc(c+dx)}{d}+\frac{8a^2\csc^3(c+dx)}{3d}-\frac{12a^2\csc^5(c+dx)}{5d}\\
&\quad+\frac{8a^2\csc^7(c+dx)}{7d}-\frac{2a^2\csc^9(c+dx)}{9d}+a^2\int \cot^2(c+dx)\,dx\\
&= -\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^7(c+dx)}{7d}\\
&\quad-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{2a^2\csc(c+dx)}{d}+\frac{8a^2\csc^3(c+dx)}{3d}\\
&\quad-\frac{12a^2\csc^5(c+dx)}{5d}+\frac{8a^2\csc^7(c+dx)}{7d}-\frac{2a^2\csc^9(c+dx)}{9d}-a^2\int 1\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -a^2x - \frac{a^2\cot(c+dx)}{d} + \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} \\
&\quad + \frac{a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{2a^2\csc(c+dx)}{d} + \frac{8a^2\csc^3(c+dx)}{3d} \\
&\quad - \frac{12a^2\csc^5(c+dx)}{5d} + \frac{8a^2\csc^7(c+dx)}{7d} - \frac{2a^2\csc^9(c+dx)}{9d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.77 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.79

$$
\begin{aligned}
&\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx \\
&= -\frac{a^2\cot^9(c+dx)}{9d} - \frac{2a^2\csc(c+dx)}{d} + \frac{8a^2\csc^3(c+dx)}{3d} \\
&\quad - \frac{12a^2\csc^5(c+dx)}{5d} + \frac{8a^2\csc^7(c+dx)}{7d} - \frac{2a^2\csc^9(c+dx)}{9d} \\
&\quad - \frac{a^2\cot^9(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(c+dx)\right)}{9d}
\end{aligned}
$$

```
[In] Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/9*(a^2*Cot[c + d*x]^9)/d - (2*a^2*Csc[c + d*x])/d + (8*a^2*Csc[c + d*x]^
3)/(3*d) - (12*a^2*Csc[c + d*x]^5)/(5*d) + (8*a^2*Csc[c + d*x]^7)/(7*d) - (
2*a^2*Csc[c + d*x]^9)/(9*d) - (a^2*Cot[c + d*x]^9*Hypergeometric2F1[-9/2, 1
, -7/2, -Tan[c + d*x]^2])/(9*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.48 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.05

method	result
risch	$-a^2x-\frac{4ia^2\left(315\,e^{13i(dx+c)}-315\,e^{12i(dx+c)}-1470\,e^{11i(dx+c)}+3360\,e^{10i(dx+c)}+1113\,e^{9i(dx+c)}-6447\,e^{8i(dx+c)}+202\right.}{315d\left(e^{i(dx+c)}-\right.}$
derivativedivides	$-\frac{a^2\cos(dx+c)^9}{9\sin(dx+c)^9}+2a^2\left(-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)^9}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^7}-\frac{\cos(dx+c)^{10}}{105\sin(dx+c)^5}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^3}-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8}{}\right.}{}\right.$
default	$-\frac{a^2\cos(dx+c)^9}{9\sin(dx+c)^9}+2a^2\left(-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)^9}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^7}-\frac{\cos(dx+c)^{10}}{105\sin(dx+c)^5}+\frac{\cos(dx+c)^{10}}{63\sin(dx+c)^3}-\frac{\cos(dx+c)^{10}}{9\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8}{}\right.}{}\right.$

```
[In] int(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*x-4/315*I*a^2*(315*exp(13*I*(d*x+c))-315*exp(12*I*(d*x+c))-1470*exp(11
*I*(d*x+c))+3360*exp(10*I*(d*x+c))+1113*exp(9*I*(d*x+c))-6447*exp(8*I*(d*x+
c))+2028*exp(7*I*(d*x+c))+7008*exp(6*I*(d*x+c))-4867*exp(5*I*(d*x+c))-2321*
exp(4*I*(d*x+c))+3314*exp(3*I*(d*x+c))-16*exp(2*I*(d*x+c))-881*exp(I*(d*x+c
))+299)/d/(exp(I*(d*x+c))-1)^9/(exp(I*(d*x+c))+1)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.53

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{598\,a^2\cos\left(dx+c\right)^7 - 566\,a^2\cos\left(dx+c\right)^6 - 1212\,a^2\cos\left(dx+c\right)^5 + 1310\,a^2\cos\left(dx+c\right)^4 + 860\,a^2\cos\left(d\right.}{315\left(d\,c\right.}$$

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/315*(598*a^2*cos(d*x + c)^7 - 566*a^2*cos(d*x + c)^6 - 1212*a^2*cos(d*x
+ c)^5 + 1310*a^2*cos(d*x + c)^4 + 860*a^2*cos(d*x + c)^3 - 1014*a^2*cos(d*
x + c)^2 - 197*a^2*cos(d*x + c) + 256*a^2 + 315*(a^2*d*x*cos(d*x + c)^6 - 2
*a^2*d*x*cos(d*x + c)^5 - a^2*d*x*cos(d*x + c)^4 + 4*a^2*d*x*cos(d*x + c)^3
 - a^2*d*x*cos(d*x + c)^2 - 2*a^2*d*x*cos(d*x + c) + a^2*d*x)*sin(d*x + c))
/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^5 - d*cos(d*x + c)^4 + 4*d*cos(d*x +
 c)^3 - d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**10*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.77

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{\left(315\,dx+315\,c+\frac{315\,\tan(dx+c)^8-105\,\tan(dx+c)^6+63\,\tan(dx+c)^4-45\,\tan(dx+c)^2+35}{\tan(dx+c)^9}\right)a^2+\frac{2\left(315\,\sin(dx+c)^8-420\,\sin(dx+c\right.}{}}{315\,d}$$

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*ta
n(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a^2 + 2*(315*sin(d*x
 + c)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 + 35
)*a^2/sin(d*x + c)^9 + 35*a^2/tan(d*x + c)^9)/d
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.81

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$=\frac{63\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-945\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40320\,(dx+c)a^2+11655\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{51345\,a}{}}{40320\,d}$$

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40320*(63*a^2*tan(1/2*d*x + 1/2*c)^5 - 945*a^2*tan(1/2*d*x + 1/2*c)^3 - 4
0320*(d*x + c)*a^2 + 11655*a^2*tan(1/2*d*x + 1/2*c) - (51345*a^2*tan(1/2*d*
x + 1/2*c)^8 - 9765*a^2*tan(1/2*d*x + 1/2*c)^6 + 2331*a^2*tan(1/2*d*x + 1/2
*c)^4 - 405*a^2*tan(1/2*d*x + 1/2*c)^2 + 35*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.28

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^2\,dx =$$

$$-\frac{a^2\left(35\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-63\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+945\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-11655\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\right.}{}$$

```
[In] int(cot(c + d*x)^10*(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(a^2*(35*cos(c/2 + (d*x)/2)^14 - 63*sin(c/2 + (d*x)/2)^14 + 945*cos(c/2 +
(d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 11655*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*
x)/2)^10 + 51345*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 9765*cos(c/2 +
 (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 2331*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*
x)/2)^4 - 405*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 40320*cos(c/2 +
(d*x)/2)^5*sin(c/2 + (d*x)/2)^9*(c + d*x)))/(40320*d*cos(c/2 + (d*x)/2)^5*s
in(c/2 + (d*x)/2)^9)
```

3.37 $\int (a + a\sec(c + dx))^3 \tan^9(c + dx)\, dx$

Optimal result

Integrand size = 21, antiderivative size = 210

$$\begin{aligned}\int (a+a\sec(c+dx))^3\tan^9(c+dx)\,dx = {}& -\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d} \\ & -\frac{a^3\sec^2(c+dx)}{2d}-\frac{11a^3\sec^3(c+dx)}{3d} \\ & -\frac{3a^3\sec^4(c+dx)}{2d}+\frac{14a^3\sec^5(c+dx)}{5d} \\ & +\frac{7a^3\sec^6(c+dx)}{3d}-\frac{6a^3\sec^7(c+dx)}{7d} \\ & -\frac{11a^3\sec^8(c+dx)}{8d}-\frac{a^3\sec^9(c+dx)}{9d} \\ & +\frac{3a^3\sec^{10}(c+dx)}{10d}+\frac{a^3\sec^{11}(c+dx)}{11d}\end{aligned}$$

```
[Out] -a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d-1/2*a^3*sec(d*x+c)^2/d-11/3*a^3*se
c(d*x+c)^3/d-3/2*a^3*sec(d*x+c)^4/d+14/5*a^3*sec(d*x+c)^5/d+7/3*a^3*sec(d*x
+c)^6/d-6/7*a^3*sec(d*x+c)^7/d-11/8*a^3*sec(d*x+c)^8/d-1/9*a^3*sec(d*x+c)^9
/d+3/10*a^3*sec(d*x+c)^10/d+1/11*a^3*sec(d*x+c)^11/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used

$= \{3964, 90\}$

$$\int (a+a\sec(c+dx))^3\tan^9(c+dx)\,dx = \frac{a^3\sec^{11}(c+dx)}{11d}+\frac{3a^3\sec^{10}(c+dx)}{10d} - \frac{a^3\sec^9(c+dx)}{9d}-\frac{11a^3\sec^8(c+dx)}{8d} - \frac{6a^3\sec^7(c+dx)}{7d}+\frac{7a^3\sec^6(c+dx)}{3d} + \frac{14a^3\sec^5(c+dx)}{5d}-\frac{3a^3\sec^4(c+dx)}{2d} - \frac{11a^3\sec^3(c+dx)}{3d}-\frac{a^3\sec^2(c+dx)}{2d} + \frac{3a^3\sec(c+dx)}{d}-\frac{a^3\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^9,x]
```

```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d - (a^3*Sec[c + d*x]^2
)/(2*d) - (11*a^3*Sec[c + d*x]^3)/(3*d) - (3*a^3*Sec[c + d*x]^4)/(2*d) + (1
4*a^3*Sec[c + d*x]^5)/(5*d) + (7*a^3*Sec[c + d*x]^6)/(3*d) - (6*a^3*Sec[c +
 d*x]^7)/(7*d) - (11*a^3*Sec[c + d*x]^8)/(8*d) - (a^3*Sec[c + d*x]^9)/(9*d)
 + (3*a^3*Sec[c + d*x]^10)/(10*d) + (a^3*Sec[c + d*x]^11)/(11*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^7}{x^{12}}\,dx, x, \cos(c+dx)\right)}{a^8d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a^{11}}{x^{12}}+\frac{3a^{11}}{x^{11}}-\frac{a^{11}}{x^{10}}-\frac{11a^{11}}{x^9}-\frac{6a^{11}}{x^8}+\frac{14a^{11}}{x^7}+\frac{14a^{11}}{x^6}-\frac{6a^{11}}{x^5}-\frac{11a^{11}}{x^4}-\frac{a^{11}}{x^3}+\frac{3a^{11}}{x^2}+\frac{a^{11}}{x}\right)dx, x\right.}{a^8d}$$

$$
\begin{aligned}
&= -\frac{a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d} - \frac{a^3\sec^2(c+dx)}{2d} - \frac{11a^3\sec^3(c+dx)}{3d} \\
&\quad - \frac{3a^3\sec^4(c+dx)}{2d} + \frac{14a^3\sec^5(c+dx)}{5d} + \frac{7a^3\sec^6(c+dx)}{3d} - \frac{6a^3\sec^7(c+dx)}{7d} \\
&\quad - \frac{11a^3\sec^8(c+dx)}{8d} - \frac{a^3\sec^9(c+dx)}{9d} + \frac{3a^3\sec^{10}(c+dx)}{10d} + \frac{a^3\sec^{11}(c+dx)}{11d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.02

$$
\int (a+a\sec(c+dx))^3\tan^9(c+dx)\,dx = -\frac{a^3(-1151740 - 1613260\cos(2(c+dx)) + 960960\cos(3(c+dx)) - 1131504\cos(4(c+dx)) + 314160\,\mathrm{co}}{}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^9,x]
```

```
[Out] -1/3548160*(a^3*(-1151740 - 1613260*Cos[2*(c + d*x)] + 960960*Cos[3*(c + d*
x)] - 1131504*Cos[4*(c + d*x)] + 314160*Cos[5*(c + d*x)] - 432894*Cos[6*(c
+ d*x)] + 145530*Cos[7*(c + d*x)] - 106260*Cos[8*(c + d*x)] + 6930*Cos[9*(c
 + d*x)] - 20790*Cos[10*(c + d*x)] + 1143450*Cos[3*(c + d*x)]*Log[Cos[c + d
*x]] + 571725*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 190575*Cos[7*(c + d*x)]*
Log[Cos[c + d*x]] + 38115*Cos[9*(c + d*x)]*Log[Cos[c + d*x]] + 3465*Cos[11*
(c + d*x)]*Log[Cos[c + d*x]] + 462*Cos[c + d*x]*(2606 + 3465*Log[Cos[c + d*
x]]))*Sec[c + d*x]^11)/d
```

Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)^{11}}{11}+\frac{3\sec(dx+c)^{10}}{10}-\frac{\sec(dx+c)^9}{9}-\frac{11\sec(dx+c)^8}{8}-\frac{6\sec(dx+c)^7}{7}+\frac{7\sec(dx+c)^6}{3}+\frac{14\sec(dx+c)^5}{5}-\frac{3\sec(dx+c)^4}{2}-\cdots\right.}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)^{11}}{11}+\frac{3\sec(dx+c)^{10}}{10}-\frac{\sec(dx+c)^9}{9}-\frac{11\sec(dx+c)^8}{8}-\frac{6\sec(dx+c)^7}{7}+\frac{7\sec(dx+c)^6}{3}+\frac{14\sec(dx+c)^5}{5}-\frac{3\sec(dx+c)^4}{2}-\cdots\right.}{d}$
parts	$\frac{a^3\left(\frac{\tan(dx+c)^8}{8}-\frac{\tan(dx+c)^6}{6}+\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a^3\left(\frac{\sec(dx+c)^{11}}{11}-\frac{4\sec(dx+c)^9}{9}+\frac{6\sec(dx\cdots}{7}\right.}{d}$
risch	$ia^3x+\frac{2ia^3c}{d}+\frac{2a^3\left(10395\,\mathrm{e}^{21i(dx+c)}-3465\,\mathrm{e}^{20i(dx+c)}+53130\,\mathrm{e}^{19i(dx+c)}-72765\,\mathrm{e}^{18i(dx+c)}+216447\,\mathrm{e}^{17i(dx+c)}-15\cdots\right.}{}$

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/d*(1/11*sec(d*x+c)^11+3/10*sec(d*x+c)^10-1/9*sec(d*x+c)^9-11/8*sec(d*x+
c)^8-6/7*sec(d*x+c)^7+7/3*sec(d*x+c)^6+14/5*sec(d*x+c)^5-3/2*sec(d*x+c)^4-1
1/3*sec(d*x+c)^3-1/2*sec(d*x+c)^2+3*sec(d*x+c)+ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.80

$$\int (a+a\sec(c+dx))^3\tan^9(c+dx)\,dx =$$
$$-\frac{27720\,a^3\cos\left(dx+c\right)^{11}\log\left(-\cos\left(dx+c\right)\right)-83160\,a^3\cos\left(dx+c\right)^{10}+13860\,a^3\cos\left(dx+c\right)^{9}+101640\,a$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/27720*(27720*a^3*cos(d*x + c)^11*log(-cos(d*x + c)) - 83160*a^3*cos(d*x
+ c)^10 + 13860*a^3*cos(d*x + c)^9 + 101640*a^3*cos(d*x + c)^8 + 41580*a^3*
cos(d*x + c)^7 - 77616*a^3*cos(d*x + c)^6 - 64680*a^3*cos(d*x + c)^5 + 2376
0*a^3*cos(d*x + c)^4 + 38115*a^3*cos(d*x + c)^3 + 3080*a^3*cos(d*x + c)^2 -
 8316*a^3*cos(d*x + c) - 2520*a^3)/(d*cos(d*x + c)^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(190) = 380.

Time = 3.40 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.09

$$\int (a+a\sec(c+dx))^3\tan^9(c+dx)\,dx$$
$$=\begin{cases}\frac{a^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^3\tan^8\left(c+dx\right)\sec^3\left(c+dx\right)}{11d}+\frac{3a^3\tan^8\left(c+dx\right)\sec^2\left(c+dx\right)}{10d}+\frac{a^3\tan^8\left(c+dx\right)\sec\left(c+dx\right)}{3d}+\frac{a^3\tan^8\left(c+dx\right)}{8d}-\\ x\left(a\sec\left(c\right)+a\right)^3\tan^9\left(c\right)\end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**8*sec(c
 + d*x)**3/(11*d) + 3*a**3*tan(c + d*x)**8*sec(c + d*x)**2/(10*d) + a**3*ta
n(c + d*x)**8*sec(c + d*x)/(3*d) + a**3*tan(c + d*x)**8/(8*d) - 8*a**3*tan(
c + d*x)**6*sec(c + d*x)**3/(99*d) - 3*a**3*tan(c + d*x)**6*sec(c + d*x)**2
/(10*d) - 8*a**3*tan(c + d*x)**6*sec(c + d*x)/(21*d) - a**3*tan(c + d*x)**6
/(6*d) + 16*a**3*tan(c + d*x)**4*sec(c + d*x)**3/(231*d) + 3*a**3*tan(c + d
*x)**4*sec(c + d*x)**2/(10*d) + 16*a**3*tan(c + d*x)**4*sec(c + d*x)/(35*d)
 + a**3*tan(c + d*x)**4/(4*d) - 64*a**3*tan(c + d*x)**2*sec(c + d*x)**3/(11
55*d) - 3*a**3*tan(c + d*x)**2*sec(c + d*x)**2/(10*d) - 64*a**3*tan(c + d*x
```

```
)**2*sec(c + d*x)/(105*d) - a**3*tan(c + d*x)**2/(2*d) + 128*a**3*sec(c + d
*x)**3/(3465*d) + 3*a**3*sec(c + d*x)**2/(10*d) + 128*a**3*sec(c + d*x)/(10
5*d), Ne(d, 0)), (x*(a*sec(c) + a)**3*tan(c)**9, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.77

$$\int (a + a\sec(c + dx))^3 \tan^9(c + dx)\, dx = -\frac{27720\, a^3 \log\left(\cos\left(dx + c\right)\right) - \frac{83160\, a^3 \cos(dx+c)^{10} - 13860\, a^3 \cos(dx+c)^9 - 101640\, a^3 \cos(dx+c)^8 - 41580\, a^3 \cos(dx+c)^7 + 77616\, a^3}{}}{27720\, d}$$

[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="maxima")

```
[Out] -1/27720*(27720*a^3*log(cos(d*x + c)) - (83160*a^3*cos(d*x + c)^10 - 13860*
a^3*cos(d*x + c)^9 - 101640*a^3*cos(d*x + c)^8 - 41580*a^3*cos(d*x + c)^7 +
 77616*a^3*cos(d*x + c)^6 + 64680*a^3*cos(d*x + c)^5 - 23760*a^3*cos(d*x +
c)^4 - 38115*a^3*cos(d*x + c)^3 - 3080*a^3*cos(d*x + c)^2 + 8316*a^3*cos(d*
x + c) + 2520*a^3)/cos(d*x + c)^11)/d
```

Giac [A] (verification not implemented)

none

Time = 5.96 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.75

$$\int (a + a\sec(c + dx))^3 \tan^9(c + dx)\, dx = \frac{27720\, a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 27720\, a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{153343\, a^3 + \frac{1742213\, a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{9043705\, a^3}{}}{}}{}$$

[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^9,x, algorithm="giac")

```
[Out] 1/27720*(27720*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2
7720*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (153343*a^3
 + 1742213*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9043705*a^3*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 + 28369275*a^3*(cos(d*x + c) - 1)^3/(cos(
d*x + c) + 1)^3 + 59954070*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 +
67458930*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 57997170*a^3*(cos(
d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 36975510*a^3*(cos(d*x + c) - 1)^7/(c
os(d*x + c) + 1)^7 + 16879995*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8
 + 5213945*a^3*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9 + 976261*a^3*(cos(
d*x + c) - 1)^10/(cos(d*x + c) + 1)^10 + 83711*a^3*(cos(d*x + c) - 1)^11/(c
os(d*x + c) + 1)^11)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^11)/d
```

Mupad [B] (verification not implemented)

Time = 17.80 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.60

$$\int (a+a\sec(c+dx))^3\tan^9(c+dx)\,dx=\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$
$$-\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}-22\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+\frac{332\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}}{3}-\frac{1012\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}}{3}+\frac{10456\,a^3\tan}{15}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{22}-11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}+55\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-165\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+330\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-\right.}$$

```
[In] int(tan(c + d*x)^9*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - ((10090*a^3*tan(c/2 + (d*x)/2)^4)/6
3 - (9334*a^3*tan(c/2 + (d*x)/2)^2)/315 - (3676*a^3*tan(c/2 + (d*x)/2)^6)/7
 + (8164*a^3*tan(c/2 + (d*x)/2)^8)/7 - (5192*a^3*tan(c/2 + (d*x)/2)^10)/5 +
 (10456*a^3*tan(c/2 + (d*x)/2)^12)/15 - (1012*a^3*tan(c/2 + (d*x)/2)^14)/3
+ (332*a^3*tan(c/2 + (d*x)/2)^16)/3 - 22*a^3*tan(c/2 + (d*x)/2)^18 + 2*a^3*
tan(c/2 + (d*x)/2)^20 + (8704*a^3)/3465)/(d*(11*tan(c/2 + (d*x)/2)^2 - 55*t
an(c/2 + (d*x)/2)^4 + 165*tan(c/2 + (d*x)/2)^6 - 330*tan(c/2 + (d*x)/2)^8 +
 462*tan(c/2 + (d*x)/2)^10 - 462*tan(c/2 + (d*x)/2)^12 + 330*tan(c/2 + (d*x
)/2)^14 - 165*tan(c/2 + (d*x)/2)^16 + 55*tan(c/2 + (d*x)/2)^18 - 11*tan(c/2
 + (d*x)/2)^20 + tan(c/2 + (d*x)/2)^22 - 1))
```

3.38 $\int (a+a\sec(c+dx))^3\tan^7(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\int (a+a\sec(c+dx))^3\tan^7(c+dx)\,dx = \frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^3\sec(c+dx)}{d}+\frac{8a^3\sec^3(c+dx)}{3d}+\frac{3a^3\sec^4(c+dx)}{2d}-\frac{6a^3\sec^5(c+dx)}{5d}-\frac{4a^3\sec^6(c+dx)}{3d}+\frac{3a^3\sec^8(c+dx)}{8d}+\frac{a^3\sec^9(c+dx)}{9d}$$

```
[Out] a^3*ln(cos(d*x+c))/d-3*a^3*sec(d*x+c)/d+8/3*a^3*sec(d*x+c)^3/d+3/2*a^3*sec(
d*x+c)^4/d-6/5*a^3*sec(d*x+c)^5/d-4/3*a^3*sec(d*x+c)^6/d+3/8*a^3*sec(d*x+c)
^8/d+1/9*a^3*sec(d*x+c)^9/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int (a+a\sec(c+dx))^3\tan^7(c+dx)\,dx = \frac{a^3\sec^9(c+dx)}{9d}+\frac{3a^3\sec^8(c+dx)}{8d}-\frac{4a^3\sec^6(c+dx)}{3d}-\frac{6a^3\sec^5(c+dx)}{5d}+\frac{3a^3\sec^4(c+dx)}{2d}+\frac{8a^3\sec^3(c+dx)}{3d}-\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^3*Sec[c + d*x])/d + (8*a^3*Sec[c + d*x]^3)
/(3*d) + (3*a^3*Sec[c + d*x]^4)/(2*d) - (6*a^3*Sec[c + d*x]^5)/(5*d) - (4*a
^3*Sec[c + d*x]^6)/(3*d) + (3*a^3*Sec[c + d*x]^8)/(8*d) + (a^3*Sec[c + d*x]
^9)/(9*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^6}{x^{10}}\,dx, x, \cos(c+dx)\right)}{a^6d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^9}{x^{10}}+\frac{3a^9}{x^9}-\frac{8a^9}{x^7}-\frac{6a^9}{x^6}+\frac{6a^9}{x^5}+\frac{8a^9}{x^4}-\frac{3a^9}{x^2}-\frac{a^9}{x}\right)\,dx, x, \cos(c+dx)\right)}{a^6d}\\
&= \frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^3\sec(c+dx)}{d}+\frac{8a^3\sec^3(c+dx)}{3d}+\frac{3a^3\sec^4(c+dx)}{2d}\\
&\quad -\frac{6a^3\sec^5(c+dx)}{5d}-\frac{4a^3\sec^6(c+dx)}{3d}+\frac{3a^3\sec^8(c+dx)}{8d}+\frac{a^3\sec^9(c+dx)}{9d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.22

$$\int (a+a\sec(c+dx))^3\tan^7(c+dx)\,dx$$
$$=\frac{a^3(-3754-7632\cos(2(c+dx))+1560\cos(3(c+dx))-3528\cos(4(c+dx))+1080\cos(5(c+dx))-120}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (a^3*(-3754 - 7632*Cos[2*(c + d*x)] + 1560*Cos[3*(c + d*x)] - 3528*Cos[4*(c
 + d*x)] + 1080*Cos[5*(c + d*x)] - 1200*Cos[6*(c + d*x)] - 270*Cos[8*(c + d
*x)] + 3780*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + 1620*Cos[5*(c + d*x)]*Log[
Cos[c + d*x]] + 405*Cos[7*(c + d*x)]*Log[Cos[c + d*x]] + 45*Cos[9*(c + d*x)
]*Log[Cos[c + d*x]] + 90*Cos[c + d*x]*(40 + 63*Log[Cos[c + d*x]]))*Sec[c +
d*x]^9)/(11520*d)
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)^9}{9}+\frac{3\sec(dx+c)^8}{8}-\frac{4\sec(dx+c)^6}{3}-\frac{6\sec(dx+c)^5}{5}+\frac{3\sec(dx+c)^4}{2}+\frac{8\sec(dx+c)^3}{3}-3\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)^9}{9}+\frac{3\sec(dx+c)^8}{8}-\frac{4\sec(dx+c)^6}{3}-\frac{6\sec(dx+c)^5}{5}+\frac{3\sec(dx+c)^4}{2}+\frac{8\sec(dx+c)^3}{3}-3\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a^3\left(\frac{\tan(dx+c)^6}{6}-\frac{\tan(dx+c)^4}{4}+\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{a^3\left(\frac{\sec(dx+c)^9}{9}-\frac{3\sec(dx+c)^7}{7}+\frac{3\sec(dx+c)^5}{5}-\frac{\sec(dx+}{3}\right.}{d}$
risch	$-ia^3x-\frac{2ia^3c}{d}-\frac{2a^3\left(135\,\mathrm{e}^{17i(dx+c)}+600\,\mathrm{e}^{15i(dx+c)}-540\,\mathrm{e}^{14i(dx+c)}+1764\,\mathrm{e}^{13i(dx+c)}-780\,\mathrm{e}^{12i(dx+c)}+3816\,\mathrm{e}^{11i(}\right.}{}$

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/d*(1/9*sec(d*x+c)^9+3/8*sec(d*x+c)^8-4/3*sec(d*x+c)^6-6/5*sec(d*x+c)^5+
3/2*sec(d*x+c)^4+8/3*sec(d*x+c)^3-3*sec(d*x+c)-ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int (a+a\sec(c+dx))^3\tan^7(c+dx)\,dx = \frac{360\,a^3\cos(dx+c)^9\log(-\cos(dx+c))-1080\,a^3\cos(dx+c)^8+960\,a^3\cos(dx+c)^6+540\,a^3\cos(dx+c}{360\,d\cos(dx+c)^9}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/360*(360*a^3*cos(d*x + c)^9*log(-cos(d*x + c)) - 1080*a^3*cos(d*x + c)^8
+ 960*a^3*cos(d*x + c)^6 + 540*a^3*cos(d*x + c)^5 - 432*a^3*cos(d*x + c)^4
- 480*a^3*cos(d*x + c)^3 + 135*a^3*cos(d*x + c) + 40*a^3)/(d*cos(d*x + c)^9
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(126) = 252$.

Time = 1.67 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.55

$$\int (a + a\sec(c + dx))^3 \tan^7(c + dx)\, dx$$
$$= \begin{cases} -\frac{a^3 \log\left(\tan^2(c+dx)+1\right)}{2d} + \frac{a^3 \tan^6(c+dx)\sec^3(c+dx)}{9d} + \frac{3a^3 \tan^6(c+dx)\sec^2(c+dx)}{8d} + \frac{3a^3 \tan^6(c+dx)\sec(c+dx)}{7d} + \frac{a^3 \tan^6(c+dx)}{6d} \\ x(a\sec(c) + a)^3 \tan^7(c) \end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**6*sec(
c + d*x)**3/(9*d) + 3*a**3*tan(c + d*x)**6*sec(c + d*x)**2/(8*d) + 3*a**3*t
an(c + d*x)**6*sec(c + d*x)/(7*d) + a**3*tan(c + d*x)**6/(6*d) - 2*a**3*tan
(c + d*x)**4*sec(c + d*x)**3/(21*d) - 3*a**3*tan(c + d*x)**4*sec(c + d*x)**
2/(8*d) - 18*a**3*tan(c + d*x)**4*sec(c + d*x)/(35*d) - a**3*tan(c + d*x)**
4/(4*d) + 8*a**3*tan(c + d*x)**2*sec(c + d*x)**3/(105*d) + 3*a**3*tan(c + d
*x)**2*sec(c + d*x)**2/(8*d) + 24*a**3*tan(c + d*x)**2*sec(c + d*x)/(35*d)
+ a**3*tan(c + d*x)**2/(2*d) - 16*a**3*sec(c + d*x)**3/(315*d) - 3*a**3*sec
(c + d*x)**2/(8*d) - 48*a**3*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a*sec(c) +
 a)**3*tan(c)**7, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.80

$$\int (a + a\sec(c + dx))^3 \tan^7(c + dx)\, dx$$
$$= \frac{360\, a^3 \log\left(\cos\left(dx + c\right)\right) - \frac{1080\, a^3 \cos(dx+c)^8 - 960\, a^3 \cos(dx+c)^6 - 540\, a^3 \cos(dx+c)^5 + 432\, a^3 \cos(dx+c)^4 + 480\, a^3 \cos(dx+c)^3 - 135\, a^3 \cos}{\cos(dx+c)^9}}{360\, d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/360*(360*a^3*log(cos(d*x + c)) - (1080*a^3*cos(d*x + c)^8 - 960*a^3*cos(d
*x + c)^6 - 540*a^3*cos(d*x + c)^5 + 432*a^3*cos(d*x + c)^4 + 480*a^3*cos(d
*x + c)^3 - 135*a^3*cos(d*x + c) - 40*a^3)/cos(d*x + c)^9)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(125) = 250$.

Time = 3.83 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.31

$$\int (a + a\sec(c+dx))^3 \tan^7(c+dx)\, dx =$$

$$-\frac{2520\, a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 2520\, a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{14297\, a^3 + \frac{133713\, a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{560052\, a^3(\cdots}{(\cos(d\cdots}}{\cdots}}{\cdots}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/2520*(2520*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 25
20*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (14297*a^3 +
133713*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 560052*a^3*(cos(d*x + c)
 - 1)^2/(cos(d*x + c) + 1)^2 + 1384068*a^3*(cos(d*x + c) - 1)^3/(cos(d*x +
c) + 1)^3 + 1594782*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1336734
*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 781956*a^3*(cos(d*x + c) -
 1)^6/(cos(d*x + c) + 1)^6 + 302004*a^3*(cos(d*x + c) - 1)^7/(cos(d*x + c)
+ 1)^7 + 69201*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 + 7129*a^3*(co
s(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 1)^9)/d
```

Mupad [B] (verification not implemented)

Time = 17.91 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.03

$$\int (a + a\sec(c+dx))^3 \tan^7(c+dx)\, dx$$

$$= \frac{2\, a^3 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{16} - 18\, a^3 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{14} + \frac{218\, a^3 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{12}}{3} - 174\, a^3 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{10} + \frac{1382\, a^3 \tan\left(\frac{c}{2}\cdots\right.}{5}\cdots}{d\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{18} - 9\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{16} + 36\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{14} - 84\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{12} + 126\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{10} - 12\cdots\right.}$$
$$- \frac{2\, a^3 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2\right)}{d}$$

```
[In] int(tan(c + d*x)^7*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((602*a^3*tan(c/2 + (d*x)/2)^4)/5 - (138*a^3*tan(c/2 + (d*x)/2)^2)/5 - (155
8*a^3*tan(c/2 + (d*x)/2)^6)/5 + (1382*a^3*tan(c/2 + (d*x)/2)^8)/5 - 174*a^3
*tan(c/2 + (d*x)/2)^10 + (218*a^3*tan(c/2 + (d*x)/2)^12)/3 - 18*a^3*tan(c/2
 + (d*x)/2)^14 + 2*a^3*tan(c/2 + (d*x)/2)^16 + (128*a^3)/45)/(d*(9*tan(c/2
+ (d*x)/2)^2 - 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/2 + (d*x)/2)^6 - 126*tan(
c/2 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 - 84*tan(c/2 + (d*x)/2)^12 + 3
6*tan(c/2 + (d*x)/2)^14 - 9*tan(c/2 + (d*x)/2)^16 + tan(c/2 + (d*x)/2)^18 -
 1)) - (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.39 $\int (a+a\sec(c+dx))^3\tan^5(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int (a+a\sec(c+dx))^3\tan^5(c+dx)\,dx=-\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}-\frac{5a^3\sec^3(c+dx)}{3d}-\frac{5a^3\sec^4(c+dx)}{4d}+\frac{a^3\sec^5(c+dx)}{5d}+\frac{a^3\sec^6(c+dx)}{2d}+\frac{a^3\sec^7(c+dx)}{7d}$$

```
[Out] -a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d-5/3*a^3*sec
(d*x+c)^3/d-5/4*a^3*sec(d*x+c)^4/d+1/5*a^3*sec(d*x+c)^5/d+1/2*a^3*sec(d*x+c
)^6/d+1/7*a^3*sec(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int (a+a\sec(c+dx))^3\tan^5(c+dx)\,dx=\frac{a^3\sec^7(c+dx)}{7d}+\frac{a^3\sec^6(c+dx)}{2d}+\frac{a^3\sec^5(c+dx)}{5d}-\frac{5a^3\sec^4(c+dx)}{4d}-\frac{5a^3\sec^3(c+dx)}{3d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2
)/(2*d) - (5*a^3*Sec[c + d*x]^3)/(3*d) - (5*a^3*Sec[c + d*x]^4)/(4*d) + (a^
3*Sec[c + d*x]^5)/(5*d) + (a^3*Sec[c + d*x]^6)/(2*d) + (a^3*Sec[c + d*x]^7)
/(7*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)^5}{x^8}\,dx, x, \cos(c+dx)\right)}{a^4d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}+\frac{3a^7}{x^7}+\frac{a^7}{x^6}-\frac{5a^7}{x^5}-\frac{5a^7}{x^4}+\frac{a^7}{x^3}+\frac{3a^7}{x^2}+\frac{a^7}{x}\right)dx, x, \cos(c+dx)\right)}{a^4d}\\
&= -\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}-\frac{5a^3\sec^3(c+dx)}{3d}\\
&\quad -\frac{5a^3\sec^4(c+dx)}{4d}+\frac{a^3\sec^5(c+dx)}{5d}+\frac{a^3\sec^6(c+dx)}{2d}+\frac{a^3\sec^7(c+dx)}{7d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

$$
\int (a+a\sec(c+dx))^3\tan^5(c+dx)\,dx = -\frac{a^3(-3732-4522\cos(2(c+dx))+1050\cos(3(c+dx))-2380\cos(4(c+dx))-210\cos(5(c+dx))-6
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^5,x]
```

[Out] -1/6720*(a^3*(-3732 - 4522*Cos[2*(c + d*x)] + 1050*Cos[3*(c + d*x)] - 2380*Cos[4*(c + d*x)] - 210*Cos[5*(c + d*x)] - 630*Cos[6*(c + d*x)] + 2205*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + 735*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 105*Cos[7*(c + d*x)]*Log[Cos[c + d*x]] + 105*Cos[c + d*x]*(8 + 35*Log[Cos[c + d*x]]))*Sec[c + d*x]^7)/d

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.61

method	result
derivativedivides	$\frac{a^{3}\left(\frac{\sec(dx+c)^{7}}{7}+\frac{\sec(dx+c)^{6}}{2}+\frac{\sec(dx+c)^{5}}{5}-\frac{5\sec(dx+c)^{4}}{4}-\frac{5\sec(dx+c)^{3}}{3}+\frac{\sec(dx+c)^{2}}{2}+3\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^{3}\left(\frac{\sec(dx+c)^{7}}{7}+\frac{\sec(dx+c)^{6}}{2}+\frac{\sec(dx+c)^{5}}{5}-\frac{5\sec(dx+c)^{4}}{4}-\frac{5\sec(dx+c)^{3}}{3}+\frac{\sec(dx+c)^{2}}{2}+3\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a^{3}\left(\frac{\tan(dx+c)^{4}}{4}-\frac{\tan(dx+c)^{2}}{2}+\frac{\ln\left(1+\tan(dx+c)^{2}\right)}{2}\right)}{d}+\frac{a^{3}\left(\frac{\sec(dx+c)^{7}}{7}-\frac{2\sec(dx+c)^{5}}{5}+\frac{\sec(dx+c)^{3}}{3}\right)}{d}+\frac{3a^{3}\left(\frac{\sec(dx+c)^{5}}{5}-\right.}{}$
risch	$ia^{3}x+\frac{2ia^{3}c}{d}+\frac{2a^{3}\left(315\,e^{13i(dx+c)}+105\,e^{12i(dx+c)}+1190\,e^{11i(dx+c)}-525\,e^{10i(dx+c)}+2261\,e^{9i(dx+c)}-420\,e^{8i(dx+c)}+\right.}{105d(e}$

[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x,method=_RETURNVERBOSE)

[Out] a^3/d*(1/7*sec(d*x+c)^7+1/2*sec(d*x+c)^6+1/5*sec(d*x+c)^5-5/4*sec(d*x+c)^4-5/3*sec(d*x+c)^3+1/2*sec(d*x+c)^2+3*sec(d*x+c)+ln(sec(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int (a+a\sec(c+dx))^3\tan^5(c+dx)\,dx = -\frac{420\,a^3\cos(dx+c)^7\log(-\cos(dx+c)) - 1260\,a^3\cos(dx+c)^6 - 210\,a^3\cos(dx+c)^5 + 700\,a^3\cos(dx+}{420\,d\cos(dx+c)^7}$$

[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="fricas")

[Out] -1/420*(420*a^3*cos(d*x + c)^7*log(-cos(d*x + c)) - 1260*a^3*cos(d*x + c)^6 - 210*a^3*cos(d*x + c)^5 + 700*a^3*cos(d*x + c)^4 + 525*a^3*cos(d*x + c)^3 - 84*a^3*cos(d*x + c)^2 - 210*a^3*cos(d*x + c) - 60*a^3)/(d*cos(d*x + c)^7)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(121) = 242$.

Time = 0.87 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.85

$$\int (a + a\sec(c + dx))^3 \tan^5(c + dx)\,dx$$
$$= \begin{cases} \frac{a^3 \log(\tan^2(c+dx)+1)}{2d} + \frac{a^3 \tan^4(c+dx)\sec^3(c+dx)}{7d} + \frac{a^3 \tan^4(c+dx)\sec^2(c+dx)}{2d} + \frac{3a^3 \tan^4(c+dx)\sec(c+dx)}{5d} + \frac{a^3 \tan^4(c+dx)}{4d} - \\ x(a\sec(c) + a)^3 \tan^5(c) \end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**4*sec(c
 + d*x)**3/(7*d) + a**3*tan(c + d*x)**4*sec(c + d*x)**2/(2*d) + 3*a**3*tan(
c + d*x)**4*sec(c + d*x)/(5*d) + a**3*tan(c + d*x)**4/(4*d) - 4*a**3*tan(c
+ d*x)**2*sec(c + d*x)**3/(35*d) - a**3*tan(c + d*x)**2*sec(c + d*x)**2/(2*
d) - 4*a**3*tan(c + d*x)**2*sec(c + d*x)/(5*d) - a**3*tan(c + d*x)**2/(2*d)
 + 8*a**3*sec(c + d*x)**3/(105*d) + a**3*sec(c + d*x)**2/(2*d) + 8*a**3*sec
(c + d*x)/(5*d), Ne(d, 0)), (x*(a*sec(c) + a)**3*tan(c)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.80

$$\int (a + a\sec(c + dx))^3 \tan^5(c + dx)\,dx =$$
$$-\frac{420\,a^3 \log(\cos(dx + c)) - \frac{1260\,a^3 \cos(dx+c)^6 + 210\,a^3 \cos(dx+c)^5 - 700\,a^3 \cos(dx+c)^4 - 525\,a^3 \cos(dx+c)^3 + 84\,a^3 \cos(dx+c)^2 + 210}{\cos(dx+c)^7}}{420\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/420*(420*a^3*log(cos(d*x + c)) - (1260*a^3*cos(d*x + c)^6 + 210*a^3*cos(
d*x + c)^5 - 700*a^3*cos(d*x + c)^4 - 525*a^3*cos(d*x + c)^3 + 84*a^3*cos(d
*x + c)^2 + 210*a^3*cos(d*x + c) + 60*a^3)/cos(d*x + c)^7)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(126) = 252$.

Time = 2.03 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.93

$$\int (a + a\sec(c + dx))^3 \tan^5(c + dx)\, dx$$
$$= \frac{420\, a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 420\, a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{2497\, a^3 + \frac{18319\, a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{58317\, a^3(\cos(dx+c)-1)}{(\cos(dx+c)+1)^2}}{}}{420\, d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/420*(420*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a
^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2497*a^3 + 18319
*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 58317*a^3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 69475*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
 + 56035*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a^3*(cos(d*x
 + c) - 1)^5/(cos(d*x + c) + 1)^5 + 8463*a^3*(cos(d*x + c) - 1)^6/(cos(d*x
+ c) + 1)^6 + 1089*a^3*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.60

$$\int (a + a\sec(c + dx))^3 \tan^5(c + dx)\, dx = \frac{2\, a^3\, \mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2\right)}{d}$$
$$- \frac{2\, a^3 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{12} - 14\, a^3 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{10} + \frac{128\, a^3 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3} - \frac{224\, a^3 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3} + \frac{422\, a^3 \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}}{d\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{14} - 7\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{12} + 21\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^{10} - 35\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^8 + 35\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^6 - 21\,\mathrm{ta}\right.}$$

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - ((422*a^3*tan(c/2 + (d*x)/2)^4)/5 -
 (382*a^3*tan(c/2 + (d*x)/2)^2)/15 - (224*a^3*tan(c/2 + (d*x)/2)^6)/3 + (12
8*a^3*tan(c/2 + (d*x)/2)^8)/3 - 14*a^3*tan(c/2 + (d*x)/2)^10 + 2*a^3*tan(c/
2 + (d*x)/2)^12 + (352*a^3)/105)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 +
(d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2
 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.40 $\int (a+a\sec(c+dx))^3\tan^3(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int (a+a\sec(c+dx))^3\tan^3(c+dx)\,dx = \frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\sec^2(c+dx)}{d}+\frac{2a^3\sec^3(c+dx)}{3d}+\frac{3a^3\sec^4(c+dx)}{4d}+\frac{a^3\sec^5(c+dx)}{5d}$$

```
[Out] a^3*ln(cos(d*x+c))/d-3*a^3*sec(d*x+c)/d-a^3*sec(d*x+c)^2/d+2/3*a^3*sec(d*x+
c)^3/d+3/4*a^3*sec(d*x+c)^4/d+1/5*a^3*sec(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 76}

$$\int (a+a\sec(c+dx))^3\tan^3(c+dx)\,dx = \frac{a^3\sec^5(c+dx)}{5d}+\frac{3a^3\sec^4(c+dx)}{4d}+\frac{2a^3\sec^3(c+dx)}{3d}-\frac{a^3\sec^2(c+dx)}{d}-\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^3*Sec[c + d*x])/d - (a^3*Sec[c + d*x]^2)/d
 + (2*a^3*Sec[c + d*x]^3)/(3*d) + (3*a^3*Sec[c + d*x]^4)/(4*d) + (a^3*Sec[c
 + d*x]^5)/(5*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)(a+ax)^4}{x^6}\,dx, x, \cos(c+dx)\right)}{a^2 d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{3a^5}{x^5}+\frac{2a^5}{x^4}-\frac{2a^5}{x^3}-\frac{3a^5}{x^2}-\frac{a^5}{x}\right)\,dx, x, \cos(c+dx)\right)}{a^2 d}\\
&= \frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\sec^2(c+dx)}{d}\\
&\quad+\frac{2a^3\sec^3(c+dx)}{3d}+\frac{3a^3\sec^4(c+dx)}{4d}+\frac{a^3\sec^5(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$
\int (a+a\sec(c+dx))^3\tan^3(c+dx)\,dx = -\frac{a^3(142+280\cos(2(c+dx))+90\cos(4(c+dx))+\cos(3(c+dx))(60-75\log(\cos(c+dx)))-150\cos(c}{240d}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] -1/240*(a^3*(142 + 280*Cos[2*(c + d*x)] + 90*Cos[4*(c + d*x)] + Cos[3*(c +
d*x)]*(60 - 75*Log[Cos[c + d*x]]) - 150*Cos[c + d*x]*Log[Cos[c + d*x]] - 15
*Cos[5*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^5)/d
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{a^{3}\left(\frac{\sec(dx+c)^{5}}{5}+\frac{3\sec(dx+c)^{4}}{4}+\frac{2\sec(dx+c)^{3}}{3}-\sec(dx+c)^{2}-3\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^{3}\left(\frac{\sec(dx+c)^{5}}{5}+\frac{3\sec(dx+c)^{4}}{4}+\frac{2\sec(dx+c)^{3}}{3}-\sec(dx+c)^{2}-3\sec(dx+c)-\ln(\sec(dx+c))\right)}{d}$
parts	$\frac{a^{3}\left(\frac{\tan(dx+c)^{2}}{2}-\frac{\ln\left(1+\tan(dx+c)^{2}\right)}{2}\right)}{d}+\frac{a^{3}\left(\frac{\sec(dx+c)^{5}}{5}-\frac{\sec(dx+c)^{3}}{3}\right)}{d}+\frac{3a^{3}\left(\frac{\sec(dx+c)^{3}}{3}-\sec(dx+c)\right)}{d}+\frac{3a^{3}\tan(}{4d}$
risch	$-ia^{3}x-\frac{2ia^{3}c}{d}-\frac{2a^{3}\left(45\,\mathrm{e}^{9i(dx+c)}+30\,\mathrm{e}^{8i(dx+c)}+140\,\mathrm{e}^{7i(dx+c)}+142\,\mathrm{e}^{5i(dx+c)}+140\,\mathrm{e}^{3i(dx+c)}+30\,\mathrm{e}^{2i(dx+c)}+45\,\mathrm{e}^{i(}\right.}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}$

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/d*(1/5*sec(d*x+c)^5+3/4*sec(d*x+c)^4+2/3*sec(d*x+c)^3-sec(d*x+c)^2-3*se
c(d*x+c)-ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int (a+a\sec(c+dx))^{3}\tan^{3}(c+dx)\,dx$$
$$=\frac{60\,a^{3}\cos\left(dx+c\right)^{5}\log\left(-\cos\left(dx+c\right)\right)-180\,a^{3}\cos\left(dx+c\right)^{4}-60\,a^{3}\cos\left(dx+c\right)^{3}+40\,a^{3}\cos\left(dx+c\right)^{2}+}{60\,d\cos\left(dx+c\right)^{5}}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a^3*cos(d*x + c)^5*log(-cos(d*x + c)) - 180*a^3*cos(d*x + c)^4 - 6
0*a^3*cos(d*x + c)^3 + 40*a^3*cos(d*x + c)^2 + 45*a^3*cos(d*x + c) + 12*a^3
)/(d*cos(d*x + c)^5)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.67

$$\int (a+a\sec(c+dx))^3\tan^3(c+dx)\,dx$$
$$=\begin{cases}-\frac{a^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^3\tan^2\left(c+dx\right)\sec^3\left(c+dx\right)}{5d}+\frac{3a^3\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)}{4d}+\frac{a^3\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{d}+\frac{a^3\tan^2\left(c+dx\right)}{2d}-\\ x\left(a\sec\left(c\right)+a\right)^3\tan^3\left(c\right)\end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*tan(c + d*x)**2*sec(
c + d*x)**3/(5*d) + 3*a**3*tan(c + d*x)**2*sec(c + d*x)**2/(4*d) + a**3*tan
(c + d*x)**2*sec(c + d*x)/d + a**3*tan(c + d*x)**2/(2*d) - 2*a**3*sec(c + d
*x)**3/(15*d) - 3*a**3*sec(c + d*x)**2/(4*d) - 2*a**3*sec(c + d*x)/d, Ne(d,
 0)), (x*(a*sec(c) + a)**3*tan(c)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int (a+a\sec(c+dx))^3\tan^3(c+dx)\,dx$$
$$=\frac{60\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{180\,a^3\cos(dx+c)^4+60\,a^3\cos(dx+c)^3-40\,a^3\cos(dx+c)^2-45\,a^3\cos(dx+c)-12\,a^3}{\cos(dx+c)^5}}{60\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/60*(60*a^3*log(cos(d*x + c)) - (180*a^3*cos(d*x + c)^4 + 60*a^3*cos(d*x +
 c)^3 - 40*a^3*cos(d*x + c)^2 - 45*a^3*cos(d*x + c) - 12*a^3)/cos(d*x + c)^
5)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(93) = 186.

Time = 0.93 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.19

$$\int (a+a\sec(c+dx))^3\tan^3(c+dx)\,dx =$$
$$-\frac{60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{393\,a^3+\frac{2085\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2610\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+}{\left(\frac{c}{c}\right.}}{60\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^3
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (393*a^3 + 2085*a^3
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2610*a^3*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 + 1970*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805
*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^3*(cos(d*x + c) - 1)
^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 18.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.64

$$\int (a+a\sec(c+dx))^3\tan^3(c+dx)\,dx$$
$$=\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{62\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{70\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{64\,a^3}{15}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$
$$-\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((62*a^3*tan(c/2 + (d*x)/2)^4)/3 - (70*a^3*tan(c/2 + (d*x)/2)^2)/3 - 10*a^3
*tan(c/2 + (d*x)/2)^6 + 2*a^3*tan(c/2 + (d*x)/2)^8 + (64*a^3)/15)/(d*(5*tan
(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*t
an(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - (2*a^3*atanh(tan(c/2 +
(d*x)/2)^2))/d
```

3.41 $\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx=-\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{3a^3\sec^2(c+dx)}{2d}+\frac{a^3\sec^3(c+dx)}{3d}$$

```
[Out] -a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+3/2*a^3*sec(d*x+c)^2/d+1/3*a^3*sec
(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 45}

$$\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx=\frac{a^3\sec^3(c+dx)}{3d}+\frac{3a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -((a^3*Log[Cos[c + d*x]])/d) + (3*a^3*Sec[c + d*x])/d + (3*a^3*Sec[c + d*x]
^2)/(2*d) + (a^3*Sec[c + d*x]^3)/(3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a+ax)^3}{x^4}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{3a^3}{x^3}+\frac{3a^3}{x^2}+\frac{a^3}{x}\right)\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{3a^3\sec^2(c+dx)}{2d}+\frac{a^3\sec^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx = -\frac{a^3(-22-18\cos(2(c+dx))+9\cos(c+dx)(-2+\log(\cos(c+dx)))+3\cos(3(c+dx))\log(\cos(c+dx)}{12d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -1/12*(a^3*(-22 - 18*Cos[2*(c + d*x)] + 9*Cos[c + d*x]*(-2 + Log[Cos[c + d*
x]]) + 3*Cos[3*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^3)/d
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)^3}{3}+\frac{3\sec(dx+c)^2}{2}+3\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$	44
default	$\frac{a^3\left(\frac{\sec(dx+c)^3}{3}+\frac{3\sec(dx+c)^2}{2}+3\sec(dx+c)+\ln(\sec(dx+c))\right)}{d}$	44
parts	$\frac{a^3\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{a^3\sec(dx+c)^3}{3d}+\frac{3a^3\sec(dx+c)}{d}+\frac{3a^3\sec(dx+c)^2}{2d}$	67
risch	$ia^3x+\frac{2ia^3c}{d}+\frac{2a^3\left(9\,\mathrm{e}^{5i(dx+c)}+9\,\mathrm{e}^{4i(dx+c)}+22\,\mathrm{e}^{3i(dx+c)}+9\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}-\frac{a^3\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	116

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^3*(1/3*sec(d*x+c)^3+3/2*sec(d*x+c)^2+3*sec(d*x+c)+ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx$$
$$=-\frac{6\,a^3\cos\left(dx+c\right)^3\log\left(-\cos\left(dx+c\right)\right)-18\,a^3\cos\left(dx+c\right)^2-9\,a^3\cos\left(dx+c\right)-2\,a^3}{6\,d\cos\left(dx+c\right)^3}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^3*cos(d*x + c)^3*log(-cos(d*x + c)) - 18*a^3*cos(d*x + c)^2 - 9*a
^3*cos(d*x + c) - 2*a^3)/(d*cos(d*x + c)^3)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx$$
$$=\begin{cases}\frac{a^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^3\sec^3\left(c+dx\right)}{3d}+\frac{3a^3\sec^2\left(c+dx\right)}{2d}+\frac{3a^3\sec\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x(a\sec\left(c\right)+a)^3\tan\left(c\right) & \text{otherwise}\end{cases}$$

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c),x)
```

```
[Out] Piecewise((a**3*log(tan(c + d*x)**2 + 1)/(2*d) + a**3*sec(c + d*x)**3/(3*d)
 + 3*a**3*sec(c + d*x)**2/(2*d) + 3*a**3*sec(c + d*x)/d, Ne(d, 0)), (x*(a*s
ec(c) + a)**3*tan(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx=-\frac{6\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{18\,a^3\cos(dx+c)^2+9\,a^3\cos(dx+c)+2\,a^3}{\cos(dx+c)^3}}{6\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")
[Out] -1/6*(6*a^3*log(cos(d*x + c)) - (18*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c)
 + 2*a^3)/cos(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(62) = 124.

Time = 0.43 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.53

$$\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx$$

$$=\frac{6\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-6\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{51\,a^3+\frac{69\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{11\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
[Out] 1/6*(6*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a^3*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (51*a^3 + 69*a^3*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 45*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 11*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 14.59 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

$$\int (a+a\sec(c+dx))^3\tan(c+dx)\,dx$$

$$=\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{20\,a^3}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a^3*tan(c/2 + (d*x)/2)^4 - 6*a^3
*tan(c/2 + (d*x)/2)^2 + (20*a^3)/3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2
+ (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.42 $\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx=\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{3a^3\log(\cos(c+dx))}{d}+\frac{a^3\sec(c+dx)}{d}$$

```
[Out] 4*a^3*ln(1-cos(d*x+c))/d-3*a^3*ln(cos(d*x+c))/d+a^3*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 90}

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx=\frac{a^3\sec(c+dx)}{d}+\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{3a^3\log(\cos(c+dx))}{d}$$

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (4*a^3*Log[1 - Cos[c + d*x]])/d - (3*a^3*Log[Cos[c + d*x]])/d + (a^3*Sec[c
+ d*x])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^2\text{Subst}\left(\int \frac{(a+ax)^2}{x^2(a-ax)}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{a^2\text{Subst}\left(\int \left(-\frac{4a}{-1+x}+\frac{a}{x^2}+\frac{3a}{x}\right)\,dx, x, \cos(c+dx)\right)}{d} \\
&= \frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{3a^3\log(\cos(c+dx))}{d}+\frac{a^3\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3\left(-3\log(\cos(c+dx))+8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec(c+dx)\right)}{d}$$

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^3,x]
[Out] (a^3*(-3*Log[Cos[c + d*x]] + 8*Log[Sin[(c + d*x)/2]] + Sec[c + d*x]))/d
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

method	result
derivativedivides	$\frac{a^3\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+3a^3\ln(\tan(dx+c))+3a^3\ln(-\cot(dx+c)+\csc(dx+c))+a^3\ln(\sin(dx+c))}{d}$
default	$\frac{a^3\left(\frac{1}{\cos(dx+c)}+\ln(-\cot(dx+c)+\csc(dx+c))\right)+3a^3\ln(\tan(dx+c))+3a^3\ln(-\cot(dx+c)+\csc(dx+c))+a^3\ln(\sin(dx+c))}{d}$
risch	$-ia^3x-\frac{2ia^3c}{d}+\frac{2a^3e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}+\frac{8a^3\ln(e^{i(dx+c)}-1)}{d}-\frac{3a^3\ln(e^{2i(dx+c)}+1)}{d}$

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/cos(d*x+c)+ln(-cot(d*x+c)+csc(d*x+c)))+3*a^3*ln(tan(d*x+c))+3*a
^3*ln(-cot(d*x+c)+csc(d*x+c))+a^3*ln(sin(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{3\,a^3\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-4\,a^3\cos\left(dx+c\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-a^3}{d\cos\left(dx+c\right)}$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*a^3*cos(d*x + c)*log(-cos(d*x + c)) - 4*a^3*cos(d*x + c)*log(-1/2*cos(d
*x + c) + 1/2) - a^3)/(d*cos(d*x + c))
```

Sympy [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cot\left(c+dx\right)\sec\left(c+dx\right)dx + \int 3\cot\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cot\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int \cot\left(c+dx\right)dx\right)$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)*sec(c + d*x), x) + Integral(3*cot(c + d*x)*se
c(c + d*x)**2, x) + Integral(cot(c + d*x)*sec(c + d*x)**3, x) + Integral(co
t(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{4\,a^3\log\left(\cos\left(dx+c\right)-1\right)-3\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{a^3}{\cos(dx+c)}}{d}$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] (4*a^3*log(cos(d*x + c) - 1) - 3*a^3*log(cos(d*x + c)) + a^3/cos(d*x + c))/
d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(48) = 96.

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.02

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{4\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-3\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{5\,a^3+\frac{3\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
[Out] (4*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^3*log(abs(-(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 3*a^3*log(abs(-(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 1)) + (5*a^3 + 3*a^3*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.79

$$\int \cot(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{8\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a^3}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}-\frac{3\,a^3\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}{d}-\frac{a^3\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}$$

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^3,x)

[Out] (8*a^3*log(tan(c/2 + (d*x)/2)))/d - (2*a^3)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
- (3*a^3*log(tan(c/2 + (d*x)/2)^2 - 1))/d - (a^3*log(tan(c/2 + (d*x)/2)^2 +
 1))/d
```

3.43 $\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx=-\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3\log(1-\cos(c+dx))}{d}$$

```
[Out] -2*a^3/d/(1-cos(d*x+c))-a^3*ln(1-cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 45}

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx=-\frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3\log(1-\cos(c+dx))}{d}$$

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a^3)/(d*(1 - Cos[c + d*x])) - (a^3*Log[1 - Cos[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
```

```
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^4 \text{Subst}\left(\int \frac{a+ax}{(a-ax)^2} \, dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{a^4 \text{Subst}\left(\int \left(\frac{2}{a(-1+x)^2} + \frac{1}{a(-1+x)}\right) \, dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{2a^3}{d(1-\cos(c+dx))} - \frac{a^3 \log(1-\cos(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3 \, dx = -\frac{a^3\left(\cot^2\left(\frac{1}{2}(c+dx)\right)+2\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{d}$$

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
[Out] -((a^3*(Cot[(c + d*x)/2]^2 + 2*(Log[Cos[(c + d*x)/2]] + Log[Tan[(c + d*x)/2
]])))/d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

method	result
risch	$i a^3 x + \frac{2 i a^3 c}{d} + \frac{4 a^3 \mathrm{e}^{i(dx+c)}}{d\left(\mathrm{e}^{i(dx+c)}-1\right)^2} - \frac{2 a^3 \ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d}$
derivativedivides	$\frac{a^3\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)-\frac{3a^3}{2\sin(dx+c)^2}+3a^3\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc}{2}\right.}{d}$
default	$\frac{a^3\left(-\frac{\cot(dx+c)\csc(dx+c)}{2}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)-\frac{3a^3}{2\sin(dx+c)^2}+3a^3\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc}{2}\right.}{d}$

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

[Out] I*a^3*x+2*I/d*a^3*c+4*a^3*exp(I*(d*x+c))/d/(exp(I*(d*x+c))-1)^2-2/d*a^3*ln(
exp(I*(d*x+c))-1)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{2\,a^3 - (a^3\cos(dx+c) - a^3)\log\left(-\frac{1}{2}\,\cos(dx+c) + \frac{1}{2}\right)}{d\cos(dx+c) - d}$$

[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] (2*a^3 - (a^3*cos(d*x + c) - a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x
+ c) - d)

Sympy [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cot^3(c+dx)\sec(c+dx)\,dx + \int 3\cot^3(c+dx)\sec^2(c+dx)\,dx + \int \cot^3(c+dx)\sec^3(c+dx)\,dx + \int \cot^3(c+dx)\,dx\right)$$

[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**3,x)

[Out] a**3*(Integral(3*cot(c + d*x)**3*sec(c + d*x), x) + Integral(3*cot(c + d*x)
3*sec(c + d*x)2, x) + Integral(cot(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**3, x))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3\log(\cos(dx+c) - 1) - \frac{2\,a^3}{\cos(dx+c)-1}}{d}$$

[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] -(a^3*log(cos(d*x + c) - 1) - 2*a^3/(cos(d*x + c) - 1))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(38) = 76$.

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.72

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{\left(a^3+\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{d}$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^3*log(abs(-(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^3 + a^3*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 14.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3\left(\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{d}$$

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*(2*log(tan(c/2 + (d*x)/2)) - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2
+ (d*x)/2)^2))/d
```

3.44 $\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3}{2d(1-\cos(c+dx))^2}+\frac{2a^3}{d(1-\cos(c+dx))} + \frac{a^3\log(1-\cos(c+dx))}{d}$$

```
[Out] -1/2*a^3/d/(1-cos(d*x+c))^2+2*a^3/d/(1-cos(d*x+c))+a^3*ln(1-cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 45}

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{2a^3}{d(1-\cos(c+dx))}-\frac{a^3}{2d(1-\cos(c+dx))^2} + \frac{a^3\log(1-\cos(c+dx))}{d}$$

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*a^3/(d*(1 - Cos[c + d*x])^2) + (2*a^3)/(d*(1 - Cos[c + d*x])) + (a^3*L
og[1 - Cos[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a^6\text{Subst}\left(\int \frac{x^2}{(a-ax)^3}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^6\text{Subst}\left(\int \left(-\frac{1}{a^3(-1+x)^3}-\frac{2}{a^3(-1+x)^2}-\frac{1}{a^3(-1+x)}\right)\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^3}{2d(1-\cos(c+dx))^2}+\frac{2a^3}{d(1-\cos(c+dx))}+\frac{a^3\log(1-\cos(c+dx))}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$
\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3(1+\cos(c+dx))^3\left(-8\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)-16\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^6\left(\frac{1}{2}(c+dx\right.}{64d}
$$

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
[Out] -1/64*(a^3*(1 + Cos[c + d*x])^3*(-8*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^4
 - 16*Log[Sin[(c + d*x)/2]])*Sec[(c + d*x)/2]^6)/d
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.54

method	result
risch	$-ia^3x-\frac{2ia^3c}{d}-\frac{2a^3\left(2\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{i(dx+c)}\right)}{d\left(\mathrm{e}^{i(dx+c)}-1\right)^4}+\frac{2a^3\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d}$
derivativedivides	$\frac{a^3\left(-\frac{\cos(dx+c)^3}{4\sin(dx+c)^4}-\frac{\cos(dx+c)^3}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)-\frac{3a^3\cos(dx+c)^4}{4\sin(dx+c)^4}+3a^3\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c}{8\sin(dx+c}\right.}{d}$
default	$\frac{a^3\left(-\frac{\cos(dx+c)^3}{4\sin(dx+c)^4}-\frac{\cos(dx+c)^3}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{8}\right)-\frac{3a^3\cos(dx+c)^4}{4\sin(dx+c)^4}+3a^3\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c}{8\sin(dx+c}\right.}{d}$

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -I*a^3*x-2*I/d*a^3*c-2*a^3/d/(exp(I*(d*x+c))-1)^4*(2*exp(3*I*(d*x+c))-3*exp
(2*I*(d*x+c))+2*exp(I*(d*x+c)))+2/d*a^3*ln(exp(I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{4\,a^3\cos\left(dx+c\right)-3\,a^3-2\,\left(a^3\cos\left(dx+c\right)^2-2\,a^3\cos\left(dx+c\right)+a^3\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,\left(d\cos\left(dx+c\right)^2-2\,d\cos\left(dx+c\right)+d\right)}$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*cos(d*x + c) - 3*a^3 - 2*(a^3*cos(d*x + c)^2 - 2*a^3*cos(d*x +
c) + a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - 2*d*cos(d*x + c
) + d)
```

Sympy [F]

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cot^5\left(c+dx\right)\sec\left(c+dx\right)dx + \int 3\cot^5\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cot^5\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int \cot^5\left(c+dx\right)dx\right)$$

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)**5*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**5*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**5*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**5, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx=\frac{2\,a^3\log\left(\cos\left(dx+c\right)-1\right)-\frac{4\,a^3\cos(dx+c)-3\,a^3}{\cos(dx+c)^2-2\,\cos(dx+c)+1}}{2\,d}$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^3*log(cos(d*x + c) - 1) - (4*a^3*cos(d*x + c) - 3*a^3)/(cos(d*x +
c)^2 - 2*cos(d*x + c) + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(55) = 110.

Time = 0.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.26

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{8\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-8\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a^3+\frac{6\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}}{8\,d}$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*a^3*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a^3 + 6*a^3*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 12*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^3\,dx=\frac{2\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}-\frac{a^3}{8}}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}-\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*a^3*log(tan(c/2 + (d*x)/2)))/d + ((3*a^3*tan(c/2 + (d*x)/2)^2)/4 - a^3/8
)/(d*tan(c/2 + (d*x)/2)^4) - (a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.45 $\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3}{6d(1-\cos(c+dx))^3}+\frac{7a^3}{8d(1-\cos(c+dx))^2} - \frac{17a^3}{8d(1-\cos(c+dx))}-\frac{15a^3\log(1-\cos(c+dx))}{16d} - \frac{a^3\log(1+\cos(c+dx))}{16d}$$

```
[Out] -1/6*a^3/d/(1-cos(d*x+c))^3+7/8*a^3/d/(1-cos(d*x+c))^2-17/8*a^3/d/(1-cos(d*
x+c))-15/16*a^3*ln(1-cos(d*x+c))/d-1/16*a^3*ln(1+cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{17a^3}{8d(1-\cos(c+dx))}+\frac{7a^3}{8d(1-\cos(c+dx))^2} - \frac{a^3}{6d(1-\cos(c+dx))^3}-\frac{15a^3\log(1-\cos(c+dx))}{16d} - \frac{a^3\log(\cos(c+dx)+1)}{16d}$$

```
[In] Int[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/6*a^3/(d*(1 - Cos[c + d*x])^3) + (7*a^3)/(8*d*(1 - Cos[c + d*x])^2) - (1
7*a^3)/(8*d*(1 - Cos[c + d*x])) - (15*a^3*Log[1 - Cos[c + d*x]])/(16*d) - (
a^3*Log[1 + Cos[c + d*x]])/(16*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^8\text{Subst}\left(\int \frac{x^4}{(a-ax)^4(a+ax)}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^8\text{Subst}\left(\int \left(\frac{1}{2a^5(-1+x)^4}+\frac{7}{4a^5(-1+x)^3}+\frac{17}{8a^5(-1+x)^2}+\frac{15}{16a^5(-1+x)}+\frac{1}{16a^5(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^3}{6d(1-\cos(c+dx))^3}+\frac{7a^3}{8d(1-\cos(c+dx))^2}-\frac{17a^3}{8d(1-\cos(c+dx))}\\
&\quad -\frac{15a^3\log(1-\cos(c+dx))}{16d}-\frac{a^3\log(1+\cos(c+dx))}{16d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{a^3(1+\cos(c+dx))^3\left(102\csc^2\left(\frac{1}{2}(c+dx)\right)-21\csc^4\left(\frac{1}{2}(c+dx)\right)+2\csc^6\left(\frac{1}{2}(c+dx)\right)+12\left(\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.\right.}{768d}$$

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/768*(a^3*(1 + Cos[c + d*x])^3*(102*Csc[(c + d*x)/2]^2 - 21*Csc[(c + d*x)
/2]^4 + 2*Csc[(c + d*x)/2]^6 + 12*(Log[Cos[(c + d*x)/2]] + 15*Log[Sin[(c +
d*x)/2]]))*Sec[(c + d*x)/2]^6)/d
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.27

method	result
risch	$i a^{3} x+\frac{2 i a^{3} c}{d}+\frac{a^{3}\left(51 \mathrm{e}^{5 i(d x+c)}-162 \mathrm{e}^{4 i(d x+c)}+238 \mathrm{e}^{3 i(d x+c)}-162 \mathrm{e}^{2 i(d x+c)}+51 \mathrm{e}^{i(d x+c)}\right)}{12 d\left(\mathrm{e}^{i(d x+c)}-1\right)^{6}}-\frac{15 a^{3} \ln \left(\mathrm{e}^{i(d x+c)}-1\right)}{8 d}$
derivativedivides	$\frac{a^{3}\left(-\frac{\cos(dx+c)^{5}}{6\sin(dx+c)^{6}}-\frac{\cos(dx+c)^{5}}{24\sin(dx+c)^{4}}+\frac{\cos(dx+c)^{5}}{48\sin(dx+c)^{2}}+\frac{\cos(dx+c)^{3}}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right)-\frac{a^{3}\cos(dx+c)^{6}}{2\sin(dx+c)^{6}}+3a^{3}}{}$
default	$\frac{a^{3}\left(-\frac{\cos(dx+c)^{5}}{6\sin(dx+c)^{6}}-\frac{\cos(dx+c)^{5}}{24\sin(dx+c)^{4}}+\frac{\cos(dx+c)^{5}}{48\sin(dx+c)^{2}}+\frac{\cos(dx+c)^{3}}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{16}\right)-\frac{a^{3}\cos(dx+c)^{6}}{2\sin(dx+c)^{6}}+3a^{3}}{}$

```
[In] int(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] I*a^3*x+2*I/d*a^3*c+1/12*a^3/d/(exp(I*(d*x+c))-1)^6*(51*exp(5*I*(d*x+c))-16
2*exp(4*I*(d*x+c))+238*exp(3*I*(d*x+c))-162*exp(2*I*(d*x+c))+51*exp(I*(d*x+
c)))-15/8/d*a^3*ln(exp(I*(d*x+c))-1)-1/8/d*a^3*ln(exp(I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.66

$$\int \cot^{7}(c+dx)(a+a\sec(c+dx))^{3}\,dx$$
$$=\frac{102\,a^{3}\cos\left(dx+c\right)^{2}-162\,a^{3}\cos\left(dx+c\right)+68\,a^{3}-3\,\left(a^{3}\cos\left(dx+c\right)^{3}-3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+\right.\right.}{48\,\left(d\cos\left(dx+c\right)^{3}-\right.}$$

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(102*a^3*cos(d*x + c)^2 - 162*a^3*cos(d*x + c) + 68*a^3 - 3*(a^3*cos(d
*x + c)^3 - 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) - a^3)*log(1/2*cos(d*
x + c) + 1/2) - 45*(a^3*cos(d*x + c)^3 - 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d
*x + c) - a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - 3*d*cos(d*
x + c)^2 + 3*d*cos(d*x + c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**7*(a+a*sec(d*x+c))**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{3\,a^3\log\left(\cos\left(dx+c\right)+1\right)+45\,a^3\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(51\,a^3\cos(dx+c)^2-81\,a^3\cos(dx+c)+34\,a^3\right)}{\cos(dx+c)^3-3\,\cos(dx+c)^2+3\,\cos(dx+c)-1}}{48\,d}$$

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] -1/48*(3*a^3*log(cos(d*x + c) + 1) + 45*a^3*log(cos(d*x + c) - 1) - 2*(51*a
^3*cos(d*x + c)^2 - 81*a^3*cos(d*x + c) + 34*a^3)/(cos(d*x + c)^3 - 3*cos(d
*x + c)^2 + 3*cos(d*x + c) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.54

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{90\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-96\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(2\,a^3+\frac{15\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{66\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{165\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\cos(dx+c)+1)^3}{(\cos(dx+c)-1)^3}}{96\,d}$$

```
[In] integrate(cot(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="giac")
[Out] -1/96*(90*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 96*a^3*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (2*a^3 + 15*a^3*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 66*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 165*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c)
+ 1)^3/(cos(d*x + c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.88

$$\int \cot^7(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3\ \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d} - \frac{\frac{11\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}-\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}+\frac{a^3}{6}}{8\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6} - \frac{15\,a^3\ \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

```
[In] int(cot(c + d*x)^7*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - ((11*a^3*tan(c/2 + (d*x)/2)^4)/2 -
(5*a^3*tan(c/2 + (d*x)/2)^2)/4 + a^3/6)/(8*d*tan(c/2 + (d*x)/2)^6) - (15*a^
3*log(tan(c/2 + (d*x)/2)))/(8*d)
```

3.46 $\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\begin{aligned}\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx = &-\frac{a^3}{16d(1-\cos(c+dx))^4}+\frac{5a^3}{12d(1-\cos(c+dx))^3}\\ &-\frac{39a^3}{32d(1-\cos(c+dx))^2}+\frac{9a^3}{4d(1-\cos(c+dx))}\\ &+\frac{a^3}{32d(1+\cos(c+dx))}\\ &+\frac{57a^3\log(1-\cos(c+dx))}{64d}\\ &+\frac{7a^3\log(1+\cos(c+dx))}{64d}\end{aligned}$$

```
[Out] -1/16*a^3/d/(1-cos(d*x+c))^4+5/12*a^3/d/(1-cos(d*x+c))^3-39/32*a^3/d/(1-cos
(d*x+c))^2+9/4*a^3/d/(1-cos(d*x+c))+1/32*a^3/d/(1+cos(d*x+c))+57/64*a^3*ln(
1-cos(d*x+c))/d+7/64*a^3*ln(1+cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used

$= \{3964, 90\}$

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{9a^3}{4d(1-\cos(c+dx))}+\frac{a^3}{32d(\cos(c+dx)+1)} - \frac{39a^3}{32d(1-\cos(c+dx))^2}+\frac{5a^3}{12d(1-\cos(c+dx))^3} - \frac{a^3}{16d(1-\cos(c+dx))^4} + \frac{57a^3\log(1-\cos(c+dx))}{64d} + \frac{7a^3\log(\cos(c+dx)+1)}{64d}$$

```
[In] Int[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/16*a^3/(d*(1 - Cos[c + d*x])^4) + (5*a^3)/(12*d*(1 - Cos[c + d*x])^3) -
(39*a^3)/(32*d*(1 - Cos[c + d*x])^2) + (9*a^3)/(4*d*(1 - Cos[c + d*x])) + a
^3/(32*d*(1 + Cos[c + d*x])) + (57*a^3*Log[1 - Cos[c + d*x]])/(64*d) + (7*a
^3*Log[1 + Cos[c + d*x]])/(64*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\text{integral} = -\frac{a^{10}\text{Subst}\left(\int \frac{x^6}{(a-ax)^5(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^{10}\text{Subst}\left(\int \left(-\frac{1}{4a^7(-1+x)^5}-\frac{5}{4a^7(-1+x)^4}-\frac{39}{16a^7(-1+x)^3}-\frac{9}{4a^7(-1+x)^2}-\frac{57}{64a^7(-1+x)}+\frac{1}{32a^7(1+x)^2}-\frac{7}{64a^7(\right.\right.}{d}$$

$$= -\frac{a^3}{16d(1-\cos(c+dx))^4} + \frac{5a^3}{12d(1-\cos(c+dx))^3} - \frac{39a^3}{32d(1-\cos(c+dx))^2} + \frac{9a^3}{4d(1-\cos(c+dx))} + \frac{a^3}{32d(1+\cos(c+dx))} + \frac{57a^3\log(1-\cos(c+dx))}{64d} + \frac{7a^3\log(1+\cos(c+dx))}{64d}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(864\csc^2\left(\frac{1}{2}(c+dx)\right) - 234\csc^4\left(\frac{1}{2}(c+dx)\right) + 40\csc^6\left(\frac{1}{2}(c+dx)\right) +\right.}{6144d}$$

```
[In] Integrate[Cot[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(864*Csc[(c + d*x)/2]^2 - 234*
Csc[(c + d*x)/2]^4 + 40*Csc[(c + d*x)/2]^6 - 3*Csc[(c + d*x)/2]^8 + 12*(14*
Log[Cos[(c + d*x)/2]] + 114*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2)))/(
6144*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.23 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30

method	result
risch	$-ia^3x - \frac{2ia^3c}{d} - \frac{a^3\left(213\,\mathrm{e}^{9i(dx+c)} - 606\,\mathrm{e}^{8i(dx+c)} + 472\,\mathrm{e}^{7i(dx+c)} + 846\,\mathrm{e}^{6i(dx+c)} - 1658\,\mathrm{e}^{5i(dx+c)} + 846\,\mathrm{e}^{4i(dx+c)} + 4\right.}{48d\left(\mathrm{e}^{i(dx+c)}-1\right)^8\left(\mathrm{e}^{i(dx+c)}+1\right)^2}$
derivativedivides	$\frac{a^3\left(-\frac{\cos(dx+c)^7}{8\sin(dx+c)^8} - \frac{\cos(dx+c)^7}{48\sin(dx+c)^6} + \frac{\cos(dx+c)^7}{192\sin(dx+c)^4} - \frac{\cos(dx+c)^7}{128\sin(dx+c)^2} - \frac{\cos(dx+c)^5}{128} - \frac{5\cos(dx+c)^3}{384} - \frac{5\cos(dx+c)}{128} - \frac{5\ln(-\cot(dx+\right.}{1}$
default	$\frac{a^3\left(-\frac{\cos(dx+c)^7}{8\sin(dx+c)^8} - \frac{\cos(dx+c)^7}{48\sin(dx+c)^6} + \frac{\cos(dx+c)^7}{192\sin(dx+c)^4} - \frac{\cos(dx+c)^7}{128\sin(dx+c)^2} - \frac{\cos(dx+c)^5}{128} - \frac{5\cos(dx+c)^3}{384} - \frac{5\cos(dx+c)}{128} - \frac{5\ln(-\cot(dx+\right.}{1}$

```
[In] int(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -I*a^3*x-2*I/d*a^3*c-1/48*a^3/d/(exp(I*(d*x+c))-1)^8/(exp(I*(d*x+c))+1)^2*(
213*exp(9*I*(d*x+c))-606*exp(8*I*(d*x+c))+472*exp(7*I*(d*x+c))+846*exp(6*I*
(d*x+c))-1658*exp(5*I*(d*x+c))+846*exp(4*I*(d*x+c))+472*exp(3*I*(d*x+c))-60
6*exp(2*I*(d*x+c))+213*exp(I*(d*x+c)))+7/32/d*a^3*ln(exp(I*(d*x+c))+1)+57/3
2/d*a^3*ln(exp(I*(d*x+c))-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(127) = 254$.

Time = 0.29 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.83

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{426\,a^3\cos\left(dx+c\right)^4 - 606\,a^3\cos\left(dx+c\right)^3 - 190\,a^3\cos\left(dx+c\right)^2 + 666\,a^3\cos\left(dx+c\right) - 272\,a^3 - 21\,\left(a^3\right.}{}$$

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(426*a^3*cos(d*x + c)^4 - 606*a^3*cos(d*x + c)^3 - 190*a^3*cos(d*x +
 c)^2 + 666*a^3*cos(d*x + c) - 272*a^3 - 21*(a^3*cos(d*x + c)^5 - 3*a^3*cos
(d*x + c)^4 + 2*a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x +
 c) + a^3)*log(1/2*cos(d*x + c) + 1/2) - 171*(a^3*cos(d*x + c)^5 - 3*a^3*co
s(d*x + c)^4 + 2*a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x
+ c) + a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^5 - 3*d*cos(d*x +
 c)^4 + 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - 3*d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**9*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{21\,a^3\log\left(\cos\left(dx+c\right)+1\right) + 171\,a^3\log\left(\cos\left(dx+c\right)-1\right) - \frac{2\left(213\,a^3\cos(dx+c)^4 - 303\,a^3\cos(dx+c)^3 - 95\,a^3\cos(dx+c)^2 + 3\right.}{\cos(dx+c)^5 - 3\,\cos(dx+c)^4 + 2\,\cos(dx+c)^3 + 2\,\cos(dx+c)}}{192\,d}$$

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/192*(21*a^3*log(cos(d*x + c) + 1) + 171*a^3*log(cos(d*x + c) - 1) - 2*(21
3*a^3*cos(d*x + c)^4 - 303*a^3*cos(d*x + c)^3 - 95*a^3*cos(d*x + c)^2 + 333
*a^3*cos(d*x + c) - 136*a^3)/(cos(d*x + c)^5 - 3*cos(d*x + c)^4 + 2*cos(d*x
 + c)^3 + 2*cos(d*x + c)^2 - 3*cos(d*x + c) + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.43

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{684\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 768\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{\left(3\,a^3+\frac{28\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a^3}{(\cos}\right.}{}}{768\,d}$$

```
[In] integrate(cot(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(684*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 768*a^3*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^3*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - (3*a^3 + 28*a^3*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 132*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 504*a^3*(cos(d*
x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1425*a^3*(cos(d*x + c) - 1)^4/(cos(d*x
 + c) + 1)^4)*(cos(d*x + c) + 1)^4/(cos(d*x + c) - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 14.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

$$\int \cot^9(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64\,d} + \frac{57\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{32\,d}$$
$$+ \frac{21\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - \frac{11\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2} + \frac{7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6} - \frac{a^3}{8}}{32\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}$$
$$- \frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

```
[In] int(cot(c + d*x)^9*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(64*d) + (57*a^3*log(tan(c/2 + (d*x)/2)))/(32*d)
 + ((7*a^3*tan(c/2 + (d*x)/2)^2)/6 - (11*a^3*tan(c/2 + (d*x)/2)^4)/2 + 21*a
^3*tan(c/2 + (d*x)/2)^6 - a^3/8)/(32*d*tan(c/2 + (d*x)/2)^8) - (a^3*log(tan
(c/2 + (d*x)/2)^2 + 1))/d
```

3.47 $\int (a+a\sec(c+dx))^3\tan^6(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 237

$$\begin{aligned}\int (a+a\sec(c+dx))^3\tan^6(c+dx)\,dx = {} & -a^3x-\frac{125a^3\operatorname{arctanh}(\sin(c+dx))}{128d}\\ & +\frac{a^3\tan(c+dx)}{d}+\frac{115a^3\sec(c+dx)\tan(c+dx)}{128d}\\ & +\frac{5a^3\sec^3(c+dx)\tan(c+dx)}{64d}\\ & -\frac{a^3\tan^3(c+dx)}{3d}-\frac{5a^3\sec(c+dx)\tan^3(c+dx)}{8d}\\ & -\frac{5a^3\sec^3(c+dx)\tan^3(c+dx)}{48d}\\ & +\frac{a^3\tan^5(c+dx)}{5d}+\frac{a^3\sec(c+dx)\tan^5(c+dx)}{2d}\\ & +\frac{a^3\sec^3(c+dx)\tan^5(c+dx)}{8d}+\frac{3a^3\tan^7(c+dx)}{7d}\end{aligned}$$

```
[Out] -a^3*x-125/128*a^3*arctanh(sin(d*x+c))/d+a^3*tan(d*x+c)/d+115/128*a^3*sec(d
*x+c)*tan(d*x+c)/d+5/64*a^3*sec(d*x+c)^3*tan(d*x+c)/d-1/3*a^3*tan(d*x+c)^3/
d-5/8*a^3*sec(d*x+c)*tan(d*x+c)^3/d-5/48*a^3*sec(d*x+c)^3*tan(d*x+c)^3/d+1/
5*a^3*tan(d*x+c)^5/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)^5/d+1/8*a^3*sec(d*x+c)^3
*tan(d*x+c)^5/d+3/7*a^3*tan(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$\int (a+a\sec(c+dx))^3\tan^6(c+dx)\,dx = -\frac{125a^3\text{arctanh}(\sin(c+dx))}{128d}+\frac{3a^3\tan^7(c+dx)}{7d}+\frac{a^3\tan^5(c+dx)}{5d}-\frac{a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan^5(c+dx)\sec^3(c+dx)}{8d}-\frac{5a^3\tan^3(c+dx)\sec^3(c+dx)}{48d}+\frac{5a^3\tan(c+dx)\sec^3(c+dx)}{64d}+\frac{a^3\tan^5(c+dx)\sec(c+dx)}{2d}-\frac{5a^3\tan^3(c+dx)\sec(c+dx)}{8d}+\frac{115a^3\tan(c+dx)\sec(c+dx)}{128d}-a^3x$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^6,x]
```

```
[Out] -(a^3*x) - (125*a^3*ArcTanh[Sin[c + d*x]])/(128*d) + (a^3*Tan[c + d*x])/d +
 (115*a^3*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*a^3*Sec[c + d*x]^3*Tan[c
+ d*x])/(64*d) - (a^3*Tan[c + d*x]^3)/(3*d) - (5*a^3*Sec[c + d*x]*Tan[c + d
*x]^3)/(8*d) - (5*a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(48*d) + (a^3*Tan[c +
d*x]^5)/(5*d) + (a^3*Sec[c + d*x]*Tan[c + d*x]^5)/(2*d) + (a^3*Sec[c + d*x]
^3*Tan[c + d*x]^5)/(8*d) + (3*a^3*Tan[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3\tan^6(c+dx)+3a^3\sec(c+dx)\tan^6(c+dx)+3a^3\sec^2(c+dx)\tan^6(c+dx)\right.\\
&\qquad\left.+a^3\sec^3(c+dx)\tan^6(c+dx)\right)\,dx\\
&= a^3\int \tan^6(c+dx)\,dx+a^3\int \sec^3(c+dx)\tan^6(c+dx)\,dx\\
&\quad+\left(3a^3\right)\int \sec(c+dx)\tan^6(c+dx)\,dx+\left(3a^3\right)\int \sec^2(c+dx)\tan^6(c+dx)\,dx
\end{aligned}$$

$$= \frac{a^3 \tan^5(c+dx)}{5d} + \frac{a^3 \sec(c+dx)\tan^5(c+dx)}{2d} + \frac{a^3 \sec^3(c+dx)\tan^5(c+dx)}{8d} - \frac{1}{8}(5a^3)\int \sec^3(c+dx)\tan^4(c+dx)\,dx - a^3\int \tan^4(c+dx)\,dx - \frac{1}{2}(5a^3)\int \sec(c+dx)\tan^4(c+dx)\,dx + \frac{(3a^3)\,\mathrm{Subst}\left(\int x^6\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{a^3 \tan^3(c+dx)}{3d} - \frac{5a^3 \sec(c+dx)\tan^3(c+dx)}{8d} - \frac{5a^3 \sec^3(c+dx)\tan^3(c+dx)}{48d} + \frac{a^3 \tan^5(c+dx)}{5d} + \frac{a^3 \sec(c+dx)\tan^5(c+dx)}{2d} + \frac{a^3 \sec^3(c+dx)\tan^5(c+dx)}{8d} + \frac{3a^3 \tan^7(c+dx)}{7d} + \frac{1}{16}(5a^3)\int \sec^3(c+dx)\tan^2(c+dx)\,dx + a^3\int \tan^2(c+dx)\,dx + \frac{1}{8}(15a^3)\int \sec(c+dx)\tan^2(c+dx)\,dx$$

$$= \frac{a^3 \tan(c+dx)}{d} + \frac{15a^3 \sec(c+dx)\tan(c+dx)}{16d} + \frac{5a^3 \sec^3(c+dx)\tan(c+dx)}{64d} - \frac{a^3 \tan^3(c+dx)}{3d} - \frac{5a^3 \sec(c+dx)\tan^3(c+dx)}{8d} - \frac{5a^3 \sec^3(c+dx)\tan^3(c+dx)}{48d} + \frac{a^3 \tan^5(c+dx)}{5d} + \frac{a^3 \sec(c+dx)\tan^5(c+dx)}{2d} + \frac{a^3 \sec^3(c+dx)\tan^5(c+dx)}{8d} + \frac{3a^3 \tan^7(c+dx)}{7d} - \frac{1}{64}(5a^3)\int \sec^3(c+dx)\,dx - \frac{1}{16}(15a^3)\int \sec(c+dx)\,dx - a^3\int 1\,dx$$

$$= -a^3x - \frac{15a^3\operatorname{arctanh}(\sin(c+dx))}{16d} + \frac{a^3 \tan(c+dx)}{d} + \frac{115a^3 \sec(c+dx)\tan(c+dx)}{128d} + \frac{5a^3 \sec^3(c+dx)\tan(c+dx)}{64d} - \frac{a^3 \tan^3(c+dx)}{3d} - \frac{5a^3 \sec(c+dx)\tan^3(c+dx)}{8d} - \frac{5a^3 \sec^3(c+dx)\tan^3(c+dx)}{48d} + \frac{a^3 \tan^5(c+dx)}{5d} + \frac{a^3 \sec(c+dx)\tan^5(c+dx)}{2d} + \frac{a^3 \sec^3(c+dx)\tan^5(c+dx)}{8d} + \frac{3a^3 \tan^7(c+dx)}{7d} - \frac{1}{128}(5a^3)\int \sec(c+dx)\,dx$$

$$= -a^3x - \frac{125a^3\operatorname{arctanh}(\sin(c+dx))}{128d} + \frac{a^3 \tan(c+dx)}{d} + \frac{115a^3 \sec(c+dx)\tan(c+dx)}{128d} + \frac{5a^3 \sec^3(c+dx)\tan(c+dx)}{64d} - \frac{a^3 \tan^3(c+dx)}{3d} - \frac{5a^3 \sec(c+dx)\tan^3(c+dx)}{8d} - \frac{5a^3 \sec^3(c+dx)\tan^3(c+dx)}{48d} + \frac{a^3 \tan^5(c+dx)}{5d} + \frac{a^3 \sec(c+dx)\tan^5(c+dx)}{2d} + \frac{a^3 \sec^3(c+dx)\tan^5(c+dx)}{8d} + \frac{3a^3 \tan^7(c+dx)}{7d}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.23

$$\int (a+a\sec(c+dx))^3\tan^6(c+dx)\,dx = -\frac{a^3\arctan(\tan(c+dx))}{d} - \frac{125a^3\text{arctanh}(\sin(c+dx))}{128d} + \frac{a^3\tan(c+dx)}{d} - \frac{125a^3\sec(c+dx)\tan(c+dx)}{128d} - \frac{125a^3\sec^3(c+dx)\tan(c+dx)}{192d} + \frac{119a^3\sec^5(c+dx)\tan(c+dx)}{48d} + \frac{a^3\sec^7(c+dx)\tan(c+dx)}{8d} - \frac{a^3\tan^3(c+dx)}{3d} - \frac{5a^3\sec^3(c+dx)\tan^3(c+dx)}{d} - \frac{a^3\sec^5(c+dx)\tan^3(c+dx)}{3d} + \frac{a^3\tan^5(c+dx)}{5d} + \frac{3a^3\sec(c+dx)\tan^5(c+dx)}{d} + \frac{a^3\sec^3(c+dx)\tan^5(c+dx)}{3d} + \frac{3a^3\tan^7(c+dx)}{7d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^6,x]
```

```
[Out] -((a^3*ArcTan[Tan[c + d*x]])/d) - (125*a^3*ArcTanh[Sin[c + d*x]])/(128*d) +
 (a^3*Tan[c + d*x])/d - (125*a^3*Sec[c + d*x]*Tan[c + d*x])/(128*d) - (125*
a^3*Sec[c + d*x]^3*Tan[c + d*x])/(192*d) + (119*a^3*Sec[c + d*x]^5*Tan[c +
d*x])/(48*d) + (a^3*Sec[c + d*x]^7*Tan[c + d*x])/(8*d) - (a^3*Tan[c + d*x]^
3)/(3*d) - (5*a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/d - (a^3*Sec[c + d*x]^5*Ta
n[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d) + (3*a^3*Sec[c + d*x]*Tan[
c + d*x]^5)/d + (a^3*Sec[c + d*x]^3*Tan[c + d*x]^5)/(3*d) + (3*a^3*Tan[c +
d*x]^7)/(7*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.93 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.96

method	result
risch	$-a^3x-\frac{ia^3(27195\,\mathrm{e}^{15i(dx+c)}+65135\,\mathrm{e}^{13i(dx+c)}-161280\,\mathrm{e}^{12i(dx+c)}+63595\,\mathrm{e}^{11i(dx+c)}-286720\,\mathrm{e}^{10i(dx+c)}+133175\,\mathrm{e}^{9}}{}$
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^7}{8\cos(dx+c)^8}+\frac{\sin(dx+c)^7}{48\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{192\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{128\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{128}+\frac{5\sin(dx+c)^3}{384}+\frac{5\sin(dx+c)}{128}-\frac{5\ln(\sec(dx+c)+}{128}\right.}{}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^7}{8\cos(dx+c)^8}+\frac{\sin(dx+c)^7}{48\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{192\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{128\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{128}+\frac{5\sin(dx+c)^3}{384}+\frac{5\sin(dx+c)}{128}-\frac{5\ln(\sec(dx+c)+}{128}\right.}{}$
parts	$\frac{a^3\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{a^3\left(\frac{\sin(dx+c)^7}{8\cos(dx+c)^8}+\frac{\sin(dx+c)^7}{48\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{192\cos(dx+c)^4}+\right.}{}$

[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x,method=_RETURNVERBOSE)

[Out] -a^3*x-1/6720*I*a^3*(27195*exp(15*I*(d*x+c))+65135*exp(13*I*(d*x+c))-161280
*exp(12*I*(d*x+c))+63595*exp(11*I*(d*x+c))-286720*exp(10*I*(d*x+c))+133175*
exp(9*I*(d*x+c))-519680*exp(8*I*(d*x+c))-133175*exp(7*I*(d*x+c))-544768*exp
(6*I*(d*x+c))-63595*exp(5*I*(d*x+c))-254464*exp(4*I*(d*x+c))-65135*exp(3*I*
(d*x+c))-118784*exp(2*I*(d*x+c))-27195*exp(I*(d*x+c))-14848)/d/(exp(2*I*(d*
x+c))+1)^8-125/128*a^3/d*ln(exp(I*(d*x+c))+I)+125/128*a^3/d*ln(exp(I*(d*x+c
))-I)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.75

$$\int (a+a\sec(c+dx))^3\tan^6(c+dx)\,dx =$$

$$-\frac{26880\,a^3dx\cos(dx+c)^8+13125\,a^3\cos(dx+c)^8\log(\sin(dx+c)+1)-13125\,a^3\cos(dx+c)^8\log(-\mathrm{s}}{}$$

[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="fricas")

[Out] -1/26880*(26880*a^3*d*x*cos(d*x + c)^8 + 13125*a^3*cos(d*x + c)^8*log(sin(d
*x + c) + 1) - 13125*a^3*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(14848*a
^3*cos(d*x + c)^7 + 27195*a^3*cos(d*x + c)^6 + 7424*a^3*cos(d*x + c)^5 - 17
710*a^3*cos(d*x + c)^4 - 14592*a^3*cos(d*x + c)^3 + 1960*a^3*cos(d*x + c)^2
 + 5760*a^3*cos(d*x + c) + 1680*a^3)*sin(d*x + c))/(d*cos(d*x + c)^8)

Sympy [F]

$$\int (a + a\sec(c + dx))^3 \tan^6(c + dx)\, dx = a^3 \left(\int 3\tan^6(c + dx)\sec(c + dx)\, dx + \int 3\tan^6(c + dx)\sec^2(c + dx)\, dx + \int \tan^6(c + dx)\sec^3(c + dx)\, dx + \int \tan^6(c + dx)\, dx \right)$$

[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**6,x)

[Out] a**3*(Integral(3*tan(c + d*x)**6*sec(c + d*x), x) + Integral(3*tan(c + d*x)**6*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**6*sec(c + d*x)**3, x) + Integral(tan(c + d*x)**6, x))

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.11

$$\int (a + a\sec(c + dx))^3 \tan^6(c + dx)\, dx$$

$$= \frac{11520\, a^3 \tan(dx + c)^7 + 1792\left(3\, \tan(dx + c)^5 - 5\, \tan(dx + c)^3 - 15\, dx - 15\, c + 15\, \tan(dx + c)\right)a^3 + 35}{}$$

[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="maxima")

[Out] 1/26880*(11520*a^3*tan(d*x + c)^7 + 1792*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^3 + 35*a^3*(2*(15*sin(d*x + c)^7 + 73*sin(d*x + c)^5 - 55*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 840*a^3*(2*(33*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d

Giac [A] (verification not implemented)

none

Time = 2.69 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.83

$$\int (a + a\sec(c+dx))^3 \tan^6(c+dx)\,dx =$$

$$-\frac{13440\,(dx+c)a^3 + 13125\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 13125\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\,(315\cdots}{\cdots}}{\cdots}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/13440*(13440*(d*x + c)*a^3 + 13125*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - 13125*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(315*a^3*tan(1/2*d*x +
 1/2*c)^15 - 11375*a^3*tan(1/2*d*x + 1/2*c)^13 + 79723*a^3*tan(1/2*d*x + 1/
2*c)^11 - 269879*a^3*tan(1/2*d*x + 1/2*c)^9 + 550089*a^3*tan(1/2*d*x + 1/2*
c)^7 - 749973*a^3*tan(1/2*d*x + 1/2*c)^5 + 212625*a^3*tan(1/2*d*x + 1/2*c)^
3 - 26565*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 15.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.11

$$\int (a + a\sec(c+dx))^3 \tan^6(c+dx)\,dx = -a^3\,x - \frac{125\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,d}$$

$$-\frac{\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64} - \frac{325\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192} + \frac{11389\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{960} - \frac{269879\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{6720} + \frac{183363\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2240} - \cdots}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16} - 8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} + 28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 56\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + 70\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} - 56\cdots\right)}$$

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

```
[Out] - a^3*x - (125*a^3*atanh(tan(c/2 + (d*x)/2)))/(64*d) - ((2025*a^3*tan(c/2 +
 (d*x)/2)^3)/64 - (35713*a^3*tan(c/2 + (d*x)/2)^5)/320 + (183363*a^3*tan(c/
2 + (d*x)/2)^7)/2240 - (269879*a^3*tan(c/2 + (d*x)/2)^9)/6720 + (11389*a^3*
tan(c/2 + (d*x)/2)^11)/960 - (325*a^3*tan(c/2 + (d*x)/2)^13)/192 + (3*a^3*t
an(c/2 + (d*x)/2)^15)/64 - (253*a^3*tan(c/2 + (d*x)/2))/64)/(d*(28*tan(c/2
+ (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/
2 + (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 - 8*ta
n(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1))
```

3.48 $\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 169

$$\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx = a^3x+\frac{19a^3\text{arctanh}(\sin(c+dx))}{16d} -\frac{a^3\tan(c+dx)}{d}-\frac{17a^3\sec(c+dx)\tan(c+dx)}{16d} -\frac{a^3\sec^3(c+dx)\tan(c+dx)}{8d}+\frac{a^3\tan^3(c+dx)}{3d} +\frac{3a^3\sec(c+dx)\tan^3(c+dx)}{4d} +\frac{a^3\sec^3(c+dx)\tan^3(c+dx)}{6d}+\frac{3a^3\tan^5(c+dx)}{5d}$$

```
[Out] a^3*x+19/16*a^3*arctanh(sin(d*x+c))/d-a^3*tan(d*x+c)/d-17/16*a^3*sec(d*x+c)
*tan(d*x+c)/d-1/8*a^3*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a^3*tan(d*x+c)^3/d+3/4*
a^3*sec(d*x+c)*tan(d*x+c)^3/d+1/6*a^3*sec(d*x+c)^3*tan(d*x+c)^3/d+3/5*a^3*t
an(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

$= \{3971, 3554, 8, 2691, 3855, 2687, 30, 3853\}$

$$\int (a + a\sec(c+dx))^3 \tan^4(c+dx)\,dx = \frac{19a^3\text{arctanh}(\sin(c+dx))}{16d} + \frac{3a^3\tan^5(c+dx)}{5d} + \frac{a^3\tan^3(c+dx)}{3d} - \frac{a^3\tan(c+dx)}{d} + \frac{a^3\tan^3(c+dx)\sec^3(c+dx)}{6d} - \frac{a^3\tan(c+dx)\sec^3(c+dx)}{8d} + \frac{3a^3\tan^3(c+dx)\sec(c+dx)}{4d} - \frac{17a^3\tan(c+dx)\sec(c+dx)}{16d} + a^3x$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] a^3*x + (19*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (a^3*Tan[c + d*x])/d - (17*
a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(
8*d) + (a^3*Tan[c + d*x]^3)/(3*d) + (3*a^3*Sec[c + d*x]*Tan[c + d*x]^3)/(4*
d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*d) + (3*a^3*Tan[c + d*x]^5)/(5*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3\tan^4(c+dx)+3a^3\sec(c+dx)\tan^4(c+dx)+3a^3\sec^2(c+dx)\tan^4(c+dx)\right.\\
&\qquad \left.+a^3\sec^3(c+dx)\tan^4(c+dx)\right)\,dx\\
&= a^3\int \tan^4(c+dx)\,dx+a^3\int \sec^3(c+dx)\tan^4(c+dx)\,dx\\
&\quad +\left(3a^3\right)\int \sec(c+dx)\tan^4(c+dx)\,dx+\left(3a^3\right)\int \sec^2(c+dx)\tan^4(c+dx)\,dx\\
&= \frac{a^3\tan^3(c+dx)}{3d}+\frac{3a^3\sec(c+dx)\tan^3(c+dx)}{4d}+\frac{a^3\sec^3(c+dx)\tan^3(c+dx)}{6d}\\
&\quad -\frac{1}{2}a^3\int \sec^3(c+dx)\tan^2(c+dx)\,dx-a^3\int \tan^2(c+dx)\,dx\\
&\quad -\frac{1}{4}\left(9a^3\right)\int \sec(c+dx)\tan^2(c+dx)\,dx+\frac{\left(3a^3\right)\operatorname{Subst}\left(\int x^4\,dx,x,\tan(c+dx)\right)}{d}\\
&= -\frac{a^3\tan(c+dx)}{d}-\frac{9a^3\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^3\sec^3(c+dx)\tan(c+dx)}{8d}\\
&\quad +\frac{a^3\tan^3(c+dx)}{3d}+\frac{3a^3\sec(c+dx)\tan^3(c+dx)}{4d}+\frac{a^3\sec^3(c+dx)\tan^3(c+dx)}{6d}\\
&\quad +\frac{3a^3\tan^5(c+dx)}{5d}+\frac{1}{8}a^3\int \sec^3(c+dx)\,dx+a^3\int 1\,dx+\frac{1}{8}\left(9a^3\right)\int \sec(c+dx)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= a^3x+\frac{9a^3\operatorname{arctanh}(\sin(c+dx))}{8d}-\frac{a^3\tan(c+dx)}{d}-\frac{17a^3\sec(c+dx)\tan(c+dx)}{16d}\\
&\quad-\frac{a^3\sec^3(c+dx)\tan(c+dx)}{8d}+\frac{a^3\tan^3(c+dx)}{3d}+\frac{3a^3\sec(c+dx)\tan^3(c+dx)}{4d}\\
&\quad+\frac{a^3\sec^3(c+dx)\tan^3(c+dx)}{6d}+\frac{3a^3\tan^5(c+dx)}{5d}+\frac{1}{16}a^3\int\sec(c+dx)\,dx\\
&= a^3x+\frac{19a^3\operatorname{arctanh}(\sin(c+dx))}{16d}-\frac{a^3\tan(c+dx)}{d}-\frac{17a^3\sec(c+dx)\tan(c+dx)}{16d}\\
&\quad-\frac{a^3\sec^3(c+dx)\tan(c+dx)}{8d}+\frac{a^3\tan^3(c+dx)}{3d}+\frac{3a^3\sec(c+dx)\tan^3(c+dx)}{4d}\\
&\quad+\frac{a^3\sec^3(c+dx)\tan^3(c+dx)}{6d}+\frac{3a^3\tan^5(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.18

$$
\begin{aligned}
\int(a+a\sec(c+dx))^3\tan^4(c+dx)\,dx &= \frac{a^3\arctan(\tan(c+dx))}{d}+\frac{19a^3\operatorname{arctanh}(\sin(c+dx))}{16d}\\
&\quad-\frac{a^3\tan(c+dx)}{d}+\frac{19a^3\sec(c+dx)\tan(c+dx)}{16d}\\
&\quad-\frac{53a^3\sec^3(c+dx)\tan(c+dx)}{24d}\\
&\quad-\frac{a^3\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{a^3\tan^3(c+dx)}{3d}\\
&\quad+\frac{3a^3\sec(c+dx)\tan^3(c+dx)}{d}\\
&\quad+\frac{a^3\sec^3(c+dx)\tan^3(c+dx)}{3d}+\frac{3a^3\tan^5(c+dx)}{5d}
\end{aligned}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (a^3*ArcTan[Tan[c + d*x]])/d + (19*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (a^3
*Tan[c + d*x])/d + (19*a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (53*a^3*Sec[
c + d*x]^3*Tan[c + d*x])/(24*d) - (a^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) +
 (a^3*Tan[c + d*x]^3)/(3*d) + (3*a^3*Sec[c + d*x]*Tan[c + d*x]^3)/d + (a^3*
Sec[c + d*x]^3*Tan[c + d*x]^3)/(3*d) + (3*a^3*Tan[c + d*x]^5)/(5*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.15

method	result
risch	$a^3x+\frac{ia^3(435\,\mathrm{e}^{11i(dx+c)}+240\,\mathrm{e}^{10i(dx+c)}+865\,\mathrm{e}^{9i(dx+c)}-1200\,\mathrm{e}^{8i(dx+c)}-210\,\mathrm{e}^{7i(dx+c)}-1760\,\mathrm{e}^{6i(dx+c)}+210\,\mathrm{e}^{5i(dx+c)}}{120d(\mathrm{e}^{2i(dx+c)}+1)^6}$
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^5}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^5}{24\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{48\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{3a^3\sin(dx+c)^5}{5\cos(dx+c)^5}+3a^3(}{d}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^5}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^5}{24\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{48\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{3a^3\sin(dx+c)^5}{5\cos(dx+c)^5}+3a^3(}{d}$
parts	$\frac{a^3\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{a^3\left(\frac{\sin(dx+c)^5}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^5}{24\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{48\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{48}-\frac{\sin(d}{16}\right.}{d}$

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+1/120*I*a^3*(435*exp(11*I*(d*x+c))+240*exp(10*I*(d*x+c))+865*exp(9*I*
(d*x+c))-1200*exp(8*I*(d*x+c))-210*exp(7*I*(d*x+c))-1760*exp(6*I*(d*x+c))+2
10*exp(5*I*(d*x+c))-1440*exp(4*I*(d*x+c))-865*exp(3*I*(d*x+c))-1296*exp(2*I
*(d*x+c))-435*exp(I*(d*x+c))-176)/d/(exp(2*I*(d*x+c))+1)^6+19/16*a^3/d*ln(e
xp(I*(d*x+c))+I)-19/16*a^3/d*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.90

$$\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx$$
$$=\frac{480\,a^3dx\cos(dx+c)^6+285\,a^3\cos(dx+c)^6\log(\sin(dx+c)+1)-285\,a^3\cos(dx+c)^6\log(-\sin(dx+c)}{}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/480*(480*a^3*d*x*cos(d*x + c)^6 + 285*a^3*cos(d*x + c)^6*log(sin(d*x + c)
 + 1) - 285*a^3*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(176*a^3*cos(d*x
+ c)^5 + 435*a^3*cos(d*x + c)^4 + 208*a^3*cos(d*x + c)^3 - 110*a^3*cos(d*x
+ c)^2 - 144*a^3*cos(d*x + c) - 40*a^3)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx = a^3\left(\int 3\tan^4(c+dx)\sec(c+dx)\,dx + \int 3\tan^4(c+dx)\sec^2(c+dx)\,dx + \int \tan^4(c+dx)\sec^3(c+dx)\,dx + \int \tan^4(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**4,x)
```

```
[Out] a**3*(Integral(3*tan(c + d*x)**4*sec(c + d*x), x) + Integral(3*tan(c + d*x)
**4*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**4*sec(c + d*x)**3, x) + In
tegral(tan(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.24

$$\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx$$

$$= \frac{288\,a^3\tan(dx+c)^5 + 160\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a^3 - 5\,a^3\left(\frac{2\left(3\,\sin(dx+c)^5 + 8\,\sin(dx+c)^3\right.}{\sin(dx+c)^6 - 3\,\sin(dx+c)^4 + 3}\right.}{}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/480*(288*a^3*tan(d*x + c)^5 + 160*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d
*x + c))*a^3 - 5*a^3*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x +
c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 90*a^3*(2*(5*sin(d*x + c)^3 - 3*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c)
+ 1) - 3*log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 1.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.97

$$\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx$$
$$=\frac{240\,(dx+c)a^3+285\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-285\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{240\,d}$$

[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")

[Out] 1/240*(240*(d*x + c)*a^3 + 285*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 285 *a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*a^3*tan(1/2*d*x + 1/2*c)^11 - 95*a^3*tan(1/2*d*x + 1/2*c)^9 - 366*a^3*tan(1/2*d*x + 1/2*c)^7 + 1746*a^3*tan(1/2*d*x + 1/2*c)^5 - 3135*a^3*tan(1/2*d*x + 1/2*c)^3 + 525*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d

Mupad [B] (verification not implemented)

Time = 14.83 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.20

$$\int (a+a\sec(c+dx))^3\tan^4(c+dx)\,dx=a^3\,x+\frac{19\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$
$$+\frac{-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{19\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{24}+\frac{61\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{20}-\frac{291\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20}+\frac{209\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8}-\frac{35\,a^3\,\mathrm{ta}}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-6\tan\right.}$$

[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^3,x)

[Out] a^3*x + (19*a^3*atanh(tan(c/2 + (d*x)/2)))/(8*d) + ((209*a^3*tan(c/2 + (d*x)/2)^3)/8 - (291*a^3*tan(c/2 + (d*x)/2)^5)/20 + (61*a^3*tan(c/2 + (d*x)/2)^7)/20 + (19*a^3*tan(c/2 + (d*x)/2)^9)/24 - (3*a^3*tan(c/2 + (d*x)/2)^11)/8 - (35*a^3*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))

3.49 $\int (a+a\sec(c+dx))^3\tan^2(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (a+a\sec(c+dx))^3\tan^2(c+dx)\,dx = -a^3x-\frac{13a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)}{d}+\frac{11a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a^3\tan^3(c+dx)}{d}$$

```
[Out] -a^3*x-13/8*a^3*arctanh(sin(d*x+c))/d+a^3*tan(d*x+c)/d+11/8*a^3*sec(d*x+c)*
tan(d*x+c)/d+1/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+a^3*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$\int (a+a\sec(c+dx))^3\tan^2(c+dx)\,dx = -\frac{13a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\tan^3(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{11a^3\tan(c+dx)\sec(c+dx)}{8d}-a^3x$$

```
[In] Int[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -(a^3*x) - (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*Tan[c + d*x])/d + (1
1*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/
(4*d) + (a^3*Tan[c + d*x]^3)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^3\tan^2(c+dx)+3a^3\sec(c+dx)\tan^2(c+dx)+3a^3\sec^2(c+dx)\tan^2(c+dx)\right.\\
&\qquad \left.+a^3\sec^3(c+dx)\tan^2(c+dx)\right)\,dx\\
&= a^3\int \tan^2(c+dx)\,dx+a^3\int \sec^3(c+dx)\tan^2(c+dx)\,dx\\
&\quad+\left(3a^3\right)\int \sec(c+dx)\tan^2(c+dx)\,dx+\left(3a^3\right)\int \sec^2(c+dx)\tan^2(c+dx)\,dx\\
&= \frac{a^3\tan(c+dx)}{d}+\frac{3a^3\sec(c+dx)\tan(c+dx)}{2d}\\
&\quad+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}-\frac{1}{4}a^3\int \sec^3(c+dx)\,dx-a^3\int 1\,dx\\
&\quad-\frac{1}{2}\left(3a^3\right)\int \sec(c+dx)\,dx+\frac{\left(3a^3\right)\operatorname{Subst}\left(\int x^2\,dx,x,\tan(c+dx)\right)}{d}\\
&= -a^3x-\frac{3a^3\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{a^3\tan(c+dx)}{d}+\frac{11a^3\sec(c+dx)\tan(c+dx)}{8d}\\
&\quad+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a^3\tan^3(c+dx)}{d}-\frac{1}{8}a^3\int \sec(c+dx)\,dx\\
&= -a^3x-\frac{13a^3\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)}{d}\\
&\quad+\frac{11a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a^3\tan^3(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09

$$
\begin{aligned}
\int (a+a\sec(c+dx))^3\tan^2(c+dx)\,dx &= -\frac{a^3\arctan(\tan(c+dx))}{d}\\
&\quad-\frac{13a^3\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)}{d}\\
&\quad+\frac{11a^3\sec(c+dx)\tan(c+dx)}{8d}\\
&\quad+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a^3\tan^3(c+dx)}{d}
\end{aligned}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -((a^3*ArcTan[Tan[c + d*x]])/d) - (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (a
^3*Tan[c + d*x])/d + (11*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c
+ d*x]^3*Tan[c + d*x])/(4*d) + (a^3*Tan[c + d*x]^3)/d
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.42

method	result
risch	$-a^3x-\frac{ia^3\left(11\,\mathrm{e}^{7i(dx+c)}+16\,\mathrm{e}^{6i(dx+c)}+19\,\mathrm{e}^{5i(dx+c)}-19\,\mathrm{e}^{3i(dx+c)}-16\,\mathrm{e}^{2i(dx+c)}-11\,\mathrm{e}^{i(dx+c)}\right)}{4d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}-\frac{13a^3\ln\left(\mathrm{e}^{i(dx+c)}+i\right.}{8d}$
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a^3\sin(dx+c)^3}{\cos(dx+c)^3}+3a^3\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\mathrm{se}}{}\right.}{d}$
default	$\frac{a^3\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a^3\sin(dx+c)^3}{\cos(dx+c)^3}+3a^3\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\mathrm{se}}{}\right.}{d}$
parts	$\frac{a^3(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{a^3\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{3a^3\left(\frac{\mathrm{s}}{2}\right.}{}$

```
[In] int((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*x-1/4*I*a^3/d/(exp(2*I*(d*x+c))+1)^4*(11*exp(7*I*(d*x+c))+16*exp(6*I*(
d*x+c))+19*exp(5*I*(d*x+c))-19*exp(3*I*(d*x+c))-16*exp(2*I*(d*x+c))-11*exp(
I*(d*x+c)))-13/8*a^3/d*ln(exp(I*(d*x+c))+I)+13/8*a^3/d*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int(a+a\sec(c+dx))^3\tan^2(c+dx)\,dx=$$
$$-\frac{16\,a^3dx\cos(dx+c)^4+13\,a^3\cos(dx+c)^4\log(\sin(dx+c)+1)-13\,a^3\cos(dx+c)^4\log(-\sin(dx+c)+}{16\,d\cos(dx+c)^4}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^3*d*x*cos(d*x + c)^4 + 13*a^3*cos(d*x + c)^4*log(sin(d*x + c) +
 1) - 13*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(11*a^3*cos(d*x + c)
^2 + 8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^3\tan^2(c+dx)\,dx = a^3\left(\int 3\tan^2(c+dx)\sec(c+dx)\,dx + \int 3\tan^2(c+dx)\sec^2(c+dx)\,dx + \int \tan^2(c+dx)\sec^3(c+dx)\,dx + \int \tan^2(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**3*tan(d*x+c)**2,x)
```

```
[Out] a**3*(Integral(3*tan(c + d*x)**2*sec(c + d*x), x) + Integral(3*tan(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(tan(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(tan(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.50

$$\int (a+a\sec(c+dx))^3\tan^2(c+dx)\,dx = \frac{16\,a^3\tan(dx+c)^3 - 16\,(dx+c-\tan(dx+c))a^3 + a^3\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \log(\sin(dx+c)+1) + \right.}{16\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^3*tan(d*x + c)^3 - 16*(d*x + c - tan(d*x + c))*a^3 + a^3*(2*(sin
(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.74 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.35

$$\int (a + a\sec(c + dx))^3 \tan^2(c + dx)\, dx =$$

$$-\frac{8\,(dx + c)a^3 + 13\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 13\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 \cdots\right.}{\cdots}}{8\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*(8*(d*x + c)*a^3 + 13*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^3*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 13*a
^3*tan(1/2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*a^3*tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 14.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.49

$$\int (a + a\sec(c + dx))^3 \tan^2(c + dx)\, dx$$

$$= \frac{\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} - \frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4} + \frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4} + \frac{21\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + 6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)}$$
$$- a^3\,x - \frac{13\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((3*a^3*tan(c/2 + (d*x)/2)^3)/4 - (13*a^3*tan(c/2 + (d*x)/2)^5)/4 + (5*a^3*
tan(c/2 + (d*x)/2)^7)/4 + (21*a^3*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d
*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x
)/2)^8 + 1)) - a^3*x - (13*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.50 $\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx=-a^3x+\frac{a^3\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3\cot(c+dx)}{d}-\frac{4a^3\csc(c+dx)}{d}$$

```
[Out] -a^3*x+a^3*arctanh(sin(d*x+c))/d-4*a^3*cot(d*x+c)/d-4*a^3*csc(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3971, 3554, 8, 2686, 3852, 2701, 327, 213}

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx=\frac{a^3\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3\cot(c+dx)}{d}-\frac{4a^3\csc(c+dx)}{d}+a^3(-x)$$

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (a^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*Cot[c + d*x])/d - (4*a^3*
Csc[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3\cot^2(c+dx)+3a^3\cot(c+dx)\csc(c+dx)+3a^3\csc^2(c+dx)\right.\\
&\qquad\left.+a^3\csc^2(c+dx)\sec(c+dx)\right)\,dx\\
&= a^3\int \cot^2(c+dx)\,dx+a^3\int \csc^2(c+dx)\sec(c+dx)\,dx\\
&\quad+\left(3a^3\right)\int \cot(c+dx)\csc(c+dx)\,dx+\left(3a^3\right)\int \csc^2(c+dx)\,dx\\
&= -\frac{a^3\cot(c+dx)}{d}-a^3\int 1\,dx-\frac{a^3\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad-\frac{\left(3a^3\right)\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{d}-\frac{\left(3a^3\right)\text{Subst}(\int 1\,dx,x,\csc(c+dx))}{d}\\
&= -a^3x-\frac{4a^3\cot(c+dx)}{d}-\frac{4a^3\csc(c+dx)}{d}-\frac{a^3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}\\
&= -a^3x+\frac{a^3\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3\cot(c+dx)}{d}-\frac{4a^3\csc(c+dx)}{d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(49) = 98.

Time = 0.63 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.22

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(dx+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+d\right.\right.\right.}{8d}$$

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/8*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(d*x + Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 4*Csc[c
/2]*Csc[(c + d*x)/2]*Sin[(d*x)/2]))/d
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

method	result	size
risch	$-a^{3} x -\frac{8 i a^{3}}{d \left({\mathrm e}^{i \left(d x +c \right)}-1\right)}-\frac{a^{3} \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{d}+\frac{a^{3} \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{d}$	71
derivativedivides	$\frac{a^{3} \left(-\frac{1}{\sin \left(d x +c \right)}+\ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)\right)-3 a^{3} \cot \left(d x +c \right)-\frac{3 a^{3}}{\sin \left(d x +c \right)}+a^{3} \left(-\cot \left(d x +c \right)-d x -c \right)}{d}$	79
default	$\frac{a^{3} \left(-\frac{1}{\sin \left(d x +c \right)}+\ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)\right)-3 a^{3} \cot \left(d x +c \right)-\frac{3 a^{3}}{\sin \left(d x +c \right)}+a^{3} \left(-\cot \left(d x +c \right)-d x -c \right)}{d}$	79

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -a^3*x-8*I*a^3/d/(exp(I*(d*x+c))-1)-a^3/d*ln(exp(I*(d*x+c))-I)+a^3/d*ln(exp
(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.71

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{2\,a^{3} d x \sin\left(d x + c\right) - a^{3} \log\left(\sin\left(d x + c\right) + 1\right) \sin\left(d x + c\right) + a^{3} \log\left(-\sin\left(d x + c\right) + 1\right) \sin\left(d x + c\right) + 8\,a}{2\,d \sin\left(d x + c\right)}$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
[Out] -1/2*(2*a^3*d*x*sin(d*x + c) - a^3*log(sin(d*x + c) + 1)*sin(d*x + c) + a^3
*log(-sin(d*x + c) + 1)*sin(d*x + c) + 8*a^3*cos(d*x + c) + 8*a^3)/(d*sin(d
*x + c))
```

Sympy [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx = a^{3} \left(\int 3 \cot^{2}{\left(c + d x \right)} \sec{\left(c + d x \right)}\, dx + \int 3 \cot^{2}{\left(c + d x \right)} \sec^{2}{\left(c + d x \right)}\, dx + \int \cot^{2}{\left(c + d x \right)} \sec^{3}{\left(c + d x \right)}\, dx + \int \cot^{2}{\left(c + d x \right)}\, dx\right)$$

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)**2*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3+a^3\left(\frac{2}{\sin(dx+c)}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{6\,a^3}{\sin(dx+c)}+\cdots}{2\,d}$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a^3 + a^3*(2/sin(d*x + c) - log(sin(d*x
+ c) + 1) + log(sin(d*x + c) - 1)) + 6*a^3/sin(d*x + c) + 6*a^3/tan(d*x + c
))/d
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{(dx+c)a^3-a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((d*x + c)*a^3 - a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + a^3*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) + 4*a^3/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^3\,dx=-\frac{a^3\left(4\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+d\,x\right)}{d}$$

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
[Out] -(a^3*(4*cot(c/2 + (d*x)/2) - 2*atanh(tan(c/2 + (d*x)/2)) + d*x))/d
```

3.51 $\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx = a^3x+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}-\frac{4a^3\csc^3(c+dx)}{3d}$$

```
[Out] a^3*x+a^3*cot(d*x+c)/d-4/3*a^3*cot(d*x+c)^3/d+3*a^3*csc(d*x+c)/d-4/3*a^3*cs
c(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3971, 3554, 8, 2686, 2687, 30}

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{4a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}+a^3x$$

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (4*a^3*Cot[c + d*x]^3)/(3*d) + (3*a^3*Csc[c
+ d*x])/d - (4*a^3*Csc[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^3\cot^4(c+dx)+3a^3\cot^3(c+dx)\csc(c+dx)+3a^3\cot^2(c+dx)\csc^2(c+dx)\right.\\
&\qquad \left.+a^3\cot(c+dx)\csc^3(c+dx)\right)\,dx\\
&= a^3\int\cot^4(c+dx)\,dx+a^3\int\cot(c+dx)\csc^3(c+dx)\,dx\\
&\quad +\left(3a^3\right)\int\cot^3(c+dx)\csc(c+dx)\,dx+\left(3a^3\right)\int\cot^2(c+dx)\csc^2(c+dx)\,dx\\
&= -\frac{a^3\cot^3(c+dx)}{3d}-a^3\int\cot^2(c+dx)\,dx-\frac{a^3\text{Subst}\left(\int x^2\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad +\frac{\left(3a^3\right)\text{Subst}\left(\int x^2\,dx,x,-\cot(c+dx)\right)}{d}-\frac{\left(3a^3\right)\text{Subst}\left(\int\left(-1+x^2\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&= \frac{a^3\cot(c+dx)}{d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}-\frac{4a^3\csc^3(c+dx)}{3d}+a^3\int 1\,dx
\end{aligned}
$$

$$= a^3x + \frac{a^3\cot(c+dx)}{d} - \frac{4a^3\cot^3(c+dx)}{3d} + \frac{3a^3\csc(c+dx)}{d} - \frac{4a^3\csc^3(c+dx)}{3d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\begin{aligned}&\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx\\&= -\frac{a^3\cot^3(c+dx)}{d} + \frac{3a^3\csc(c+dx)}{d} - \frac{4a^3\csc^3(c+dx)}{3d}\\&\quad - \frac{a^3\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d}\end{aligned}$$

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -((a^3*Cot[c + d*x]^3)/d) + (3*a^3*Csc[c + d*x])/d - (4*a^3*Csc[c + d*x]^3)
/(3*d) - (a^3*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]
^2])/(3*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

method	result
risch	$a^3x + \frac{2ia^3\left(9\,e^{2i(dx+c)}-12\,e^{i(dx+c)}+7\right)}{3d\left(e^{i(dx+c)}-1\right)^3}$
derivativedivides	$\frac{-\frac{a^3}{3\sin(dx+c)^3}-\frac{a^3\cos(dx+c)^3}{\sin(dx+c)^3}+3a^3\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)+a^3\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx\right.}{d}$
default	$\frac{-\frac{a^3}{3\sin(dx+c)^3}-\frac{a^3\cos(dx+c)^3}{\sin(dx+c)^3}+3a^3\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)+a^3\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx\right.}{d}$

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+2/3*I*a^3*(9*exp(2*I*(d*x+c))-12*exp(I*(d*x+c))+7)/d/(exp(I*(d*x+c))-
1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{7\,a^3\cos\left(dx+c\right)^2+2\,a^3\cos\left(dx+c\right)-5\,a^3+3\left(a^3dx\cos\left(dx+c\right)-a^3dx\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
[Out] 1/3*(7*a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) - 5*a^3 + 3*(a^3*d*x*cos(d*x
 + c) - a^3*d*x)*sin(d*x + c))/((d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cot^4\left(c+dx\right)\sec\left(c+dx\right)dx + \int 3\cot^4\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cot^4\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int \cot^4\left(c+dx\right)dx\right)$$

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
[Out] a**3*(Integral(3*cot(c + d*x)**4*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**4*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**4*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^2-1}{\tan\left(dx+c\right)^3}\right)a^3+\frac{3\left(3\,\sin\left(dx+c\right)^2-1\right)a^3}{\sin\left(dx+c\right)^3}-\frac{a^3}{\sin\left(dx+c\right)^3}-\frac{3\,a^3}{\tan\left(dx+c\right)^3}}{3\,d}$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 + 3*(3*sin(d
*x + c)^2 - 1)*a^3/sin(d*x + c)^3 - a^3/sin(d*x + c)^3 - 3*a^3/tan(d*x + c)
^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx=\frac{3\,(dx+c)a^3+\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{3\,d}$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^3 + (6*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3)/tan(1/2*d*x + 1
/2*c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^3\,dx=a^3\,x+\frac{a^3\,\left(6\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}{3\,d}$$

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

```
[Out] a^3*x + (a^3*(6*cot(c/2 + (d*x)/2) - cot(c/2 + (d*x)/2)^3))/(3*d)
```

3.52 $\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx = -a^3x-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{3a^3\csc(c+dx)}{d}+\frac{7a^3\csc^3(c+dx)}{3d}-\frac{4a^3\csc^5(c+dx)}{5d}$$

```
[Out] -a^3*x-a^3*cot(d*x+c)/d+1/3*a^3*cot(d*x+c)^3/d-4/5*a^3*cot(d*x+c)^5/d-3*a^3
*csc(d*x+c)/d+7/3*a^3*csc(d*x+c)^3/d-4/5*a^3*csc(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3971, 3554, 8, 2686, 200, 2687, 30, 14}

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{4a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^5(c+dx)}{5d}+\frac{7a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc(c+dx)}{d}-a^3x$$

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*x) - (a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/(3*d) - (4*a^3*Cot[c
 + d*x]^5)/(5*d) - (3*a^3*Csc[c + d*x])/d + (7*a^3*Csc[c + d*x]^3)/(3*d) -
(4*a^3*Csc[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3\cot^6(c+dx)+3a^3\cot^5(c+dx)\csc(c+dx)+3a^3\cot^4(c+dx)\csc^2(c+dx)\right.\\
&\qquad \left.+a^3\cot^3(c+dx)\csc^3(c+dx)\right)\,dx\\
&= a^3\int \cot^6(c+dx)\,dx+a^3\int \cot^3(c+dx)\csc^3(c+dx)\,dx\\
&\quad+\left(3a^3\right)\int \cot^5(c+dx)\csc(c+dx)\,dx+\left(3a^3\right)\int \cot^4(c+dx)\csc^2(c+dx)\,dx\\
&= -\frac{a^3\cot^5(c+dx)}{5d}-a^3\int \cot^4(c+dx)\,dx-\frac{a^3\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad+\frac{\left(3a^3\right)\text{Subst}\left(\int x^4\,dx,x,-\cot(c+dx)\right)}{d}-\frac{\left(3a^3\right)\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{d}\\
&= \frac{a^3\cot^3(c+dx)}{3d}-\frac{4a^3\cot^5(c+dx)}{5d}+a^3\int \cot^2(c+dx)\,dx\\
&\quad-\frac{a^3\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad-\frac{\left(3a^3\right)\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&= -\frac{a^3\cot(c+dx)}{d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{4a^3\cot^5(c+dx)}{5d}\\
&\quad-\frac{3a^3\csc(c+dx)}{d}+\frac{7a^3\csc^3(c+dx)}{3d}-\frac{4a^3\csc^5(c+dx)}{5d}-a^3\int 1\,dx\\
&= -a^3x-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{4a^3\cot^5(c+dx)}{5d}\\
&\quad-\frac{3a^3\csc(c+dx)}{d}+\frac{7a^3\csc^3(c+dx)}{3d}-\frac{4a^3\csc^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\begin{aligned}
&\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx\\
&= -\frac{3a^3\cot^5(c+dx)}{5d}-\frac{3a^3\csc(c+dx)}{d}+\frac{7a^3\csc^3(c+dx)}{3d}-\frac{4a^3\csc^5(c+dx)}{5d}\\
&\quad-\frac{a^3\cot^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5d}
\end{aligned}$$

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Cot[c + d*x]^5)/(5*d) - (3*a^3*Csc[c + d*x])/d + (7*a^3*Csc[c + d*x
]^3)/(3*d) - (4*a^3*Csc[c + d*x]^5)/(5*d) - (a^3*Cot[c + d*x]^5*Hypergeomet
ric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(5*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.89 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.71

method	result
risch	$-a^3x - \frac{2ia^3\left(45\,\mathrm{e}^{4i(dx+c)}-135\,\mathrm{e}^{3i(dx+c)}+185\,\mathrm{e}^{2i(dx+c)}-115\,\mathrm{e}^{i(dx+c)}+32\right)}{15d\left(\mathrm{e}^{i(dx+c)}-1\right)^5}$
derivativedivides	$\frac{a^3\left(-\frac{\cos(dx+c)^4}{5\sin(dx+c)^5}-\frac{\cos(dx+c)^4}{15\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{15\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)-\frac{3a^3\cos(dx+c)^5}{5\sin(dx+c)^5}+3a^3\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos}{15\sin}\right.}{d}$
default	$\frac{a^3\left(-\frac{\cos(dx+c)^4}{5\sin(dx+c)^5}-\frac{\cos(dx+c)^4}{15\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{15\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)-\frac{3a^3\cos(dx+c)^5}{5\sin(dx+c)^5}+3a^3\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos}{15\sin}\right.}{d}$

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*x-2/15*I*a^3*(45*exp(4*I*(d*x+c))-135*exp(3*I*(d*x+c))+185*exp(2*I*(d*
x+c))-115*exp(I*(d*x+c))+32)/d/(exp(I*(d*x+c))-1)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.10

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{32\,a^3\cos\left(dx+c\right)^3-19\,a^3\cos\left(dx+c\right)^2-29\,a^3\cos\left(dx+c\right)+22\,a^3+15\,\left(a^3dx\cos\left(dx+c\right)^2-2\,a^3dx\,c\right.}{15\,\left(d\cos\left(dx+c\right)^2-2\,d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(32*a^3*cos(d*x + c)^3 - 19*a^3*cos(d*x + c)^2 - 29*a^3*cos(d*x + c)
+ 22*a^3 + 15*(a^3*d*x*cos(d*x + c)^2 - 2*a^3*d*x*cos(d*x + c) + a^3*d*x)*s
in(d*x + c))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F]

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cot^6(c+dx)\sec(c+dx)\,dx + \int 3\cot^6(c+dx)\sec^2(c+dx)\,dx + \int \cot^6(c+dx)\sec^3(c+dx)\,dx + \int \cot^6(c+dx)\,dx\right)$$

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cot(c + d*x)**6*sec(c + d*x), x) + Integral(3*cot(c + d*x)
**6*sec(c + d*x)**2, x) + Integral(cot(c + d*x)**6*sec(c + d*x)**3, x) + In
tegral(cot(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3+\frac{3\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)a^3}{\sin(dx+c)^5}-\frac{\left(5\,\sin(dx+c)^2-3\right)a^3}{\sin(dx+c)^5}+\frac{9\,a^3}{\tan(dx+c)^5}}{15\,d}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a^3 + 3*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a^3/sin(d*x + c
)^5 - (5*sin(d*x + c)^2 - 3)*a^3/sin(d*x + c)^5 + 9*a^3/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{60\,(dx+c)a^3+\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{60\,d}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*(d*x + c)*a^3 + (105*a^3*tan(1/2*d*x + 1/2*c)^4 - 20*a^3*tan(1/2*
d*x + 1/2*c)^2 + 3*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,d} - a^3\,x - \frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20\,d} - \frac{7\,a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d}$$

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*cot(c/2 + (d*x)/2)^3)/(3*d) - a^3*x - (a^3*cot(c/2 + (d*x)/2)^5)/(20*d
) - (7*a^3*cot(c/2 + (d*x)/2))/(4*d)
```

3.53 $\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 141

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx = a^3x+\frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{3a^3\csc(c+dx)}{d}-\frac{10a^3\csc^3(c+dx)}{3d}+\frac{11a^3\csc^5(c+dx)}{5d}-\frac{4a^3\csc^7(c+dx)}{7d}$$

```
[Out] a^3*x+a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d+1/5*a^3*cot(d*x+c)^5/d-4/7*a^
3*cot(d*x+c)^7/d+3*a^3*csc(d*x+c)/d-10/3*a^3*csc(d*x+c)^3/d+11/5*a^3*csc(d*
x+c)^5/d-4/7*a^3*csc(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3971, 3554, 8, 2686, 200, 2687, 30, 276}

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{4a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^7(c+dx)}{7d}+\frac{11a^3\csc^5(c+dx)}{5d}-\frac{10a^3\csc^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}+a^3x$$

```
[In] Int[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*
x]^5)/(5*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) + (3*a^3*Csc[c + d*x])/d - (10*a
^3*Csc[c + d*x]^3)/(3*d) + (11*a^3*Csc[c + d*x]^5)/(5*d) - (4*a^3*Csc[c + d
*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3\cot^8(c+dx)+3a^3\cot^7(c+dx)\csc(c+dx)+3a^3\cot^6(c+dx)\csc^2(c+dx)\right.\\
&\qquad\left.+a^3\cot^5(c+dx)\csc^3(c+dx)\right)\,dx\\
&= a^3\int \cot^8(c+dx)\,dx+a^3\int \cot^5(c+dx)\csc^3(c+dx)\,dx\\
&\quad+\left(3a^3\right)\int \cot^7(c+dx)\csc(c+dx)\,dx+\left(3a^3\right)\int \cot^6(c+dx)\csc^2(c+dx)\,dx\\
&= -\frac{a^3\cot^7(c+dx)}{7d}-a^3\int \cot^6(c+dx)\,dx-\frac{a^3\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad+\frac{\left(3a^3\right)\text{Subst}\left(\int x^6\,dx,x,-\cot(c+dx)\right)}{d}-\frac{\left(3a^3\right)\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\csc(c+dx)\right)}{d}\\
&= \frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^7(c+dx)}{7d}+a^3\int \cot^4(c+dx)\,dx\\
&\quad-\frac{a^3\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad-\frac{\left(3a^3\right)\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&= -\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{3a^3\csc(c+dx)}{d}\\
&\quad-\frac{10a^3\csc^3(c+dx)}{3d}+\frac{11a^3\csc^5(c+dx)}{5d}-\frac{4a^3\csc^7(c+dx)}{7d}-a^3\int \cot^2(c+dx)\,dx\\
&= \frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot^5(c+dx)}{5d}\\
&\quad-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{3a^3\csc(c+dx)}{d}-\frac{10a^3\csc^3(c+dx)}{3d}\\
&\quad+\frac{11a^3\csc^5(c+dx)}{5d}-\frac{4a^3\csc^7(c+dx)}{7d}+a^3\int 1\,dx\\
&= a^3x+\frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^7(c+dx)}{7d}\\
&\quad+\frac{3a^3\csc(c+dx)}{d}-\frac{10a^3\csc^3(c+dx)}{3d}+\frac{11a^3\csc^5(c+dx)}{5d}-\frac{4a^3\csc^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.76 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= -\frac{3a^3\cot^7(c+dx)}{7d} + \frac{3a^3\csc(c+dx)}{d} - \frac{10a^3\csc^3(c+dx)}{3d} + \frac{11a^3\csc^5(c+dx)}{5d} - \frac{4a^3\csc^7(c+dx)}{7d} - \frac{a^3\cot^7(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7d}$$

```
[In] Integrate[Cot[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Cot[c + d*x]^7)/(7*d) + (3*a^3*Csc[c + d*x])/d - (10*a^3*Csc[c + d*
x]^3)/(3*d) + (11*a^3*Csc[c + d*x]^5)/(5*d) - (4*a^3*Csc[c + d*x]^7)/(7*d)
- (a^3*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[c + d*x]^2])/(7
*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.80 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

method	result
risch	$a^3x + \frac{2ia^3\left(315\,e^{7i(dx+c)}-1155\,e^{6i(dx+c)}+1715\,e^{5i(dx+c)}-525\,e^{4i(dx+c)}-1379\,e^{3i(dx+c)}+1939\,e^{2i(dx+c)}-1011\,e^{i(dx}\right.}{105d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)}$
derivativedivides	$\frac{a^3\left(-\frac{\cos(dx+c)^6}{7\sin(dx+c)^7}-\frac{\cos(dx+c)^6}{35\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{105\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{35}\right)-\frac{3a^3\cos(dx-}{7\sin(dx+c}}{}$
default	$\frac{a^3\left(-\frac{\cos(dx+c)^6}{7\sin(dx+c)^7}-\frac{\cos(dx+c)^6}{35\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{105\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{35}\right)-\frac{3a^3\cos(dx-}{7\sin(dx+c}}{}$

```
[In] int(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+2/105*I*a^3*(315*exp(7*I*(d*x+c))-1155*exp(6*I*(d*x+c))+1715*exp(5*I*
(d*x+c))-525*exp(4*I*(d*x+c))-1379*exp(3*I*(d*x+c))+1939*exp(2*I*(d*x+c))-1
011*exp(I*(d*x+c))+221)/d/(exp(I*(d*x+c))-1)^7/(exp(I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{221\,a^3\cos\left(dx+c\right)^4-348\,a^3\cos\left(dx+c\right)^3-25\,a^3\cos\left(dx+c\right)^2+303\,a^3\cos\left(dx+c\right)-136\,a^3+105\,\left(a^3dx\right.}{105\,\left(d\cos\left(dx+c\right)^3-3\,d\cos\left(dx+c\right)^2+3\,d\cos\left(dx\right.\right.}$$

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/105*(221*a^3*cos(d*x + c)^4 - 348*a^3*cos(d*x + c)^3 - 25*a^3*cos(d*x + c
)^2 + 303*a^3*cos(d*x + c) - 136*a^3 + 105*(a^3*d*x*cos(d*x + c)^3 - 3*a^3*
d*x*cos(d*x + c)^2 + 3*a^3*d*x*cos(d*x + c) - a^3*d*x)*sin(d*x + c))/((d*co
s(d*x + c)^3 - 3*d*cos(d*x + c)^2 + 3*d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**8*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.08

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{\tan(dx+c)^7}\right)a^3+\frac{9\left(35\,\sin(dx+c)^6-35\,\sin(dx+c)^4+21\,\sin(dx+c)^2-5\right)a^3}{\sin(dx+c)^7}}{105\,d}$$

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a^3 + 9*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*a^3/sin(d*x + c)^7 - (35*sin(d*x + c)^4 - 42*
sin(d*x + c)^2 + 15)*a^3/sin(d*x + c)^7 - 45*a^3/tan(d*x + c)^7)/d
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{1680\,(dx+c)a^3-105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2730\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-560\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+126\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{1680\,d}$$

```
[In] integrate(cot(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1680*(1680*(d*x + c)*a^3 - 105*a^3*tan(1/2*d*x + 1/2*c) + (2730*a^3*tan(1
/2*d*x + 1/2*c)^6 - 560*a^3*tan(1/2*d*x + 1/2*c)^4 + 126*a^3*tan(1/2*d*x +
1/2*c)^2 - 15*a^3)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\int \cot^8(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{a^3\left(126\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-560\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2730\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-105\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+1680\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\right.}{1680\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

```
[In] int(cot(c + d*x)^8*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*(126*tan(c/2 + (d*x)/2)^2 - 560*tan(c/2 + (d*x)/2)^4 + 2730*tan(c/2 +
(d*x)/2)^6 - 105*tan(c/2 + (d*x)/2)^8 + 1680*tan(c/2 + (d*x)/2)^7*(c + d*x)
 - 15))/(1680*d*tan(c/2 + (d*x)/2)^7)
```

3.54 $\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = -a^3x - \frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{a^3\cot^7(c+dx)}{7d} - \frac{4a^3\cot^9(c+dx)}{9d} - \frac{3a^3\csc(c+dx)}{d} + \frac{13a^3\csc^3(c+dx)}{3d} - \frac{21a^3\csc^5(c+dx)}{5d} + \frac{15a^3\csc^7(c+dx)}{7d} - \frac{4a^3\csc^9(c+dx)}{9d}$$

```
[Out] -a^3*x-a^3*cot(d*x+c)/d+1/3*a^3*cot(d*x+c)^3/d-1/5*a^3*cot(d*x+c)^5/d+1/7*a
^3*cot(d*x+c)^7/d-4/9*a^3*cot(d*x+c)^9/d-3*a^3*csc(d*x+c)/d+13/3*a^3*csc(d*
x+c)^3/d-21/5*a^3*csc(d*x+c)^5/d+15/7*a^3*csc(d*x+c)^7/d-4/9*a^3*csc(d*x+c)
^9/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

$= \{3971, 3554, 8, 2686, 200, 2687, 30, 276\}$

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{4a^3\cot^9(c+dx)}{9d}+\frac{a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\csc^9(c+dx)}{9d}+\frac{15a^3\csc^7(c+dx)}{7d}-\frac{21a^3\csc^5(c+dx)}{5d}+\frac{13a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc(c+dx)}{d}-a^3x$$

```
[In] Int[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*x) - (a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/(3*d) - (a^3*Cot[c +
 d*x]^5)/(5*d) + (a^3*Cot[c + d*x]^7)/(7*d) - (4*a^3*Cot[c + d*x]^9)/(9*d)
- (3*a^3*Csc[c + d*x])/d + (13*a^3*Csc[c + d*x]^3)/(3*d) - (21*a^3*Csc[c +
d*x]^5)/(5*d) + (15*a^3*Csc[c + d*x]^7)/(7*d) - (4*a^3*Csc[c + d*x]^9)/(9*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3\cot^{10}(c+dx)+3a^3\cot^9(c+dx)\csc(c+dx)+3a^3\cot^8(c+dx)\csc^2(c+dx)\right.\\
&\qquad\left.+a^3\cot^7(c+dx)\csc^3(c+dx)\right)\,dx\\
&= a^3\int\cot^{10}(c+dx)\,dx+a^3\int\cot^7(c+dx)\csc^3(c+dx)\,dx\\
&\quad+\left(3a^3\right)\int\cot^9(c+dx)\csc(c+dx)\,dx+\left(3a^3\right)\int\cot^8(c+dx)\csc^2(c+dx)\,dx\\
&= -\frac{a^3\cot^9(c+dx)}{9d}-a^3\int\cot^8(c+dx)\,dx-\frac{a^3\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad+\frac{\left(3a^3\right)\text{Subst}\left(\int x^8\,dx,x,-\cot(c+dx)\right)}{d}-\frac{\left(3a^3\right)\text{Subst}\left(\int\left(-1+x^2\right)^4\,dx,x,\csc(c+dx)\right)}{d}\\
&= \frac{a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^9(c+dx)}{9d}+a^3\int\cot^6(c+dx)\,dx\\
&\quad-\frac{a^3\text{Subst}\left(\int\left(-x^2+3x^4-3x^6+x^8\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad-\frac{\left(3a^3\right)\text{Subst}\left(\int\left(1-4x^2+6x^4-4x^6+x^8\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&= -\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^9(c+dx)}{9d}-\frac{3a^3\csc(c+dx)}{d}+\frac{13a^3\csc^3(c+dx)}{3d}\\
&\quad-\frac{21a^3\csc^5(c+dx)}{5d}+\frac{15a^3\csc^7(c+dx)}{7d}-\frac{4a^3\csc^9(c+dx)}{9d}-a^3\int\cot^4(c+dx)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a^3 \cot^3(c+dx)}{3d} - \frac{a^3 \cot^5(c+dx)}{5d} + \frac{a^3 \cot^7(c+dx)}{7d} - \frac{4a^3 \cot^9(c+dx)}{9d} \\
&\quad - \frac{3a^3 \csc(c+dx)}{d} + \frac{13a^3 \csc^3(c+dx)}{3d} - \frac{21a^3 \csc^5(c+dx)}{5d} \\
&\quad + \frac{15a^3 \csc^7(c+dx)}{7d} - \frac{4a^3 \csc^9(c+dx)}{9d} + a^3 \int \cot^2(c+dx)\,dx \\
&= -\frac{a^3 \cot(c+dx)}{d} + \frac{a^3 \cot^3(c+dx)}{3d} - \frac{a^3 \cot^5(c+dx)}{5d} + \frac{a^3 \cot^7(c+dx)}{7d} \\
&\quad - \frac{4a^3 \cot^9(c+dx)}{9d} - \frac{3a^3 \csc(c+dx)}{d} + \frac{13a^3 \csc^3(c+dx)}{3d} \\
&\quad - \frac{21a^3 \csc^5(c+dx)}{5d} + \frac{15a^3 \csc^7(c+dx)}{7d} - \frac{4a^3 \csc^9(c+dx)}{9d} - a^3 \int 1\,dx \\
&= -a^3 x - \frac{a^3 \cot(c+dx)}{d} + \frac{a^3 \cot^3(c+dx)}{3d} - \frac{a^3 \cot^5(c+dx)}{5d} \\
&\quad + \frac{a^3 \cot^7(c+dx)}{7d} - \frac{4a^3 \cot^9(c+dx)}{9d} - \frac{3a^3 \csc(c+dx)}{d} + \frac{13a^3 \csc^3(c+dx)}{3d} \\
&\quad - \frac{21a^3 \csc^5(c+dx)}{5d} + \frac{15a^3 \csc^7(c+dx)}{7d} - \frac{4a^3 \csc^9(c+dx)}{9d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.79 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.79

$$
\begin{aligned}
&\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx \\
&= -\frac{a^3 \cot^9(c+dx)}{3d} - \frac{3a^3 \csc(c+dx)}{d} + \frac{13a^3 \csc^3(c+dx)}{3d} \\
&\quad - \frac{21a^3 \csc^5(c+dx)}{5d} + \frac{15a^3 \csc^7(c+dx)}{7d} - \frac{4a^3 \csc^9(c+dx)}{9d} \\
&\quad - \frac{a^3 \cot^9(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(c+dx)\right)}{9d}
\end{aligned}
$$

```
[In] Integrate[Cot[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/3*(a^3*Cot[c + d*x]^9)/d - (3*a^3*Csc[c + d*x])/d + (13*a^3*Csc[c + d*x]
^3)/(3*d) - (21*a^3*Csc[c + d*x]^5)/(5*d) + (15*a^3*Csc[c + d*x]^7)/(7*d) -
 (4*a^3*Csc[c + d*x]^9)/(9*d) - (a^3*Cot[c + d*x]^9*Hypergeometric2F1[-9/2,
 1, -7/2, -Tan[c + d*x]^2])/(9*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.81 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

method	result
risch	$-a^3x-\frac{2ia^3(945\,e^{11i(dx+c)}-3150\,e^{10i(dx+c)}+2625\,e^{9i(dx+c)}+6300\,e^{8i(dx+c)}-13482\,e^{7i(dx+c)}+5292\,e^{6i(dx+c)}+1056}{315d(e^{i(dx+c)}-1)^9(e^{i(dx+c)}+}$
derivativedivides	$a^3\left(-\frac{\cos(dx+c)^8}{9\sin(dx+c)^9}-\frac{\cos(dx+c)^8}{63\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{315\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{315\sin(dx+c)^3}+\frac{\cos(dx+c)^8}{63\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+}{5}\right.}{63}\right.$
default	$a^3\left(-\frac{\cos(dx+c)^8}{9\sin(dx+c)^9}-\frac{\cos(dx+c)^8}{63\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{315\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{315\sin(dx+c)^3}+\frac{\cos(dx+c)^8}{63\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+}{5}\right.}{63}\right.$

```
[In] int(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*x-2/315*I*a^3*(945*exp(11*I*(d*x+c))-3150*exp(10*I*(d*x+c))+2625*exp(9
*I*(d*x+c))+6300*exp(8*I*(d*x+c))-13482*exp(7*I*(d*x+c))+5292*exp(6*I*(d*x+
c))+10566*exp(5*I*(d*x+c))-11736*exp(4*I*(d*x+c))+1289*exp(3*I*(d*x+c))+486
6*exp(2*I*(d*x+c))-3063*exp(I*(d*x+c))+668)/d/(exp(I*(d*x+c))-1)^9/(exp(I*(
d*x+c))+1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.31

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{668\,a^3\cos(dx+c)^6-1059\,a^3\cos(dx+c)^5-573\,a^3\cos(dx+c)^4+1813\,a^3\cos(dx+c)^3-393\,a^3\cos(d}{315\left(d\cos(dx+c)^5-\right.}$$

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/315*(668*a^3*cos(d*x + c)^6 - 1059*a^3*cos(d*x + c)^5 - 573*a^3*cos(d*x
+ c)^4 + 1813*a^3*cos(d*x + c)^3 - 393*a^3*cos(d*x + c)^2 - 789*a^3*cos(d*x
 + c) + 368*a^3 + 315*(a^3*d*x*cos(d*x + c)^5 - 3*a^3*d*x*cos(d*x + c)^4 +
2*a^3*d*x*cos(d*x + c)^3 + 2*a^3*d*x*cos(d*x + c)^2 - 3*a^3*d*x*cos(d*x + c
) + a^3*d*x)*sin(d*x + c))/((d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4 + 2*d*co
s(d*x + c)^3 + 2*d*cos(d*x + c)^2 - 3*d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**10*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.02

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{\left(315\,dx+315\,c+\frac{315\,\tan(dx+c)^8-105\,\tan(dx+c)^6+63\,\tan(dx+c)^4-45\,\tan(dx+c)^2+35}{\tan(dx+c)^9}\right)a^3+\frac{3\left(315\,\sin(dx+c)^8-420\,\sin(dx+c\cdots\right.}{\cdots}}{315\,d}$$

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/315*((315*d*x + 315*c + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 63*ta
n(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/tan(d*x + c)^9)*a^3 + 3*(315*sin(d*x
 + c)^8 - 420*sin(d*x + c)^6 + 378*sin(d*x + c)^4 - 180*sin(d*x + c)^2 + 35
)*a^3/sin(d*x + c)^9 - (105*sin(d*x + c)^6 - 189*sin(d*x + c)^4 + 135*sin(d
*x + c)^2 - 35)*a^3/sin(d*x + c)^9 + 105*a^3/tan(d*x + c)^9)/d
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.72

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20160\,(dx+c)a^3-2520\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{31185\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-6720\,a^3\tan\left(\frac{1}{2}\cdots\right.}{\cdots}}{20160\,d}$$

```
[In] integrate(cot(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20160*(105*a^3*tan(1/2*d*x + 1/2*c)^3 + 20160*(d*x + c)*a^3 - 2520*a^3*t
an(1/2*d*x + 1/2*c) + (31185*a^3*tan(1/2*d*x + 1/2*c)^8 - 6720*a^3*tan(1/2*
d*x + 1/2*c)^6 + 1827*a^3*tan(1/2*d*x + 1/2*c)^4 - 360*a^3*tan(1/2*d*x + 1/
2*c)^2 + 35*a^3)/tan(1/2*d*x + 1/2*c)^9)/d
```

Mupad [B] (verification not implemented)

Time = 15.08 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.15

$$\int \cot^{10}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{a^3\left(35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+105\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-2520\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+31185\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\right.}{}$$

```
[In] int(cot(c + d*x)^10*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*(35*cos(c/2 + (d*x)/2)^12 + 105*sin(c/2 + (d*x)/2)^12 - 2520*cos(c/2
+ (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 31185*cos(c/2 + (d*x)/2)^4*sin(c/2 + (
d*x)/2)^8 - 6720*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6 + 1827*cos(c/2 +
 (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 360*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x
)/2)^2 + 20160*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^9*(c + d*x)))/(20160
*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^9)
```

3.55 $\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 213

$$\begin{aligned}\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx = {} & a^3x+\frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}\\ & +\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^7(c+dx)}{7d}\\ & +\frac{a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^{11}(c+dx)}{11d}\\ & +\frac{3a^3\csc(c+dx)}{d}-\frac{16a^3\csc^3(c+dx)}{3d}\\ & +\frac{34a^3\csc^5(c+dx)}{5d}-\frac{36a^3\csc^7(c+dx)}{7d}\\ & +\frac{19a^3\csc^9(c+dx)}{9d}-\frac{4a^3\csc^{11}(c+dx)}{11d}\end{aligned}$$

```
[Out] a^3*x+a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d+1/5*a^3*cot(d*x+c)^5/d-1/7*a^
3*cot(d*x+c)^7/d+1/9*a^3*cot(d*x+c)^9/d-4/11*a^3*cot(d*x+c)^11/d+3*a^3*csc(
d*x+c)/d-16/3*a^3*csc(d*x+c)^3/d+34/5*a^3*csc(d*x+c)^5/d-36/7*a^3*csc(d*x+c
)^7/d+19/9*a^3*csc(d*x+c)^9/d-4/11*a^3*csc(d*x+c)^11/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

$= \{3971, 3554, 8, 2686, 200, 2687, 30, 276\}$

$$\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{4a^3\cot^{11}(c+dx)}{11d}+\frac{a^3\cot^9(c+dx)}{9d} -\frac{a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^5(c+dx)}{5d} -\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d} -\frac{4a^3\csc^{11}(c+dx)}{11d}+\frac{19a^3\csc^9(c+dx)}{9d} -\frac{36a^3\csc^7(c+dx)}{7d}+\frac{34a^3\csc^5(c+dx)}{5d} -\frac{16a^3\csc^3(c+dx)}{3d}+\frac{3a^3\csc(c+dx)}{d}+a^3x$$

```
[In] Int[Cot[c + d*x]^12*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*
x]^5)/(5*d) - (a^3*Cot[c + d*x]^7)/(7*d) + (a^3*Cot[c + d*x]^9)/(9*d) - (4*
a^3*Cot[c + d*x]^11)/(11*d) + (3*a^3*Csc[c + d*x])/d - (16*a^3*Csc[c + d*x]
^3)/(3*d) + (34*a^3*Csc[c + d*x]^5)/(5*d) - (36*a^3*Csc[c + d*x]^7)/(7*d) +
 (19*a^3*Csc[c + d*x]^9)/(9*d) - (4*a^3*Csc[c + d*x]^11)/(11*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3\cot^{12}(c+dx)+3a^3\cot^{11}(c+dx)\csc(c+dx)+3a^3\cot^{10}(c+dx)\csc^2(c+dx)\right.\\
&\qquad \left.+a^3\cot^9(c+dx)\csc^3(c+dx)\right)\,dx\\
&= a^3\int \cot^{12}(c+dx)\,dx+a^3\int \cot^9(c+dx)\csc^3(c+dx)\,dx\\
&\quad+\left(3a^3\right)\int \cot^{11}(c+dx)\csc(c+dx)\,dx+\left(3a^3\right)\int \cot^{10}(c+dx)\csc^2(c+dx)\,dx\\
&= -\frac{a^3\cot^{11}(c+dx)}{11d}-a^3\int \cot^{10}(c+dx)\,dx-\frac{a^3\text{Subst}\left(\int x^2\left(-1+x^2\right)^4\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad+\frac{\left(3a^3\right)\text{Subst}\left(\int x^{10}\,dx,x,-\cot(c+dx)\right)}{d}-\frac{\left(3a^3\right)\text{Subst}\left(\int \left(-1+x^2\right)^5\,dx,x,\csc(c+dx)\right)}{d}\\
&= \frac{a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^{11}(c+dx)}{11d}+a^3\int \cot^8(c+dx)\,dx\\
&\quad-\frac{a^3\text{Subst}\left(\int \left(x^2-4x^4+6x^6-4x^8+x^{10}\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad-\frac{\left(3a^3\right)\text{Subst}\left(\int \left(-1+5x^2-10x^4+10x^6-5x^8+x^{10}\right)\,dx,x,\csc(c+dx)\right)}{d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^{11}(c+dx)}{11d}+\frac{3a^3\csc(c+dx)}{d} \\
&\quad -\frac{16a^3\csc^3(c+dx)}{3d}+\frac{34a^3\csc^5(c+dx)}{5d}-\frac{36a^3\csc^7(c+dx)}{7d} \\
&\quad +\frac{19a^3\csc^9(c+dx)}{9d}-\frac{4a^3\csc^{11}(c+dx)}{11d}-a^3\int\cot^6(c+dx)\,dx \\
&= \frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^{11}(c+dx)}{11d} \\
&\quad +\frac{3a^3\csc(c+dx)}{d}-\frac{16a^3\csc^3(c+dx)}{3d}+\frac{34a^3\csc^5(c+dx)}{5d}-\frac{36a^3\csc^7(c+dx)}{7d} \\
&\quad +\frac{19a^3\csc^9(c+dx)}{9d}-\frac{4a^3\csc^{11}(c+dx)}{11d}+a^3\int\cot^4(c+dx)\,dx \\
&= -\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^7(c+dx)}{7d}+\frac{a^3\cot^9(c+dx)}{9d} \\
&\quad -\frac{4a^3\cot^{11}(c+dx)}{11d}+\frac{3a^3\csc(c+dx)}{d}-\frac{16a^3\csc^3(c+dx)}{3d}+\frac{34a^3\csc^5(c+dx)}{5d} \\
&\quad -\frac{36a^3\csc^7(c+dx)}{7d}+\frac{19a^3\csc^9(c+dx)}{9d}-\frac{4a^3\csc^{11}(c+dx)}{11d}-a^3\int\cot^2(c+dx)\,dx \\
&= \frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^7(c+dx)}{7d} \\
&\quad +\frac{a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^{11}(c+dx)}{11d}+\frac{3a^3\csc(c+dx)}{d} \\
&\quad -\frac{16a^3\csc^3(c+dx)}{3d}+\frac{34a^3\csc^5(c+dx)}{5d}-\frac{36a^3\csc^7(c+dx)}{7d} \\
&\quad +\frac{19a^3\csc^9(c+dx)}{9d}-\frac{4a^3\csc^{11}(c+dx)}{11d}+a^3\int 1\,dx \\
&= a^3x+\frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^7(c+dx)}{7d} \\
&\quad +\frac{a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^{11}(c+dx)}{11d}+\frac{3a^3\csc(c+dx)}{d}-\frac{16a^3\csc^3(c+dx)}{3d} \\
&\quad +\frac{34a^3\csc^5(c+dx)}{5d}-\frac{36a^3\csc^7(c+dx)}{7d}+\frac{19a^3\csc^9(c+dx)}{9d}-\frac{4a^3\csc^{11}(c+dx)}{11d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.67 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.75

$$\begin{aligned}&\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx\\&=-\frac{3a^3\cot^{11}(c+dx)}{11d}+\frac{3a^3\csc(c+dx)}{d}-\frac{16a^3\csc^3(c+dx)}{3d}+\frac{34a^3\csc^5(c+dx)}{5d}\\&\quad-\frac{36a^3\csc^7(c+dx)}{7d}+\frac{19a^3\csc^9(c+dx)}{9d}-\frac{4a^3\csc^{11}(c+dx)}{11d}\\&\quad-\frac{a^3\cot^{11}(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{11}{2},1,-\frac{9}{2},-\tan^2(c+dx)\right)}{11d}\end{aligned}$$

```
[In] Integrate[Cot[c + d*x]^12*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Cot[c + d*x]^11)/(11*d) + (3*a^3*Csc[c + d*x])/d - (16*a^3*Csc[c +
d*x]^3)/(3*d) + (34*a^3*Csc[c + d*x]^5)/(5*d) - (36*a^3*Csc[c + d*x]^7)/(7*
d) + (19*a^3*Csc[c + d*x]^9)/(9*d) - (4*a^3*Csc[c + d*x]^11)/(11*d) - (a^3*
Cot[c + d*x]^11*Hypergeometric2F1[-11/2, 1, -9/2, -Tan[c + d*x]^2])/(11*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 12.94 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.98

method	result
risch	$a^3x+\frac{2ia^3\left(10395\,\mathrm{e}^{15i(dx+c)}-31185\,\mathrm{e}^{14i(dx+c)}+1155\,\mathrm{e}^{13i(dx+c)}+148995\,\mathrm{e}^{12i(dx+c)}-190113\,\mathrm{e}^{11i(dx+c)}-117117\,\mathrm{e}^{10i}\right.}{}$
derivativedivides	$\frac{a^3\left(-\frac{\cos(dx+c)^{10}}{11\sin(dx+c)^{11}}-\frac{\cos(dx+c)^{10}}{99\sin(dx+c)^9}+\frac{\cos(dx+c)^{10}}{693\sin(dx+c)^7}-\frac{\cos(dx+c)^{10}}{1155\sin(dx+c)^5}+\frac{\cos(dx+c)^{10}}{693\sin(dx+c)^3}-\frac{\cos(dx+c)^{10}}{99\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos(dx+c)^8+\right.}{}\right.}{}$
default	$\frac{a^3\left(-\frac{\cos(dx+c)^{10}}{11\sin(dx+c)^{11}}-\frac{\cos(dx+c)^{10}}{99\sin(dx+c)^9}+\frac{\cos(dx+c)^{10}}{693\sin(dx+c)^7}-\frac{\cos(dx+c)^{10}}{1155\sin(dx+c)^5}+\frac{\cos(dx+c)^{10}}{693\sin(dx+c)^3}-\frac{\cos(dx+c)^{10}}{99\sin(dx+c)}-\frac{\left(\frac{128}{35}+\cos(dx+c)^8+\right.}{}\right.}{}$

```
[In] int(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+2/3465*I*a^3*(10395*exp(15*I*(d*x+c))-31185*exp(14*I*(d*x+c))+1155*ex
p(13*I*(d*x+c))+148995*exp(12*I*(d*x+c))-190113*exp(11*I*(d*x+c))-117117*ex
p(10*I*(d*x+c))+434775*exp(9*I*(d*x+c))-138105*exp(8*I*(d*x+c))-385055*exp(
7*I*(d*x+c))+374781*exp(6*I*(d*x+c))+63289*exp(5*I*(d*x+c))-223655*exp(4*I*
(d*x+c))+75685*exp(3*I*(d*x+c))+43345*exp(2*I*(d*x+c))-34323*exp(I*(d*x+c))
+7453)/d/(exp(I*(d*x+c))-1)^11/(exp(I*(d*x+c))+1)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.47

$$\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{7453\,a^3\cos(dx+c)^8 - 11964\,a^3\cos(dx+c)^7 - 11866\,a^3\cos(dx+c)^6 + 30542\,a^3\cos(dx+c)^5 + 90\,a^3\cos}{}$$

```
[In] integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(7453*a^3*cos(d*x + c)^8 - 11964*a^3*cos(d*x + c)^7 - 11866*a^3*cos(
d*x + c)^6 + 30542*a^3*cos(d*x + c)^5 + 90*a^3*cos(d*x + c)^4 - 26438*a^3*c
os(d*x + c)^3 + 8539*a^3*cos(d*x + c)^2 + 7671*a^3*cos(d*x + c) - 3712*a^3
+ 3465*(a^3*d*x*cos(d*x + c)^7 - 3*a^3*d*x*cos(d*x + c)^6 + a^3*d*x*cos(d*x
 + c)^5 + 5*a^3*d*x*cos(d*x + c)^4 - 5*a^3*d*x*cos(d*x + c)^3 - a^3*d*x*cos
(d*x + c)^2 + 3*a^3*d*x*cos(d*x + c) - a^3*d*x)*sin(d*x + c))/((d*cos(d*x +
 c)^7 - 3*d*cos(d*x + c)^6 + d*cos(d*x + c)^5 + 5*d*cos(d*x + c)^4 - 5*d*co
s(d*x + c)^3 - d*cos(d*x + c)^2 + 3*d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**12*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{\left(3465\,dx + 3465\,c + \frac{3465\,\tan(dx+c)^{10} - 1155\,\tan(dx+c)^8 + 693\,\tan(dx+c)^6 - 495\,\tan(dx+c)^4 + 385\,\tan(dx+c)^2 - 315}{\tan(dx+c)^{11}}\right)a^3 + \frac{15\left(693\,\sin\right.}{}}{}$$

```
[In] integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3465*((3465*d*x + 3465*c + (3465*tan(d*x + c)^10 - 1155*tan(d*x + c)^8 +
693*tan(d*x + c)^6 - 495*tan(d*x + c)^4 + 385*tan(d*x + c)^2 - 315)/tan(d*x
 + c)^11)*a^3 + 15*(693*sin(d*x + c)^10 - 1155*sin(d*x + c)^8 + 1386*sin(d*
x + c)^6 - 990*sin(d*x + c)^4 + 385*sin(d*x + c)^2 - 63)*a^3/sin(d*x + c)^1
1 - (1155*sin(d*x + c)^8 - 2772*sin(d*x + c)^6 + 2970*sin(d*x + c)^4 - 1540
*sin(d*x + c)^2 + 315)*a^3/sin(d*x + c)^11 - 945*a^3/tan(d*x + c)^11)/d
```

Giac [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.76

$$\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{693\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-11550\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-887040\,(dx+c)a^3+159390\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{88}$$

```
[In] integrate(cot(d*x+c)^12*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/887040*(693*a^3*tan(1/2*d*x + 1/2*c)^5 - 11550*a^3*tan(1/2*d*x + 1/2*c)^
3 - 887040*(d*x + c)*a^3 + 159390*a^3*tan(1/2*d*x + 1/2*c) - 5*(264726*a^3*
tan(1/2*d*x + 1/2*c)^10 - 59136*a^3*tan(1/2*d*x + 1/2*c)^8 + 18018*a^3*tan(
1/2*d*x + 1/2*c)^6 - 4554*a^3*tan(1/2*d*x + 1/2*c)^4 + 770*a^3*tan(1/2*d*x
+ 1/2*c)^2 - 63*a^3)/tan(1/2*d*x + 1/2*c)^11)/d
```

Mupad [B] (verification not implemented)

Time = 15.85 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.19

$$\int \cot^{12}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{a^3\left(315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+693\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-11550\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+159390\cos\left(\frac{c}{2}+\frac{dx}{2}\right.\right.}{}$$

```
[In] int(cot(c + d*x)^12*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*(315*cos(c/2 + (d*x)/2)^16 + 693*sin(c/2 + (d*x)/2)^16 - 11550*cos(c/
2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 159390*cos(c/2 + (d*x)/2)^4*sin(c/2
+ (d*x)/2)^12 - 1323630*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 295680
*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8 - 90090*cos(c/2 + (d*x)/2)^10*si
n(c/2 + (d*x)/2)^6 + 22770*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 385
0*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2 - 887040*cos(c/2 + (d*x)/2)^5*
sin(c/2 + (d*x)/2)^11*(c + d*x)))/(887040*d*cos(c/2 + (d*x)/2)^5*sin(c/2 +
(d*x)/2)^11)
```

3.56 $\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\log(\cos(c+dx))}{ad}-\frac{\sec(c+dx)}{ad}-\frac{3\sec^2(c+dx)}{2ad}+\frac{\sec^3(c+dx)}{ad}+\frac{3\sec^4(c+dx)}{4ad}-\frac{3\sec^5(c+dx)}{5ad}-\frac{\sec^6(c+dx)}{6ad}+\frac{\sec^7(c+dx)}{7ad}$$

```
[Out] -ln(cos(d*x+c))/a/d-sec(d*x+c)/a/d-3/2*sec(d*x+c)^2/a/d+sec(d*x+c)^3/a/d+3/
4*sec(d*x+c)^4/a/d-3/5*sec(d*x+c)^5/a/d-1/6*sec(d*x+c)^6/a/d+1/7*sec(d*x+c)
^7/a/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sec^7(c+dx)}{7ad}-\frac{\sec^6(c+dx)}{6ad}-\frac{3\sec^5(c+dx)}{5ad}+\frac{3\sec^4(c+dx)}{4ad}+\frac{\sec^3(c+dx)}{ad}-\frac{3\sec^2(c+dx)}{2ad}-\frac{\sec(c+dx)}{ad}-\frac{\log(\cos(c+dx))}{ad}$$

```
[In] Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Sec[c + d*x]/(a*d) - (3*Sec[c + d*x]^2)/(2*a*d
) + Sec[c + d*x]^3/(a*d) + (3*Sec[c + d*x]^4)/(4*a*d) - (3*Sec[c + d*x]^5)/
(5*a*d) - Sec[c + d*x]^6/(6*a*d) + Sec[c + d*x]^7/(7*a*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^3}{x^8}\,dx, x, \cos(c+dx)\right)}{a^8d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}-\frac{a^7}{x^7}-\frac{3a^7}{x^6}+\frac{3a^7}{x^5}+\frac{3a^7}{x^4}-\frac{3a^7}{x^3}-\frac{a^7}{x^2}+\frac{a^7}{x}\right)dx, x, \cos(c+dx)\right)}{a^8d}\\
&= -\frac{\log(\cos(c+dx))}{ad}-\frac{\sec(c+dx)}{ad}-\frac{3\sec^2(c+dx)}{2ad}+\frac{\sec^3(c+dx)}{ad}\\
&\quad+\frac{3\sec^4(c+dx)}{4ad}-\frac{3\sec^5(c+dx)}{5ad}-\frac{\sec^6(c+dx)}{6ad}+\frac{\sec^7(c+dx)}{7ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{(35\cos(c+dx)(104+105\log(\cos(c+dx)))+3(212+602\cos(2(c+dx))+140\cos(4(c+dx))+210\,\mathrm{c}$$

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/6720*((35*Cos[c + d*x]*(104 + 105*Log[Cos[c + d*x]]) + 3*(212 + 602*Cos[
2*(c + d*x)] + 140*Cos[4*(c + d*x)] + 210*Cos[5*(c + d*x)] + 70*Cos[6*(c +
d*x)] + 245*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 35*Cos[7*(c + d*x)]*Log[Co
s[c + d*x]] + 105*Cos[3*(c + d*x)]*(6 + 7*Log[Cos[c + d*x]])))*Sec[c + d*x]
^7)/(a*d)
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{-\frac{3}{2\cos(dx+c)^2}-\ln(\cos(dx+c))+\frac{1}{\cos(dx+c)^3}+\frac{3}{4\cos(dx+c)^4}-\frac{3}{5\cos(dx+c)^5}-\frac{1}{6\cos(dx+c)^6}+\frac{1}{7\cos(dx+c)^7}-\frac{1}{\cos(dx+c)}}{da}$
default	$\frac{-\frac{3}{2\cos(dx+c)^2}-\ln(\cos(dx+c))+\frac{1}{\cos(dx+c)^3}+\frac{3}{4\cos(dx+c)^4}-\frac{3}{5\cos(dx+c)^5}-\frac{1}{6\cos(dx+c)^6}+\frac{1}{7\cos(dx+c)^7}-\frac{1}{\cos(dx+c)}}{da}$
risch	$\frac{ix}{a}+\frac{2ic}{da}-\frac{2\left(105\,\mathrm{e}^{13i(dx+c)}+315\,\mathrm{e}^{12i(dx+c)}+210\,\mathrm{e}^{11i(dx+c)}+945\,\mathrm{e}^{10i(dx+c)}+903\,\mathrm{e}^{9i(dx+c)}+1820\,\mathrm{e}^{8i(dx+c)}+636\,\mathrm{e}^{7i(d}\right.}{105da\left(\mathrm{e}^{2i(dx+c)}\right.}$

[In] int(tan(d*x+c)^9/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d/a*(-3/2/cos(d*x+c)^2-ln(cos(d*x+c))+1/cos(d*x+c)^3+3/4/cos(d*x+c)^4-3/5
/cos(d*x+c)^5-1/6/cos(d*x+c)^6+1/7/cos(d*x+c)^7-1/cos(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{420\cos(dx+c)^7\log(-\cos(dx+c))+420\cos(dx+c)^6+630\cos(dx+c)^5-420\cos(dx+c)^4-315}{420\,ad\cos(dx+c)^7}$$

[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/420*(420*cos(d*x + c)^7*log(-cos(d*x + c)) + 420*cos(d*x + c)^6 + 630*co
s(d*x + c)^5 - 420*cos(d*x + c)^4 - 315*cos(d*x + c)^3 + 252*cos(d*x + c)^2
 + 70*cos(d*x + c) - 60)/(a*d*cos(d*x + c)^7)

Sympy [F]

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^9(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

[In] integrate(tan(d*x+c)**9/(a+a*sec(d*x+c)),x)

[Out] Integral(tan(c + d*x)**9/(sec(c + d*x) + 1), x)/a

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{420 \log(\cos(dx+c))}{a} + \frac{420 \cos(dx+c)^6+630 \cos(dx+c)^5-420 \cos(dx+c)^4-315 \cos(dx+c)^3+252 \cos(dx+c)^2+70 \cos(dx+c)-60}{a\cos(dx+c)^7}}{420\,d}$$

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(420*log(cos(d*x + c))/a + (420*cos(d*x + c)^6 + 630*cos(d*x + c)^5
- 420*cos(d*x + c)^4 - 315*cos(d*x + c)^3 + 252*cos(d*x + c)^2 + 70*cos(d*x
 + c) - 60)/(a*cos(d*x + c)^7))/d
```

Giac [A] (verification not implemented)

none

Time = 5.98 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.81

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{420 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} - \frac{420 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a} + \frac{\frac{5775\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{20685\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{42595\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{56035\,(\cos(dx}{(\cos(dx+c}}{a\left(\frac{\cos(dx+}{\cos(dx+}\right.}}{420\,d}$$

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*(420*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 420*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (5775*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 20685*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
42595*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*(cos(d*x + c) - 1)^
4/(cos(d*x + c) + 1)^4 + 28749*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 +
8463*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1089*(cos(d*x + c) - 1)^7/
(cos(d*x + c) + 1)^7 + 705)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^
7))/d
```

Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.54

$$\int \frac{\tan^9(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a\,d}$$

$$-\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}+\frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{26\tan\left(\frac{c}{2}+\right.}{5}}{d\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-21\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+35\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-35\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

```
[In] int(tan(c + d*x)^9/(a + a/cos(c + d*x)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d) - ((26*tan(c/2 + (d*x)/2)^4)/5 - (22*
tan(c/2 + (d*x)/2)^2)/5 + (32*tan(c/2 + (d*x)/2)^6)/3 - (128*tan(c/2 + (d*x
)/2)^8)/3 + 14*tan(c/2 + (d*x)/2)^10 - 2*tan(c/2 + (d*x)/2)^12 + 32/35)/(d*
(a - 7*a*tan(c/2 + (d*x)/2)^2 + 21*a*tan(c/2 + (d*x)/2)^4 - 35*a*tan(c/2 +
(d*x)/2)^6 + 35*a*tan(c/2 + (d*x)/2)^8 - 21*a*tan(c/2 + (d*x)/2)^10 + 7*a*t
an(c/2 + (d*x)/2)^12 - a*tan(c/2 + (d*x)/2)^14))
```

3.57 $\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^2(c+dx)}{ad} - \frac{2\sec^3(c+dx)}{3ad}-\frac{\sec^4(c+dx)}{4ad}+\frac{\sec^5(c+dx)}{5ad}$$

```
[Out] ln(cos(d*x+c))/a/d+sec(d*x+c)/a/d+sec(d*x+c)^2/a/d-2/3*sec(d*x+c)^3/a/d-1/4
*sec(d*x+c)^4/a/d+1/5*sec(d*x+c)^5/a/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sec^5(c+dx)}{5ad}-\frac{\sec^4(c+dx)}{4ad}-\frac{2\sec^3(c+dx)}{3ad} +\frac{\sec^2(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\log(\cos(c+dx))}{ad}$$

```
[In] Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^2/(a*d) - (2*Se
c[c + d*x]^3)/(3*a*d) - Sec[c + d*x]^4/(4*a*d) + Sec[c + d*x]^5/(5*a*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)^2}{x^6}\,dx, x, \cos(c+dx)\right)}{a^6d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}-\frac{a^5}{x^5}-\frac{2a^5}{x^4}+\frac{2a^5}{x^3}+\frac{a^5}{x^2}-\frac{a^5}{x}\right)dx, x, \cos(c+dx)\right)}{a^6d}\\
&= \frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^2(c+dx)}{ad}\\
&\quad -\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec^4(c+dx)}{4ad}+\frac{\sec^5(c+dx)}{5ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{(58+40\cos(2(c+dx))+60\cos(3(c+dx))+30\cos(4(c+dx))+75\cos(3(c+dx))\log(\cos(c+dx))+15}{240ad}$$

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((58 + 40*Cos[2*(c + d*x)] + 60*Cos[3*(c + d*x)] + 30*Cos[4*(c + d*x)] + 75
*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + 15*Cos[5*(c + d*x)]*Log[Cos[c + d*x]]
 + 30*Cos[c + d*x]*(4 + 5*Log[Cos[c + d*x]]))*Sec[c + d*x]^5)/(240*a*d)
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{-\frac{2}{3\cos(dx+c)^3}+\ln(\cos(dx+c))+\frac{1}{\cos(dx+c)^2}+\frac{1}{5\cos(dx+c)^5}-\frac{1}{4\cos(dx+c)^4}+\frac{1}{\cos(dx+c)}}{da}$
default	$\frac{-\frac{2}{3\cos(dx+c)^3}+\ln(\cos(dx+c))+\frac{1}{\cos(dx+c)^2}+\frac{1}{5\cos(dx+c)^5}-\frac{1}{4\cos(dx+c)^4}+\frac{1}{\cos(dx+c)}}{da}$
risch	$-\frac{ix}{a}-\frac{2ic}{da}+\frac{2\,\mathrm{e}^{9i(dx+c)}+4\,\mathrm{e}^{8i(dx+c)}+\frac{8\,\mathrm{e}^{7i(dx+c)}}{3}+8\,\mathrm{e}^{6i(dx+c)}+\frac{116\,\mathrm{e}^{5i(dx+c)}}{15}+8\,\mathrm{e}^{4i(dx+c)}+\frac{8\,\mathrm{e}^{3i(dx+c)}}{3}+4\,\mathrm{e}^{2i(dx+c)}}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$

```
[In] int(tan(d*x+c)^7/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-2/3/cos(d*x+c)^3+ln(cos(d*x+c))+1/cos(d*x+c)^2+1/5/cos(d*x+c)^5-1/4
/cos(d*x+c)^4+1/cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{60\cos(dx+c)^5\log(-\cos(dx+c))+60\cos(dx+c)^4+60\cos(dx+c)^3-40\cos(dx+c)^2-15\cos(dx}{60\,ad\cos(dx+c)^5}$$

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(60*cos(d*x + c)^5*log(-cos(d*x + c)) + 60*cos(d*x + c)^4 + 60*cos(d*x
 + c)^3 - 40*cos(d*x + c)^2 - 15*cos(d*x + c) + 12)/(a*d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^7(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(tan(d*x+c)**7/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**7/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{60\,\log(\cos(dx+c))}{a}+\frac{60\,\cos(dx+c)^4+60\,\cos(dx+c)^3-40\,\cos(dx+c)^2-15\,\cos(dx+c)+12}{a\cos(dx+c)^5}}{60\,d}$$

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*log(cos(d*x + c))/a + (60*cos(d*x + c)^4 + 60*cos(d*x + c)^3 - 40*
cos(d*x + c)^2 - 15*cos(d*x + c) + 12)/(a*cos(d*x + c)^5))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(91) = 182.

Time = 3.49 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.07

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx =$$

$$-\frac{\frac{60\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}-\frac{60\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a}+\frac{\frac{485\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1330\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{1970\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{805\,(\cos(dx+c)-1}{(\cos(dx+c)+1)^4}}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^5}}{60\,d}$$

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 60*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (485*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 1330*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*(cos(d*x + c) - 1)^4/(cos(
d*x + c) + 1)^4 + 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 73)/(a*((
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 18.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int \frac{\tan^7(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= \frac{-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}-\frac{10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+\frac{16}{15}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-10\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\right.}$$

$$-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a\,d}$$

```
[In] int(tan(c + d*x)^7/(a + a/cos(c + d*x)),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^4)/3 - (10*tan(c/2 + (d*x)/2)^2)/3 + 10*tan(c/2 + (d
*x)/2)^6 - 2*tan(c/2 + (d*x)/2)^8 + 16/15)/(d*(a - 5*a*tan(c/2 + (d*x)/2)^2
 + 10*a*tan(c/2 + (d*x)/2)^4 - 10*a*tan(c/2 + (d*x)/2)^6 + 5*a*tan(c/2 + (d
*x)/2)^8 - a*tan(c/2 + (d*x)/2)^10)) - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d
)
```

3.58 $\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\log(\cos(c+dx))}{ad} - \frac{\sec(c+dx)}{ad} - \frac{\sec^2(c+dx)}{2ad} + \frac{\sec^3(c+dx)}{3ad}$$

```
[Out] -ln(cos(d*x+c))/a/d-sec(d*x+c)/a/d-1/2*sec(d*x+c)^2/a/d+1/3*sec(d*x+c)^3/a/
d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 76}

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sec^3(c+dx)}{3ad} - \frac{\sec^2(c+dx)}{2ad} - \frac{\sec(c+dx)}{ad} - \frac{\log(\cos(c+dx))}{ad}$$

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Sec[c + d*x]/(a*d) - Sec[c + d*x]^2/(2*a*d) +
Sec[c + d*x]^3/(3*a*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)}{x^4}\,dx, x, \cos(c+dx)\right)}{a^4 d} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{a^3}{x^3}-\frac{a^3}{x^2}+\frac{a^3}{x}\right)\,dx, x, \cos(c+dx)\right)}{a^4 d} \\
&= -\frac{\log(\cos(c+dx))}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^2(c+dx)}{2ad}+\frac{\sec^3(c+dx)}{3ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$
\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{(2+6\cos(2(c+dx))+3\cos(3(c+dx))\log(\cos(c+dx))+\cos(c+dx)(6+9\log(\cos(c+dx))))\sec^3(c}{12ad}
$$

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/12*((2 + 6*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)]*Log[Cos[c + d*x]] + Cos
[c + d*x]*(6 + 9*Log[Cos[c + d*x]]))*Sec[c + d*x]^3)/(a*d)
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{-\ln(\cos(dx+c))+\frac{1}{3\cos(dx+c)^3}-\frac{1}{2\cos(dx+c)^2}-\frac{1}{\cos(dx+c)}}{da}$	48
default	$\frac{-\ln(\cos(dx+c))+\frac{1}{3\cos(dx+c)^3}-\frac{1}{2\cos(dx+c)^2}-\frac{1}{\cos(dx+c)}}{da}$	48
risch	$\frac{ix}{a}+\frac{2ic}{da}-\frac{2\left(3\,e^{5i(dx+c)}+3\,e^{4i(dx+c)}+2\,e^{3i(dx+c)}+3\,e^{2i(dx+c)}+3\,e^{i(dx+c)}\right)}{3da\left(e^{2i(dx+c)}+1\right)^3}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{da}$	116

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-ln(cos(d*x+c))+1/3/cos(d*x+c)^3-1/2/cos(d*x+c)^2-1/cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= -\frac{6\cos(dx+c)^3\log(-\cos(dx+c)) + 6\cos(dx+c)^2 + 3\cos(dx+c) - 2}{6\,ad\cos(dx+c)^3}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/6*(6*cos(d*x + c)^3*log(-cos(d*x + c)) + 6*cos(d*x + c)^2 + 3*cos(d*x +
c) - 2)/(a*d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c)),x)
[Out] Integral(tan(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{6\log(\cos(dx+c))}{a} + \frac{6\cos(dx+c)^2+3\cos(dx+c)-2}{a\cos(dx+c)^3}}{6\,d}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/6*(6*log(cos(d*x + c))/a + (6*cos(d*x + c)^2 + 3*cos(d*x + c) - 2)/(a*co
s(d*x + c)^3))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(62) = 124$.

Time = 1.49 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.38

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} - \frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a} + \frac{\frac{21\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{11\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+3}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - 6*log(abs(-
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + (21*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 45*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*(cos(d*x
 + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3)/(a*((cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

$$\int \frac{\tan^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a\,d} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-\frac{4}{3}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}$$

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d) + (2*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2
 + (d*x)/2)^4 - 4/3)/(d*(a - 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)
/2)^4 - a*tan(c/2 + (d*x)/2)^6))
```

3.59 $\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 28

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\log(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad}$$

```
[Out] ln(cos(d*x+c))/a/d+sec(d*x+c)/a/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 45}

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sec(c+dx)}{ad} + \frac{\log(\cos(c+dx))}{ad}$$

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
```

```
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{a-ax}{x^2}\,dx, x, \cos(c+dx)\right)}{a^2 d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{a}{x}\right)\,dx, x, \cos(c+dx)\right)}{a^2 d}\\ &= \frac{\log(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\log(\cos(c+dx))+\sec(c+dx)}{ad}$$

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x]),x]
[Out] (Log[Cos[c + d*x]] + Sec[c + d*x])/(a*d)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\ln(\cos(dx+c))+\frac{1}{\cos(dx+c)}}{da}$	24
default	$\frac{\ln(\cos(dx+c))+\frac{1}{\cos(dx+c)}}{da}$	24
risch	$-\frac{ix}{a}-\frac{2ic}{da}+\frac{2\,\mathrm{e}^{i(dx+c)}}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{da}$	68

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(ln(cos(d*x+c))+1/cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos(dx+c)\log(-\cos(dx+c))+1}{ad\cos(dx+c)}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] (cos(d*x + c)*log(-cos(d*x + c)) + 1)/(a*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c)),x)
[Out] Integral(tan(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\log(\cos(dx+c))}{a}+\frac{1}{a\cos(dx+c)}}{d}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] (log(cos(d*x + c))/a + 1/(a*cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(28) = 56.

Time = 0.60 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.96

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}-\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a}+\frac{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{d}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] -(log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - log(abs(-(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) - 1))/a + ((cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 1)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{\tan^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2}{d\left(a-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a\,d}$$

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x)),x)
[Out] 2/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a*d)
```

3.60 $\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\log(1+\cos(c+dx))}{ad}$$

```
[Out] -ln(1+cos(d*x+c))/a/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 31}

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\log(\cos(c+dx)+1)}{ad}$$

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Log[1 + Cos[c + d*x]]/(a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
```

[n]

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{a+ax}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{\log(1+\cos(c+dx))}{ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$
\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}
$$

[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x]),x]

[Out] (-2*Log[Cos[(c + d*x)/2]])/(a*d)

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

method	result	size
derivativedivides	$\frac{-\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{da}$	27
default	$\frac{-\ln(1+\sec(dx+c))+\ln(\sec(dx+c))}{da}$	27
risch	$\frac{ix}{a}+\frac{2ic}{da}-\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{da}$	39

[In] int(tan(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d/a*(-ln(1+sec(d*x+c))+ln(sec(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{ad}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -log(1/2*cos(d*x + c) + 1/2)/(a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

Time = 2.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.41

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = \begin{cases} \frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2ad} - \frac{\log\left(\sec\left(c+dx\right)+1\right)}{ad} & \text{for}\: d \neq 0 \\ \frac{x\tan\left(c\right)}{a\sec\left(c\right)+a} & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x)
[Out] Piecewise((log(tan(c + d*x)**2 + 1)/(2*a*d) - log(sec(c + d*x) + 1)/(a*d),
Ne(d, 0)), (x*tan(c)/(a*sec(c) + a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\log\left(\cos\left(dx+c\right)+1\right)}{ad}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -log(cos(d*x + c) + 1)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{ad}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 14.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{\tan(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}$$

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x)),x)
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

3.61 $\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{1}{2ad(1+\cos(c+dx))}+\frac{\log(1-\cos(c+dx))}{4ad}+\frac{3\log(1+\cos(c+dx))}{4ad}$$

```
[Out] 1/2/a/d/(1+cos(d*x+c))+1/4*ln(1-cos(d*x+c))/a/d+3/4*ln(1+cos(d*x+c))/a/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 90}

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{1}{2ad(\cos(c+dx)+1)}+\frac{\log(1-\cos(c+dx))}{4ad}+\frac{3\log(\cos(c+dx)+1)}{4ad}$$

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] 1/(2*a*d*(1 + Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(4*a*d) + (3*Log[1 + C
os[c + d*x]])/(4*a*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a^2\text{Subst}\left(\int \frac{x^2}{(a-ax)(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^2\text{Subst}\left(\int \left(-\frac{1}{4a^3(-1+x)}+\frac{1}{2a^3(1+x)^2}-\frac{3}{4a^3(1+x)}\right)\,dx, x, \cos(c+dx)\right)}{d}\\
&= \frac{1}{2ad(1+\cos(c+dx))}+\frac{\log(1-\cos(c+dx))}{4ad}+\frac{3\log(1+\cos(c+dx))}{4ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$
\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\left(1+2\cos^2\left(\frac{1}{2}(c+dx)\right)\left(3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec(c+dx)}{2ad(1+\sec(c+dx))}
$$

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((1 + 2*Cos[(c + d*x)/2]^2*(3*Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]
))*Sec[c + d*x])/(2*a*d*(1 + Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{4}+\frac{1}{2\cos(dx+c)+2}+\frac{3\ln(\cos(dx+c)+1)}{4}}{da}$	43
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{4}+\frac{1}{2\cos(dx+c)+2}+\frac{3\ln(\cos(dx+c)+1)}{4}}{da}$	43
risch	$-\frac{ix}{a}-\frac{2ic}{da}+\frac{e^{i(dx+c)}}{ad\left(e^{i(dx+c)}+1\right)^2}+\frac{3\ln\left(e^{i(dx+c)}+1\right)}{2da}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2da}$	88

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/4*ln(cos(d*x+c)-1)+1/2/(cos(d*x+c)+1)+3/4*ln(cos(d*x+c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$=\frac{3\left(\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\cos\left(dx+c\right)+1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2}{4\left(ad\cos\left(dx+c\right)+ad\right)}$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c) + 1)*
log(-1/2*cos(d*x + c) + 1/2) + 2)/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\int \frac{\cot\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\frac{3\log(\cos(dx+c)+1)}{a}+\frac{\log(\cos(dx+c)-1)}{a}+\frac{2}{a\cos(dx+c)+a}}{4\,d}$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(3*log(cos(d*x + c) + 1)/a + log(cos(d*x + c) - 1)/a + 2/(a*cos(d*x + c
) + a))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.41

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} - \frac{4\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} - \frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{4\,d}$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a - 4*log(abs(-(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a - (cos(d*x + c) - 1)/(a*(cos(d*x +
 c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{\cot(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2} - \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right) + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}}{a\,d}$$

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x)),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))/2 - log(tan(c/2 + (d*x)/2)^2 + 1) + tan(c/2 + (d*x
)/2)^2/4)/(a*d)
```

3.62 $\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{1}{8ad(1-\cos(c+dx))}+\frac{1}{8ad(1+\cos(c+dx))^2} - \frac{3}{4ad(1+\cos(c+dx))}-\frac{5\log(1-\cos(c+dx))}{16ad} - \frac{11\log(1+\cos(c+dx))}{16ad}$$

```
[Out] -1/8/a/d/(1-cos(d*x+c))+1/8/a/d/(1+cos(d*x+c))^2-3/4/a/d/(1+cos(d*x+c))-5/1
6*ln(1-cos(d*x+c))/a/d-11/16*ln(1+cos(d*x+c))/a/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{1}{8ad(1-\cos(c+dx))}-\frac{3}{4ad(\cos(c+dx)+1)} + \frac{1}{8ad(\cos(c+dx)+1)^2}-\frac{5\log(1-\cos(c+dx))}{16ad} - \frac{11\log(\cos(c+dx)+1)}{16ad}$$

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*1/(a*d*(1 - Cos[c + d*x])) + 1/(8*a*d*(1 + Cos[c + d*x])^2) - 3/(4*a*d
*(1 + Cos[c + d*x])) - (5*Log[1 - Cos[c + d*x]])/(16*a*d) - (11*Log[1 + Cos
[c + d*x]])/(16*a*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^4\text{Subst}\left(\int \frac{x^4}{(a-ax)^2(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^4\text{Subst}\left(\int \left(\frac{1}{8a^5(-1+x)^2}+\frac{5}{16a^5(-1+x)}+\frac{1}{4a^5(1+x)^3}-\frac{3}{4a^5(1+x)^2}+\frac{11}{16a^5(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{1}{8ad(1-\cos(c+dx))}+\frac{1}{8ad(1+\cos(c+dx))^2}-\frac{3}{4ad(1+\cos(c+dx))}\\
&\quad-\frac{5\log(1-\cos(c+dx))}{16ad}-\frac{11\log(1+\cos(c+dx))}{16ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{\left(12+2\cot^2\left(\frac{1}{2}(c+dx)\right)+4\cos^2\left(\frac{1}{2}(c+dx)\right)\left(11\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\text{sec}}{16ad(1+\sec(c+dx))}$$

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/16*((12 + 2*Cot[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^2*(11*Log[Cos[(c + d
*x)/2]] + 5*Log[Sin[(c + d*x)/2]]) - Sec[(c + d*x)/2]^2)*Sec[c + d*x])/(a*d
*(1 + Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{\frac{1}{8\cos(dx+c)-8}-\frac{5\ln(\cos(dx+c)-1)}{16}+\frac{1}{8(\cos(dx+c)+1)^2}-\frac{3}{4(\cos(dx+c)+1)}-\frac{11\ln(\cos(dx+c)+1)}{16}}{da}$
default	$\frac{\frac{1}{8\cos(dx+c)-8}-\frac{5\ln(\cos(dx+c)-1)}{16}+\frac{1}{8(\cos(dx+c)+1)^2}-\frac{3}{4(\cos(dx+c)+1)}-\frac{11\ln(\cos(dx+c)+1)}{16}}{da}$
risch	$\frac{ix}{a}+\frac{2ic}{da}-\frac{5\,\mathrm{e}^{5i(dx+c)}-6\,\mathrm{e}^{4i(dx+c)}-14\,\mathrm{e}^{3i(dx+c)}-6\,\mathrm{e}^{2i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}}{4da(\mathrm{e}^{i(dx+c)}+1)^4(\mathrm{e}^{i(dx+c)}-1)^2}-\frac{11\ln(\mathrm{e}^{i(dx+c)}+1)}{8da}-\frac{5\ln(\mathrm{e}^{i(dx+c)}-1)}{8da}$

[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d/a*(1/8/(cos(d*x+c)-1)-5/16*ln(cos(d*x+c)-1)+1/8/(cos(d*x+c)+1)^2-3/4/(c
os(d*x+c)+1)-11/16*ln(cos(d*x+c)+1))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.35

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{10\cos(dx+c)^2+11\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+5\left(\cos\right.}{16\left(ad\cos(dx+c)^3+ad\cos(dx+c\right.}$$

[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/16*(10*cos(d*x + c)^2 + 11*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x +
c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 5*(cos(d*x + c)^3 + cos(d*x + c)^2 -
cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 6*cos(d*x + c) - 12)/(a*d*
cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)

Sympy [F]

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cot^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c)),x)

[Out] Integral(cot(c + d*x)**3/(sec(c + d*x) + 1), x)/a

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{2\left(5\cos(dx+c)^2-3\cos(dx+c)-6\right)}{a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a}+\frac{11\log(\cos(dx+c)+1)}{a}+\frac{5\log(\cos(dx+c)-1)}{a}}{16\,d}$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(2*(5*cos(d*x + c)^2 - 3*cos(d*x + c) - 6)/(a*cos(d*x + c)^3 + a*cos(
d*x + c)^2 - a*cos(d*x + c) - a) + 11*log(cos(d*x + c) + 1)/a + 5*log(cos(d
*x + c) - 1)/a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.52

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{2\left(\frac{5(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a(\cos(dx+c)-1)}-\frac{10\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{32\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}+\frac{\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}}{32\,d}$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(2*(5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(a
*(cos(d*x + c) - 1)) - 10*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))
/a + 32*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a + (10*a*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2)/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 15.83 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \frac{\cot^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{5\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8}-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32}}{a\,d}$$

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

```
[Out] -((5*log(tan(c/2 + (d*x)/2)))/8 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2 +
 (d*x)/2)^2/16 + (5*tan(c/2 + (d*x)/2)^2)/16 - tan(c/2 + (d*x)/2)^4/32)/(a*
d)
```

3.63 $\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{1}{32ad(1-\cos(c+dx))^2}+\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{24ad(1+\cos(c+dx))^3}-\frac{9}{32ad(1+\cos(c+dx))^2}+\frac{15}{16ad(1+\cos(c+dx))}+\frac{11\log(1-\cos(c+dx))}{32ad}+\frac{21\log(1+\cos(c+dx))}{32ad}$$

```
[Out] -1/32/a/d/(1-cos(d*x+c))^2+1/4/a/d/(1-cos(d*x+c))+1/24/a/d/(1+cos(d*x+c))^3
-9/32/a/d/(1+cos(d*x+c))^2+15/16/a/d/(1+cos(d*x+c))+11/32*ln(1-cos(d*x+c))/
a/d+21/32*ln(1+cos(d*x+c))/a/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{1}{4ad(1-\cos(c+dx))}+\frac{15}{16ad(\cos(c+dx)+1)}-\frac{1}{32ad(1-\cos(c+dx))^2}-\frac{9}{32ad(\cos(c+dx)+1)^2}+\frac{1}{24ad(\cos(c+dx)+1)^3}+\frac{11\log(1-\cos(c+dx))}{32ad}+\frac{21\log(\cos(c+dx)+1)}{32ad}$$

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/32*1/(a*d*(1 - Cos[c + d*x])^2) + 1/(4*a*d*(1 - Cos[c + d*x])) + 1/(24*a
*d*(1 + Cos[c + d*x])^3) - 9/(32*a*d*(1 + Cos[c + d*x])^2) + 15/(16*a*d*(1
+ Cos[c + d*x])) + (11*Log[1 - Cos[c + d*x]])/(32*a*d) + (21*Log[1 + Cos[c
+ d*x]])/(32*a*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a^6 \text{Subst}\left(\int \frac{x^6}{(a-ax)^3(a+ax)^4}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^6 \text{Subst}\left(\int \left(-\frac{1}{16a^7(-1+x)^3}-\frac{1}{4a^7(-1+x)^2}-\frac{11}{32a^7(-1+x)}+\frac{1}{8a^7(1+x)^4}-\frac{9}{16a^7(1+x)^3}+\frac{15}{16a^7(1+x)^2}-\frac{21}{32a^7(1+x)}\right.\right.}{d}\\
&= -\frac{1}{32ad(1-\cos(c+dx))^2}+\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{24ad(1+\cos(c+dx))^3}\\
&\quad-\frac{9}{32ad(1+\cos(c+dx))^2}+\frac{15}{16ad(1+\cos(c+dx))}\\
&\quad+\frac{11\log(1-\cos(c+dx))}{32ad}+\frac{21\log(1+\cos(c+dx))}{32ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.93

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-48\csc^2\left(\frac{1}{2}(c+dx)\right)+3\csc^4\left(\frac{1}{2}(c+dx)\right)-504\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-264\log\left(\sin\right.\right.}{192ad(1+\sec(c\,-}$$

[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x]),x]

[Out] -1/192*(Cos[(c + d*x)/2]^2*(-48*Csc[(c + d*x)/2]^2 + 3*Csc[(c + d*x)/2]^4 - 504*Log[Cos[(c + d*x)/2]] - 264*Log[Sin[(c + d*x)/2]] - 180*Sec[(c + d*x)/2]^2 + 27*Sec[(c + d*x)/2]^4 - 2*Sec[(c + d*x)/2]^6)*Sec[c + d*x])/(a*d*(1 + Sec[c + d*x]))

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{-\frac{1}{32(\cos(dx+c)-1)^2}-\frac{1}{4(\cos(dx+c)-1)}+\frac{11\ln(\cos(dx+c)-1)}{32}+\frac{1}{24(\cos(dx+c)+1)^3}-\frac{9}{32(\cos(dx+c)+1)^2}+\frac{15}{16(\cos(dx+c)+1)}+\frac{21\ln(c}{}}{da}$
default	$\frac{-\frac{1}{32(\cos(dx+c)-1)^2}-\frac{1}{4(\cos(dx+c)-1)}+\frac{11\ln(\cos(dx+c)-1)}{32}+\frac{1}{24(\cos(dx+c)+1)^3}-\frac{9}{32(\cos(dx+c)+1)^2}+\frac{15}{16(\cos(dx+c)+1)}+\frac{21\ln(c}{}}{da}$
risch	$-\frac{ix}{a}-\frac{2ic}{da}+\frac{33\,e^{9i(dx+c)}-78\,e^{8i(dx+c)}-184\,e^{7i(dx+c)}-2\,e^{6i(dx+c)}+270\,e^{5i(dx+c)}-2\,e^{4i(dx+c)}-184\,e^{3i(dx+c)}-78\,e}{24da\left(e^{i(dx+c)}+1\right)^6\left(e^{i(dx+c)}-1\right)^4}$

[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d/a*(-1/32/(cos(d*x+c)-1)^2-1/4/(cos(d*x+c)-1)+11/32*ln(cos(d*x+c)-1)+1/24/(cos(d*x+c)+1)^3-9/32/(cos(d*x+c)+1)^2+15/16/(cos(d*x+c)+1)+21/32*ln(cos(d*x+c)+1))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.50

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{66\,\cos(dx+c)^4-78\,\cos(dx+c)^3-158\,\cos(dx+c)^2+63\left(\cos(dx+c)^5+\cos(dx+c)^4-2\,\cos(dx+\right.}{9}$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(66*cos(d*x + c)^4 - 78*cos(d*x + c)^3 - 158*cos(d*x + c)^2 + 63*(cos(
d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x
 + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 33*(cos(d*x + c)^5 + cos(d*x + c)^
4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*log(-1/2*cos(d*
x + c) + 1/2) + 58*cos(d*x + c) + 88)/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c
)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cot^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{2\left(33\cos(dx+c)^4-39\cos(dx+c)^3-79\cos(dx+c)^2+29\cos(dx+c)+44\right)}{a\cos(dx+c)^5+a\cos(dx+c)^4-2\,a\cos(dx+c)^3-2\,a\cos(dx+c)^2+a\cos(dx+c)+a}+\frac{63\log(\cos(dx+c)+1)}{a}+\frac{33\log(\cos(dx+c)-1)}{a}}{96\,d}$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(2*(33*cos(d*x + c)^4 - 39*cos(d*x + c)^3 - 79*cos(d*x + c)^2 + 29*cos
(d*x + c) + 44)/(a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 -
 2*a*cos(d*x + c)^2 + a*cos(d*x + c) + a) + 63*log(cos(d*x + c) + 1)/a + 33
*log(cos(d*x + c) - 1)/a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.46

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{3\left(\frac{14(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{66(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{a(\cos(dx+c)-1)^2} - \frac{132\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} + \frac{384\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a} + \frac{132a^2(\cos(dx+c)}{\cos(dx+c}}{384\,d}$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/384*(3*(14*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 66*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2)
- 132*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + 384*log(abs(-(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a + (132*a^2*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 21*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2*a^
2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^3)/d
```

Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.91

$$\int \frac{\cot^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{32\,a\,d} - \frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{128\,a\,d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{192\,a\,d} + \frac{11\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{16\,a\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d} + \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}-\frac{1}{4}\right)}{32\,a\,d}$$

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x)),x)
```

```
[Out] (11*tan(c/2 + (d*x)/2)^2)/(32*a*d) - (7*tan(c/2 + (d*x)/2)^4)/(128*a*d) + t
an(c/2 + (d*x)/2)^6/(192*a*d) + (11*log(tan(c/2 + (d*x)/2)))/(16*a*d) - log
(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) + (cot(c/2 + (d*x)/2)^4*((7*tan(c/2 + (d*x
)/2)^2)/2 - 1/4))/(32*a*d)
```

3.64 $\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{x}{a}-\frac{5\text{arctanh}(\sin(c+dx))}{16ad}-\frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad}+\frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad}-\frac{(6-5\sec(c+dx))\tan^5(c+dx)}{30ad}$$

```
[Out] x/a-5/16*arctanh(sin(d*x+c))/a/d-1/16*(16-5*sec(d*x+c))*tan(d*x+c)/a/d+1/24
*(8-5*sec(d*x+c))*tan(d*x+c)^3/a/d-1/30*(6-5*sec(d*x+c))*tan(d*x+c)^5/a/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3973, 3966, 3855}

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{5\text{arctanh}(\sin(c+dx))}{16ad}-\frac{\tan^5(c+dx)(6-5\sec(c+dx))}{30ad}+\frac{\tan^3(c+dx)(8-5\sec(c+dx))}{24ad}-\frac{\tan(c+dx)(16-5\sec(c+dx))}{16ad}+\frac{x}{a}$$

```
[In] Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] x/a - (5*ArcTanh[Sin[c + d*x]])/(16*a*d) - ((16 - 5*Sec[c + d*x])*Tan[c + d
*x])/(16*a*d) + ((8 - 5*Sec[c + d*x])*Tan[c + d*x]^3)/(24*a*d) - ((6 - 5*Se
c[c + d*x])*Tan[c + d*x]^5)/(30*a*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (-a+a\sec(c+dx))\tan^6(c+dx)\,dx}{a^2}\\
&= -\frac{(6-5\sec(c+dx))\tan^5(c+dx)}{30ad} - \frac{\int (-6a+5a\sec(c+dx))\tan^4(c+dx)\,dx}{6a^2}\\
&= \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad} - \frac{(6-5\sec(c+dx))\tan^5(c+dx)}{30ad}\\
&\quad + \frac{\int (-24a+15a\sec(c+dx))\tan^2(c+dx)\,dx}{24a^2}\\
&= -\frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad} + \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad}\\
&\quad - \frac{(6-5\sec(c+dx))\tan^5(c+dx)}{30ad} - \frac{\int (-48a+15a\sec(c+dx))\,dx}{48a^2}\\
&= \frac{x}{a} - \frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad} + \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad}\\
&\quad - \frac{(6-5\sec(c+dx))\tan^5(c+dx)}{30ad} - \frac{5\int \sec(c+dx)\,dx}{16a}\\
&= \frac{x}{a} - \frac{5\text{arctanh}(\sin(c+dx))}{16ad} - \frac{(16-5\sec(c+dx))\tan(c+dx)}{16ad}\\
&\quad + \frac{(8-5\sec(c+dx))\tan^3(c+dx)}{24ad} - \frac{(6-5\sec(c+dx))\tan^5(c+dx)}{30ad}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 301 vs. $2(105) = 210$.

Time = 2.33 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.87

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(2400\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}\right.\right.\right.}{}$$

```
[In] Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(2400*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c + d*x]^6*(
2400*d*x*Cos[c] + 1800*d*x*Cos[c + 2*d*x] + 1800*d*x*Cos[3*c + 2*d*x] + 720
*d*x*Cos[3*c + 4*d*x] + 720*d*x*Cos[5*c + 4*d*x] + 120*d*x*Cos[5*c + 6*d*x]
 + 120*d*x*Cos[7*c + 6*d*x] + 3680*Sin[c] + 450*Sin[d*x] + 450*Sin[2*c + d*
x] - 3360*Sin[c + 2*d*x] + 2160*Sin[3*c + 2*d*x] - 25*Sin[2*c + 3*d*x] - 25
*Sin[4*c + 3*d*x] - 1488*Sin[3*c + 4*d*x] + 720*Sin[5*c + 4*d*x] + 165*Sin[
4*c + 5*d*x] + 165*Sin[6*c + 5*d*x] - 368*Sin[5*c + 6*d*x])))/(3840*a*d*(1
+ Sec[c + d*x]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.85

method	result
risch	$\frac{x}{a} - \frac{i\left(165\,e^{11i(dx+c)}+720\,e^{10i(dx+c)}-25\,e^{9i(dx+c)}+2160\,e^{8i(dx+c)}+450\,e^{7i(dx+c)}+3680\,e^{6i(dx+c)}-450\,e^{5i(dx+c)}+3360\right.}{120da\left(e^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}+\frac{7}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{3}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{5}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{9}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{21}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{}$
default	$\frac{\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}+\frac{7}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{3}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{5}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{9}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{21}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{}$

```
[In] int(tan(d*x+c)^8/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] x/a-1/120*I*(165*exp(11*I*(d*x+c))+720*exp(10*I*(d*x+c))-25*exp(9*I*(d*x+c)
)+2160*exp(8*I*(d*x+c))+450*exp(7*I*(d*x+c))+3680*exp(6*I*(d*x+c))-450*exp(
5*I*(d*x+c))+3360*exp(4*I*(d*x+c))+25*exp(3*I*(d*x+c))+1488*exp(2*I*(d*x+c)
)-165*exp(I*(d*x+c))+368)/d/a/(exp(2*I*(d*x+c))+1)^6+5/16/a/d*ln(exp(I*(d*x
+c))-I)-5/16/a/d*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.21

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{480\,dx\cos(dx+c)^6 - 75\cos(dx+c)^6\log(\sin(dx+c)+1) + 75\cos(dx+c)^6\log(-\sin(dx+c)+1) - \ldots}{480\ldots}$$

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/480*(480*d*x*cos(d*x + c)^6 - 75*cos(d*x + c)^6*log(sin(d*x + c) + 1) + 7
5*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*cos(d*x + c)^5 - 165*cos(d
*x + c)^4 - 176*cos(d*x + c)^3 + 130*cos(d*x + c)^2 + 48*cos(d*x + c) - 40)
*sin(d*x + c))/(a*d*cos(d*x + c)^6)
```

Sympy [F]

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^8(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(tan(d*x+c)**8/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**8/(sec(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(97) = 194$.

Time = 0.31 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.13

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{2\left(\frac{165\sin(dx+c)}{\cos(dx+c)+1} - \frac{1095\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3138\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{5118\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{1945\sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{315\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a - \frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}} - \frac{480\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \ldots}{240\,d}$$

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(2*(165*sin(d*x + c)/(cos(d*x + c) + 1) - 1095*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3138*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5118*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 1945*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 315*
```

```
sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/(a - 6*a*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a*sin(d*
x + c)^10/(cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)
- 480*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 75*log(sin(d*x + c)/(cos(
d*x + c) + 1) + 1)/a - 75*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Giac [A] (verification not implemented)

none

Time = 4.80 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.42

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= \frac{\frac{240\,(dx+c)}{a} - \frac{75\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} + \frac{75\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} + \frac{2\left(315\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-1945\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+5118\,\tan\left(\frac{1}{2}\,dx\cdots\right.\right.}{\left(\tan\left(\frac{1}{2}\cdots\right.\right.}}{240\,d}$$

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(240*(d*x + c)/a - 75*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + 75*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a + 2*(315*tan(1/2*d*x + 1/2*c)^11 - 1945*tan
(1/2*d*x + 1/2*c)^9 + 5118*tan(1/2*d*x + 1/2*c)^7 - 3138*tan(1/2*d*x + 1/2*
c)^5 + 1095*tan(1/2*d*x + 1/2*c)^3 - 165*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*
x + 1/2*c)^2 - 1)^6*a))/d
```

Mupad [B] (verification not implemented)

Time = 15.58 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.84

$$\int \frac{\tan^8(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{x}{a} - \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}$$

$$- \frac{-\frac{21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} + \frac{389\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} - \frac{853\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20} + \frac{523\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20} - \frac{73\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8} + \frac{11\,\mathrm{ta}\cdots}{\cdots}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + 15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} - 20\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} + 15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\cdots\right.}$$

```
[In] int(tan(c + d*x)^8/(a + a/cos(c + d*x)),x)
```

```
[Out] x/a - (5*atanh(tan(c/2 + (d*x)/2)))/(8*a*d) - ((11*tan(c/2 + (d*x)/2))/8 -
(73*tan(c/2 + (d*x)/2)^3)/8 + (523*tan(c/2 + (d*x)/2)^5)/20 - (853*tan(c/2
+ (d*x)/2)^7)/20 + (389*tan(c/2 + (d*x)/2)^9)/24 - (21*tan(c/2 + (d*x)/2)^1
1)/8)/(d*(a - 6*a*tan(c/2 + (d*x)/2)^2 + 15*a*tan(c/2 + (d*x)/2)^4 - 20*a*t
an(c/2 + (d*x)/2)^6 + 15*a*tan(c/2 + (d*x)/2)^8 - 6*a*tan(c/2 + (d*x)/2)^10
 + a*tan(c/2 + (d*x)/2)^12))
```

3.65 $\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{x}{a}+\frac{3\operatorname{arctanh}(\sin(c+dx))}{8ad}+\frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad} - \frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad}$$

```
[Out] -x/a+3/8*arctanh(sin(d*x+c))/a/d+1/8*(8-3*sec(d*x+c))*tan(d*x+c)/a/d-1/12*(
4-3*sec(d*x+c))*tan(d*x+c)^3/a/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3973, 3966, 3855}

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3\operatorname{arctanh}(\sin(c+dx))}{8ad}-\frac{\tan^3(c+dx)(4-3\sec(c+dx))}{12ad} + \frac{\tan(c+dx)(8-3\sec(c+dx))}{8ad}-\frac{x}{a}$$

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (3*ArcTanh[Sin[c + d*x]])/(8*a*d) + ((8 - 3*Sec[c + d*x])*Tan[c +
d*x])/(8*a*d) - ((4 - 3*Sec[c + d*x])*Tan[c + d*x]^3)/(12*a*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (-a + a\sec(c+dx))\tan^4(c+dx)\,dx}{a^2} \\
&= -\frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad} - \frac{\int (-4a+3a\sec(c+dx))\tan^2(c+dx)\,dx}{4a^2} \\
&= \frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad} \\
&\quad - \frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad} + \frac{\int (-8a+3a\sec(c+dx))\,dx}{8a^2} \\
&= -\frac{x}{a} + \frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad} - \frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad} + \frac{3\int \sec(c+dx)\,dx}{8a} \\
&= -\frac{x}{a} + \frac{3\operatorname{arctanh}(\sin(c+dx))}{8ad} + \frac{(8-3\sec(c+dx))\tan(c+dx)}{8ad} \\
&\quad - \frac{(4-3\sec(c+dx))\tan^3(c+dx)}{12ad}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 893 vs. $2(78) = 156$.

Time = 7.10 (sec) , antiderivative size = 893, normalized size of antiderivative = 11.45

$$
\begin{aligned}
&\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx \\
&= -\frac{2x\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)}{a+a\sec(c+dx)} \\
&\quad -\frac{3\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec(c+dx)}{4d(a+a\sec(c+dx))} \\
&\quad +\frac{3\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec(c+dx)}{4d(a+a\sec(c+dx))} \\
&\quad +\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)}{8d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^4} \\
&\quad -\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3} \\
&\quad +\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\left(-19\cos\left(\frac{c}{2}\right)+11\sin\left(\frac{c}{2}\right)\right)}{24d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} \\
&\quad +\frac{8\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} \\
&\quad -\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)}{8d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^4} \\
&\quad -\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3} \\
&\quad +\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\left(19\cos\left(\frac{c}{2}\right)+11\sin\left(\frac{c}{2}\right)\right)}{24d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} \\
&\quad +\frac{8\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}
\end{aligned}
$$

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*x*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x])/(a + a*Sec[c + d*x]) - (3*Cos[c/2
+ (d*x)/2]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x])/(4*
d*(a + a*Sec[c + d*x])) + (3*Cos[c/2 + (d*x)/2]^2*Log[Cos[c/2 + (d*x)/2] +
Sin[c/2 + (d*x)/2]]*Sec[c + d*x])/(4*d*(a + a*Sec[c + d*x])) + (Cos[c/2 + (
d*x)/2]^2*Sec[c + d*x])/(8*d*(a + a*Sec[c + d*x])*(Cos[c/2 + (d*x)/2] - Sin
[c/2 + (d*x)/2])^4) - (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d
*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 +
 (d*x)/2])^3) + (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*(-19*Cos[c/2] + 11*Sin[c
/2]))/(24*d*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2]
- Sin[c/2 + (d*x)/2])^2) + (8*Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2
```

])/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Si n[c/2 + (d*x)/2])) - (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x])/(8*d*(a + a*Sec[c + d*x])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) - (Cos[c/2 + (d*x)/2]^ 2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*(19*Cos[c/2] + 11*Sin[c/2]))/(24*d*(a + a*Sec[c + d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (8*Cos[c/2 + (d* x)/2]^2*Sec[c + d*x]*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])*(Cos[c/2] + Si n[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.94

method	result
risch	$-\frac{x}{a}+\frac{i\left(15\,\mathrm{e}^{7i(dx+c)}+48\,\mathrm{e}^{6i(dx+c)}-9\,\mathrm{e}^{5i(dx+c)}+96\,\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{3i(dx+c)}+80\,\mathrm{e}^{2i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}+32\right)}{12da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}}+\frac{3\ln\left(\mathrm{e}^{i(dx}\right.}{8a}$
derivativedivides	$\frac{\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{5}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{11}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}-2\arctan\left(\tan\left(\frac{dx}{2}\right.\right.}{da}$
default	$\frac{\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{5}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{11}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}-2\arctan\left(\tan\left(\frac{dx}{2}\right.\right.}{da}$

[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -x/a+1/12*I*(15*exp(7*I*(d*x+c))+48*exp(6*I*(d*x+c))-9*exp(5*I*(d*x+c))+96* exp(4*I*(d*x+c))+9*exp(3*I*(d*x+c))+80*exp(2*I*(d*x+c))-15*exp(I*(d*x+c))+3 2)/d/a/(exp(2*I*(d*x+c))+1)^4+3/8/a/d*ln(exp(I*(d*x+c))+I)-3/8/a/d*ln(exp(I *(d*x+c))-I)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.37

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{48\,dx\cos\left(dx+c\right)^4-9\,\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)+9\,\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-2}{48\,ad\cos\left(dx+c\right)^4}$$

[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")

```
[Out] -1/48*(48*d*x*cos(d*x + c)^4 - 9*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 9*c
os(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 - 15*cos(d*x +
c)^2 - 8*cos(d*x + c) + 6)*sin(d*x + c))/(a*d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(72) = 144$.

Time = 0.31 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.17

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\frac{2\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{71\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{137\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{33\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a-\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{48\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{9\log\left(\frac{\sin(dx}{\cos(dx+}\right.}{a}}{24\,d}$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/24*(2*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 71*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 137*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 33*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 48*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - 9*log(sin(d*x +
 c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Giac [A] (verification not implemented)

none

Time = 2.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.58

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx =$$
$$-\frac{\frac{24\,(dx+c)}{a}-\frac{9\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}+\frac{9\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2\left(33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-137\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+71\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4 a}}{24\,d}$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(24*(d*x + c)/a - 9*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + 9*log(abs(
tan(1/2*d*x + 1/2*c) - 1))/a + 2*(33*tan(1/2*d*x + 1/2*c)^7 - 137*tan(1/2*d
*x + 1/2*c)^5 + 71*tan(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 - 1)^4*a))/d
```

Mupad [B] (verification not implemented)

Time = 15.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.78

$$\int \frac{\tan^6(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a\,d}-\frac{x}{a}$$
$$+\frac{-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{137\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}-\frac{71\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}$$

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x)),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d) - x/a + ((5*tan(c/2 + (d*x)/2))/4 - (
71*tan(c/2 + (d*x)/2)^3)/12 + (137*tan(c/2 + (d*x)/2)^5)/12 - (11*tan(c/2 +
 (d*x)/2)^7)/4)/(d*(a - 4*a*tan(c/2 + (d*x)/2)^2 + 6*a*tan(c/2 + (d*x)/2)^4
 - 4*a*tan(c/2 + (d*x)/2)^6 + a*tan(c/2 + (d*x)/2)^8))
```

3.66 $\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{x}{a}-\frac{\operatorname{arctanh}(\sin(c+dx))}{2ad}-\frac{(2-\sec(c+dx))\tan(c+dx)}{2ad}$$

```
[Out] x/a-1/2*arctanh(sin(d*x+c))/a/d-1/2*(2-sec(d*x+c))*tan(d*x+c)/a/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3973, 3966, 3855}

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{\operatorname{arctanh}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)(2-\sec(c+dx))}{2ad}+\frac{x}{a}$$

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] x/a - ArcTanh[Sin[c + d*x]]/(2*a*d) - ((2 - Sec[c + d*x])*Tan[c + d*x])/(2*
a*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
```

```
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (-a+a\sec(c+dx))\tan^2(c+dx)\,dx}{a^2}\\
&= -\frac{(2-\sec(c+dx))\tan(c+dx)}{2ad} - \frac{\int (-2a+a\sec(c+dx))\,dx}{2a^2}\\
&= \frac{x}{a} - \frac{(2-\sec(c+dx))\tan(c+dx)}{2ad} - \frac{\int \sec(c+dx)\,dx}{2a}\\
&= \frac{x}{a} - \frac{\operatorname{arctanh}(\sin(c+dx))}{2ad} - \frac{(2-\sec(c+dx))\tan(c+dx)}{2ad}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(49) = 98$.

Time = 1.20 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.92

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(4x+\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{}{d(\cos(\frac{1}{2}(c+dx}\right.}{2a}$$

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(4*x + (2*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]])/d - (2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + 1/(d*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])^2) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2) - (4*Sin[d*x])/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(2
*a*(1 + Sec[c + d*x]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.12

method	result
risch	$\frac{x}{a}-\frac{i\left(\mathrm{e}^{3i(dx+c)}+2\,\mathrm{e}^{2i(dx+c)}-\mathrm{e}^{i(dx+c)}+2\right)}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2ad}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2ad}$
derivativedivides	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{2}}{da}$
default	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{2}}{da}$

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] x/a-I*(exp(3*I*(d*x+c))+2*exp(2*I*(d*x+c))-exp(I*(d*x+c))+2)/d/a/(exp(2*I*(
d*x+c))+1)^2+1/2/a/d*ln(exp(I*(d*x+c))-I)-1/2/a/d*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{4\,dx\cos\left(dx+c\right)^2-\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)-2\left(2\cos\right.}{4\,ad\cos\left(dx+c\right)^2}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*x*cos(d*x + c)^2 - cos(d*x + c)^2*log(sin(d*x + c) + 1) + cos(d*x
+ c)^2*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) - 1)*sin(d*x + c))/(a*d*c
os(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(43) = 86$.

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.33

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2d}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4) - 4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/a)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(43) = 86$.

Time = 0.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.96

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{2(dx+c)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a}}{2d}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a + 2*(3*tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{\tan^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{x}{a} - \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}$$

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

```
[Out] x/a - atanh(tan(c/2 + (d*x)/2))/(a*d) - (tan(c/2 + (d*x)/2) - 3*tan(c/2 + (
d*x)/2)^3)/(d*(a - 2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4))
```

3.67 $\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)} dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{x}{a}+\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}$$

```
[Out] -x/a+arctanh(sin(d*x+c))/a/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3973, 3855}

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}-\frac{x}{a}$$

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x]),x]
[Out] -(x/a) + ArcTanh[Sin[c + d*x]]/(a*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
```

```
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\int(-a+a\sec(c+dx))\,dx}{a^2}\\&=-\frac{x}{a}+\frac{\int\sec(c+dx)\,dx}{a}\\&=-\frac{x}{a}+\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 60 vs. $2(21) = 42$.

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.86

$$\int\frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx\\=-\frac{dx+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x]),x]
[Out] -((d*x + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]])/(a*d))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

method	result	size
risch	$-\frac{x}{a}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{ad}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{ad}$	49
derivativedivides	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	50
default	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	50

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -x/a-1/a/d*ln(exp(I*(d*x+c))-I)+1/a/d*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{2\,dx - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(-\sin\left(dx+c\right)+1\right)}{2\,ad}$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(2*d*x - log(sin(d*x + c) + 1) + log(-sin(d*x + c) + 1))/(a*d)
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\tan^2\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c)),x)
[Out] Integral(tan(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(21) = 42.

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.71

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{2\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} + \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{d}$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - log(sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)/a + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(21) = 42$.

Time = 0.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.38

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{dx+c}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}}{d}$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a)/d
```

Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\tan^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} - \frac{x}{a}$$

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a*d) - x/a
```

3.68 $\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{x}{a}-\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad}+\frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}$$

```
[Out] -x/a-1/3*cot(d*x+c)*(3-2*sec(d*x+c))/a/d+1/3*cot(d*x+c)^3*(1-sec(d*x+c))/a/
d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3973, 3967, 8}

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}-\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad}-\frac{x}{a}$$

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) - (Cot[c + d*x]*(3 - 2*Sec[c + d*x]))/(3*a*d) + (Cot[c + d*x]^3*(1 -
 Sec[c + d*x]))/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \cot^4(c+dx)(-a+a\sec(c+dx))\,dx}{a^2}\\
&= \frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad} + \frac{\int \cot^2(c+dx)(3a-2a\sec(c+dx))\,dx}{3a^2}\\
&= -\frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad} + \frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad} + \frac{\int -3a\,dx}{3a^2}\\
&= -\frac{x}{a} - \frac{\cot(c+dx)(3-2\sec(c+dx))}{3ad} + \frac{\cot^3(c+dx)(1-\sec(c+dx))}{3ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.64

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sec(c+dx)\left(-12dx\cos^2\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(3\cot\left(\frac{1}{2}(c+dx)\right)\csc\left(\frac{c}{2}\right)+13\sec\left(\frac{c}{2}\right)\right)\sin\left(\frac{dx}{2}\right)-\right.}{6ad(1+\sec(c+dx))}$$

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(-12*d*x*Cos[(c + d*x)/2]^2 + Cos[(c + d*x)/2]*(3*Cot[(c + d*
x)/2]*Csc[c/2] + 13*Sec[c/2])*Sin[(d*x)/2] - Tan[(c + d*x)/2]))/(6*a*d*(1 +
 Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	59
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	59
risch	$-\frac{x}{a}+\frac{2i\left(3\,\mathrm{e}^{3i(dx+c)}-5\,\mathrm{e}^{i(dx+c)}-4\right)}{3da\left(\mathrm{e}^{i(dx+c)}+1\right)^3\left(\mathrm{e}^{i(dx+c)}-1\right)}$	67

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/4/d/a*(-1/3*tan(1/2*d*x+1/2*c)^3+4*tan(1/2*d*x+1/2*c)-8*arctan(tan(1/2*d*
x+1/2*c))-1/tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{4\cos(dx+c)^2+3\,(dx\cos(dx+c)+dx)\sin(dx+c)+\cos(dx+c)-2}{3\,(ad\cos(dx+c)+ad)\sin(dx+c)}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/3*(4*cos(d*x + c)^2 + 3*(d*x*cos(d*x + c) + d*x)*sin(d*x + c) + cos(d*x
+ c) - 2)/((a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cot^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c)),x)
[Out] Integral(cot(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{3(\cos(dx+c)+1)}{a\sin(dx+c)}}{12\,d}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 3*(cos(d*x + c) +
1)/(a*sin(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{12(dx+c)}{a}+\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{3}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{12\,d}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(12*(d*x + c)/a + (a^2*tan(1/2*d*x + 1/2*c)^3 - 12*a^2*tan(1/2*d*x +
1/2*c))/a^3 + 3/(a*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 14.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{\cot^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{x}{a}-\frac{\frac{4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{7\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6}+\frac{1}{12}}{a\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

```
[Out] - x/a - ((4*cos(c/2 + (d*x)/2)^4)/3 - (7*cos(c/2 + (d*x)/2)^2)/6 + 1/12)/(a
*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2))
```

3.69 $\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{x}{a}+\frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad} - \frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad}+\frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}$$

```
[Out] x/a+1/15*cot(d*x+c)*(15-8*sec(d*x+c))/a/d-1/15*cot(d*x+c)^3*(5-4*sec(d*x+c)
)/a/d+1/5*cot(d*x+c)^5*(1-sec(d*x+c))/a/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3973, 3967, 8}

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}-\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad} + \frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad}+\frac{x}{a}$$

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] x/a + (Cot[c + d*x]*(15 - 8*Sec[c + d*x]))/(15*a*d) - (Cot[c + d*x]^3*(5 -
4*Sec[c + d*x]))/(15*a*d) + (Cot[c + d*x]^5*(1 - Sec[c + d*x]))/(5*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \cot^6(c+dx)(-a+a\sec(c+dx))\,dx}{a^2}\\
&= \frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}+\frac{\int \cot^4(c+dx)(5a-4a\sec(c+dx))\,dx}{5a^2}\\
&= -\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad}+\frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}\\
&\quad+\frac{\int \cot^2(c+dx)(-15a+8a\sec(c+dx))\,dx}{15a^2}\\
&= \frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad}-\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad}\\
&\quad+\frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}+\frac{\int 15a\,dx}{15a^2}\\
&= \frac{x}{a}+\frac{\cot(c+dx)(15-8\sec(c+dx))}{15ad}\\
&\quad-\frac{\cot^3(c+dx)(5-4\sec(c+dx))}{15ad}+\frac{\cot^5(c+dx)(1-\sec(c+dx))}{5ad}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 254 vs. 2(88) = 176.

Time = 1.07 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.89

$$
\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx
$$

$$
= \frac{\csc\left(\frac{c}{2}\right)\csc^3(c+dx)\sec\left(\frac{c}{2}\right)\sec(c+dx)(360dx\cos(dx)-360dx\cos(2c+dx)+120dx\cos(c+2dx)-120d}{}
$$

[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x]),x]

[Out] (Csc[c/2]*Csc[c + d*x]^3*Sec[c/2]*Sec[c + d*x]*(360*d*x*Cos[d*x] - 360*d*x*Cos[2*c + d*x] + 120*d*x*Cos[c + 2*d*x] - 120*d*x*Cos[3*c + 2*d*x] - 120*d*x*Cos[2*c + 3*d*x] + 120*d*x*Cos[4*c + 3*d*x] - 60*d*x*Cos[3*c + 4*d*x] + 60*d*x*Cos[5*c + 4*d*x] - 200*Sin[c] - 584*Sin[d*x] + 534*Sin[c + d*x] + 178*Sin[2*(c + d*x)] - 178*Sin[3*(c + d*x)] - 89*Sin[4*(c + d*x)] - 520*Sin[2*c + d*x] - 248*Sin[c + 2*d*x] - 120*Sin[3*c + 2*d*x] + 248*Sin[2*c + 3*d*x] + 120*Sin[4*c + 3*d*x] + 184*Sin[3*c + 4*d*x]))/(1920*a*d*(1 + Sec[c + d*x]))

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16da}$	85
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16da}$	85
risch	$\frac{x}{a}-\frac{2i\left(15\,\mathrm{e}^{7i(dx+c)}-15\,\mathrm{e}^{6i(dx+c)}-65\,\mathrm{e}^{5i(dx+c)}-25\,\mathrm{e}^{4i(dx+c)}+73\,\mathrm{e}^{3i(dx+c)}+31\,\mathrm{e}^{2i(dx+c)}-31\,\mathrm{e}^{i(dx+c)}-23\right)}{15da\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}\left(\mathrm{e}^{i(dx+c)}-1\right)^{3}}$	121

[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/16/d/a*(-1/5*tan(1/2*d*x+1/2*c)^5+2*tan(1/2*d*x+1/2*c)^3-16*tan(1/2*d*x+1/2*c)+32*arctan(tan(1/2*d*x+1/2*c))-1/3/tan(1/2*d*x+1/2*c)^3+6/tan(1/2*d*x+1/2*c))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{23\,\cos\left(dx+c\right)^{4}+8\,\cos\left(dx+c\right)^{3}-27\,\cos\left(dx+c\right)^{2}+15\,\left(dx\cos\left(dx+c\right)^{3}+dx\cos\left(dx+c\right)^{2}-dx\cos\left(dx\right.\right.}{15\,\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}-ad\cos\left(dx+c\right)-ad\right)\sin\left(d\right.}$$

[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/15*(23*cos(d*x + c)^4 + 8*cos(d*x + c)^3 - 27*cos(d*x + c)^2 + 15*(d*x*cos(d*x + c)^3 + d*x*cos(d*x + c)^2 - d*x*cos(d*x + c) - d*x)*sin(d*x + c) - 7*cos(d*x + c) + 8)/((a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))

Sympy [F]

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cot^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c)),x)
[Out] Integral(cot(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.56

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{3\left(\frac{80\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a}-\frac{480\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{5\left(\frac{18\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{240\,d}$$

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/240*(3*(80*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 480*arctan(sin(d*x +
 c)/(cos(d*x + c) + 1))/a - 5*(18*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*
(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.11

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{240\,(dx+c)}{a}+\frac{5\left(18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}-\frac{3\left(a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-10a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+80a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^5}}{240\,d}$$

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] 1/240*(240*(d*x + c)/a + 5*(18*tan(1/2*d*x + 1/2*c)^2 - 1)/(a*tan(1/2*d*x +
 1/2*c)^3) - 3*(a^4*tan(1/2*d*x + 1/2*c)^5 - 10*a^4*tan(1/2*d*x + 1/2*c)^3
+ 80*a^4*tan(1/2*d*x + 1/2*c))/a^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.80

$$\int \frac{\cot^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8+3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8-30\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-9}{240\,a\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}$$

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

```
[Out] -(5*cos(c/2 + (d*x)/2)^8 + 3*sin(c/2 + (d*x)/2)^8 - 30*cos(c/2 + (d*x)/2)^2
*sin(c/2 + (d*x)/2)^6 + 240*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4 - 90*
cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 - 240*cos(c/2 + (d*x)/2)^5*sin(c/
2 + (d*x)/2)^3*(c + d*x))/(240*a*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^
3)
```

3.70 $\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 117

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx = &-\frac{x}{a}+\frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad}\\ &-\frac{\cot(c+dx)(35-16\sec(c+dx))}{35ad}\\ &-\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad}+\frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad}\end{aligned}$$

```
[Out] -x/a+1/105*cot(d*x+c)^3*(35-24*sec(d*x+c))/a/d-1/35*cot(d*x+c)*(35-16*sec(d
*x+c))/a/d-1/35*cot(d*x+c)^5*(7-6*sec(d*x+c))/a/d+1/7*cot(d*x+c)^7*(1-sec(d
*x+c))/a/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3973, 3967, 8}

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx = &\frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad}-\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad}\\ &+\frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad}\\ &-\frac{\cot(c+dx)(35-16\sec(c+dx))}{35ad}-\frac{x}{a}\end{aligned}$$

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (Cot[c + d*x]^3*(35 - 24*Sec[c + d*x]))/(105*a*d) - (Cot[c + d*x]*
(35 - 16*Sec[c + d*x]))/(35*a*d) - (Cot[c + d*x]^5*(7 - 6*Sec[c + d*x]))/(3
5*a*d) + (Cot[c + d*x]^7*(1 - Sec[c + d*x]))/(7*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \cot^8(c+dx)(-a+a\sec(c+dx))\,dx}{a^2}\\
&= \frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad}+\frac{\int \cot^6(c+dx)(7a-6a\sec(c+dx))\,dx}{7a^2}\\
&= -\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad}+\frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad}\\
&\quad+\frac{\int \cot^4(c+dx)(-35a+24a\sec(c+dx))\,dx}{35a^2}\\
&= \frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad}-\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad}\\
&\quad+\frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad}+\frac{\int \cot^2(c+dx)(105a-48a\sec(c+dx))\,dx}{105a^2}\\
&= \frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad}-\frac{\cot(c+dx)(35-16\sec(c+dx))}{35ad}\\
&\quad-\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad}+\frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad}+\frac{\int -105a\,dx}{105a^2}\\
&= -\frac{x}{a}+\frac{\cot^3(c+dx)(35-24\sec(c+dx))}{105ad}-\frac{\cot(c+dx)(35-16\sec(c+dx))}{35ad}\\
&\quad-\frac{\cot^5(c+dx)(7-6\sec(c+dx))}{35ad}+\frac{\cot^7(c+dx)(1-\sec(c+dx))}{7ad}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 359 vs. $2(117) = 234$.

Time = 1.63 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.07

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\csc\left(\frac{c}{2}\right)\csc^5(c+dx)\sec\left(\frac{c}{2}\right)\sec(c+dx)(-16800dx\cos(dx)+16800dx\cos(2c+dx)-4200dx\cos(c+2dx}{}$$

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x]^5*Sec[c/2]*Sec[c + d*x]*(-16800*d*x*Cos[d*x] + 16800
*d*x*Cos[2*c + d*x] - 4200*d*x*Cos[c + 2*d*x] + 4200*d*x*Cos[3*c + 2*d*x] +
 8400*d*x*Cos[2*c + 3*d*x] - 8400*d*x*Cos[4*c + 3*d*x] + 3360*d*x*Cos[3*c +
 4*d*x] - 3360*d*x*Cos[5*c + 4*d*x] - 1680*d*x*Cos[4*c + 5*d*x] + 1680*d*x*
Cos[6*c + 5*d*x] - 840*d*x*Cos[5*c + 6*d*x] + 840*d*x*Cos[7*c + 6*d*x] + 31
36*Sin[c] + 30112*Sin[d*x] - 22860*Sin[c + d*x] - 5715*Sin[2*(c + d*x)] + 1
1430*Sin[3*(c + d*x)] + 4572*Sin[4*(c + d*x)] - 2286*Sin[5*(c + d*x)] - 114
3*Sin[6*(c + d*x)] + 26208*Sin[2*c + d*x] + 14080*Sin[c + 2*d*x] - 16400*Si
n[2*c + 3*d*x] - 11760*Sin[4*c + 3*d*x] - 7904*Sin[3*c + 4*d*x] - 3360*Sin[
5*c + 4*d*x] + 3952*Sin[4*c + 5*d*x] + 1680*Sin[6*c + 5*d*x] + 2816*Sin[5*c
 + 6*d*x]))/(107520*a*d*(1 + Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{8}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{29}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-128}{64da}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{8}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{29}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-128}{64da}$
risch	$-\frac{x}{a}+\frac{2i\left(105\,e^{11i(dx+c)}-210\,e^{10i(dx+c)}-735\,e^{9i(dx+c)}+1638\,e^{7i(dx+c)}+196\,e^{6i(dx+c)}-1882\,e^{5i(dx+c)}-880\,e^{4i(dx+c)}\right.}{105da\left(e^{i(dx+c)}+1\right)^7\left(e^{i(dx+c)}-1\right)^5}$

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d/a*(-1/7*tan(1/2*d*x+1/2*c)^7+8/5*tan(1/2*d*x+1/2*c)^5-29/3*tan(1/2*d
*x+1/2*c)^3+64*tan(1/2*d*x+1/2*c)-1/5/tan(1/2*d*x+1/2*c)^5+8/3/tan(1/2*d*x+
1/2*c)^3-29/tan(1/2*d*x+1/2*c)-128*arctan(tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.69

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{176\cos(dx+c)^6+71\cos(dx+c)^5-335\cos(dx+c)^4-125\cos(dx+c)^3+225\cos(dx+c)^2+105\,(}{105\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4-\right.}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/105*(176*cos(d*x + c)^6 + 71*cos(d*x + c)^5 - 335*cos(d*x + c)^4 - 125*c
os(d*x + c)^3 + 225*cos(d*x + c)^2 + 105*(d*x*cos(d*x + c)^5 + d*x*cos(d*x
+ c)^4 - 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(d*x + c)^2 + d*x*cos(d*x + c) + d
*x)*sin(d*x + c) + 57*cos(d*x + c) - 48)/((a*d*cos(d*x + c)^5 + a*d*cos(d*x
 + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) +
a*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cot^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c)),x)
[Out] Integral(cot(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.51

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\frac{6720\sin(dx+c)}{\cos(dx+c)+1}-\frac{1015\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{168\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a}-\frac{13440\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{7\left(\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{435\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-3\right)(\cos}{a\sin(dx+c)^5}}{6720\,d}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/6720*((6720*sin(d*x + c)/(cos(d*x + c) + 1) - 1015*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 168*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a - 13440*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 7*(40*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 435*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a*sin(d*x + c)^5))/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{6720\,(dx+c)}{a} + \frac{7\left(435\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5} + \frac{15\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 168\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 1015\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{a^7}}{6720\,d}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(6720*(d*x + c)/a + 7*(435*tan(1/2*d*x + 1/2*c)^4 - 40*tan(1/2*d*x
+ 1/2*c)^2 + 3)/(a*tan(1/2*d*x + 1/2*c)^5) + (15*a^6*tan(1/2*d*x + 1/2*c)^7
 - 168*a^6*tan(1/2*d*x + 1/2*c)^5 + 1015*a^6*tan(1/2*d*x + 1/2*c)^3 - 6720*
a^6*tan(1/2*d*x + 1/2*c))/a^7)/d
```

Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.76

$$\int \frac{\cot^6(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{21\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} + 15\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 168\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + 1015\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\right.}{}$$

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x)),x)
```

```
[Out] -(21*cos(c/2 + (d*x)/2)^12 + 15*sin(c/2 + (d*x)/2)^12 - 168*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2)^10 + 1015*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^
8 - 6720*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6 + 3045*cos(c/2 + (d*x)/2
)^8*sin(c/2 + (d*x)/2)^4 - 280*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2 +
 6720*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5*(c + d*x))/(6720*a*d*cos(c/
2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)
```

3.71 $\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\log(\cos(c+dx))}{a^2d} - \frac{2\sec(c+dx)}{a^2d} - \frac{\sec^2(c+dx)}{2a^2d} + \frac{4\sec^3(c+dx)}{3a^2d} - \frac{\sec^4(c+dx)}{4a^2d} - \frac{2\sec^5(c+dx)}{5a^2d} + \frac{\sec^6(c+dx)}{6a^2d}$$

```
[Out] -ln(cos(d*x+c))/a^2/d-2*sec(d*x+c)/a^2/d-1/2*sec(d*x+c)^2/a^2/d+4/3*sec(d*x
+c)^3/a^2/d-1/4*sec(d*x+c)^4/a^2/d-2/5*sec(d*x+c)^5/a^2/d+1/6*sec(d*x+c)^6/
a^2/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sec^6(c+dx)}{6a^2d} - \frac{2\sec^5(c+dx)}{5a^2d} - \frac{\sec^4(c+dx)}{4a^2d} + \frac{4\sec^3(c+dx)}{3a^2d} - \frac{\sec^2(c+dx)}{2a^2d} - \frac{2\sec(c+dx)}{a^2d} - \frac{\log(\cos(c+dx))}{a^2d}$$

```
[In] Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) - Sec[c + d*x]^2/(2
*a^2*d) + (4*Sec[c + d*x]^3)/(3*a^2*d) - Sec[c + d*x]^4/(4*a^2*d) - (2*Sec[
c + d*x]^5)/(5*a^2*d) + Sec[c + d*x]^6/(6*a^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)^2}{x^7}\,dx, x, \cos(c+dx)\right)}{a^8 d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^6}{x^7}-\frac{2a^6}{x^6}-\frac{a^6}{x^5}+\frac{4a^6}{x^4}-\frac{a^6}{x^3}-\frac{2a^6}{x^2}+\frac{a^6}{x}\right)dx, x, \cos(c+dx)\right)}{a^8 d}\\ &= -\frac{\log(\cos(c+dx))}{a^2 d}-\frac{2\sec(c+dx)}{a^2 d}-\frac{\sec^2(c+dx)}{2a^2 d}\\ &\quad+\frac{4\sec^3(c+dx)}{3a^2 d}-\frac{\sec^4(c+dx)}{4a^2 d}-\frac{2\sec^5(c+dx)}{5a^2 d}+\frac{\sec^6(c+dx)}{6a^2 d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(312\cos(c+dx)+5(14+28\cos(3(c+dx))+6\cos(4(c+dx))+12\cos(5(c+dx))+30\log(\cos(c+dx}{}$$

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/480*((312*Cos[c + d*x] + 5*(14 + 28*Cos[3*(c + d*x)] + 6*Cos[4*(c + d*x)
] + 12*Cos[5*(c + d*x)] + 30*Log[Cos[c + d*x]] + 18*Cos[4*(c + d*x)]*Log[Co
s[c + d*x]] + 3*Cos[6*(c + d*x)]*Log[Cos[c + d*x]] + 9*Cos[2*(c + d*x)]*(4
+ 5*Log[Cos[c + d*x]])))*Sec[c + d*x]^6)/(a^2*d)
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{-\ln(\cos(dx+c))+\frac{4}{3\cos(dx+c)^3}-\frac{1}{2\cos(dx+c)^2}-\frac{2}{5\cos(dx+c)^5}-\frac{2}{\cos(dx+c)}+\frac{1}{6\cos(dx+c)^6}-\frac{1}{4\cos(dx+c)^4}}{d\,a^2}$
default	$\frac{-\ln(\cos(dx+c))+\frac{4}{3\cos(dx+c)^3}-\frac{1}{2\cos(dx+c)^2}-\frac{2}{5\cos(dx+c)^5}-\frac{2}{\cos(dx+c)}+\frac{1}{6\cos(dx+c)^6}-\frac{1}{4\cos(dx+c)^4}}{d\,a^2}$
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2\left(30\,\mathrm{e}^{11i(dx+c)}+15\,\mathrm{e}^{10i(dx+c)}+70\,\mathrm{e}^{9i(dx+c)}+90\,\mathrm{e}^{8i(dx+c)}+156\,\mathrm{e}^{7i(dx+c)}+70\,\mathrm{e}^{6i(dx+c)}+156\,\mathrm{e}^{5i(dx+c)}+90\right.}{15d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}$

```
[In] int(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-ln(cos(d*x+c))+4/3/cos(d*x+c)^3-1/2/cos(d*x+c)^2-2/5/cos(d*x+c)^5
-2/cos(d*x+c)+1/6/cos(d*x+c)^6-1/4/cos(d*x+c)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{60\,\cos\left(dx+c\right)^6\log\left(-\cos\left(dx+c\right)\right)+120\,\cos\left(dx+c\right)^5+30\,\cos\left(dx+c\right)^4-80\,\cos\left(dx+c\right)^3+15\,\cos\left(\right.}{60\,a^2d\cos\left(dx+c\right)^6}$$

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(60*cos(d*x + c)^6*log(-cos(d*x + c)) + 120*cos(d*x + c)^5 + 30*cos(d
*x + c)^4 - 80*cos(d*x + c)^3 + 15*cos(d*x + c)^2 + 24*cos(d*x + c) - 10)/(
a^2*d*cos(d*x + c)^6)
```

Sympy [F]

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^9\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

```
[In] integrate(tan(d*x+c)**9/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**9/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{60\,\log(\cos(dx+c))}{a^2}+\frac{120\,\cos(dx+c)^5+30\,\cos(dx+c)^4-80\,\cos(dx+c)^3+15\,\cos(dx+c)^2+24\,\cos(dx+c)-10}{a^2\cos(dx+c)^6}}{60\,d}$$

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*(60*log(cos(d*x + c))/a^2 + (120*cos(d*x + c)^5 + 30*cos(d*x + c)^4 -
 80*cos(d*x + c)^3 + 15*cos(d*x + c)^2 + 24*cos(d*x + c) - 10)/(a^2*cos(d*x
 + c)^6))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(110) = 220.

Time = 5.81 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.86

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{60\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{60\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2}+\frac{\frac{234\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1005\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2220\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{2925\,(\cos(dx+c)-1)}{(\cos(dx+c)+1)^4}\cdots}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^6}}{60\,d}$$

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 60*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 + (234*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 1005*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2
220*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 + 1002*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 147*
(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 19)/(a^2*((cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 1)^6))/d
```

Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.61

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^2\,d}$$

$$+\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{54\tan\left(\right.}{}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\,a^2\tan\left(\frac{c}{2}+\right.\right.}$$

```
[In] int(tan(c + d*x)^9/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^2*d) + ((54*tan(c/2 + (d*x)/2)^2)/5 - 20
*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 + 12*tan(c/2 + (d*x)/2)^8 -
 2*tan(c/2 + (d*x)/2)^10 - 32/15)/(d*(15*a^2*tan(c/2 + (d*x)/2)^4 - 6*a^2*t
an(c/2 + (d*x)/2)^2 - 20*a^2*tan(c/2 + (d*x)/2)^6 + 15*a^2*tan(c/2 + (d*x)/
2)^8 - 6*a^2*tan(c/2 + (d*x)/2)^10 + a^2*tan(c/2 + (d*x)/2)^12 + a^2))
```

3.72 $\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\frac{\log(\cos(c+dx))}{a^2d}+\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}+\frac{\sec^4(c+dx)}{4a^2d}$$

```
[Out] ln(cos(d*x+c))/a^2/d+2*sec(d*x+c)/a^2/d-2/3*sec(d*x+c)^3/a^2/d+1/4*sec(d*x+
c)^4/a^2/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 76}

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\frac{\sec^4(c+dx)}{4a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}+\frac{2\sec(c+dx)}{a^2d}+\frac{\log(\cos(c+dx))}{a^2d}$$

```
[In] Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + (2*Sec[c + d*x])/(a^2*d) - (2*Sec[c + d*x]^3)/(
3*a^2*d) + Sec[c + d*x]^4/(4*a^2*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^3(a+ax)}{x^5}\,dx, x, \cos(c+dx)\right)}{a^6 d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^4}{x^5}-\frac{2a^4}{x^4}+\frac{2a^4}{x^2}-\frac{a^4}{x}\right)dx, x, \cos(c+dx)\right)}{a^6 d}\\
&= \frac{\log(\cos(c+dx))}{a^2 d}+\frac{2\sec(c+dx)}{a^2 d}-\frac{2\sec^3(c+dx)}{3a^2 d}+\frac{\sec^4(c+dx)}{4a^2 d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{(20\cos(c+dx)+3(2+4\cos(3(c+dx))+3\log(\cos(c+dx))+4\cos(2(c+dx))\log(\cos(c+dx))+\cos(4(c}{24a^2 d}$$

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((20*Cos[c + d*x] + 3*(2 + 4*Cos[3*(c + d*x)] + 3*Log[Cos[c + d*x]] + 4*Cos
[2*(c + d*x)]*Log[Cos[c + d*x]] + Cos[4*(c + d*x)]*Log[Cos[c + d*x]]))*Sec[
c + d*x]^4)/(24*a^2*d)
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\frac{2}{\cos(dx+c)}+\frac{1}{4\cos(dx+c)^4}-\frac{2}{3\cos(dx+c)^3}+\ln(\cos(dx+c))}{d\,a^2}$	46
default	$\frac{\frac{2}{\cos(dx+c)}+\frac{1}{4\cos(dx+c)^4}-\frac{2}{3\cos(dx+c)^3}+\ln(\cos(dx+c))}{d\,a^2}$	46
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2 d}+\frac{4\,e^{7i(dx+c)}+\frac{20\,e^{5i(dx+c)}}{3}+4\,e^{4i(dx+c)}+\frac{20\,e^{3i(dx+c)}}{3}+4\,e^{i(dx+c)}}{d\,a^2\left(e^{2i(dx+c)}+1\right)^4}+\frac{\ln\left(e^{2i(dx+c)}+1\right)}{a^2 d}$	115

```
[In] int(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(2/cos(d*x+c)+1/4/cos(d*x+c)^4-2/3/cos(d*x+c)^3+ln(cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{12\cos(dx+c)^4\log(-\cos(dx+c)) + 24\cos(dx+c)^3 - 8\cos(dx+c) + 3}{12\,a^2d\cos(dx+c)^4}$$

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] 1/12*(12*cos(d*x + c)^4*log(-cos(d*x + c)) + 24*cos(d*x + c)^3 - 8*cos(d*x
+ c) + 3)/(a^2*d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^7(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(tan(d*x+c)**7/(a+a*sec(d*x+c))**2,x)
[Out] Integral(tan(c + d*x)**7/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{12\log(\cos(dx+c))}{a^2} + \frac{24\cos(dx+c)^3-8\cos(dx+c)+3}{a^2\cos(dx+c)^4}}{12\,d}$$

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] 1/12*(12*log(cos(d*x + c))/a^2 + (24*cos(d*x + c)^3 - 8*cos(d*x + c) + 3)/(
a^2*cos(d*x + c)^4))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(61) = 122$.

Time = 3.47 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.77

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{12\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{12\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2}-\frac{\frac{4(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{54(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{124(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{25(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+7}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^4}}{12\,d}$$

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*(12*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 12*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 - (4*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 54*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 124*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 25*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 + 7)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^4))/d
```

Mupad [B] (verification not implemented)

Time = 15.77 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.08

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{26\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{8}{3}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^2\,d}$$

```
[In] int(tan(c + d*x)^7/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (8*tan(c/2 + (d*x)/2)^4 - (26*tan(c/2 + (d*x)/2)^2)/3 + 2*tan(c/2 + (d*x)/2
)^6 + 8/3)/(d*(6*a^2*tan(c/2 + (d*x)/2)^4 - 4*a^2*tan(c/2 + (d*x)/2)^2 - 4*
a^2*tan(c/2 + (d*x)/2)^6 + a^2*tan(c/2 + (d*x)/2)^8 + a^2)) - (2*atanh(tan(
c/2 + (d*x)/2)^2))/(a^2*d)
```

3.73 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\log(\cos(c+dx))}{a^2d} - \frac{2\sec(c+dx)}{a^2d} + \frac{\sec^2(c+dx)}{2a^2d}$$

```
[Out] -ln(cos(d*x+c))/a^2/d-2*sec(d*x+c)/a^2/d+1/2*sec(d*x+c)^2/a^2/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 45}

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sec^2(c+dx)}{2a^2d} - \frac{2\sec(c+dx)}{a^2d} - \frac{\log(\cos(c+dx))}{a^2d}$$

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) + Sec[c + d*x]^2/(2
*a^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^2}{x^3}\,dx, x, \cos(c+dx)\right)}{a^4d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3}-\frac{2a^2}{x^2}+\frac{a^2}{x}\right)\,dx, x, \cos(c+dx)\right)}{a^4d}\\ &= -\frac{\log(\cos(c+dx))}{a^2d}-\frac{2\sec(c+dx)}{a^2d}+\frac{\sec^2(c+dx)}{2a^2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(-1+4\cos(c+dx)+\log(\cos(c+dx))+\cos(2(c+dx))\log(\cos(c+dx)))\sec^2(c+dx)}{2a^2d}$$

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
[Out] -1/2*((-1 + 4*Cos[c + d*x] + Log[Cos[c + d*x]] + Cos[2*(c + d*x)]*Log[Cos[c
 + d*x]])*Sec[c + d*x]^2)/(a^2*d)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{1}{2\cos(dx+c)^2}-\ln(\cos(dx+c))-\frac{2}{\cos(dx+c)}}{d\,a^2}$	38
default	$\frac{\frac{1}{2\cos(dx+c)^2}-\ln(\cos(dx+c))-\frac{2}{\cos(dx+c)}}{d\,a^2}$	38
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2\left(2\,e^{3i(dx+c)}-e^{2i(dx+c)}+2\,e^{i(dx+c)}\right)}{d\,a^2\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{a^2d}$	94

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/2/cos(d*x+c)^2-ln(cos(d*x+c))-2/cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\cos(dx+c)^2\log(-\cos(dx+c))+4\cos(dx+c)-1}{2a^2d\cos(dx+c)^2}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -1/2*(2*cos(d*x + c)^2*log(-cos(d*x + c)) + 4*cos(d*x + c) - 1)/(a^2*d*cos(
d*x + c)^2)
```

Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
[Out] Integral(tan(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{2\log(\cos(dx+c))}{a^2}+\frac{4\cos(dx+c)-1}{a^2\cos(dx+c)^2}}{2d}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -1/2*(2*log(cos(d*x + c))/a^2 + (4*cos(d*x + c) - 1)/(a^2*cos(d*x + c)^2))/
d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(46) = 92$.

Time = 1.58 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.83

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{2\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2} - \frac{2\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2} - \frac{\frac{6\,(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{3\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+5}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 2*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^2 - (6*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5)/(a^2*(
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)} + \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^2\,d}$$

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (6*tan(c/2 + (d*x)/2)^2 - 4)/(d*(a^2*tan(c/2 + (d*x)/2)^4 - 2*a^2*tan(c/2 +
 (d*x)/2)^2 + a^2)) + (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^2*d)
```

3.74 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 33

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\log(\cos(c+dx))}{a^2d}+\frac{2\log(1+\cos(c+dx))}{a^2d}$$

```
[Out] -ln(cos(d*x+c))/a^2/d+2*ln(1+cos(d*x+c))/a^2/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 78}

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\log(\cos(c+dx)+1)}{a^2d}-\frac{\log(\cos(c+dx))}{a^2d}$$

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + (2*Log[1 + Cos[c + d*x]])/(a^2*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{a-ax}{x(a+ax)}\,dx, x, \cos(c+dx)\right)}{a^2 d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{x}-\frac{2}{1+x}\right)\,dx, x, \cos(c+dx)\right)}{a^2 d}\\ &= -\frac{\log(\cos(c+dx))}{a^2 d}+\frac{2\log(1+\cos(c+dx))}{a^2 d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log(\cos(c+dx))}{a^2 d}$$

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
[Out] (4*Log[Cos[(c + d*x)/2]] - Log[Cos[c + d*x]])/(a^2*d)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\ln(\cos(dx+c))+2\ln(\cos(dx+c)+1)}{d\,a^2}$	29
default	$\frac{-\ln(\cos(dx+c))+2\ln(\cos(dx+c)+1)}{d\,a^2}$	29
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2 d}+\frac{4\ln(\mathrm{e}^{i(dx+c)}+1)}{a^2 d}-\frac{\ln(\mathrm{e}^{2i(dx+c)}+1)}{a^2 d}$	59

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(-ln(cos(d*x+c))+2*ln(cos(d*x+c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\log\left(-\cos\left(dx+c\right)\right)-2\,\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{a^2d}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -(log(-cos(d*x + c)) - 2*log(1/2*cos(d*x + c) + 1/2))/(a^2*d)
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^3\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
[Out] Integral(tan(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{2\,\log(\cos(dx+c)+1)}{a^2}-\frac{\log(\cos(dx+c))}{a^2}}{d}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] (2*log(cos(d*x + c) + 1)/a^2 - log(cos(d*x + c))/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\log\left(\left|\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right|\right)}{a^2d}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] -log(abs((cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1))/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 14.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}{a^2\,d}$$

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
[Out] -log(tan(c/2 + (d*x)/2)^4 - 1)/(a^2*d)
```

3.75 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{1}{a^2d(1+\cos(c+dx))}-\frac{\log(1+\cos(c+dx))}{a^2d}$$

```
[Out] -1/a^2/d/(1+cos(d*x+c))-ln(1+cos(d*x+c))/a^2/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 45}

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{1}{a^2d(\cos(c+dx)+1)}-\frac{\log(\cos(c+dx)+1)}{a^2d}$$

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^2,x]
[Out] -(1/(a^2*d*(1 + Cos[c + d*x]))) - Log[1 + Cos[c + d*x]]/(a^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
```

```
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x}{(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^2(1+x)^2}+\frac{1}{a^2(1+x)}\right)\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{1}{a^2d(1+\cos(c+dx))}-\frac{\log(1+\cos(c+dx))}{a^2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\left(1+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{2a^2d}$$

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^2,x]
[Out] -1/2*((1 + 2*Log[Cos[(c + d*x)/2]] + 2*Cos[c + d*x]*Log[Cos[(c + d*x)/2]])*
Sec[(c + d*x)/2]^2)/(a^2*d)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{\ln(\sec(dx+c))+\frac{1}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))}{d\,a^2}$	37
default	$\frac{\ln(\sec(dx+c))+\frac{1}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))}{d\,a^2}$	37
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2\,e^{i(dx+c)}}{a^2d\left(e^{i(dx+c)}+1\right)^2}-\frac{2\ln\left(e^{i(dx+c)}+1\right)}{a^2d}$	69

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(ln(sec(d*x+c))+1/(1+sec(d*x+c))-ln(1+sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+1}{a^2d\cos(dx+c)+a^2d}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 1)/(a^2*d*cos(d*x + c) +
 a^2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(31) = 62$.

Time = 11.78 (sec) , antiderivative size = 177, normalized size of antiderivative = 4.92

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \begin{cases} \frac{\log(\tan^2(c+dx)+1)\sec(c+dx)}{2a^2d\sec(c+dx)+2a^2d} + \frac{\log(\tan^2(c+dx)+1)}{2a^2d\sec(c+dx)+2a^2d} - \frac{2\log(\sec(c+dx)+1)\sec(c+dx)}{2a^2d\sec(c+dx)+2a^2d} - \frac{2\log(\sec(c+dx)+1)}{2a^2d\sec(c+dx)+2a^2d} + \frac{2}{2a^2d\sec(c+dx} \\ \frac{x\tan(c)}{(a\sec(c)+a)^2} \end{cases}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**2,x)
[Out] Piecewise((log(tan(c + d*x)**2 + 1)*sec(c + d*x)/(2*a**2*d*sec(c + d*x) + 2
*a**2*d) + log(tan(c + d*x)**2 + 1)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) - 2*
log(sec(c + d*x) + 1)*sec(c + d*x)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) - 2*l
og(sec(c + d*x) + 1)/(2*a**2*d*sec(c + d*x) + 2*a**2*d) + 2/(2*a**2*d*sec(c
 + d*x) + 2*a**2*d), Ne(d, 0)), (x*tan(c)/(a*sec(c) + a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{1}{a^2\cos(dx+c)+a^2} + \frac{\log(\cos(dx+c)+1)}{a^2}}{d}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -(1/(a^2*cos(d*x + c) + a^2) + log(cos(d*x + c) + 1)/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}+\frac{\cos(dx+c)-1}{a^2(\cos(dx+c)+1)}}{2d}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (cos(d*x
+ c) - 1)/(a^2*(cos(d*x + c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}}{a^2\,d}$$

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1) - tan(c/2 + (d*x)/2)^2/2)/(a^2*d)
```

3.76 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{1}{4a^2d(1+\cos(c+dx))^2}+\frac{5}{4a^2d(1+\cos(c+dx))}+\frac{\log(1-\cos(c+dx))}{8a^2d}+\frac{7\log(1+\cos(c+dx))}{8a^2d}$$

```
[Out] -1/4/a^2/d/(1+cos(d*x+c))^2+5/4/a^2/d/(1+cos(d*x+c))+1/8*ln(1-cos(d*x+c))/a
^2/d+7/8*ln(1+cos(d*x+c))/a^2/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 90}

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{5}{4a^2d(\cos(c+dx)+1)}-\frac{1}{4a^2d(\cos(c+dx)+1)^2}+\frac{\log(1-\cos(c+dx))}{8a^2d}+\frac{7\log(\cos(c+dx)+1)}{8a^2d}$$

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*1/(a^2*d*(1 + Cos[c + d*x])^2) + 5/(4*a^2*d*(1 + Cos[c + d*x])) + Log[
1 - Cos[c + d*x]]/(8*a^2*d) + (7*Log[1 + Cos[c + d*x]])/(8*a^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^2\text{Subst}\left(\int \frac{x^3}{(a-ax)(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^2\text{Subst}\left(\int \left(-\frac{1}{8a^4(-1+x)} - \frac{1}{2a^4(1+x)^3} + \frac{5}{4a^4(1+x)^2} - \frac{7}{8a^4(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{1}{4a^2d(1+\cos(c+dx))^2} + \frac{5}{4a^2d(1+\cos(c+dx))}\\
&\quad + \frac{\log(1-\cos(c+dx))}{8a^2d} + \frac{7\log(1+\cos(c+dx))}{8a^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\left(-1+10\cos^2\left(\frac{1}{2}(c+dx)\right)+4\cos^4\left(\frac{1}{2}(c+dx)\right)\left(7\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec^2(c+}{4a^2d(1+\sec(c+dx))^2}$$

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((-1 + 10*Cos[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^4*(7*Log[Cos[(c + d*x)/2]
] + Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^2)/(4*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{8}-\frac{1}{4(\cos(dx+c)+1)^2}+\frac{5}{4(\cos(dx+c)+1)}+\frac{7\ln(\cos(dx+c)+1)}{8}}{d\,a^2}$	55
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{8}-\frac{1}{4(\cos(dx+c)+1)^2}+\frac{5}{4(\cos(dx+c)+1)}+\frac{7\ln(\cos(dx+c)+1)}{8}}{d\,a^2}$	55
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}+\frac{5\,\mathrm{e}^{3i(dx+c)}+8\,\mathrm{e}^{2i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}}{2d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^4}+\frac{7\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{4a^2d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{4a^2d}$	114

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/8*ln(cos(d*x+c)-1)-1/4/(cos(d*x+c)+1)^2+5/4/(cos(d*x+c)+1)+7/8*l
n(cos(d*x+c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.31

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$=\frac{7\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\log\left(\right.}{8\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(7*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) +
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 10*cos
(d*x + c) + 8)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\frac{\int \frac{\cot\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{2\,(5\,\cos(dx+c)+4)}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2} + \frac{7\,\log(\cos(dx+c)+1)}{a^2} + \frac{\log(\cos(dx+c)-1)}{a^2}}{8\,d}$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(5*cos(d*x + c) + 4)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
 + 7*log(cos(d*x + c) + 1)/a^2 + log(cos(d*x + c) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.44

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{2\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} - \frac{16\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2} - \frac{\frac{8\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^4}}{16\,d}$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 - 16*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - (8*a^2*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/a^4)/
d
```

Mupad [B] (verification not implemented)

Time = 14.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4} - \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right) + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16}}{a^2\,d}$$

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))/4 - log(tan(c/2 + (d*x)/2)^2 + 1) + tan(c/2 + (d*x
)/2)^2/2 - tan(c/2 + (d*x)/2)^4/16)/(a^2*d)
```

3.77 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{1}{16a^2d(1-\cos(c+dx))}-\frac{1}{12a^2d(1+\cos(c+dx))^3}+\frac{1}{2a^2d(1+\cos(c+dx))^2}-\frac{23}{16a^2d(1+\cos(c+dx))}-\frac{3\log(1-\cos(c+dx))}{16a^2d}-\frac{13\log(1+\cos(c+dx))}{16a^2d}$$

```
[Out] -1/16/a^2/d/(1-cos(d*x+c))-1/12/a^2/d/(1+cos(d*x+c))^3+1/2/a^2/d/(1+cos(d*x
+c))^2-23/16/a^2/d/(1+cos(d*x+c))-3/16*ln(1-cos(d*x+c))/a^2/d-13/16*ln(1+co
s(d*x+c))/a^2/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{1}{16a^2d(1-\cos(c+dx))}-\frac{23}{16a^2d(\cos(c+dx)+1)}+\frac{1}{2a^2d(\cos(c+dx)+1)^2}-\frac{1}{12a^2d(\cos(c+dx)+1)^3}-\frac{3\log(1-\cos(c+dx))}{16a^2d}-\frac{13\log(\cos(c+dx)+1)}{16a^2d}$$

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/16*1/(a^2*d*(1 - Cos[c + d*x])) - 1/(12*a^2*d*(1 + Cos[c + d*x])^3) + 1/
(2*a^2*d*(1 + Cos[c + d*x])^2) - 23/(16*a^2*d*(1 + Cos[c + d*x])) - (3*Log[
1 - Cos[c + d*x]])/(16*a^2*d) - (13*Log[1 + Cos[c + d*x]])/(16*a^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a^4\text{Subst}\left(\int \frac{x^5}{(a-ax)^2(a+ax)^4}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^4\text{Subst}\left(\int \left(\frac{1}{16a^6(-1+x)^2}+\frac{3}{16a^6(-1+x)}-\frac{1}{4a^6(1+x)^4}+\frac{1}{a^6(1+x)^3}-\frac{23}{16a^6(1+x)^2}+\frac{13}{16a^6(1+x)}\right)dx, x, \cos(c+
}{d}\\
&= -\frac{1}{16a^2d(1-\cos(c+dx))}-\frac{1}{12a^2d(1+\cos(c+dx))^3}+\frac{1}{2a^2d(1+\cos(c+dx))^2}\\
&\quad-\frac{23}{16a^2d(1+\cos(c+dx))}-\frac{3\log(1-\cos(c+dx))}{16a^2d}-\frac{13\log(1+\cos(c+dx))}{16a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$
\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \\
-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(3\csc^2\left(\frac{1}{2}(c+dx)\right)+156\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+36\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+69\sec^2\left(\frac{1}{2}(c
}{24a^2d(1+\sec(c+dx))^2}
$$

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/24*(Cos[(c + d*x)/2]^4*(3*Csc[(c + d*x)/2]^2 + 156*Log[Cos[(c + d*x)/2]]
 + 36*Log[Sin[(c + d*x)/2]] + 69*Sec[(c + d*x)/2]^2 - 12*Sec[(c + d*x)/2]^4
 + Sec[(c + d*x)/2]^6)*Sec[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{\frac{1}{16\cos(dx+c)-16}-\frac{3\ln(\cos(dx+c)-1)}{16}-\frac{1}{12(\cos(dx+c)+1)^3}+\frac{1}{2(\cos(dx+c)+1)^2}-\frac{23}{16(\cos(dx+c)+1)}-\frac{13\ln(\cos(dx+c)+1)}{16}}{d\,a^2}$
default	$\frac{\frac{1}{16\cos(dx+c)-16}-\frac{3\ln(\cos(dx+c)-1)}{16}-\frac{1}{12(\cos(dx+c)+1)^3}+\frac{1}{2(\cos(dx+c)+1)^2}-\frac{23}{16(\cos(dx+c)+1)}-\frac{13\ln(\cos(dx+c)+1)}{16}}{d\,a^2}$
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{33\,\mathrm{e}^{7i(dx+c)}+36\,\mathrm{e}^{6i(dx+c)}-49\,\mathrm{e}^{5i(dx+c)}-136\,\mathrm{e}^{4i(dx+c)}-49\,\mathrm{e}^{3i(dx+c)}+36\,\mathrm{e}^{2i(dx+c)}+33\,\mathrm{e}^{i(dx+c)}}{12d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^6\left(\mathrm{e}^{i(dx+c)}-1\right)^2}-\frac{3\ln}{}$

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/16/(cos(d*x+c)-1)-3/16*ln(cos(d*x+c)-1)-1/12/(cos(d*x+c)+1)^3+1/
2/(cos(d*x+c)+1)^2-23/16/(cos(d*x+c)+1)-13/16*ln(cos(d*x+c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.32

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{66\cos(dx+c)^3+36\cos(dx+c)^2+39\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\log\left(\frac{1}{2}\,\mathrm{c}\right.}{48\left(a^2d\cos(dx+c)^4+2\right.}$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(66*cos(d*x + c)^3 + 36*cos(d*x + c)^2 + 39*(cos(d*x + c)^4 + 2*cos(d
*x + c)^3 - 2*cos(d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 9*(cos(d*x +
c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2)
- 74*cos(d*x + c) - 52)/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*
a^2*d*cos(d*x + c) - a^2*d)
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cot^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{2\left(33\cos(dx+c)^3+18\cos(dx+c)^2-37\cos(dx+c)-26\right)}{a^2\cos(dx+c)^4+2a^2\cos(dx+c)^3-2a^2\cos(dx+c)-a^2}+\frac{39\log(\cos(dx+c)+1)}{a^2}+\frac{9\log(\cos(dx+c)-1)}{a^2}}{48\,d}$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(33*cos(d*x + c)^3 + 18*cos(d*x + c)^2 - 37*cos(d*x + c) - 26)/(a^
2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c) - a^2) + 39*lo
g(cos(d*x + c) + 1)/a^2 + 9*log(cos(d*x + c) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.51

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{3\left(\frac{6(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)}-\frac{18\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{96\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}+\frac{\frac{48a^4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9a^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+a}{a^6}}{96\,d}$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(a
^2*(cos(d*x + c) - 1)) - 18*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1
))/a^2 + 96*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (48*
a^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^4*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 + a^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.72

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8}-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{32}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{96}}{a^2\,d}$$

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -((3*log(tan(c/2 + (d*x)/2)))/8 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2 +
 (d*x)/2)^2/32 + tan(c/2 + (d*x)/2)^2/2 - (3*tan(c/2 + (d*x)/2)^4)/32 + tan
(c/2 + (d*x)/2)^6/96)/(a^2*d)
```

3.78 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 165

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{1}{64a^2d(1-\cos(c+dx))^2}+\frac{9}{64a^2d(1-\cos(c+dx))}-\frac{1}{32a^2d(1+\cos(c+dx))^4}+\frac{11}{48a^2d(1+\cos(c+dx))^3}-\frac{3}{4a^2d(1+\cos(c+dx))^2}+\frac{51}{32a^2d(1+\cos(c+dx))}+\frac{29\log(1-\cos(c+dx))}{128a^2d}+\frac{99\log(1+\cos(c+dx))}{128a^2d}$$

```
[Out] -1/64/a^2/d/(1-cos(d*x+c))^2+9/64/a^2/d/(1-cos(d*x+c))-1/32/a^2/d/(1+cos(d*
x+c))^4+11/48/a^2/d/(1+cos(d*x+c))^3-3/4/a^2/d/(1+cos(d*x+c))^2+51/32/a^2/d
/(1+cos(d*x+c))+29/128*ln(1-cos(d*x+c))/a^2/d+99/128*ln(1+cos(d*x+c))/a^2/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{9}{64a^2d(1-\cos(c+dx))}+\frac{51}{32a^2d(\cos(c+dx)+1)}-\frac{1}{64a^2d(1-\cos(c+dx))^2}-\frac{3}{4a^2d(\cos(c+dx)+1)^2}+\frac{11}{48a^2d(\cos(c+dx)+1)^3}-\frac{1}{32a^2d(\cos(c+dx)+1)^4}+\frac{29\log(1-\cos(c+dx))}{128a^2d}+\frac{99\log(\cos(c+dx)+1)}{128a^2d}$$

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/64*1/(a^2*d*(1 - Cos[c + d*x])^2) + 9/(64*a^2*d*(1 - Cos[c + d*x])) - 1/
(32*a^2*d*(1 + Cos[c + d*x])^4) + 11/(48*a^2*d*(1 + Cos[c + d*x])^3) - 3/(4
*a^2*d*(1 + Cos[c + d*x])^2) + 51/(32*a^2*d*(1 + Cos[c + d*x])) + (29*Log[1
 - Cos[c + d*x]])/(128*a^2*d) + (99*Log[1 + Cos[c + d*x]])/(128*a^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a^6 \text{Subst}\left(\int \frac{x^7}{(a-ax)^3(a+ax)^5}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{a^6 \text{Subst}\left(\int \left(-\frac{1}{32a^8(-1+x)^3} - \frac{9}{64a^8(-1+x)^2} - \frac{29}{128a^8(-1+x)} - \frac{1}{8a^8(1+x)^5} + \frac{11}{16a^8(1+x)^4} - \frac{3}{2a^8(1+x)^3} + \frac{5}{32a^8(}\right.\right.}{d} \\
&= -\frac{1}{64a^2d(1-\cos(c+dx))^2} + \frac{9}{64a^2d(1-\cos(c+dx))} - \frac{1}{32a^2d(1+\cos(c+dx))^4} \\
&\quad + \frac{11}{48a^2d(1+\cos(c+dx))^3} - \frac{3}{4a^2d(1+\cos(c+dx))^2} \\
&\quad + \frac{51}{32a^2d(1+\cos(c+dx))} + \frac{29\log(1-\cos(c+dx))}{128a^2d} + \frac{99\log(1+\cos(c+dx))}{128a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-108\csc^2\left(\frac{1}{2}(c+dx)\right)+6\csc^4\left(\frac{1}{2}(c+dx)\right)-24\left(99\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+29\log\left(\sin\ldots\right.\right.\right.}{384a^2\ldots}$$

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/384*(Cos[(c + d*x)/2]^4*(-108*Csc[(c + d*x)/2]^2 + 6*Csc[(c + d*x)/2]^4
- 24*(99*Log[Cos[(c + d*x)/2]] + 29*Log[Sin[(c + d*x)/2]]) - 1224*Sec[(c +
d*x)/2]^2 + 288*Sec[(c + d*x)/2]^4 - 44*Sec[(c + d*x)/2]^6 + 3*Sec[(c + d*x
)/2]^8)*Sec[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{-\frac{1}{64(\cos(dx+c)-1)^2}-\frac{9}{64(\cos(dx+c)-1)}+\frac{29\ln(\cos(dx+c)-1)}{128}-\frac{1}{32(\cos(dx+c)+1)^4}+\frac{11}{48(\cos(dx+c)+1)^3}-\frac{3}{4(\cos(dx+c)+1)^2}+\frac{\ldots}{32(\cos(\ldots}}{d\,a^2}$
default	$\frac{-\frac{1}{64(\cos(dx+c)-1)^2}-\frac{9}{64(\cos(dx+c)-1)}+\frac{29\ln(\cos(dx+c)-1)}{128}-\frac{1}{32(\cos(dx+c)+1)^4}+\frac{11}{48(\cos(dx+c)+1)^3}-\frac{3}{4(\cos(dx+c)+1)^2}+\frac{\ldots}{32(\cos(\ldots}}{d\,a^2}$
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}+\frac{279\,\mathrm{e}^{11i(dx+c)}+156\,\mathrm{e}^{10i(dx+c)}-1141\,\mathrm{e}^{9i(dx+c)}-2080\,\mathrm{e}^{8i(dx+c)}+670\,\mathrm{e}^{7i(dx+c)}+2696\,\mathrm{e}^{6i(dx+c)}+670\,\mathrm{e}^{5i(\ldots}}{96d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^8\left(\mathrm{e}^{i(dx+c)}-1\right)^4}$

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/64/(cos(d*x+c)-1)^2-9/64/(cos(d*x+c)-1)+29/128*ln(cos(d*x+c)-1)
-1/32/(cos(d*x+c)+1)^4+11/48/(cos(d*x+c)+1)^3-3/4/(cos(d*x+c)+1)^2+51/32/(c
os(d*x+c)+1)+99/128*ln(cos(d*x+c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.72

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{558\cos(dx+c)^5+156\cos(dx+c)^4-1268\cos(dx+c)^3-676\cos(dx+c)^2+297\left(\cos(dx+c)^6+2\cos\ldots\right.}{\ldots}$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/384*(558*cos(d*x + c)^5 + 156*cos(d*x + c)^4 - 1268*cos(d*x + c)^3 - 676*
cos(d*x + c)^2 + 297*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 -
4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c
) + 1/2) + 87*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d
*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/
2) + 686*cos(d*x + c) + 448)/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5
 - a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2
*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cot^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{2\left(279\cos(dx+c)^5+78\cos(dx+c)^4-634\cos(dx+c)^3-338\cos(dx+c)^2+343\cos(dx+c)+224\right)}{a^2\cos(dx+c)^6+2a^2\cos(dx+c)^5-a^2\cos(dx+c)^4-4a^2\cos(dx+c)^3-a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2}+\frac{297\log(\cos(dx+c)+1)}{a^2}+\frac{87\log(\cos\cdots}{\cdots}}{384\,d}$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/384*(2*(279*cos(d*x + c)^5 + 78*cos(d*x + c)^4 - 634*cos(d*x + c)^3 - 338
*cos(d*x + c)^2 + 343*cos(d*x + c) + 224)/(a^2*cos(d*x + c)^6 + 2*a^2*cos(d
*x + c)^5 - a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2
+ 2*a^2*cos(d*x + c) + a^2) + 297*log(cos(d*x + c) + 1)/a^2 + 87*log(cos(d*
x + c) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.43

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{\frac{6\left(\frac{16\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{87\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{a^2(\cos(dx+c)-1)^2}-\frac{348\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{1536\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}+\frac{768\,a^6(\cos(dx\cdots}{\cos(dx+c)\cdots}}{1536\,d}$$

[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] -1/1536*(6*(16*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 87*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a^2*(cos(d*x + c) - 1)^
2) - 348*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + 1536*log(a
bs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 + (768*a^6*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 174*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 32*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3*a^6*(cos(d*x + c)
 - 1)^4/(cos(d*x + c) + 1)^4)/a^8)/d

Mupad [B] (verification not implemented)

Time = 14.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,a^2\,d}-\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,a^2\,d}\\&\quad+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{48\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{512\,a^2\,d}\\&\quad+\frac{29\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d}\\&\quad+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{1}{4}\right)}{64\,a^2\,d}\end{aligned}$$

[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^2,x)

[Out] tan(c/2 + (d*x)/2)^2/(2*a^2*d) - (29*tan(c/2 + (d*x)/2)^4)/(256*a^2*d) + ta
n(c/2 + (d*x)/2)^6/(48*a^2*d) - tan(c/2 + (d*x)/2)^8/(512*a^2*d) + (29*log(
tan(c/2 + (d*x)/2)))/(64*a^2*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) + (
cot(c/2 + (d*x)/2)^4*(4*tan(c/2 + (d*x)/2)^2 - 1/4))/(64*a^2*d)

3.79 $\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2} dx = \frac{x}{a^2} - \frac{3\text{arctanh}(\sin(c+dx))}{4a^2d} - \frac{\tan(c+dx)}{a^2d} + \frac{3\sec(c+dx)\tan(c+dx)}{4a^2d} + \frac{\tan^3(c+dx)}{3a^2d} - \frac{\sec(c+dx)\tan^3(c+dx)}{2a^2d} + \frac{\tan^5(c+dx)}{5a^2d}$$

```
[Out] x/a^2-3/4*arctanh(sin(d*x+c))/a^2/d-tan(d*x+c)/a^2/d+3/4*sec(d*x+c)*tan(d*x
+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d-1/2*sec(d*x+c)*tan(d*x+c)^3/a^2/d+1/5*tan(
d*x+c)^5/a^2/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3973, 3971, 3554, 8, 2691, 3855, 2687, 30}

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2} dx = -\frac{3\text{arctanh}(\sin(c+dx))}{4a^2d} + \frac{\tan^5(c+dx)}{5a^2d} + \frac{\tan^3(c+dx)}{3a^2d} - \frac{\tan(c+dx)}{a^2d} - \frac{\tan^3(c+dx)\sec(c+dx)}{2a^2d} + \frac{3\tan(c+dx)\sec(c+dx)}{4a^2d} + \frac{x}{a^2}$$

```
[In] Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (3*ArcTanh[Sin[c + d*x]])/(4*a^2*d) - Tan[c + d*x]/(a^2*d) + (3*Sec
[c + d*x]*Tan[c + d*x])/(4*a^2*d) + Tan[c + d*x]^3/(3*a^2*d) - (Sec[c + d*x
]*Tan[c + d*x]^3)/(2*a^2*d) + Tan[c + d*x]^5/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (-a+a\sec(c+dx))^2\tan^4(c+dx)\,dx}{a^4}\\
&= \frac{\int \left(a^2\tan^4(c+dx)-2a^2\sec(c+dx)\tan^4(c+dx)+a^2\sec^2(c+dx)\tan^4(c+dx)\right)\,dx}{a^4}\\
&= \frac{\int \tan^4(c+dx)\,dx}{a^2}+\frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a^2}-\frac{2\int \sec(c+dx)\tan^4(c+dx)\,dx}{a^2}\\
&= \frac{\tan^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)\tan^3(c+dx)}{2a^2d}-\frac{\int \tan^2(c+dx)\,dx}{a^2}\\
&\quad+\frac{3\int \sec(c+dx)\tan^2(c+dx)\,dx}{2a^2}+\frac{\operatorname{Subst}\left(\int x^4\,dx,x,\tan(c+dx)\right)}{a^2d}\\
&= -\frac{\tan(c+dx)}{a^2d}+\frac{3\sec(c+dx)\tan(c+dx)}{4a^2d}+\frac{\tan^3(c+dx)}{3a^2d}\\
&\quad-\frac{\sec(c+dx)\tan^3(c+dx)}{2a^2d}+\frac{\tan^5(c+dx)}{5a^2d}-\frac{3\int \sec(c+dx)\,dx}{4a^2}+\frac{\int 1\,dx}{a^2}\\
&= \frac{x}{a^2}-\frac{3\operatorname{arctanh}(\sin(c+dx))}{4a^2d}-\frac{\tan(c+dx)}{a^2d}+\frac{3\sec(c+dx)\tan(c+dx)}{4a^2d}\\
&\quad+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)\tan^3(c+dx)}{2a^2d}+\frac{\tan^5(c+dx)}{5a^2d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 495 vs. $2(119) = 238$.

Time = 5.79 (sec) , antiderivative size = 495, normalized size of antiderivative = 4.16

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(240x+\frac{180\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{180\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{(2\cdots}{}\right.}{}$$

```
[In] Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(240*x + (180*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]])/d - (180*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d - ((29
3*Cos[(d*x)/2] + 333*Cos[2*c + (3*d*x)/2] + 287*Cos[2*c + (5*d*x)/2] + 67*C
```

```
os[4*c + (7*d*x)/2] + 68*Cos[4*c + (9*d*x)/2])*Sec[c]*Sec[c + d*x]^5*Sin[(d
*x)/2])/(2*d) + (36*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^4) - (151*Sin[c/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^2) + (36*Sin[c/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^4) - (151*Sin[c/2])/(d*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (Cos[c/2]*Sec[c]*Sec[c + d*x]
^4*(308*Sin[c/2] - 43*Sin[c/2 + d*x] - 43*Sin[(3*c)/2 + d*x] - 346*Sin[(3*c
)/2 + 2*d*x] + 346*Sin[(5*c)/2 + 2*d*x] + 149*Sin[(5*c)/2 + 3*d*x] + 149*Si
n[(7*c)/2 + 3*d*x]))/(4*d)))/(60*a^2*(1 + Sec[c + d*x])^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.35

method	result
risch	$\frac{x}{a^2}-\frac{i\left(75\,\mathrm{e}^{9i(dx+c)}+60\,\mathrm{e}^{8i(dx+c)}+30\,\mathrm{e}^{7i(dx+c)}+360\,\mathrm{e}^{6i(dx+c)}+320\,\mathrm{e}^{4i(dx+c)}-30\,\mathrm{e}^{3i(dx+c)}+280\,\mathrm{e}^{2i(dx+c)}-75\,\mathrm{e}^{i(dx+c)}\right.}{30d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$
derivativedivides	$\frac{-\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{19}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{4}}{}$
default	$\frac{-\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{19}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{4}}{}$

```
[In] int(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/a^2-1/30*I*(75*exp(9*I*(d*x+c))+60*exp(8*I*(d*x+c))+30*exp(7*I*(d*x+c))+3
60*exp(6*I*(d*x+c))+320*exp(4*I*(d*x+c))-30*exp(3*I*(d*x+c))+280*exp(2*I*(d
*x+c))-75*exp(I*(d*x+c))+68)/d/a^2/(exp(2*I*(d*x+c))+1)^5+3/4/a^2/d*ln(exp(
I*(d*x+c))-I)-3/4/a^2/d*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{120\,dx\cos\left(dx+c\right)^5-45\,\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)+45\,\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)-2}{120\,a^2d\cos\left(dx+c\right.}$$

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/120*(120*d*x*cos(d*x + c)^5 - 45*cos(d*x + c)^5*log(sin(d*x + c) + 1) + 4
5*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(68*cos(d*x + c)^4 - 75*cos(d*x
 + c)^3 + 4*cos(d*x + c)^2 + 30*cos(d*x + c) - 12)*sin(d*x + c))/(a^2*d*cos
(d*x + c)^5)
```

Sympy [F]

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^8(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(tan(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**8/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(109) = 218$.

Time = 0.30 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.53

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{2\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{110\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{328\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{530\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{105\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)}{a^2-\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{10a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{45\log\left(\frac{\sin(dx+}{\cos(dx+c)}\right)}{a^2}}{60\,d}$$

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*(2*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 110*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 328*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 530*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7 + 105*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^2 - 5*a^
2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 10*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 - a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 120*ar
ctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 45*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/a^2 - 45*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 4.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.14

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{60\,(dx+c)}{a^2} - \frac{45\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} + \frac{45\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} + \frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-530\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+328\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{60\,d}$$

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)/a^2 - 45*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 45*log
(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(105*tan(1/2*d*x + 1/2*c)^9 - 530*t
an(1/2*d*x + 1/2*c)^7 + 328*tan(1/2*d*x + 1/2*c)^5 - 110*tan(1/2*d*x + 1/2*
c)^3 + 15*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 15.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.50

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{x}{a^2} - \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^2\,d}$$
$$+ \frac{\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2} - \frac{53\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3} + \frac{164\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15} - \frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + 10\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 10\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

```
[In] int(tan(c + d*x)^8/(a + a/cos(c + d*x))^2,x)
```

```
[Out] x/a^2 - (3*atanh(tan(c/2 + (d*x)/2)))/(2*a^2*d) + (tan(c/2 + (d*x)/2)/2 - (
11*tan(c/2 + (d*x)/2)^3)/3 + (164*tan(c/2 + (d*x)/2)^5)/15 - (53*tan(c/2 +
(d*x)/2)^7)/3 + (7*tan(c/2 + (d*x)/2)^9)/2)/(d*(5*a^2*tan(c/2 + (d*x)/2)^2
- 10*a^2*tan(c/2 + (d*x)/2)^4 + 10*a^2*tan(c/2 + (d*x)/2)^6 - 5*a^2*tan(c/2
 + (d*x)/2)^8 + a^2*tan(c/2 + (d*x)/2)^10 - a^2))
```

3.80 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2} dx = -\frac{x}{a^2} + \frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d} + \frac{\tan(c+dx)}{a^2d} - \frac{\sec(c+dx)\tan(c+dx)}{a^2d} + \frac{\tan^3(c+dx)}{3a^2d}$$

```
[Out] -x/a^2+arctanh(sin(d*x+c))/a^2/d+tan(d*x+c)/a^2/d-sec(d*x+c)*tan(d*x+c)/a^2
/d+1/3*tan(d*x+c)^3/a^2/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3973, 3971, 3554, 8, 2691, 3855, 2687, 30}

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d} + \frac{\tan^3(c+dx)}{3a^2d} + \frac{\tan(c+dx)}{a^2d} - \frac{\tan(c+dx)\sec(c+dx)}{a^2d} - \frac{x}{a^2}$$

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) + ArcTanh[Sin[c + d*x]]/(a^2*d) + Tan[c + d*x]/(a^2*d) - (Sec[c +
d*x]*Tan[c + d*x])/(a^2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int(-a+a\sec(c+dx))^2\tan^2(c+dx)\,dx}{a^4}\\
&= \frac{\int\left(a^2\tan^2(c+dx)-2a^2\sec(c+dx)\tan^2(c+dx)+a^2\sec^2(c+dx)\tan^2(c+dx)\right)\,dx}{a^4}\\
&= \frac{\int\tan^2(c+dx)\,dx}{a^2}+\frac{\int\sec^2(c+dx)\tan^2(c+dx)\,dx}{a^2}-\frac{2\int\sec(c+dx)\tan^2(c+dx)\,dx}{a^2}\\
&= \frac{\tan(c+dx)}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{a^2d}-\frac{\int 1\,dx}{a^2}\\
&\quad+\frac{\int\sec(c+dx)\,dx}{a^2}+\frac{\operatorname{Subst}\left(\int x^2\,dx,x,\tan(c+dx)\right)}{a^2d}\\
&= -\frac{x}{a^2}+\frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 767 vs. $2(72) = 144$.

Time = 7.08 (sec) , antiderivative size = 767, normalized size of antiderivative = 10.65

$$\begin{aligned}
&\int\frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx\\
&= -\frac{4x\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\\
&\quad-\frac{4\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^2(c+dx)}{d(a+a\sec(c+dx))^2}\\
&\quad+\frac{4\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^2(c+dx)}{d(a+a\sec(c+dx))^2}\\
&\quad+\frac{2\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}\\
&\quad+\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\left(-5\cos\left(\frac{c}{2}\right)+7\sin\left(\frac{c}{2}\right)\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}\\
&\quad+\frac{8\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}\\
&\quad+\frac{2\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}\\
&\quad+\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\left(5\cos\left(\frac{c}{2}\right)+7\sin\left(\frac{c}{2}\right)\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}\\
&\quad+\frac{8\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sin\left(\frac{dx}{2}\right)}{3d(a+a\sec(c+dx))^2\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}
\end{aligned}$$

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*x*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2)/(a + a*Sec[c + d*x])^2 - (4*Cos[
c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^
2)/(d*(a + a*Sec[c + d*x])^2) + (4*Cos[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^2)/(d*(a + a*Sec[c + d*x])^2) + (2*C
os[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*(a + a*Sec[c + d*x])^
2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos
[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(-5*Cos[c/2] + 7*Sin[c/2]))/(3*d*(a + a*Se
c[c + d*x])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2
])^2) + (8*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*(a + a*Se
c[c + d*x])^2*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2
])) + (2*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*(a + a*Sec[
c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])
^3) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(5*Cos[c/2] + 7*Sin[c/2]))/(3*d*
(a + a*Sec[c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2])^2) + (8*Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*Sin[(d*x)/2])/(3*d*
(a + a*Sec[c + d*x])^2*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{x}{a^2}+\frac{2i\left(3\,\mathrm{e}^{5i(dx+c)}+6\,\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+2\right)}{3d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^2d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^2d}$
derivativedivides	$-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$ ... $/\ a^2d$
default	$-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$ ... $/\ a^2d$

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x/a^2+2/3*I*(3*exp(5*I*(d*x+c))+6*exp(2*I*(d*x+c))-3*exp(I*(d*x+c))+2)/d/a
^2/(exp(2*I*(d*x+c))+1)^3-1/a^2/d*ln(exp(I*(d*x+c))-I)+1/a^2/d*ln(exp(I*(d*
x+c))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{6\,dx\cos{(dx+c)}^3 - 3\,\cos{(dx+c)}^3\log{(\sin{(dx+c)}+1)} + 3\,\cos{(dx+c)}^3\log{(-\sin{(dx+c)}+1)} - 2}{6\,a^2d\cos{(dx+c)}^3}$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*d*x*cos(d*x + c)^3 - 3*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*cos
(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c)^2 - 3*cos(d*x + c) +
 1)*sin(d*x + c))/(a^2*d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^6{(c+dx)}}{\sec^2{(c+dx)}+2\sec{(c+dx)}+1}\,dx}{a^2}$$

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(70) = 140$.

Time = 0.31 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.72

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{4\left(\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2 - \frac{3\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{6\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} - \frac{3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} + \frac{3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{3\,d}$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(4*(sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5)/(a^2 - 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 6
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 3*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 2.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= -\frac{\frac{3\,(dx+c)}{a^2} - \frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} + \frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} + \frac{4\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3 a^2}}{3\,d}$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^2 - 3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 3*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 4*(3*tan(1/2*d*x + 1/2*c)^5 - tan(1/2*d*
x + 1/2*c)^3)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} - \frac{x}{a^2}$$
$$+ \frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3} - 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - a^2\right)}$$

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - x/a^2 + ((4*tan(c/2 + (d*x)/2)^3)/3
 - 4*tan(c/2 + (d*x)/2)^5)/(d*(3*a^2*tan(c/2 + (d*x)/2)^2 - 3*a^2*tan(c/2 +
 (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 - a^2))
```

3.81 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{x}{a^2} - \frac{2\text{arctanh}(\sin(c+dx))}{a^2d} + \frac{\tan(c+dx)}{a^2d}$$

```
[Out] x/a^2-2*arctanh(sin(d*x+c))/a^2/d+tan(d*x+c)/a^2/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3973, 3858, 3855, 3852, 8}

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\text{arctanh}(\sin(c+dx))}{a^2d} + \frac{\tan(c+dx)}{a^2d} + \frac{x}{a^2}$$

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (2*ArcTanh[Sin[c + d*x]])/(a^2*d) + Tan[c + d*x]/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (-a+a\sec(c+dx))^2\,dx}{a^4}\\
&= \frac{x}{a^2}+\frac{\int \sec^2(c+dx)\,dx}{a^2}-\frac{2\int \sec(c+dx)\,dx}{a^2}\\
&= \frac{x}{a^2}-\frac{2\text{arctanh}(\sin(c+dx))}{a^2d}-\frac{\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{a^2d}\\
&= \frac{x}{a^2}-\frac{2\text{arctanh}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{a^2d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(34) = 68$.

Time = 1.05 (sec) , antiderivative size = 177, normalized size of antiderivative = 5.21

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(dx+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\right.\right.}{a^2d(1+\sec(c+d}$$

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c
/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

method	result	size
risch	$\frac{x}{a^{2}}+\frac{2 i}{d \,a^{2} \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)}+\frac{2 \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{a^{2} d}-\frac{2 \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{a^{2} d}$	71
derivativedivides	$\frac{-\frac{1}{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1}-2 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)+2 \arctan \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1}+2 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{a^{2} d}$	80
default	$\frac{-\frac{1}{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1}-2 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)+2 \arctan \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1}+2 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{a^{2} d}$	80

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] x/a^2+2*I/d/a^2/(exp(2*I*(d*x+c))+1)+2/a^2/d*ln(exp(I*(d*x+c))-I)-2/a^2/d*l
n(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{dx\cos(dx+c)-\cos(dx+c)\log(\sin(dx+c)+1)+\cos(dx+c)\log(-\sin(dx+c)+1)+\sin(dx+c)}{a^2d\cos(dx+c)}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] (d*x*cos(d*x + c) - cos(d*x + c)*log(sin(d*x + c) + 1) + cos(d*x + c)*log(-
sin(d*x + c) + 1) + sin(d*x + c))/(a^2*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
[Out] Integral(tan(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(34) = 68$.

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.62

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} + \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2} + \frac{\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)}{d}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^2 + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + sin(d*x
 + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))
/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(34) = 68$.

Time = 0.95 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{dx+c}{a^2} - \frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} + \frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}}{d}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^2 - 2*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + 2*log(abs(tan(1
/2*d*x + 1/2*c) - 1))/a^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 - 1)*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 14.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{x}{a^2} - \frac{4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} - \frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2\right)}$$

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^2,x)
```

```
[Out] x/a^2 - (4*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (2*tan(c/2 + (d*x)/2))/(d*(
a^2*tan(c/2 + (d*x)/2)^2 - a^2))
```

3.82 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 33

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx=-\frac{x}{a^2}+\frac{2\tan(c+dx)}{ad(a+a\sec(c+dx))}$$

```
[Out] -x/a^2+2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3973, 3971, 3554, 8, 2686, 3852}

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx=-\frac{2\cot(c+dx)}{a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*Cot[c + d*x])/(a^2*d) + (2*Csc[c + d*x])/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \cot^2(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}\\
&= \frac{\int \left(a^2\cot^2(c+dx)-2a^2\cot(c+dx)\csc(c+dx)+a^2\csc^2(c+dx)\right)\,dx}{a^4}\\
&= \frac{\int \cot^2(c+dx)\,dx}{a^2}+\frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \cot(c+dx)\csc(c+dx)\,dx}{a^2}\\
&= -\frac{\cot(c+dx)}{a^2d}-\frac{\int 1\,dx}{a^2}-\frac{\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{a^2d}+\frac{2\text{Subst}(\int 1\,dx,x,\csc(c+dx))}{a^2d}\\
&= -\frac{x}{a^2}-\frac{2\cot(c+dx)}{a^2d}+\frac{2\csc(c+dx)}{a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{-\frac{2\arctan\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}}{a^2}$$

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
[Out] ((-2*ArcTan[Tan[c/2 + (d*x)/2]])/d + (2*Tan[c/2 + (d*x)/2])/d)/a^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{x}{a^2}+\frac{4i}{a^2d\left(e^{i(dx+c)}+1\right)}$	30
derivativedivides	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	31
default	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	31

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -x/a^2+4*I/a^2/d/(exp(I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{dx\cos(dx+c)+dx-2\sin(dx+c)}{a^2d\cos(dx+c)+a^2d}$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -(d*x*cos(d*x + c) + d*x - 2*sin(d*x + c))/(a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\tan^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
[Out] Integral(tan(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} - \frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}\right)}{d}$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - sin(d*x + c)/(a^2*(cos(d*
x + c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{dx+c}{a^2} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}}{d}$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] -((d*x + c)/a^2 - 2*tan(1/2*d*x + 1/2*c)/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{d\,x}{2}\right)}{a^2\,d}$$

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
[Out] (2*(tan(c/2 + (d*x)/2) - (d*x)/2))/(a^2*d)
```

3.83 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{x}{a^2}-\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{4\csc^3(c+dx)}{3a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}$$

```
[Out] -x/a^2-cot(d*x+c)/a^2/d+1/3*cot(d*x+c)^3/a^2/d-2/5*cot(d*x+c)^5/a^2/d+2*csc
(d*x+c)/a^2/d-4/3*csc(d*x+c)^3/a^2/d+2/5*csc(d*x+c)^5/a^2/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3973, 3971, 3554, 8, 2686, 200, 2687, 30}

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{4\csc^3(c+dx)}{3a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - Cot[c + d*x]/(a^2*d) + Cot[c + d*x]^3/(3*a^2*d) - (2*Cot[c + d*x
]^5)/(5*a^2*d) + (2*Csc[c + d*x])/(a^2*d) - (4*Csc[c + d*x]^3)/(3*a^2*d) +
(2*Csc[c + d*x]^5)/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \cot^6(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^6(c+dx)-2a^2\cot^5(c+dx)\csc(c+dx)+a^2\cot^4(c+dx)\csc^2(c+dx)\right)\,dx}{a^4}$$
$$= \frac{\int \cot^6(c+dx)\,dx}{a^2}+\frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a^2}-\frac{2\int \cot^5(c+dx)\csc(c+dx)\,dx}{a^2}$$
$$= -\frac{\cot^5(c+dx)}{5a^2d}-\frac{\int \cot^4(c+dx)\,dx}{a^2}+\frac{\text{Subst}\left(\int x^4\,dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{2\text{Subst}\left(\int (-1+x^2)^2\,dx, x, \csc(c+dx)\right)}{a^2d}$$
$$= \frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\int \cot^2(c+dx)\,dx}{a^2} + \frac{2\text{Subst}\left(\int (1-2x^2+x^4)\,dx, x, \csc(c+dx)\right)}{a^2d}$$
$$= -\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot^5(c+dx)}{5a^2d} + \frac{2\csc(c+dx)}{a^2d}-\frac{4\csc^3(c+dx)}{3a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{\int 1\,dx}{a^2}$$
$$= -\frac{x}{a^2}-\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot^5(c+dx)}{5a^2d} + \frac{2\csc(c+dx)}{a^2d}-\frac{4\csc^3(c+dx)}{3a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}$$

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.39

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\sec^2(c+dx)\left(-120dx\cos^4\left(\frac{1}{2}(c+dx)\right)-31\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos^3\left(\frac{1}{2}(c+dx)\right)\left(15\cot\left(\frac{1}{2}(c\right.\right.\right.}{30a^2d(1+\sec(c+d}$$

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*(-120*d*x*Cos[(c + d*x)/2]^4 - 31*Cos[(c + d*x)/2]*Sec[c/2]
*Sin[(d*x)/2] + Cos[(c + d*x)/2]^3*(15*Cot[(c + d*x)/2]*Csc[c/2] + 193*Sec[
c/2])*Sin[(d*x)/2] - 31*Cos[(c + d*x)/2]^2*Tan[c/2] + 3*Tan[(c + d*x)/2]))/
(30*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^2}$	72
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^2}$	72
risch	$-\frac{x}{a^2}+\frac{4i\left(15\,\mathrm{e}^{5i(dx+c)}+30\,\mathrm{e}^{4i(dx+c)}+10\,\mathrm{e}^{3i(dx+c)}-35\,\mathrm{e}^{2i(dx+c)}-37\,\mathrm{e}^{i(dx+c)}-13\right)}{15d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^5\left(\mathrm{e}^{i(dx+c)}-1\right)}$	100

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^2*(1/5*tan(1/2*d*x+1/2*c)^5-5/3*tan(1/2*d*x+1/2*c)^3+11*tan(1/2*d*x
+1/2*c)-1/tan(1/2*d*x+1/2*c)-16*arctan(tan(1/2*d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{26\cos(dx+c)^3+22\cos(dx+c)^2+15\left(dx\cos(dx+c)^2+2\,dx\cos(dx+c)+dx\right)\sin(dx+c)-17\,c}{15\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)\sin(dx+c)}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(26*cos(d*x + c)^3 + 22*cos(d*x + c)^2 + 15*(d*x*cos(d*x + c)^2 + 2*d
*x*cos(d*x + c) + d*x)*sin(d*x + c) - 17*cos(d*x + c) - 16)/((a^2*d*cos(d*x
 + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cot^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\frac{165\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}-\frac{240\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{15\,(\cos(dx+c)+1)}{a^2\sin(dx+c)}}{120\,d}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/120*((165*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 - 240*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^2 - 15*(cos(d*x + c) + 1)/(a^2*sin(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{120\,(dx+c)}{a^2}+\frac{15}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{3\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-25\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+165\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{10}}}{120\,d}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/120*(120*(d*x + c)/a^2 + 15/(a^2*tan(1/2*d*x + 1/2*c)) - (3*a^8*tan(1/2*
d*x + 1/2*c)^5 - 25*a^8*tan(1/2*d*x + 1/2*c)^3 + 165*a^8*tan(1/2*d*x + 1/2*
c))/a^10)/d
```

Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.73

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{x}{a^2}-\frac{\frac{26\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{15}-\frac{28\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15}+\frac{17\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{60}-\frac{1}{40}}{a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
```

```
[Out] - x/a^2 - ((17*cos(c/2 + (d*x)/2)^2)/60 - (28*cos(c/2 + (d*x)/2)^4)/15 + (2
6*cos(c/2 + (d*x)/2)^6)/15 - 1/40)/(a^2*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d
*x)/2))
```

3.84 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{x}{a^2}+\frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{2\csc(c+dx)}{a^2d}+\frac{2\csc^3(c+dx)}{a^2d}-\frac{6\csc^5(c+dx)}{5a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}$$

```
[Out] x/a^2+cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d+1/5*cot(d*x+c)^5/a^2/d-2/7*co
t(d*x+c)^7/a^2/d-2*csc(d*x+c)/a^2/d+2*csc(d*x+c)^3/a^2/d-6/5*csc(d*x+c)^5/a
^2/d+2/7*csc(d*x+c)^7/a^2/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3973, 3971, 3554, 8, 2686, 200, 2687, 30}

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\cot^7(c+dx)}{7a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{6\csc^5(c+dx)}{5a^2d}+\frac{2\csc^3(c+dx)}{a^2d}-\frac{2\csc(c+dx)}{a^2d}+\frac{x}{a^2}$$

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 + Cot[c + d*x]/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d) + Cot[c + d*x]^5/(5
*a^2*d) - (2*Cot[c + d*x]^7)/(7*a^2*d) - (2*Csc[c + d*x])/(a^2*d) + (2*Csc[
c + d*x]^3)/(a^2*d) - (6*Csc[c + d*x]^5)/(5*a^2*d) + (2*Csc[c + d*x]^7)/(7*
a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \cot^8(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}\\
&= \frac{\int \left(a^2\cot^8(c+dx)-2a^2\cot^7(c+dx)\csc(c+dx)+a^2\cot^6(c+dx)\csc^2(c+dx)\right)\,dx}{a^4}\\
&= \frac{\int \cot^8(c+dx)\,dx}{a^2}+\frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a^2}-\frac{2\int \cot^7(c+dx)\csc(c+dx)\,dx}{a^2}\\
&= -\frac{\cot^7(c+dx)}{7a^2d}-\frac{\int \cot^6(c+dx)\,dx}{a^2}+\frac{\text{Subst}\left(\int x^6\,dx,x,-\cot(c+dx)\right)}{a^2d}\\
&\quad+\frac{2\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\csc(c+dx)\right)}{a^2d}\\
&= \frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}+\frac{\int \cot^4(c+dx)\,dx}{a^2}\\
&\quad+\frac{2\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx,x,\csc(c+dx)\right)}{a^2d}\\
&= -\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{2\csc(c+dx)}{a^2d}\\
&\quad+\frac{2\csc^3(c+dx)}{a^2d}-\frac{6\csc^5(c+dx)}{5a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{\int \cot^2(c+dx)\,dx}{a^2}\\
&= \frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{2\csc(c+dx)}{a^2d}\\
&\quad+\frac{2\csc^3(c+dx)}{a^2d}-\frac{6\csc^5(c+dx)}{5a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}+\frac{\int 1\,dx}{a^2}\\
&= \frac{x}{a^2}+\frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}\\
&\quad-\frac{2\csc(c+dx)}{a^2d}+\frac{2\csc^3(c+dx)}{a^2d}-\frac{6\csc^5(c+dx)}{5a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 314 vs. $2(139) = 278$.

Time = 1.36 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.26

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\csc\left(\frac{c}{2}\right)\csc^3(c+dx)\sec\left(\frac{c}{2}\right)\sec^2(c+dx)(5880dx\cos(dx)-5880dx\cos(2c+dx)+3360dx\cos(c+2dx)-3}{}$$

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x]^3*Sec[c/2]*Sec[c + d*x]^2*(5880*d*x*Cos[d*x] - 5880*
d*x*Cos[2*c + d*x] + 3360*d*x*Cos[c + 2*d*x] - 3360*d*x*Cos[3*c + 2*d*x] -
1260*d*x*Cos[2*c + 3*d*x] + 1260*d*x*Cos[4*c + 3*d*x] - 1680*d*x*Cos[3*c +
4*d*x] + 1680*d*x*Cos[5*c + 4*d*x] - 420*d*x*Cos[4*c + 5*d*x] + 420*d*x*Cos
[6*c + 5*d*x] - 4032*Sin[c] - 9632*Sin[d*x] + 16002*Sin[c + d*x] + 9144*Sin
[2*(c + d*x)] - 3429*Sin[3*(c + d*x)] - 4572*Sin[4*(c + d*x)] - 1143*Sin[5*
(c + d*x)] - 11760*Sin[2*c + d*x] - 8864*Sin[c + 2*d*x] - 3360*Sin[3*c + 2*
d*x] + 2064*Sin[2*c + 3*d*x] + 2520*Sin[4*c + 3*d*x] + 4432*Sin[3*c + 4*d*x
] + 1680*Sin[5*c + 4*d*x] + 1528*Sin[4*c + 5*d*x]))/(26880*a^2*d*(1 + Sec[c
 + d*x])^2)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{22\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-42\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{7}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{32d\,a^2}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{22\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-42\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{7}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{32d\,a^2}$
risch	$\frac{x}{a^2}-\frac{2i\left(210\,e^{9i(dx+c)}+315\,e^{8i(dx+c)}-420\,e^{7i(dx+c)}-1470\,e^{6i(dx+c)}-504\,e^{5i(dx+c)}+1204\,e^{4i(dx+c)}+1108\,e^{3i(dx+c)}-25}{105d\,a^2\left(e^{i(dx+c)}+1\right)^7\left(e^{i(dx+c)}-1\right)^3}$

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a^2*(1/7*tan(1/2*d*x+1/2*c)^7-7/5*tan(1/2*d*x+1/2*c)^5+22/3*tan(1/2*
d*x+1/2*c)^3-42*tan(1/2*d*x+1/2*c)+64*arctan(tan(1/2*d*x+1/2*c))-1/3/tan(1/
2*d*x+1/2*c)^3+7/tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$=\frac{191\,\cos\left(dx+c\right)^5+172\,\cos\left(dx+c\right)^4-253\,\cos\left(dx+c\right)^3-258\,\cos\left(dx+c\right)^2+105\,\left(dx\cos\left(dx+c\right)^4+2\right.}{105\,\left(a^2d\cos\left(dx+c\right)^4+2\,a^2d\cos\left(dx+c\right)^3-2\,a^2d\cos\right.}$$

[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] 1/105*(191*cos(d*x + c)^5 + 172*cos(d*x + c)^4 - 253*cos(d*x + c)^3 - 258*cos(d*x + c)^2 + 105*(d*x*cos(d*x + c)^4 + 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(d*x + c) - d*x)*sin(d*x + c) + 87*cos(d*x + c) + 96)/((a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))

Sympy [F]

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\frac{\int \frac{\cot^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**2,x)

[Out] Integral(cot(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.13

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx=$$
$$-\frac{\frac{\frac{4410\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{770\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2}-\frac{6720\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{35\left(\frac{21\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)}{a^2\sin(dx+c)^3}}{3360\,d}$$

[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] -1/3360*((4410*sin(d*x + c)/(cos(d*x + c) + 1) - 770*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^2 - 6720*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 35*(21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a^2*sin(d*x + c)^3))/d

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.82

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{3360\,(dx+c)}{a^2} + \frac{35\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3} + \frac{15\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-147\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+770\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4410\,a^{12}\tan\left(\frac{1}{2}\,dx+\right.}{a^{14}}}{3360\,d}$$

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3360*(3360*(d*x + c)/a^2 + 35*(21*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^2*tan(1/
2*d*x + 1/2*c)^3) + (15*a^12*tan(1/2*d*x + 1/2*c)^7 - 147*a^12*tan(1/2*d*x
+ 1/2*c)^5 + 770*a^12*tan(1/2*d*x + 1/2*c)^3 - 4410*a^12*tan(1/2*d*x + 1/2*
c))/a^14)/d
```

Mupad [B] (verification not implemented)

Time = 14.93 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.31

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{15\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-147\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+770\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\ldots}{3360\,a^2\,d\cos\left(\frac{c}{2}\ldots\right.}$$

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (15*sin(c/2 + (d*x)/2)^10 - 35*cos(c/2 + (d*x)/2)^10 - 147*cos(c/2 + (d*x)/
2)^2*sin(c/2 + (d*x)/2)^8 + 770*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 -
 4410*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 735*cos(c/2 + (d*x)/2)^8*
sin(c/2 + (d*x)/2)^2 + 3360*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3*(c +
d*x))/(3360*a^2*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3)
```

3.85 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{x}{a^2}-\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{8\csc^3(c+dx)}{3a^2d}+\frac{12\csc^5(c+dx)}{5a^2d}-\frac{8\csc^7(c+dx)}{7a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}$$

```
[Out] -x/a^2-cot(d*x+c)/a^2/d+1/3*cot(d*x+c)^3/a^2/d-1/5*cot(d*x+c)^5/a^2/d+1/7*c
ot(d*x+c)^7/a^2/d-2/9*cot(d*x+c)^9/a^2/d+2*csc(d*x+c)/a^2/d-8/3*csc(d*x+c)^
3/a^2/d+12/5*csc(d*x+c)^5/a^2/d-8/7*csc(d*x+c)^7/a^2/d+2/9*csc(d*x+c)^9/a^2
/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3973, 3971, 3554, 8, 2686, 200, 2687, 30}

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\cot^9(c+dx)}{9a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-\frac{8\csc^7(c+dx)}{7a^2d}+\frac{12\csc^5(c+dx)}{5a^2d}-\frac{8\csc^3(c+dx)}{3a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - Cot[c + d*x]/(a^2*d) + Cot[c + d*x]^3/(3*a^2*d) - Cot[c + d*x]^5
/(5*a^2*d) + Cot[c + d*x]^7/(7*a^2*d) - (2*Cot[c + d*x]^9)/(9*a^2*d) + (2*C
sc[c + d*x])/(a^2*d) - (8*Csc[c + d*x]^3)/(3*a^2*d) + (12*Csc[c + d*x]^5)/(
5*a^2*d) - (8*Csc[c + d*x]^7)/(7*a^2*d) + (2*Csc[c + d*x]^9)/(9*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \cot^{10}(c+dx)(-a+a\sec(c+dx))^2\,dx}{a^4}\\
&= \frac{\int \left(a^2\cot^{10}(c+dx)-2a^2\cot^9(c+dx)\csc(c+dx)+a^2\cot^8(c+dx)\csc^2(c+dx)\right)\,dx}{a^4}\\
&= \frac{\int \cot^{10}(c+dx)\,dx}{a^2}+\frac{\int \cot^8(c+dx)\csc^2(c+dx)\,dx}{a^2}-\frac{2\int \cot^9(c+dx)\csc(c+dx)\,dx}{a^2}\\
&= -\frac{\cot^9(c+dx)}{9a^2d}-\frac{\int \cot^8(c+dx)\,dx}{a^2}+\frac{\text{Subst}\left(\int x^8\,dx,x,-\cot(c+dx)\right)}{a^2d}\\
&\quad+\frac{2\text{Subst}\left(\int \left(-1+x^2\right)^4\,dx,x,\csc(c+dx)\right)}{a^2d}\\
&= \frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{\int \cot^6(c+dx)\,dx}{a^2}\\
&\quad+\frac{2\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\,dx,x,\csc(c+dx)\right)}{a^2d}\\
&= -\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{2\csc(c+dx)}{a^2d}-\frac{8\csc^3(c+dx)}{3a^2d}\\
&\quad+\frac{12\csc^5(c+dx)}{5a^2d}-\frac{8\csc^7(c+dx)}{7a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-\frac{\int \cot^4(c+dx)\,dx}{a^2}\\
&= \frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}+\frac{2\csc(c+dx)}{a^2d}\\
&\quad-\frac{8\csc^3(c+dx)}{3a^2d}+\frac{12\csc^5(c+dx)}{5a^2d}-\frac{8\csc^7(c+dx)}{7a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}\\
&\quad+\frac{\int \cot^2(c+dx)\,dx}{a^2}\\
&= -\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}\\
&\quad+\frac{2\csc(c+dx)}{a^2d}-\frac{8\csc^3(c+dx)}{3a^2d}+\frac{12\csc^5(c+dx)}{5a^2d}-\frac{8\csc^7(c+dx)}{7a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}\\
&\quad-\frac{\int 1\,dx}{a^2}\\
&= -\frac{x}{a^2}-\frac{\cot(c+dx)}{a^2d}+\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}\\
&\quad+\frac{2\csc(c+dx)}{a^2d}-\frac{8\csc^3(c+dx)}{3a^2d}+\frac{12\csc^5(c+dx)}{5a^2d}-\frac{8\csc^7(c+dx)}{7a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 6.50 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.54

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-63\cot^2\left(\frac{c}{2}\right)\csc^2\left(\frac{1}{2}(c+dx)\right)\left(-17+\csc^2\left(\frac{1}{2}(c+dx)\right)\right)-\frac{63}{8}\cot\left(\frac{c}{2}\right)(5120dx}{}$$

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(-63*Cot[c/2]^2*Csc[(c + d*x)/2]^2*(-17
+ Csc[(c + d*x)/2]^2) - (63*Cot[c/2]*(5120*d*x + (-543 + 736*Cos[c + d*x] -
 201*Cos[2*(c + d*x)])*Csc[c/2]*Csc[(c + d*x)/2]^5*Sin[(d*x)/2]))/8 + (Csc[
c/2]*Sec[(c + d*x)/2]^9*(4360986*Sin[(d*x)/2] - 3688020*Sin[c + (d*x)/2] +
3365964*Sin[c + (3*d*x)/2] - 2000040*Sin[2*c + (3*d*x)/2] + 1660896*Sin[2*c
 + (5*d*x)/2] - 638820*Sin[3*c + (5*d*x)/2] + 479484*Sin[3*c + (7*d*x)/2] -
 95445*Sin[4*c + (7*d*x)/2] + 63881*Sin[4*c + (9*d*x)/2]))/256)*Tan[c/2])/(
10080*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{37\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+163\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-256\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{5\tan\left(\frac{d}{2}\right.}}{128d\,a^2}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{37\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+163\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-256\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{5\tan\left(\frac{d}{2}\right.}}{128d\,a^2}$
risch	$-\frac{x}{a^2}+\frac{4i\left(315\,e^{13i(dx+c)}+315\,e^{12i(dx+c)}-1470\,e^{11i(dx+c)}-3360\,e^{10i(dx+c)}+1113\,e^{9i(dx+c)}+6447\,e^{8i(dx+c)}+2028\,e^{7i(c}\right.}{315d\,a^2\left(e^{i(dx+c)}+1\right)^9}$

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/d/a^2*(1/9*tan(1/2*d*x+1/2*c)^9-9/7*tan(1/2*d*x+1/2*c)^7+37/5*tan(1/2
*d*x+1/2*c)^5-31*tan(1/2*d*x+1/2*c)^3+163*tan(1/2*d*x+1/2*c)-256*arctan(tan
(1/2*d*x+1/2*c))-1/5/tan(1/2*d*x+1/2*c)^5+3/tan(1/2*d*x+1/2*c)^3-37/tan(1/2
*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.40

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{598\cos(dx+c)^7+566\cos(dx+c)^6-1212\cos(dx+c)^5-1310\cos(dx+c)^4+860\cos(dx+c)^3+\cdots}{315\left(a^2d\cos(dx+c)^6+2a^2d\cdots\right.}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/315*(598*cos(d*x + c)^7 + 566*cos(d*x + c)^6 - 1212*cos(d*x + c)^5 - 131
0*cos(d*x + c)^4 + 860*cos(d*x + c)^3 + 1014*cos(d*x + c)^2 + 315*(d*x*cos(
d*x + c)^6 + 2*d*x*cos(d*x + c)^5 - d*x*cos(d*x + c)^4 - 4*d*x*cos(d*x + c)
^3 - d*x*cos(d*x + c)^2 + 2*d*x*cos(d*x + c) + d*x)*sin(d*x + c) - 197*cos(
d*x + c) - 256)/((a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos
(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cot^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.10

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\frac{51345\sin(dx+c)}{\cos(dx+c)+1}-\frac{9765\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2331\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{405\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^2}-\frac{80640\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{63\left(\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\cdots\right.}{a^{\cdots}}}{40320\,d}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/40320*((51345*sin(d*x + c)/(cos(d*x + c) + 1) - 9765*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 2331*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 405*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^2 - 8
0640*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 63*(15*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 185*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1)*(cos(d*x
+ c) + 1)^5/(a^2*sin(d*x + c)^5))/d
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.80

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{40320\,(dx+c)}{a^2} + \frac{63\left(185\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5} - \frac{35\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-405\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+2331\,a^{16}\tan\left(\frac{1}{2}\,dx+\right.}{a^{18}}}{40320\,d}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/40320*(40320*(d*x + c)/a^2 + 63*(185*tan(1/2*d*x + 1/2*c)^4 - 15*tan(1/2
*d*x + 1/2*c)^2 + 1)/(a^2*tan(1/2*d*x + 1/2*c)^5) - (35*a^16*tan(1/2*d*x +
1/2*c)^9 - 405*a^16*tan(1/2*d*x + 1/2*c)^7 + 2331*a^16*tan(1/2*d*x + 1/2*c)
^5 - 9765*a^16*tan(1/2*d*x + 1/2*c)^3 + 51345*a^16*tan(1/2*d*x + 1/2*c))/a^
18)/d
```

Mupad [B] (verification not implemented)

Time = 15.78 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.28

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} - 35\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + 405\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 2331\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d}{2}\right.}{}$$

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(63*cos(c/2 + (d*x)/2)^14 - 35*sin(c/2 + (d*x)/2)^14 + 405*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2)^12 - 2331*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^
10 + 9765*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 51345*cos(c/2 + (d*x)
/2)^8*sin(c/2 + (d*x)/2)^6 + 11655*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)
^4 - 945*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 40320*cos(c/2 + (d*x)
/2)^9*sin(c/2 + (d*x)/2)^5*(c + d*x))/(40320*a^2*d*cos(c/2 + (d*x)/2)^9*sin
(c/2 + (d*x)/2)^5)
```

3.86 $\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\log(\cos(c+dx))}{a^3d}+\frac{3\sec(c+dx)}{a^3d}-\frac{\sec^2(c+dx)}{2a^3d}-\frac{5\sec^3(c+dx)}{3a^3d}+\frac{5\sec^4(c+dx)}{4a^3d}+\frac{\sec^5(c+dx)}{5a^3d}-\frac{\sec^6(c+dx)}{2a^3d}+\frac{\sec^7(c+dx)}{7a^3d}$$

```
[Out] ln(cos(d*x+c))/a^3/d+3*sec(d*x+c)/a^3/d-1/2*sec(d*x+c)^2/a^3/d-5/3*sec(d*x+
c)^3/a^3/d+5/4*sec(d*x+c)^4/a^3/d+1/5*sec(d*x+c)^5/a^3/d-1/2*sec(d*x+c)^6/a
^3/d+1/7*sec(d*x+c)^7/a^3/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sec^7(c+dx)}{7a^3d}-\frac{\sec^6(c+dx)}{2a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{5\sec^4(c+dx)}{4a^3d}-\frac{5\sec^3(c+dx)}{3a^3d}-\frac{\sec^2(c+dx)}{2a^3d}+\frac{3\sec(c+dx)}{a^3d}+\frac{\log(\cos(c+dx))}{a^3d}$$

```
[In] Int[Tan[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^3*d) + (3*Sec[c + d*x])/(a^3*d) - Sec[c + d*x]^2/(2*a^
3*d) - (5*Sec[c + d*x]^3)/(3*a^3*d) + (5*Sec[c + d*x]^4)/(4*a^3*d) + Sec[c
+ d*x]^5/(5*a^3*d) - Sec[c + d*x]^6/(2*a^3*d) + Sec[c + d*x]^7/(7*a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^5(a+ax)^2}{x^8}\,dx, x, \cos(c+dx)\right)}{a^{10}d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a^7}{x^8}-\frac{3a^7}{x^7}+\frac{a^7}{x^6}+\frac{5a^7}{x^5}-\frac{5a^7}{x^4}-\frac{a^7}{x^3}+\frac{3a^7}{x^2}-\frac{a^7}{x}\right)dx, x, \cos(c+dx)\right)}{a^{10}d}\\ &= \frac{\log(\cos(c+dx))}{a^3d}+\frac{3\sec(c+dx)}{a^3d}-\frac{\sec^2(c+dx)}{2a^3d}-\frac{5\sec^3(c+dx)}{3a^3d}\\ &\quad+\frac{5\sec^4(c+dx)}{4a^3d}+\frac{\sec^5(c+dx)}{5a^3d}-\frac{\sec^6(c+dx)}{2a^3d}+\frac{\sec^7(c+dx)}{7a^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{(3732+4522\cos(2(c+dx))+1050\cos(3(c+dx))+2380\cos(4(c+dx))-210\cos(5(c+dx))+630\cos(6}{}$$

```
[In] Integrate[Tan[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((3732 + 4522*Cos[2*(c + d*x)] + 1050*Cos[3*(c + d*x)] + 2380*Cos[4*(c + d*
x)] - 210*Cos[5*(c + d*x)] + 630*Cos[6*(c + d*x)] + 2205*Cos[3*(c + d*x)]*L
og[Cos[c + d*x]] + 735*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 105*Cos[7*(c +
d*x)]*Log[Cos[c + d*x]] + 105*Cos[c + d*x]*(8 + 35*Log[Cos[c + d*x]]))*Sec[
c + d*x]^7)/(6720*a^3*d)
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{\ln(\cos(dx+c))+\frac{3}{\cos(dx+c)}+\frac{1}{5\cos(dx+c)^5}-\frac{1}{2\cos(dx+c)^2}+\frac{5}{4\cos(dx+c)^4}-\frac{1}{2\cos(dx+c)^6}+\frac{1}{7\cos(dx+c)^7}-\frac{5}{3\cos(dx+c)^3}}{d\,a^3}$
default	$\frac{\ln(\cos(dx+c))+\frac{3}{\cos(dx+c)}+\frac{1}{5\cos(dx+c)^5}-\frac{1}{2\cos(dx+c)^2}+\frac{5}{4\cos(dx+c)^4}-\frac{1}{2\cos(dx+c)^6}+\frac{1}{7\cos(dx+c)^7}-\frac{5}{3\cos(dx+c)^3}}{d\,a^3}$
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3d}+\frac{6\,e^{13i(dx+c)}-2\,e^{12i(dx+c)}+\frac{68\,e^{11i(dx+c)}}{3}+10\,e^{10i(dx+c)}+\frac{646\,e^{9i(dx+c)}}{15}+8\,e^{8i(dx+c)}+\frac{2488\,e^{7i(dx+c)}}{35}+}{d\,a^3\left(e^{2i(dx+c)}+1\right)^7}$

```
[In] int(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(ln(cos(d*x+c))+3/cos(d*x+c)+1/5/cos(d*x+c)^5-1/2/cos(d*x+c)^2+5/4/
cos(d*x+c)^4-1/2/cos(d*x+c)^6+1/7/cos(d*x+c)^7-5/3/cos(d*x+c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.69

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{420\cos(dx+c)^7\log(-\cos(dx+c))+1260\cos(dx+c)^6-210\cos(dx+c)^5-700\cos(dx+c)^4+525}{420\,a^3d\cos(dx+c)^7}$$

```
[In] integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/420*(420*cos(d*x + c)^7*log(-cos(d*x + c)) + 1260*cos(d*x + c)^6 - 210*co
s(d*x + c)^5 - 700*cos(d*x + c)^4 + 525*cos(d*x + c)^3 + 84*cos(d*x + c)^2
- 210*cos(d*x + c) + 60)/(a^3*d*cos(d*x + c)^7)
```

Sympy [F]

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^{11}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(tan(d*x+c)**11/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**11/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$=\frac{\frac{420\log(\cos(dx+c))}{a^3}+\frac{1260\cos(dx+c)^6-210\cos(dx+c)^5-700\cos(dx+c)^4+525\cos(dx+c)^3+84\cos(dx+c)^2-210\cos(dx+c)+60}{a^3\cos(dx+c)^7}}{420\,d}$$

```
[In] integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/420*(420*log(cos(d*x + c))/a^3 + (1260*cos(d*x + c)^6 - 210*cos(d*x + c)^
5 - 700*cos(d*x + c)^4 + 525*cos(d*x + c)^3 + 84*cos(d*x + c)^2 - 210*cos(d
*x + c) + 60)/(a^3*cos(d*x + c)^7))/d
```

Giac [A] (verification not implemented)

none

Time = 20.53 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.80

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{\frac{420\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}-\frac{420\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3}-\frac{\frac{1393(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{819(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{6755(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{20195(\cos(dx+}{(\cos(dx+c)+}}{a^3\left(\frac{\cos(dx}{\cos(dx}\right.}}{420\,d}$$

```
[In] integrate(tan(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/420*(420*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 420*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (1393*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 819*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 - 6755*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 20195*(cos(d*x + c) - 1
)^4/(cos(d*x + c) + 1)^4 - 28749*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5
- 8463*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089*(cos(d*x + c) - 1)^
7/(cos(d*x + c) + 1)^7 + 319)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1)^7))/d
```

Mupad [B] (verification not implemented)

Time = 18.40 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.64

$$\int \frac{\tan^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^3\,d} - \frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}-\frac{224\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{282\,\mathrm{ta}}{}}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+35\,a^3\tan\left(\frac{c}{2}\right.\right.}$$

```
[In] int(tan(c + d*x)^11/(a + a/cos(c + d*x))^3,x)
```

```
[Out] - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^3*d) - ((282*tan(c/2 + (d*x)/2)^4)/5 -
 (322*tan(c/2 + (d*x)/2)^2)/15 - (224*tan(c/2 + (d*x)/2)^6)/3 + (128*tan(c/
2 + (d*x)/2)^8)/3 + 14*tan(c/2 + (d*x)/2)^10 - 2*tan(c/2 + (d*x)/2)^12 + 35
2/105)/(d*(7*a^3*tan(c/2 + (d*x)/2)^2 - 21*a^3*tan(c/2 + (d*x)/2)^4 + 35*a^
3*tan(c/2 + (d*x)/2)^6 - 35*a^3*tan(c/2 + (d*x)/2)^8 + 21*a^3*tan(c/2 + (d*
x)/2)^10 - 7*a^3*tan(c/2 + (d*x)/2)^12 + a^3*tan(c/2 + (d*x)/2)^14 - a^3))
```

3.87 $\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\log(\cos(c+dx))}{a^3d}-\frac{3\sec(c+dx)}{a^3d}+\frac{\sec^2(c+dx)}{a^3d}+\frac{2\sec^3(c+dx)}{3a^3d}-\frac{3\sec^4(c+dx)}{4a^3d}+\frac{\sec^5(c+dx)}{5a^3d}$$

```
[Out] -ln(cos(d*x+c))/a^3/d-3*sec(d*x+c)/a^3/d+sec(d*x+c)^2/a^3/d+2/3*sec(d*x+c)^
3/a^3/d-3/4*sec(d*x+c)^4/a^3/d+1/5*sec(d*x+c)^5/a^3/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 76}

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sec^5(c+dx)}{5a^3d}-\frac{3\sec^4(c+dx)}{4a^3d}+\frac{2\sec^3(c+dx)}{3a^3d}+\frac{\sec^2(c+dx)}{a^3d}-\frac{3\sec(c+dx)}{a^3d}-\frac{\log(\cos(c+dx))}{a^3d}$$

```
[In] Int[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^3*d)) - (3*Sec[c + d*x])/(a^3*d) + Sec[c + d*x]^2/(a
^3*d) + (2*Sec[c + d*x]^3)/(3*a^3*d) - (3*Sec[c + d*x]^4)/(4*a^3*d) + Sec[c
 + d*x]^5/(5*a^3*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^4(a+ax)}{x^6}\,dx, x, \cos(c+dx)\right)}{a^8 d} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6} - \frac{3a^5}{x^5} + \frac{2a^5}{x^4} + \frac{2a^5}{x^3} - \frac{3a^5}{x^2} + \frac{a^5}{x}\right) dx, x, \cos(c+dx)\right)}{a^8 d} \\
&= -\frac{\log(\cos(c+dx))}{a^3 d} - \frac{3\sec(c+dx)}{a^3 d} + \frac{\sec^2(c+dx)}{a^3 d} \\
&\quad + \frac{2\sec^3(c+dx)}{3a^3 d} - \frac{3\sec^4(c+dx)}{4a^3 d} + \frac{\sec^5(c+dx)}{5a^3 d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(142 + 280\cos(2(c+dx)) + 90\cos(4(c+dx)) + 150\cos(c+dx)\log(\cos(c+dx)) + 15\cos(5(c+dx))\,\mathrm{l}}{240a^3 d}$$

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/240*((142 + 280*Cos[2*(c + d*x)] + 90*Cos[4*(c + d*x)] + 150*Cos[c + d*x
]*Log[Cos[c + d*x]] + 15*Cos[5*(c + d*x)]*Log[Cos[c + d*x]] + 15*Cos[3*(c +
 d*x)]*(-4 + 5*Log[Cos[c + d*x]]))*Sec[c + d*x]^5)/(a^3*d)
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{\frac{1}{5\cos(dx+c)^5}-\ln(\cos(dx+c))+\frac{2}{3\cos(dx+c)^3}-\frac{3}{4\cos(dx+c)^4}-\frac{3}{\cos(dx+c)}+\frac{1}{\cos(dx+c)^2}}{d\,a^3}$
default	$\frac{\frac{1}{5\cos(dx+c)^5}-\ln(\cos(dx+c))+\frac{2}{3\cos(dx+c)^3}-\frac{3}{4\cos(dx+c)^4}-\frac{3}{\cos(dx+c)}+\frac{1}{\cos(dx+c)^2}}{d\,a^3}$
risch	$\frac{ix}{a^3}+\frac{2ic}{a^3d}-\frac{2\left(45\,\mathrm{e}^{9i(dx+c)}-30\,\mathrm{e}^{8i(dx+c)}+140\,\mathrm{e}^{7i(dx+c)}+142\,\mathrm{e}^{5i(dx+c)}+140\,\mathrm{e}^{3i(dx+c)}-30\,\mathrm{e}^{2i(dx+c)}+45\,\mathrm{e}^{i(dx+c)}\right)}{15d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}-$

```
[In] int(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/5/cos(d*x+c)^5-ln(cos(d*x+c))+2/3/cos(d*x+c)^3-3/4/cos(d*x+c)^4-
3/cos(d*x+c)+1/cos(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.76

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{60\,\cos\left(dx+c\right)^5\log\left(-\cos\left(dx+c\right)\right)+180\,\cos\left(dx+c\right)^4-60\,\cos\left(dx+c\right)^3-40\,\cos\left(dx+c\right)^2+45\,\cos\left(\right.}{60\,a^3d\cos\left(dx+c\right)^5}$$

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(60*cos(d*x + c)^5*log(-cos(d*x + c)) + 180*cos(d*x + c)^4 - 60*cos(d
*x + c)^3 - 40*cos(d*x + c)^2 + 45*cos(d*x + c) - 12)/(a^3*d*cos(d*x + c)^5
)
```

Sympy [F]

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^9(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(tan(d*x+c)**9/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**9/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{60\,\log(\cos(dx+c))}{a^3} + \frac{180\,\cos(dx+c)^4-60\,\cos(dx+c)^3-40\,\cos(dx+c)^2+45\,\cos(dx+c)-12}{a^3\cos(dx+c)^5}}{60\,d}$$

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*log(cos(d*x + c))/a^3 + (180*cos(d*x + c)^4 - 60*cos(d*x + c)^3 -
 40*cos(d*x + c)^2 + 45*cos(d*x + c) - 12)/(a^3*cos(d*x + c)^5))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(93) = 186.

Time = 6.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.04

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{60\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} - \frac{60\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3} - \frac{\frac{475\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{590\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{50\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{805\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^5}}{60\,d}$$

```
[In] integrate(tan(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 60*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (475*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 590*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 50
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*(cos(d*x + c) - 1)^4/(cos(
d*x + c) + 1)^4 - 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 119)/(a^3
*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 18.70 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.69

$$\int \frac{\tan^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^3\,d}$$

$$-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+22\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{98\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{58\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{64}{15}}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

```
[In] int(tan(c + d*x)^9/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^3*d) - ((58*tan(c/2 + (d*x)/2)^2)/3 - (9
8*tan(c/2 + (d*x)/2)^4)/3 + 22*tan(c/2 + (d*x)/2)^6 + 2*tan(c/2 + (d*x)/2)^
8 - 64/15)/(d*(5*a^3*tan(c/2 + (d*x)/2)^2 - 10*a^3*tan(c/2 + (d*x)/2)^4 + 1
0*a^3*tan(c/2 + (d*x)/2)^6 - 5*a^3*tan(c/2 + (d*x)/2)^8 + a^3*tan(c/2 + (d*
x)/2)^10 - a^3))
```

3.88 $\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\log(\cos(c+dx))}{a^3d}+\frac{3\sec(c+dx)}{a^3d}-\frac{3\sec^2(c+dx)}{2a^3d}+\frac{\sec^3(c+dx)}{3a^3d}$$

```
[Out] ln(cos(d*x+c))/a^3/d+3*sec(d*x+c)/a^3/d-3/2*sec(d*x+c)^2/a^3/d+1/3*sec(d*x+
c)^3/a^3/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 45}

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sec^3(c+dx)}{3a^3d}-\frac{3\sec^2(c+dx)}{2a^3d}+\frac{3\sec(c+dx)}{a^3d}+\frac{\log(\cos(c+dx))}{a^3d}$$

```
[In] Int[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^3*d) + (3*Sec[c + d*x])/(a^3*d) - (3*Sec[c + d*x]^2)/(
2*a^3*d) + Sec[c + d*x]^3/(3*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^3}{x^4}\,dx, x, \cos(c+dx)\right)}{a^6 d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{3a^3}{x^3}+\frac{3a^3}{x^2}-\frac{a^3}{x}\right)\,dx, x, \cos(c+dx)\right)}{a^6 d}\\
&= \frac{\log(\cos(c+dx))}{a^3 d}+\frac{3\sec(c+dx)}{a^3 d}-\frac{3\sec^2(c+dx)}{2a^3 d}+\frac{\sec^3(c+dx)}{3a^3 d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{(22+18\cos(2(c+dx))+9\cos(c+dx)(-2+\log(\cos(c+dx)))+3\cos(3(c+dx))\log(\cos(c+dx)))\sec^3(c}{12a^3 d}$$

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((22 + 18*Cos[2*(c + d*x)] + 9*Cos[c + d*x]*(-2 + Log[Cos[c + d*x]]) + 3*Co
s[3*(c + d*x)]*Log[Cos[c + d*x]])*Sec[c + d*x]^3)/(12*a^3*d)
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\frac{1}{3\cos(dx+c)^3}+\ln(\cos(dx+c))-\frac{3}{2\cos(dx+c)^2}+\frac{3}{\cos(dx+c)}}{d\,a^3}$	46
default	$\frac{\frac{1}{3\cos(dx+c)^3}+\ln(\cos(dx+c))-\frac{3}{2\cos(dx+c)^2}+\frac{3}{\cos(dx+c)}}{d\,a^3}$	46
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3 d}+\frac{6\,e^{5i(dx+c)}-6\,e^{4i(dx+c)}+\frac{44\,e^{3i(dx+c)}}{3}-6\,e^{2i(dx+c)}+6\,e^{i(dx+c)}}{d\,a^3\left(e^{2i(dx+c)}+1\right)^3}+\frac{\ln\left(e^{2i(dx+c)}+1\right)}{a^3 d}$	115

```
[In] int(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/3/cos(d*x+c)^3+ln(cos(d*x+c))-3/2/cos(d*x+c)^2+3/cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{6\,\cos\left(dx+c\right)^3\log\left(-\cos\left(dx+c\right)\right)+18\,\cos\left(dx+c\right)^2-9\,\cos\left(dx+c\right)+2}{6\,a^3d\cos\left(dx+c\right)^3}$$

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
[Out] 1/6*(6*cos(d*x + c)^3*log(-cos(d*x + c)) + 18*cos(d*x + c)^2 - 9*cos(d*x +
c) + 2)/(a^3*d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^7(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(tan(d*x+c)**7/(a+a*sec(d*x+c))**3,x)
[Out] Integral(tan(c + d*x)**7/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{6\log(\cos(dx+c))}{a^3}+\frac{18\cos(dx+c)^2-9\cos(dx+c)+2}{a^3\cos(dx+c)^3}}{6\,d}$$

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] 1/6*(6*log(cos(d*x + c))/a^3 + (18*cos(d*x + c)^2 - 9*cos(d*x + c) + 2)/(a^
3*cos(d*x + c)^3))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(61) = 122$.

Time = 3.60 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.43

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} - \frac{6\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3} - \frac{\frac{75(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{51(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{11(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} + 29}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

```
[In] integrate(tan(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - 6*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (75*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 51*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 11*(co
s(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 29)/(a^3*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 15.50 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68

$$\int \frac{\tan^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{14\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + \frac{20}{3}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 3a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 3a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - a^3\right)} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{a^3\,d}$$

```
[In] int(tan(c + d*x)^7/(a + a/cos(c + d*x))^3,x)
```

```
[Out] - (14*tan(c/2 + (d*x)/2)^4 - 18*tan(c/2 + (d*x)/2)^2 + 20/3)/(d*(3*a^3*tan(
c/2 + (d*x)/2)^2 - 3*a^3*tan(c/2 + (d*x)/2)^4 + a^3*tan(c/2 + (d*x)/2)^6 -
a^3)) - (2*atanh(tan(c/2 + (d*x)/2)^2))/(a^3*d)
```

3.89 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\log(\cos(c+dx))}{a^3d} - \frac{4\log(1+\cos(c+dx))}{a^3d} + \frac{\sec(c+dx)}{a^3d}$$

```
[Out] 3*ln(cos(d*x+c))/a^3/d-4*ln(1+cos(d*x+c))/a^3/d+sec(d*x+c)/a^3/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sec(c+dx)}{a^3d} + \frac{3\log(\cos(c+dx))}{a^3d} - \frac{4\log(\cos(c+dx)+1)}{a^3d}$$

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Log[Cos[c + d*x]])/(a^3*d) - (4*Log[1 + Cos[c + d*x]])/(a^3*d) + Sec[c +
 d*x]/(a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^2}{x^2(a+ax)}\,dx, x, \cos(c+dx)\right)}{a^4 d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{3a}{x}+\frac{4a}{1+x}\right)\,dx, x, \cos(c+dx)\right)}{a^4 d}\\
&= \frac{3\log(\cos(c+dx))}{a^3 d}-\frac{4\log(1+\cos(c+dx))}{a^3 d}+\frac{\sec(c+dx)}{a^3 d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$
\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{-8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+3\log(\cos(c+dx))+\sec(c+dx)}{a^3 d}
$$

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
[Out] (-8*Log[Cos[(c + d*x)/2]] + 3*Log[Cos[c + d*x]] + Sec[c + d*x])/(a^3*d)
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-4\ln(\cos(dx+c)+1)+\frac{1}{\cos(dx+c)}+3\ln(\cos(dx+c))}{d\,a^3}$	37
default	$\frac{-4\ln(\cos(dx+c)+1)+\frac{1}{\cos(dx+c)}+3\ln(\cos(dx+c))}{d\,a^3}$	37
risch	$\frac{ix}{a^3}+\frac{2ic}{a^3 d}+\frac{2\,\mathrm{e}^{i(dx+c)}}{d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{8\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{a^3 d}+\frac{3\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^3 d}$	89

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d/a^3*(-4*ln(cos(d*x+c)+1)+1/cos(d*x+c)+3*ln(cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\cos(dx+c)\log(-\cos(dx+c)) - 4\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+1}{a^3 d\cos(dx+c)}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (3*cos(d*x + c)*log(-cos(d*x + c)) - 4*cos(d*x + c)*log(1/2*cos(d*x + c) +
1/2) + 1)/(a^3*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{4\log(\cos(dx+c)+1)}{a^3} - \frac{3\log(\cos(dx+c))}{a^3} - \frac{1}{a^3\cos(dx+c)}}{d}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(4*log(cos(d*x + c) + 1)/a^3 - 3*log(cos(d*x + c))/a^3 - 1/(a^3*cos(d*x +
c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(46) = 92$.

Time = 1.67 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.43

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3}-\frac{\frac{3(\cos(dx+c)-1)}{\cos(dx+c)+1}+1}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{d}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + 3*log(abs(-(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/a^3 - (3*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 1)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.65 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}{a^3\,d}-\frac{2}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^3\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^3\,d}$$

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (3*log(tan(c/2 + (d*x)/2)^2 - 1))/(a^3*d) - 2/(d*(a^3*tan(c/2 + (d*x)/2)^2
- a^3)) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^3*d)
```

3.90 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2}{a^3d(1+\cos(c+dx))} + \frac{\log(1+\cos(c+dx))}{a^3d}$$

```
[Out] 2/a^3/d/(1+cos(d*x+c))+ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3964, 45}

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2}{a^3d(\cos(c+dx)+1)} + \frac{\log(\cos(c+dx)+1)}{a^3d}$$

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 2/(a^3*d*(1 + Cos[c + d*x])) + Log[1 + Cos[c + d*x]]/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
```

```
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{a-ax}{(a+ax)^2}\,dx, x, \cos(c+dx)\right)}{a^2 d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{2}{a(1+x)^2} - \frac{1}{a(1+x)}\right)\,dx, x, \cos(c+dx)\right)}{a^2 d}\\
&= \frac{2}{a^3 d(1+\cos(c+dx))} + \frac{\log(1+\cos(c+dx))}{a^3 d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$
\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \tan^2\left(\frac{1}{2}(c+dx)\right)}{a^3 d}
$$

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
[Out] (2*Log[Cos[(c + d*x)/2]] + Tan[(c + d*x)/2]^2)/(a^3*d)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\ln(\cos(dx+c)+1)+\frac{2}{\cos(dx+c)+1}}{d\,a^3}$	30
default	$\frac{\ln(\cos(dx+c)+1)+\frac{2}{\cos(dx+c)+1}}{d\,a^3}$	30
risch	$-\frac{ix}{a^3} - \frac{2ic}{a^3 d} + \frac{4\,\mathrm{e}^{i(dx+c)}}{a^3 d\left(\mathrm{e}^{i(dx+c)}+1\right)^2} + \frac{2\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{a^3 d}$	69

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d/a^3*(ln(cos(d*x+c)+1)+2/(cos(d*x+c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2}{a^3d\cos(dx+c)+a^3d}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 2)/(a^3*d*cos(d*x + c) +
a^3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(29) = 58$.

Time = 18.80 (sec) , antiderivative size = 457, normalized size of antiderivative = 13.06

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \begin{cases} -\frac{\log(\tan^2(c+dx)+1)\sec^2(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} - \frac{2\log(\tan^2(c+dx)+1)\sec(c+dx)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} - \frac{\log(\tan^2(c+dx)+1)}{2a^3d\sec^2(c+dx)+4a^3d\sec(c+dx)+2a^3d} + \ldots \\ \frac{x\tan^3(c)}{(a\sec(c)+a)^3} \end{cases}$$

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Piecewise((-log(tan(c + d*x)**2 + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)
**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 2*log(tan(c + d*x)**2 + 1)*sec(c
+ d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - log(
tan(c + d*x)**2 + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*
a**3*d) + 2*log(sec(c + d*x) + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)**2
 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 4*log(sec(c + d*x) + 1)*sec(c + d*x)
/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 2*log(sec(
c + d*x) + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d)
 + tan(c + d*x)**2/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a*
*3*d) - 2*sec(c + d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) +
2*a**3*d) - 2/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d)
, Ne(d, 0)), (x*tan(c)**3/(a*sec(c) + a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{2}{a^3\cos(dx+c)+a^3} + \frac{\log(\cos(dx+c)+1)}{a^3}}{d}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] (2/(a^3*cos(d*x + c) + a^3) + log(cos(d*x + c) + 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.74 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} + \frac{\cos(dx+c)-1}{a^3(\cos(dx+c)+1)}}{d}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
[Out] -(log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (cos(d*x + c)
- 1)/(a^3*(cos(d*x + c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^3\,d}$$

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^3,x)
[Out] -(log(tan(c/2 + (d*x)/2)^2 + 1) - tan(c/2 + (d*x)/2)^2)/(a^3*d)
```

3.91 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{1}{2a^3d(1+\cos(c+dx))^2} - \frac{2}{a^3d(1+\cos(c+dx))} - \frac{\log(1+\cos(c+dx))}{a^3d}$$

```
[Out] 1/2/a^3/d/(1+cos(d*x+c))^2-2/a^3/d/(1+cos(d*x+c))-ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 45}

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{2}{a^3d(\cos(c+dx)+1)} + \frac{1}{2a^3d(\cos(c+dx)+1)^2} - \frac{\log(\cos(c+dx)+1)}{a^3d}$$

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 1/(2*a^3*d*(1 + Cos[c + d*x])^2) - 2/(a^3*d*(1 + Cos[c + d*x])) - Log[1 + C
os[c + d*x]]/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^2}{(a+ax)^3}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{a^3(1+x)^3}-\frac{2}{a^3(1+x)^2}+\frac{1}{a^3(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}\\ &= \frac{1}{2a^3d(1+\cos(c+dx))^2}-\frac{2}{a^3d(1+\cos(c+dx))}-\frac{\log(1+\cos(c+dx))}{a^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-1+8\cos^2\left(\frac{1}{2}(c+dx)\right)+16\cos^4\left(\frac{1}{2}(c+dx)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^3(c+dx)}{a^3d(1+\sec(c+dx))^3}$$

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -((Cos[(c + d*x)/2]^2*(-1 + 8*Cos[(c + d*x)/2]^2 + 16*Cos[(c + d*x)/2]^4*Lo
g[Cos[(c + d*x)/2]])*Sec[c + d*x]^3)/(a^3*d*(1 + Sec[c + d*x])^3))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\ln(\sec(dx+c))+\frac{1}{2(1+\sec(dx+c))^2}+\frac{1}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))}{d\,a^3}$	49
default	$\frac{\ln(\sec(dx+c))+\frac{1}{2(1+\sec(dx+c))^2}+\frac{1}{1+\sec(dx+c)}-\ln(1+\sec(dx+c))}{d\,a^3}$	49
risch	$\frac{ix}{a^3}+\frac{2ic}{a^3d}-\frac{2(2\,e^{3i(dx+c)}+3\,e^{2i(dx+c)}+2\,e^{i(dx+c)})}{d\,a^3\left(e^{i(dx+c)}+1\right)^4}-\frac{2\ln\left(e^{i(dx+c)}+1\right)}{a^3d}$	94

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(ln(sec(d*x+c))+1/2/(1+sec(d*x+c))^2+1/(1+sec(d*x+c))-ln(1+sec(d*x+
c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{2\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+4\,\cos\left(dx+c\right)+3}{2\left(a^3d\cos\left(dx+c\right)^2+2\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) +
 4*cos(d*x + c) + 3)/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(48) = 96$.

Time = 19.01 (sec) , antiderivative size = 411, normalized size of antiderivative = 7.34

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{\log\left(\tan^2\left(c+dx\right)+1\right)\sec^2\left(c+dx\right)}{2a^3d\sec^2\left(c+dx\right)+4a^3d\sec\left(c+dx\right)+2a^3d}+\frac{2\log\left(\tan^2\left(c+dx\right)+1\right)\sec\left(c+dx\right)}{2a^3d\sec^2\left(c+dx\right)+4a^3d\sec\left(c+dx\right)+2a^3d}+\frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2a^3d\sec^2\left(c+dx\right)+4a^3d\sec\left(c+dx\right)+2a^3d}- \\ \frac{x\tan\left(c\right)}{\left(a\sec\left(c\right)+a\right)^3} \end{cases}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Piecewise((log(tan(c + d*x)**2 + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)*
*2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + 2*log(tan(c + d*x)**2 + 1)*sec(c +
```

```
d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) + log(t
an(c + d*x)**2 + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a
**3*d) - 2*log(sec(c + d*x) + 1)*sec(c + d*x)**2/(2*a**3*d*sec(c + d*x)**2
+ 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 4*log(sec(c + d*x) + 1)*sec(c + d*x)/
(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d) - 2*log(sec(c
 + d*x) + 1)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d)
+ 2*sec(c + d*x)/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3
*d) + 3/(2*a**3*d*sec(c + d*x)**2 + 4*a**3*d*sec(c + d*x) + 2*a**3*d), Ne(d
, 0)), (x*tan(c)/(a*sec(c) + a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{4\cos(dx+c)+3}{a^3\cos(dx+c)^2+2a^3\cos(dx+c)+a^3}+\frac{2\log(\cos(dx+c)+1)}{a^3}}{2d}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((4*cos(d*x + c) + 3)/(a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) + a^3)
+ 2*log(cos(d*x + c) + 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{8\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{\frac{6a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^6}}{8d}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (6*a^3*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2)/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{8}}{a^3\,d}$$

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1) - (3*tan(c/2 + (d*x)/2)^2)/4 + tan(c/2 + (d*
x)/2)^4/8)/(a^3*d)
```

3.92 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{1}{6a^3d(1+\cos(c+dx))^3} - \frac{7}{8a^3d(1+\cos(c+dx))^2} + \frac{17}{8a^3d(1+\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{16a^3d} + \frac{15\log(1+\cos(c+dx))}{16a^3d}$$

```
[Out] 1/6/a^3/d/(1+cos(d*x+c))^3-7/8/a^3/d/(1+cos(d*x+c))^2+17/8/a^3/d/(1+cos(d*x
+c))+1/16*ln(1-cos(d*x+c))/a^3/d+15/16*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3964, 90}

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{17}{8a^3d(\cos(c+dx)+1)} - \frac{7}{8a^3d(\cos(c+dx)+1)^2} + \frac{1}{6a^3d(\cos(c+dx)+1)^3} + \frac{\log(1-\cos(c+dx))}{16a^3d} + \frac{15\log(\cos(c+dx)+1)}{16a^3d}$$

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 1/(6*a^3*d*(1 + Cos[c + d*x])^3) - 7/(8*a^3*d*(1 + Cos[c + d*x])^2) + 17/(8
*a^3*d*(1 + Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(16*a^3*d) + (15*Log[1 +
 Cos[c + d*x]])/(16*a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a^2\text{Subst}\left(\int \frac{x^4}{(a-ax)(a+ax)^4}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{a^2\text{Subst}\left(\int \left(-\frac{1}{16a^5(-1+x)}+\frac{1}{2a^5(1+x)^4}-\frac{7}{4a^5(1+x)^3}+\frac{17}{8a^5(1+x)^2}-\frac{15}{16a^5(1+x)}\right)dx, x, \cos(c+dx)\right)}{d}\\
&= \frac{1}{6a^3d(1+\cos(c+dx))^3}-\frac{7}{8a^3d(1+\cos(c+dx))^2}+\frac{17}{8a^3d(1+\cos(c+dx))}\\
&\quad+\frac{\log(1-\cos(c+dx))}{16a^3d}+\frac{15\log(1+\cos(c+dx))}{16a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$
\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\left(2-21\cos^2\left(\frac{1}{2}(c+dx)\right)+102\cos^4\left(\frac{1}{2}(c+dx)\right)+12\cos^6\left(\frac{1}{2}(c+dx)\right)\left(15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\right.\right.\right.}{12a^3d(1+\sec(c+dx))^3}
$$

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((2 - 21*Cos[(c + d*x)/2]^2 + 102*Cos[(c + d*x)/2]^4 + 12*Cos[(c + d*x)/2]^
6*(15*Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^3)/(12*a
^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{16}+\frac{1}{6(\cos(dx+c)+1)^3}-\frac{7}{8(\cos(dx+c)+1)^2}+\frac{17}{8(\cos(dx+c)+1)}+\frac{15\ln(\cos(dx+c)+1)}{16}}{d\,a^3}$
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{16}+\frac{1}{6(\cos(dx+c)+1)^3}-\frac{7}{8(\cos(dx+c)+1)^2}+\frac{17}{8(\cos(dx+c)+1)}+\frac{15\ln(\cos(dx+c)+1)}{16}}{d\,a^3}$
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3d}+\frac{51\,e^{5i(dx+c)}+162\,e^{4i(dx+c)}+238\,e^{3i(dx+c)}+162\,e^{2i(dx+c)}+51\,e^{i(dx+c)}}{12d\,a^3\left(e^{i(dx+c)}+1\right)^6}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{8a^3d}+\frac{15\ln(e^{i(}}{8a}$

[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d/a^3*(1/16*ln(cos(d*x+c)-1)+1/6/(cos(d*x+c)+1)^3-7/8/(cos(d*x+c)+1)^2+17/8/(cos(d*x+c)+1)+15/16*ln(cos(d*x+c)+1))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{102\,\cos\left(dx+c\right)^2+45\,\left(\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3\,(}{48\,\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx\right.\right.}$$

[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/48*(102*cos(d*x + c)^2 + 45*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 162*cos(d*x + c) + 68)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cot(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**3,x)

[Out] Integral(cot(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)/a**3

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$=\frac{\frac{2\left(51\cos(dx+c)^2+81\cos(dx+c)+34\right)}{a^3\cos(dx+c)^3+3a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3}+\frac{45\log(\cos(dx+c)+1)}{a^3}+\frac{3\log(\cos(dx+c)-1)}{a^3}}{48d}$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(2*(51*cos(d*x + c)^2 + 81*cos(d*x + c) + 34)/(a^3*cos(d*x + c)^3 + 3*
a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) + 45*log(cos(d*x + c) + 1)/a
^3 + 3*log(cos(d*x + c) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.42

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$=\frac{\frac{6\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}-\frac{96\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}-\frac{\frac{66a^6(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{15a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2a^6(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^9}}{96d}$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 - 96*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - (66*a^6*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 15*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
2*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^9)/d
```

Mupad [B] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$=\frac{\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8}-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)+\frac{11\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{16}-\frac{5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{32}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{48}}{a^3\,d}$$

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))/8 - log(tan(c/2 + (d*x)/2)^2 + 1) + (11*tan(c/2 +
(d*x)/2)^2)/16 - (5*tan(c/2 + (d*x)/2)^4)/32 + tan(c/2 + (d*x)/2)^6/48)/(a^
3*d)
```

3.93 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3} \, dx = -\frac{1}{32a^3d(1-\cos(c+dx))} + \frac{1}{16a^3d(1+\cos(c+dx))^4} - \frac{5}{12a^3d(1+\cos(c+dx))^3} + \frac{39}{32a^3d(1+\cos(c+dx))^2} - \frac{9}{4a^3d(1+\cos(c+dx))} - \frac{7\log(1-\cos(c+dx))}{64a^3d} - \frac{57\log(1+\cos(c+dx))}{64a^3d}$$

```
[Out] -1/32/a^3/d/(1-cos(d*x+c))+1/16/a^3/d/(1+cos(d*x+c))^4-5/12/a^3/d/(1+cos(d*
x+c))^3+39/32/a^3/d/(1+cos(d*x+c))^2-9/4/a^3/d/(1+cos(d*x+c))-7/64*ln(1-cos
(d*x+c))/a^3/d-57/64*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3964, 90}

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3} \, dx = -\frac{1}{32a^3d(1-\cos(c+dx))} - \frac{9}{4a^3d(\cos(c+dx)+1)} + \frac{39}{32a^3d(\cos(c+dx)+1)^2} - \frac{5}{12a^3d(\cos(c+dx)+1)^3} + \frac{1}{16a^3d(\cos(c+dx)+1)^4} - \frac{7\log(1-\cos(c+dx))}{64a^3d} - \frac{57\log(\cos(c+dx)+1)}{64a^3d}$$

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/32*1/(a^3*d*(1 - Cos[c + d*x])) + 1/(16*a^3*d*(1 + Cos[c + d*x])^4) - 5/
(12*a^3*d*(1 + Cos[c + d*x])^3) + 39/(32*a^3*d*(1 + Cos[c + d*x])^2) - 9/(4
*a^3*d*(1 + Cos[c + d*x])) - (7*Log[1 - Cos[c + d*x]])/(64*a^3*d) - (57*Log
[1 + Cos[c + d*x]])/(64*a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a^4 \text{Subst}\left(\int \frac{x^6}{(a-ax)^2(a+ax)^5}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{a^4 \text{Subst}\left(\int \left(\frac{1}{32a^7(-1+x)^2} + \frac{7}{64a^7(-1+x)} + \frac{1}{4a^7(1+x)^5} - \frac{5}{4a^7(1+x)^4} + \frac{39}{16a^7(1+x)^3} - \frac{9}{4a^7(1+x)^2} + \frac{57}{64a^7(1+x)}\right)}{d} \\
&= -\frac{1}{32a^3d(1-\cos(c+dx))} + \frac{1}{16a^3d(1+\cos(c+dx))^4} \\
&\quad - \frac{5}{12a^3d(1+\cos(c+dx))^3} + \frac{39}{32a^3d(1+\cos(c+dx))^2} \\
&\quad - \frac{9}{4a^3d(1+\cos(c+dx))} - \frac{7\log(1-\cos(c+dx))}{64a^3d} - \frac{57\log(1+\cos(c+dx))}{64a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(864+12\cot^2\left(\frac{1}{2}(c+dx)\right)+24\cos^2\left(\frac{1}{2}(c+dx)\right)\left(57\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+7\log\left(\sin\right.\right.\right.}{96a^3d(1+\sec(c}$$

[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]

[Out] -1/96*(Cos[(c + d*x)/2]^4*(864 + 12*Cot[(c + d*x)/2]^2 + 24*Cos[(c + d*x)/2]^2*(57*Log[Cos[(c + d*x)/2]] + 7*Log[Sin[(c + d*x)/2]]) - 234*Sec[(c + d*x)/2]^2 + 40*Sec[(c + d*x)/2]^4 - 3*Sec[(c + d*x)/2]^6)*Sec[c + d*x]^3)/(a^3*d*(1 + Sec[c + d*x])^3)

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{\frac{1}{16(\cos(dx+c)+1)^4}-\frac{5}{12(\cos(dx+c)+1)^3}+\frac{39}{32(\cos(dx+c)+1)^2}-\frac{9}{4(\cos(dx+c)+1)}-\frac{57\ln(\cos(dx+c)+1)}{64}+\frac{1}{32\cos(dx+c)-32}-\frac{7\ln(\cos(}{6}}{d\,a^3}$
default	$\frac{\frac{1}{16(\cos(dx+c)+1)^4}-\frac{5}{12(\cos(dx+c)+1)^3}+\frac{39}{32(\cos(dx+c)+1)^2}-\frac{9}{4(\cos(dx+c)+1)}-\frac{57\ln(\cos(dx+c)+1)}{64}+\frac{1}{32\cos(dx+c)-32}-\frac{7\ln(\cos(}{6}}{d\,a^3}$
risch	$\frac{ix}{a^3}+\frac{2ic}{a^3d}-\frac{213\,e^{9i(dx+c)}+606\,e^{8i(dx+c)}+472\,e^{7i(dx+c)}-846\,e^{6i(dx+c)}-1658\,e^{5i(dx+c)}-846\,e^{4i(dx+c)}+472\,e^{3i(dx+c}}{48d\,a^3\left(e^{i(dx+c)}+1\right)^8\left(e^{i(dx+c)}-1\right)^2}$

[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d/a^3*(1/16/(cos(d*x+c)+1)^4-5/12/(cos(d*x+c)+1)^3+39/32/(cos(d*x+c)+1)^2-9/4/(cos(d*x+c)+1)-57/64*ln(cos(d*x+c)+1)+1/32/(cos(d*x+c)-1)-7/64*ln(cos(d*x+c)-1))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.68

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{426\cos(dx+c)^4+606\cos(dx+c)^3-190\cos(dx+c)^2+171\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\,\mathrm{co}\right.}{}$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(426*cos(d*x + c)^4 + 606*cos(d*x + c)^3 - 190*cos(d*x + c)^2 + 171*
(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 -
3*cos(d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 21*(cos(d*x + c)^5 + 3*co
s(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*lo
g(-1/2*cos(d*x + c) + 1/2) - 666*cos(d*x + c) - 272)/(a^3*d*cos(d*x + c)^5
+ 3*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2
- 3*a^3*d*cos(d*x + c) - a^3*d)
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cot^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{2\left(213\cos(dx+c)^4+303\cos(dx+c)^3-95\cos(dx+c)^2-333\cos(dx+c)-136\right)}{a^3\cos(dx+c)^5+3a^3\cos(dx+c)^4+2a^3\cos(dx+c)^3-2a^3\cos(dx+c)^2-3a^3\cos(dx+c)-a^3}+\frac{171\log(\cos(dx+c)+1)}{a^3}+\frac{21\log(\cos(dx+c)-1)}{a^3}}{192\,d}$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(2*(213*cos(d*x + c)^4 + 303*cos(d*x + c)^3 - 95*cos(d*x + c)^2 - 33
3*cos(d*x + c) - 136)/(a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + 2*a^3*co
s(d*x + c)^3 - 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x + c) - a^3) + 171*log(c
os(d*x + c) + 1)/a^3 + 21*log(cos(d*x + c) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.48

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{12\left(\frac{7(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^3(\cos(dx+c)-1)} - \frac{84\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3} + \frac{768\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} + \frac{\frac{504a^9(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132a^9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{}}{768\,d}$$

[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] 1/768*(12*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(a^3*(cos(d*x + c) - 1)) - 84*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + 768*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (504*a^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 132*a^9*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 28*a^9*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3*a^9*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/a^12)/d

Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.71

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{7\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{32} - \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right) + \frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64} + \frac{21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{32} - \frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64} + \frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{192} -}{a^3\,d}$$

[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^3,x)

[Out] -((7*log(tan(c/2 + (d*x)/2)))/32 - log(tan(c/2 + (d*x)/2)^2 + 1) + cot(c/2 + (d*x)/2)^2/64 + (21*tan(c/2 + (d*x)/2)^2)/32 - (11*tan(c/2 + (d*x)/2)^4)/64 + (7*tan(c/2 + (d*x)/2)^6)/192 - tan(c/2 + (d*x)/2)^8/256)/(a^3*d)

3.94 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 185

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = &-\frac{1}{128a^3d(1-\cos(c+dx))^2}\\ &+\frac{5}{64a^3d(1-\cos(c+dx))}+\frac{1}{40a^3d(1+\cos(c+dx))^5}\\ &-\frac{13}{64a^3d(1+\cos(c+dx))^4}+\frac{35}{48a^3d(1+\cos(c+dx))^3}\\ &-\frac{99}{64a^3d(1+\cos(c+dx))^2}+\frac{303}{128a^3d(1+\cos(c+dx))}\\ &+\frac{37\log(1-\cos(c+dx))}{256a^3d}+\frac{219\log(1+\cos(c+dx))}{256a^3d}\end{aligned}$$

```
[Out] -1/128/a^3/d/(1-cos(d*x+c))^2+5/64/a^3/d/(1-cos(d*x+c))+1/40/a^3/d/(1+cos(d
*x+c))^5-13/64/a^3/d/(1+cos(d*x+c))^4+35/48/a^3/d/(1+cos(d*x+c))^3-99/64/a^
3/d/(1+cos(d*x+c))^2+303/128/a^3/d/(1+cos(d*x+c))+37/256*ln(1-cos(d*x+c))/a
^3/d+219/256*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used

$= \{3964, 90\}$

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{5}{64a^3d(1-\cos(c+dx))} + \frac{303}{128a^3d(\cos(c+dx)+1)} - \frac{1}{128a^3d(1-\cos(c+dx))^2} - \frac{99}{64a^3d(\cos(c+dx)+1)^2} + \frac{35}{48a^3d(\cos(c+dx)+1)^3} - \frac{13}{64a^3d(\cos(c+dx)+1)^4} + \frac{1}{40a^3d(\cos(c+dx)+1)^5} + \frac{37\log(1-\cos(c+dx))}{256a^3d} + \frac{219\log(\cos(c+dx)+1)}{256a^3d}$$

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/128*1/(a^3*d*(1 - Cos[c + d*x])^2) + 5/(64*a^3*d*(1 - Cos[c + d*x])) + 1
/(40*a^3*d*(1 + Cos[c + d*x])^5) - 13/(64*a^3*d*(1 + Cos[c + d*x])^4) + 35/
(48*a^3*d*(1 + Cos[c + d*x])^3) - 99/(64*a^3*d*(1 + Cos[c + d*x])^2) + 303/
(128*a^3*d*(1 + Cos[c + d*x])) + (37*Log[1 - Cos[c + d*x]])/(256*a^3*d) + (
219*Log[1 + Cos[c + d*x]])/(256*a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\text{integral} = -\frac{a^6\text{Subst}\left(\int \frac{x^8}{(a-ax)^3(a+ax)^6}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^6\text{Subst}\left(\int \left(-\frac{1}{64a^9(-1+x)^3} - \frac{5}{64a^9(-1+x)^2} - \frac{37}{256a^9(-1+x)} + \frac{1}{8a^9(1+x)^6} - \frac{13}{16a^9(1+x)^5} + \frac{35}{16a^9(1+x)^4} - \frac{}{32a^9}\right.\right.}{d}$$

$$= -\frac{1}{128a^3d(1-\cos(c+dx))^2} + \frac{5}{64a^3d(1-\cos(c+dx))} + \frac{1}{40a^3d(1+\cos(c+dx))^5} - \frac{13}{64a^3d(1+\cos(c+dx))^4} + \frac{35}{48a^3d(1+\cos(c+dx))^3} - \frac{99}{64a^3d(1+\cos(c+dx))^2} + \frac{303}{128a^3d(1+\cos(c+dx))} + \frac{37\log(1-\cos(c+dx))}{256a^3d} + \frac{219\log(1+\cos(c+dx))}{256a^3d}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\left(12-195\cos^2\left(\frac{1}{2}(c+dx)\right)+1400\cos^4\left(\frac{1}{2}(c+dx)\right)+60\cos^8\left(\frac{1}{2}(c+dx)\right)\left(303+10\cot^2\left(\frac{1}{2}(c+dx)\right)\right)-30\ldots\right.}{}$$

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((12 - 195*Cos[(c + d*x)/2]^2 + 1400*Cos[(c + d*x)/2]^4 + 60*Cos[(c + d*x)/
2]^8*(303 + 10*Cot[(c + d*x)/2]^2) - 30*Cos[(c + d*x)/2]^6*(198 + Cot[(c +
d*x)/2]^4) + 120*Cos[(c + d*x)/2]^10*(219*Log[Cos[(c + d*x)/2]] + 37*Log[Si
n[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4*Sec[c + d*x]^3)/(1920*a^3*d*(1 + Sec[c
 + d*x])^3)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{-\frac{1}{128(\cos(dx+c)-1)^2}-\frac{5}{64(\cos(dx+c)-1)}+\frac{37\ln(\cos(dx+c)-1)}{256}+\frac{1}{40(\cos(dx+c)+1)^5}-\frac{13}{64(\cos(dx+c)+1)^4}+\frac{35}{48(\cos(dx+c)+1)^3}-\frac{}{64(\cos\ldots}}{d\,a^3}$
default	$\frac{-\frac{1}{128(\cos(dx+c)-1)^2}-\frac{5}{64(\cos(dx+c)-1)}+\frac{37\ln(\cos(dx+c)-1)}{256}+\frac{1}{40(\cos(dx+c)+1)^5}-\frac{13}{64(\cos(dx+c)+1)^4}+\frac{35}{48(\cos(dx+c)+1)^3}-\frac{}{64(\cos\ldots}}{d\,a^3}$
risch	$-\frac{ix}{a^3}-\frac{2ic}{a^3d}+\frac{4395\,e^{13i(dx+c)}+11010\,e^{12i(dx+c)}-1390\,e^{11i(dx+c)}-47190\,e^{10i(dx+c)}-50987\,e^{9i(dx+c)}+25428\,e^{8i(dx+c}\ldots}{960d\,a^3\left(e^{i}\ldots\right.}$

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/128/(cos(d*x+c)-1)^2-5/64/(cos(d*x+c)-1)+37/256*ln(cos(d*x+c)-1
)+1/40/(cos(d*x+c)+1)^5-13/64/(cos(d*x+c)+1)^4+35/48/(cos(d*x+c)+1)^3-99/64
/(cos(d*x+c)+1)^2+303/128/(cos(d*x+c)+1)+219/256*ln(cos(d*x+c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.71

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{8790\cos(dx+c)^6 + 11010\cos(dx+c)^5 - 13880\cos(dx+c)^4 - 25560\cos(dx+c)^3 - 734\cos(dx+c)$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3840*(8790*cos(d*x + c)^6 + 11010*cos(d*x + c)^5 - 13880*cos(d*x + c)^4 -
 25560*cos(d*x + c)^3 - 734*cos(d*x + c)^2 + 3285*(cos(d*x + c)^7 + 3*cos(d
*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x
+ c)^2 + 3*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 555*(cos(d*x + c
)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)
^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 13
878*cos(d*x + c) + 5536)/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a
^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3
*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cot^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.02

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{2\left(4395\cos(dx+c)^6+5505\cos(dx+c)^5-6940\cos(dx+c)^4-12780\cos(dx+c)^3-367\cos(dx+c)^2+6939\cos(dx+c)+2768\right)}{a^3\cos(dx+c)^7+3a^3\cos(dx+c)^6+a^3\cos(dx+c)^5-5a^3\cos(dx+c)^4-5a^3\cos(dx+c)^3+a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3} + \frac{3285\log(\cos(dx+c}{a^3}}{3840\,d}$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3840*(2*(4395*cos(d*x + c)^6 + 5505*cos(d*x + c)^5 - 6940*cos(d*x + c)^4
- 12780*cos(d*x + c)^3 - 367*cos(d*x + c)^2 + 6939*cos(d*x + c) + 2768)/(a^
3*cos(d*x + c)^7 + 3*a^3*cos(d*x + c)^6 + a^3*cos(d*x + c)^5 - 5*a^3*cos(d*
x + c)^4 - 5*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
 a^3) + 3285*log(cos(d*x + c) + 1)/a^3 + 555*log(cos(d*x + c) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.41

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{30\left(\frac{18\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{111\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{a^3(\cos(dx+c)-1)^2} - \frac{2220\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3} + \frac{15360\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} + \frac{9780\,a^{12}(\cdots}{\cos(\cdots}\cdots}{15360\,d}$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/15360*(30*(18*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 111*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(a^3*(cos(d*x + c) -
1)^2) - 2220*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + 15360*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (9780*a^12*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 2790*a^12*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 + 740*a^12*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 135*a^12
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 12*a^12*(cos(d*x + c) - 1)^5/(
cos(d*x + c) + 1)^5)/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.92

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{163\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{256\,a^3\,d} - \frac{93\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{512\,a^3\,d} + \frac{37\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{768\,a^3\,d} - \frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{1024\,a^3\,d} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{1280\,a^3\,d} + \frac{37\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{128\,a^3\,d} - \frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a^3\,d} + \frac{{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{9\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}-\frac{1}{4}\right)}{128\,a^3\,d}$$

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (163*tan(c/2 + (d*x)/2)^2)/(256*a^3*d) - (93*tan(c/2 + (d*x)/2)^4)/(512*a^3
*d) + (37*tan(c/2 + (d*x)/2)^6)/(768*a^3*d) - (9*tan(c/2 + (d*x)/2)^8)/(102
4*a^3*d) + tan(c/2 + (d*x)/2)^10/(1280*a^3*d) + (37*log(tan(c/2 + (d*x)/2))
)/(128*a^3*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^3*d) + (cot(c/2 + (d*x)/2)
^4*((9*tan(c/2 + (d*x)/2)^2)/2 - 1/4))/(128*a^3*d)
```

3.95 $\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 237

$$\begin{aligned}\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3} dx = {} & \frac{x}{a^3} - \frac{125\text{arctanh}(\sin(c+dx))}{128a^3d} \\ & - \frac{\tan(c+dx)}{a^3d} + \frac{115\sec(c+dx)\tan(c+dx)}{128a^3d} \\ & + \frac{5\sec^3(c+dx)\tan(c+dx)}{64a^3d} + \frac{\tan^3(c+dx)}{3a^3d} \\ & - \frac{5\sec(c+dx)\tan^3(c+dx)}{8a^3d} - \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48a^3d} \\ & - \frac{\tan^5(c+dx)}{5a^3d} + \frac{\sec(c+dx)\tan^5(c+dx)}{2a^3d} \\ & + \frac{\sec^3(c+dx)\tan^5(c+dx)}{8a^3d} - \frac{3\tan^7(c+dx)}{7a^3d}\end{aligned}$$

```
[Out] x/a^3-125/128*arctanh(sin(d*x+c))/a^3/d-tan(d*x+c)/a^3/d+115/128*sec(d*x+c)
*tan(d*x+c)/a^3/d+5/64*sec(d*x+c)^3*tan(d*x+c)/a^3/d+1/3*tan(d*x+c)^3/a^3/d
-5/8*sec(d*x+c)*tan(d*x+c)^3/a^3/d-5/48*sec(d*x+c)^3*tan(d*x+c)^3/a^3/d-1/5
*tan(d*x+c)^5/a^3/d+1/2*sec(d*x+c)*tan(d*x+c)^5/a^3/d+1/8*sec(d*x+c)^3*tan(
d*x+c)^5/a^3/d-3/7*tan(d*x+c)^7/a^3/d
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3973, 3971, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{125\text{arctanh}(\sin(c+dx))}{128a^3d} - \frac{3\tan^7(c+dx)}{7a^3d} - \frac{\tan^5(c+dx)}{5a^3d} + \frac{\tan^3(c+dx)}{3a^3d} - \frac{\tan(c+dx)}{a^3d} + \frac{\tan^5(c+dx)\sec^3(c+dx)}{8a^3d} - \frac{5\tan^3(c+dx)\sec^3(c+dx)}{48a^3d} + \frac{5\tan(c+dx)\sec^3(c+dx)}{64a^3d} + \frac{\tan^5(c+dx)\sec(c+dx)}{2a^3d} - \frac{5\tan^3(c+dx)\sec(c+dx)}{8a^3d} + \frac{115\tan(c+dx)\sec(c+dx)}{128a^3d} + \frac{x}{a^3}$$

```
[In] Int[Tan[c + d*x]^12/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 - (125*ArcTanh[Sin[c + d*x]])/(128*a^3*d) - Tan[c + d*x]/(a^3*d) + (1
15*Sec[c + d*x]*Tan[c + d*x])/(128*a^3*d) + (5*Sec[c + d*x]^3*Tan[c + d*x])
/(64*a^3*d) + Tan[c + d*x]^3/(3*a^3*d) - (5*Sec[c + d*x]*Tan[c + d*x]^3)/(8
*a^3*d) - (5*Sec[c + d*x]^3*Tan[c + d*x]^3)/(48*a^3*d) - Tan[c + d*x]^5/(5*
a^3*d) + (Sec[c + d*x]*Tan[c + d*x]^5)/(2*a^3*d) + (Sec[c + d*x]^3*Tan[c +
d*x]^5)/(8*a^3*d) - (3*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (-a + a\sec(c+dx))^3 \tan^6(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(-a^3\tan^6(c+dx) + 3a^3\sec(c+dx)\tan^6(c+dx) - 3a^3\sec^2(c+dx)\tan^6(c+dx) + a^3\sec^3(c+d\right.}{a^6} \\
&= -\frac{\int \tan^6(c+dx)\,dx}{a^3} + \frac{\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{a^3} \\
&\quad + \frac{3\int \sec(c+dx)\tan^6(c+dx)\,dx}{a^3} - \frac{3\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a^3}
\end{aligned}
$$

$$= -\frac{\tan^5(c+dx)}{5a^3d} + \frac{\sec(c+dx)\tan^5(c+dx)}{2a^3d} + \frac{\sec^3(c+dx)\tan^5(c+dx)}{8a^3d} - \frac{5\int \sec^3(c+dx)\tan^4(c+dx)\,dx}{8a^3} + \frac{\int \tan^4(c+dx)\,dx}{a^3} - \frac{5\int \sec(c+dx)\tan^4(c+dx)\,dx}{2a^3} - \frac{3\text{Subst}\left(\int x^6\,dx, x, \tan(c+dx)\right)}{a^3d}$$

$$= \frac{\tan^3(c+dx)}{3a^3d} - \frac{5\sec(c+dx)\tan^3(c+dx)}{8a^3d} - \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48a^3d} - \frac{\tan^5(c+dx)}{5a^3d} + \frac{\sec(c+dx)\tan^5(c+dx)}{2a^3d} + \frac{\sec^3(c+dx)\tan^5(c+dx)}{8a^3d} - \frac{3\tan^7(c+dx)}{7a^3d} + \frac{5\int \sec^3(c+dx)\tan^2(c+dx)\,dx}{16a^3} - \frac{\int \tan^2(c+dx)\,dx}{a^3} + \frac{15\int \sec(c+dx)\tan^2(c+dx)\,dx}{8a^3}$$

$$= -\frac{\tan(c+dx)}{a^3d} + \frac{15\sec(c+dx)\tan(c+dx)}{16a^3d} + \frac{5\sec^3(c+dx)\tan(c+dx)}{64a^3d} + \frac{\tan^3(c+dx)}{3a^3d} - \frac{5\sec(c+dx)\tan^3(c+dx)}{8a^3d} - \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48a^3d} - \frac{\tan^5(c+dx)}{5a^3d} + \frac{\sec(c+dx)\tan^5(c+dx)}{2a^3d} + \frac{\sec^3(c+dx)\tan^5(c+dx)}{8a^3d} - \frac{3\tan^7(c+dx)}{7a^3d} - \frac{5\int \sec^3(c+dx)\,dx}{64a^3} - \frac{15\int \sec(c+dx)\,dx}{16a^3} + \frac{\int 1\,dx}{a^3}$$

$$= \frac{x}{a^3} - \frac{15\text{arctanh}(\sin(c+dx))}{16a^3d} - \frac{\tan(c+dx)}{a^3d} + \frac{115\sec(c+dx)\tan(c+dx)}{128a^3d} + \frac{5\sec^3(c+dx)\tan(c+dx)}{64a^3d} + \frac{\tan^3(c+dx)}{3a^3d} - \frac{5\sec(c+dx)\tan^3(c+dx)}{8a^3d} - \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48a^3d} - \frac{\tan^5(c+dx)}{5a^3d} + \frac{\sec(c+dx)\tan^5(c+dx)}{2a^3d} + \frac{\sec^3(c+dx)\tan^5(c+dx)}{8a^3d} - \frac{3\tan^7(c+dx)}{7a^3d} - \frac{5\int \sec(c+dx)\,dx}{128a^3}$$

$$= \frac{x}{a^3} - \frac{125\text{arctanh}(\sin(c+dx))}{128a^3d} - \frac{\tan(c+dx)}{a^3d} + \frac{115\sec(c+dx)\tan(c+dx)}{128a^3d} + \frac{5\sec^3(c+dx)\tan(c+dx)}{64a^3d} + \frac{\tan^3(c+dx)}{3a^3d} - \frac{5\sec(c+dx)\tan^3(c+dx)}{8a^3d} - \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48a^3d} - \frac{\tan^5(c+dx)}{5a^3d} + \frac{\sec(c+dx)\tan^5(c+dx)}{2a^3d} + \frac{\sec^3(c+dx)\tan^5(c+dx)}{8a^3d} - \frac{3\tan^7(c+dx)}{7a^3d}$$

Mathematica [A] (verified)

Time = 3.59 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.53

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(1680000\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\mathrm{si}\right.\right.\right.}{}$$

```
[In] Integrate[Tan[c + d*x]^12/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(1680000*(Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c + d*x
]^8*(470400*d*x*Cos[c] + 376320*d*x*Cos[c + 2*d*x] + 376320*d*x*Cos[3*c + 2
*d*x] + 188160*d*x*Cos[3*c + 4*d*x] + 188160*d*x*Cos[5*c + 4*d*x] + 53760*d
*x*Cos[5*c + 6*d*x] + 53760*d*x*Cos[7*c + 6*d*x] + 6720*d*x*Cos[7*c + 8*d*x
] + 6720*d*x*Cos[9*c + 8*d*x] + 519680*Sin[c] + 133175*Sin[d*x] + 133175*Si
n[2*c + d*x] - 544768*Sin[c + 2*d*x] + 286720*Sin[3*c + 2*d*x] + 63595*Sin[
2*c + 3*d*x] + 63595*Sin[4*c + 3*d*x] - 254464*Sin[3*c + 4*d*x] + 161280*Si
n[5*c + 4*d*x] + 65135*Sin[4*c + 5*d*x] + 65135*Sin[6*c + 5*d*x] - 118784*S
in[5*c + 6*d*x] + 27195*Sin[6*c + 7*d*x] + 27195*Sin[8*c + 7*d*x] - 14848*S
in[7*c + 8*d*x])))/(215040*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.02 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.96

method	result
risch	$\frac{x}{a^3}-\frac{i\left(27195\,\mathrm{e}^{15i(dx+c)}+65135\,\mathrm{e}^{13i(dx+c)}+161280\,\mathrm{e}^{12i(dx+c)}+63595\,\mathrm{e}^{11i(dx+c)}+286720\,\mathrm{e}^{10i(dx+c)}+133175\,\mathrm{e}^{9i(dx+c)}+\right.}{}$
derivativedivides	$\frac{\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^8}+\frac{13}{14\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^7}+\frac{65}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}+\frac{143}{40\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{79}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{49}{32\left(\tan\left(\frac{dx}{2}+\right.\right.}}{}$
default	$\frac{\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^8}+\frac{13}{14\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^7}+\frac{65}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}+\frac{143}{40\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{79}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{49}{32\left(\tan\left(\frac{dx}{2}+\right.\right.}}{}$

```
[In] int(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] x/a^3-1/6720*I*(27195*exp(15*I*(d*x+c))+65135*exp(13*I*(d*x+c))+161280*exp(
12*I*(d*x+c))+63595*exp(11*I*(d*x+c))+286720*exp(10*I*(d*x+c))+133175*exp(9
*I*(d*x+c))+519680*exp(8*I*(d*x+c))-133175*exp(7*I*(d*x+c))+544768*exp(6*I*
(d*x+c))-63595*exp(5*I*(d*x+c))+254464*exp(4*I*(d*x+c))-65135*exp(3*I*(d*x+
c))+118784*exp(2*I*(d*x+c))-27195*exp(I*(d*x+c))+14848)/d/a^3/(exp(2*I*(d*x
```

```
+c))+1)^8-125/128/a^3/d*ln(exp(I*(d*x+c))+I)+125/128/a^3/d*ln(exp(I*(d*x+c)
)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.62

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{26880\,dx\cos{(dx+c)}^8 - 13125\,\cos{(dx+c)}^8\log{(\sin{(dx+c)}+1)} + 13125\,\cos{(dx+c)}^8\log{(-\sin{(dx+}}{}$$

```
[In] integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/26880*(26880*d*x*cos(d*x + c)^8 - 13125*cos(d*x + c)^8*log(sin(d*x + c) +
 1) + 13125*cos(d*x + c)^8*log(-sin(d*x + c) + 1) - 2*(14848*cos(d*x + c)^7
 - 27195*cos(d*x + c)^6 + 7424*cos(d*x + c)^5 + 17710*cos(d*x + c)^4 - 1459
2*cos(d*x + c)^3 - 1960*cos(d*x + c)^2 + 5760*cos(d*x + c) - 1680)*sin(d*x
+ c))/(a^3*d*cos(d*x + c)^8)
```

Sympy [F]

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^{12}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(tan(d*x+c)**12/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**12/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.81

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{\frac{2\left(\frac{315\sin(dx+c)}{\cos(dx+c)+1} - \frac{11375\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{79723\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{269879\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{550089\sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{749973\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} + \frac{212625\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}} - \frac{26565\,\mathrm{s}}{(\cos(d}}{a^3 - \frac{8a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{28a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{56a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{70a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{56a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{28a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} - \frac{8a^3\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} + \frac{a^3\sin(d}{(\cos(dx+}}}{13440\,d}$$

```
[In] integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/13440*(2*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 11375*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 79723*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 269879*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7 + 550089*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - 749973*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 212625*sin(d*x + c)^13/(
cos(d*x + c) + 1)^13 - 26565*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^3 -
8*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 56*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8 - 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 28*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^3*sin(d*x + c)^14/(cos(
d*x + c) + 1)^14 + a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 26880*arcta
n(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + 13125*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/a^3 - 13125*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 25.91 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.74

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{13440\,(dx+c)}{a^3} - \frac{13125\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} + \frac{13125\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} + \frac{2\left(26565\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-212625\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+\right.}{}}{134}$$

```
[In] integrate(tan(d*x+c)^12/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/13440*(13440*(d*x + c)/a^3 - 13125*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3
 + 13125*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(26565*tan(1/2*d*x + 1/
2*c)^15 - 212625*tan(1/2*d*x + 1/2*c)^13 + 749973*tan(1/2*d*x + 1/2*c)^11 -
 550089*tan(1/2*d*x + 1/2*c)^9 + 269879*tan(1/2*d*x + 1/2*c)^7 - 79723*tan(
1/2*d*x + 1/2*c)^5 + 11375*tan(1/2*d*x + 1/2*c)^3 - 315*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 - 1)^8*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 15.70 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.12

$$\int \frac{\tan^{12}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3} - \frac{125\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,a^3\,d} - \frac{-\frac{253\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64} + \frac{2025\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64} - \frac{35713\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{320} + \frac{183363\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{2240} - \frac{269879\,\tan}{67}}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16} - 8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} + 28\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 56\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + 70\,a^3\tan\left(\frac{c}{2}\right.\right.}$$

```
[In] int(tan(c + d*x)^12/(a + a/cos(c + d*x))^3,x)

[Out] x/a^3 - (125*atanh(tan(c/2 + (d*x)/2)))/(64*a^3*d) - ((3*tan(c/2 + (d*x)/2)
)/64 - (325*tan(c/2 + (d*x)/2)^3)/192 + (11389*tan(c/2 + (d*x)/2)^5)/960 -
(269879*tan(c/2 + (d*x)/2)^7)/6720 + (183363*tan(c/2 + (d*x)/2)^9)/2240 - (
35713*tan(c/2 + (d*x)/2)^11)/320 + (2025*tan(c/2 + (d*x)/2)^13)/64 - (253*t
an(c/2 + (d*x)/2)^15)/64)/(d*(28*a^3*tan(c/2 + (d*x)/2)^4 - 8*a^3*tan(c/2 +
 (d*x)/2)^2 - 56*a^3*tan(c/2 + (d*x)/2)^6 + 70*a^3*tan(c/2 + (d*x)/2)^8 - 5
6*a^3*tan(c/2 + (d*x)/2)^10 + 28*a^3*tan(c/2 + (d*x)/2)^12 - 8*a^3*tan(c/2
+ (d*x)/2)^14 + a^3*tan(c/2 + (d*x)/2)^16 + a^3))
```

3.96 $\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 169

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{x}{a^3}+\frac{19\mathrm{arctanh}(\sin(c+dx))}{16a^3d}+\frac{\tan(c+dx)}{a^3d} - \frac{17\sec(c+dx)\tan(c+dx)}{16a^3d}-\frac{\sec^3(c+dx)\tan(c+dx)}{8a^3d} - \frac{\tan^3(c+dx)}{3a^3d}+\frac{3\sec(c+dx)\tan^3(c+dx)}{4a^3d} + \frac{\sec^3(c+dx)\tan^3(c+dx)}{6a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}$$

```
[Out] -x/a^3+19/16*arctanh(sin(d*x+c))/a^3/d+tan(d*x+c)/a^3/d-17/16*sec(d*x+c)*ta
n(d*x+c)/a^3/d-1/8*sec(d*x+c)^3*tan(d*x+c)/a^3/d-1/3*tan(d*x+c)^3/a^3/d+3/4
*sec(d*x+c)*tan(d*x+c)^3/a^3/d+1/6*sec(d*x+c)^3*tan(d*x+c)^3/a^3/d-3/5*tan(
d*x+c)^5/a^3/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3973, 3971, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{19\mathrm{arctanh}(\sin(c+dx))}{16a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}-\frac{\tan^3(c+dx)}{3a^3d} + \frac{\tan(c+dx)}{a^3d}+\frac{\tan^3(c+dx)\sec^3(c+dx)}{6a^3d} - \frac{\tan(c+dx)\sec^3(c+dx)}{8a^3d}+\frac{3\tan^3(c+dx)\sec(c+dx)}{4a^3d} - \frac{17\tan(c+dx)\sec(c+dx)}{16a^3d}-\frac{x}{a^3}$$

```
[In] Int[Tan[c + d*x]^10/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) + (19*ArcTanh[Sin[c + d*x]])/(16*a^3*d) + Tan[c + d*x]/(a^3*d) - (
17*Sec[c + d*x]*Tan[c + d*x])/(16*a^3*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(8
*a^3*d) - Tan[c + d*x]^3/(3*a^3*d) + (3*Sec[c + d*x]*Tan[c + d*x]^3)/(4*a^3
*d) + (Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*a^3*d) - (3*Tan[c + d*x]^5)/(5*a^3
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (-a+a\sec(c+dx))^3\tan^4(c+dx)\,dx}{a^6}\\
&= \frac{\int \left(-a^3\tan^4(c+dx)+3a^3\sec(c+dx)\tan^4(c+dx)-3a^3\sec^2(c+dx)\tan^4(c+dx)+a^3\sec^3(c+a\right.}{a^6}\\
&= -\frac{\int \tan^4(c+dx)\,dx}{a^3}+\frac{\int \sec^3(c+dx)\tan^4(c+dx)\,dx}{a^3}\\
&\quad+\frac{3\int \sec(c+dx)\tan^4(c+dx)\,dx}{a^3}-\frac{3\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a^3}\\
&= -\frac{\tan^3(c+dx)}{3a^3d}+\frac{3\sec(c+dx)\tan^3(c+dx)}{4a^3d}+\frac{\sec^3(c+dx)\tan^3(c+dx)}{6a^3d}\\
&\quad-\frac{\int \sec^3(c+dx)\tan^2(c+dx)\,dx}{2a^3}+\frac{\int \tan^2(c+dx)\,dx}{a^3}\\
&\quad-\frac{9\int \sec(c+dx)\tan^2(c+dx)\,dx}{4a^3}-\frac{3\text{Subst}\left(\int x^4\,dx,x,\tan(c+dx)\right)}{a^3d}\\
&= \frac{\tan(c+dx)}{a^3d}-\frac{9\sec(c+dx)\tan(c+dx)}{8a^3d}-\frac{\sec^3(c+dx)\tan(c+dx)}{8a^3d}\\
&\quad-\frac{\tan^3(c+dx)}{3a^3d}+\frac{3\sec(c+dx)\tan^3(c+dx)}{4a^3d}+\frac{\sec^3(c+dx)\tan^3(c+dx)}{6a^3d}\\
&\quad-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{\int \sec^3(c+dx)\,dx}{8a^3}-\frac{\int 1\,dx}{a^3}+\frac{9\int \sec(c+dx)\,dx}{8a^3}\\
&= -\frac{x}{a^3}+\frac{9\text{arctanh}(\sin(c+dx))}{8a^3d}+\frac{\tan(c+dx)}{a^3d}-\frac{17\sec(c+dx)\tan(c+dx)}{16a^3d}\\
&\quad-\frac{\sec^3(c+dx)\tan(c+dx)}{8a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+\frac{3\sec(c+dx)\tan^3(c+dx)}{4a^3d}\\
&\quad+\frac{\sec^3(c+dx)\tan^3(c+dx)}{6a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{\int \sec(c+dx)\,dx}{16a^3}
\end{aligned}$$

$$= -\frac{x}{a^3} + \frac{19\text{arctanh}(\sin(c+dx))}{16a^3d} + \frac{\tan(c+dx)}{a^3d} - \frac{17\sec(c+dx)\tan(c+dx)}{16a^3d} - \frac{\sec^3(c+dx)\tan(c+dx)}{8a^3d} - \frac{\tan^3(c+dx)}{3a^3d} + \frac{3\sec(c+dx)\tan^3(c+dx)}{4a^3d} + \frac{\sec^3(c+dx)\tan^3(c+dx)}{6a^3d} - \frac{3\tan^5(c+dx)}{5a^3d}$$

Mathematica [A] (verified)

Time = 3.63 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.79

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(9120\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\text{s}\right.\right.\right.}{}$$

```
[In] Integrate[Tan[c + d*x]^10/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/960*(Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(9120*(Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c]*Sec[c +
 d*x]^6*(2400*d*x*Cos[c] + 1800*d*x*Cos[c + 2*d*x] + 1800*d*x*Cos[3*c + 2*d
*x] + 720*d*x*Cos[3*c + 4*d*x] + 720*d*x*Cos[5*c + 4*d*x] + 120*d*x*Cos[5*c
 + 6*d*x] + 120*d*x*Cos[7*c + 6*d*x] + 1760*Sin[c] - 210*Sin[d*x] - 210*Sin
[2*c + d*x] - 1440*Sin[c + 2*d*x] + 1200*Sin[3*c + 2*d*x] + 865*Sin[2*c + 3
*d*x] + 865*Sin[4*c + 3*d*x] - 1296*Sin[3*c + 4*d*x] - 240*Sin[5*c + 4*d*x]
 + 435*Sin[4*c + 5*d*x] + 435*Sin[6*c + 5*d*x] - 176*Sin[5*c + 6*d*x])))/(a
^3*d*(1 + Sec[c + d*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{x}{a^3} + \frac{i\left(435\,e^{11i(dx+c)} - 240\,e^{10i(dx+c)} + 865\,e^{9i(dx+c)} + 1200\,e^{8i(dx+c)} - 210\,e^{7i(dx+c)} + 1760\,e^{6i(dx+c)} + 210\,e^{5i(dx+c)}\right.}{120d\,a^3\left(e^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6} + \frac{11}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5} + \frac{11}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4} + \frac{11}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3} - \frac{}{16(\tan}}{}$
default	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6} + \frac{11}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5} + \frac{11}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4} + \frac{11}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3} - \frac{}{16(\tan}}{}$

```
[In] int(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -x/a^3+1/120*I*(435*exp(11*I*(d*x+c))-240*exp(10*I*(d*x+c))+865*exp(9*I*(d*
x+c))+1200*exp(8*I*(d*x+c))-210*exp(7*I*(d*x+c))+1760*exp(6*I*(d*x+c))+210*
exp(5*I*(d*x+c))+1440*exp(4*I*(d*x+c))-865*exp(3*I*(d*x+c))+1296*exp(2*I*(d
*x+c))-435*exp(I*(d*x+c))+176)/d/a^3/(exp(2*I*(d*x+c))+1)^6+19/16/a^3/d*ln(
exp(I*(d*x+c))+I)-19/16/a^3/d*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.75

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{480\,dx\cos(dx+c)^6 - 285\,\cos(dx+c)^6\log(\sin(dx+c)+1) + 285\,\cos(dx+c)^6\log(-\sin(dx+c)+1}{4}$$

```
[In] integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/480*(480*d*x*cos(d*x + c)^6 - 285*cos(d*x + c)^6*log(sin(d*x + c) + 1) +
 285*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(176*cos(d*x + c)^5 - 435*co
s(d*x + c)^4 + 208*cos(d*x + c)^3 + 110*cos(d*x + c)^2 - 144*cos(d*x + c) +
 40)*sin(d*x + c))/(a^3*d*cos(d*x + c)^6)
```

Sympy [F]

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^{10}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(tan(d*x+c)**10/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**10/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(155) = 310.

Time = 0.30 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.03

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{2\left(\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{95\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{366\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1746\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3135\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{525\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a^3-\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{6a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}+\frac{480\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-}{240\,d}$$

```
[In] integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/240*(2*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 95*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 366*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1746*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 - 3135*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 525*sin(
d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^3 - 6*a^3*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a^3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^
3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12/(cos(d*x + c)
 + 1)^12) + 480*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 285*log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 285*log(sin(d*x + c)/(cos(d*x + c) +
1) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 12.48 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.88

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{240\,(dx+c)}{a^3}-\frac{285\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}+\frac{285\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{2\left(525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-3135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+1746\,\mathrm{ta}}{(\mathrm{t}}}{240\,d}$$

```
[In] integrate(tan(d*x+c)^10/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(240*(d*x + c)/a^3 - 285*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 28
5*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(525*tan(1/2*d*x + 1/2*c)^11 -
 3135*tan(1/2*d*x + 1/2*c)^9 + 1746*tan(1/2*d*x + 1/2*c)^7 - 366*tan(1/2*d*
x + 1/2*c)^5 - 95*tan(1/2*d*x + 1/2*c)^3 + 45*tan(1/2*d*x + 1/2*c))/((tan(1
/2*d*x + 1/2*c)^2 - 1)^6*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 15.04 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.23

$$\int \frac{\tan^{10}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{19\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^3\,d}-\frac{x}{a^3} - \frac{\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{209\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{8}+\frac{291\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}-\frac{61\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}-\frac{19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\,a^3\tan\left(\frac{c}{2}-\right.\right.}$$

```
[In] int(tan(c + d*x)^10/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (19*atanh(tan(c/2 + (d*x)/2)))/(8*a^3*d) - x/a^3 - ((3*tan(c/2 + (d*x)/2))/
8 - (19*tan(c/2 + (d*x)/2)^3)/24 - (61*tan(c/2 + (d*x)/2)^5)/20 + (291*tan(
c/2 + (d*x)/2)^7)/20 - (209*tan(c/2 + (d*x)/2)^9)/8 + (35*tan(c/2 + (d*x)/2
)^11)/8)/(d*(15*a^3*tan(c/2 + (d*x)/2)^4 - 6*a^3*tan(c/2 + (d*x)/2)^2 - 20*
a^3*tan(c/2 + (d*x)/2)^6 + 15*a^3*tan(c/2 + (d*x)/2)^8 - 6*a^3*tan(c/2 + (d
*x)/2)^10 + a^3*tan(c/2 + (d*x)/2)^12 + a^3))
```

3.97 $\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3} \, dx = \frac{x}{a^3} - \frac{13\text{arctanh}(\sin(c+dx))}{8a^3d} - \frac{\tan(c+dx)}{a^3d} + \frac{11\sec(c+dx)\tan(c+dx)}{8a^3d} + \frac{\sec^3(c+dx)\tan(c+dx)}{4a^3d} - \frac{\tan^3(c+dx)}{a^3d}$$

```
[Out] x/a^3-13/8*arctanh(sin(d*x+c))/a^3/d-tan(d*x+c)/a^3/d+11/8*sec(d*x+c)*tan(d
*x+c)/a^3/d+1/4*sec(d*x+c)^3*tan(d*x+c)/a^3/d-tan(d*x+c)^3/a^3/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3973, 3971, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3} \, dx = -\frac{13\text{arctanh}(\sin(c+dx))}{8a^3d} - \frac{\tan^3(c+dx)}{a^3d} - \frac{\tan(c+dx)}{a^3d} + \frac{\tan(c+dx)\sec^3(c+dx)}{4a^3d} + \frac{11\tan(c+dx)\sec(c+dx)}{8a^3d} + \frac{x}{a^3}$$

```
[In] Int[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 - (13*ArcTanh[Sin[c + d*x]])/(8*a^3*d) - Tan[c + d*x]/(a^3*d) + (11*S
ec[c + d*x]*Tan[c + d*x])/(8*a^3*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*a^3*
d) - Tan[c + d*x]^3/(a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (-a+a\sec(c+dx))^3\tan^2(c+dx)\,dx}{a^6}\\
&= \frac{\int \left(-a^3\tan^2(c+dx)+3a^3\sec(c+dx)\tan^2(c+dx)-3a^3\sec^2(c+dx)\tan^2(c+dx)+a^3\sec^3(c+\right.}{a^6}\\
&= -\frac{\int \tan^2(c+dx)\,dx}{a^3}+\frac{\int \sec^3(c+dx)\tan^2(c+dx)\,dx}{a^3}\\
&\quad+\frac{3\int \sec(c+dx)\tan^2(c+dx)\,dx}{a^3}-\frac{3\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a^3}\\
&= -\frac{\tan(c+dx)}{a^3d}+\frac{3\sec(c+dx)\tan(c+dx)}{2a^3d}\\
&\quad+\frac{\sec^3(c+dx)\tan(c+dx)}{4a^3d}-\frac{\int \sec^3(c+dx)\,dx}{4a^3}+\frac{\int 1\,dx}{a^3}\\
&\quad-\frac{3\int \sec(c+dx)\,dx}{2a^3}-\frac{3\text{Subst}\left(\int x^2\,dx,x,\tan(c+dx)\right)}{a^3d}\\
&= \frac{x}{a^3}-\frac{3\text{arctanh}(\sin(c+dx))}{2a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{11\sec(c+dx)\tan(c+dx)}{8a^3d}\\
&\quad+\frac{\sec^3(c+dx)\tan(c+dx)}{4a^3d}-\frac{\tan^3(c+dx)}{a^3d}-\frac{\int \sec(c+dx)\,dx}{8a^3}\\
&= \frac{x}{a^3}-\frac{13\text{arctanh}(\sin(c+dx))}{8a^3d}-\frac{\tan(c+dx)}{a^3d}\\
&\quad+\frac{11\sec(c+dx)\tan(c+dx)}{8a^3d}+\frac{\sec^3(c+dx)\tan(c+dx)}{4a^3d}-\frac{\tan^3(c+dx)}{a^3d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 230 vs. 2(99) = 198.

Time = 2.07 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.32

$$
\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx
= \frac{\sec^4(c+dx)\left(24dx+39\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\cos(2(c+dx))\left(8dx+13\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.}{}
$$

[In] Integrate[Tan[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]

[Out] (Sec[c + d*x]^4*(24*d*x + 39*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*C
os[2*(c + d*x)]*(8*d*x + 13*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 13*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[4*(c + d*x)]*(8*d*x + 13*Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 13*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) - 39*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 38*Sin[c + d*x] -
32*Sin[2*(c + d*x)] + 22*Sin[3*(c + d*x)]))/(64*a^3*d)

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.39

method	result
risch	$\frac{x}{a^3} - \frac{i\left(11\,\mathrm{e}^{7i(dx+c)}-16\,\mathrm{e}^{6i(dx+c)}+19\,\mathrm{e}^{5i(dx+c)}-19\,\mathrm{e}^{3i(dx+c)}+16\,\mathrm{e}^{2i(dx+c)}-11\,\mathrm{e}^{i(dx+c)}\right)}{4d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^4} - \frac{13\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8a^3d} + 13\,\mathrm{l}$
derivativedivides	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{27}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{21}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{13\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{a^3d}$
default	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{27}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{21}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{13\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{a^3d}$

[In] int(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] x/a^3-1/4*I/d/a^3/(exp(2*I*(d*x+c))+1)^4*(11*exp(7*I*(d*x+c))-16*exp(6*I*(d
*x+c))+19*exp(5*I*(d*x+c))-19*exp(3*I*(d*x+c))+16*exp(2*I*(d*x+c))-11*exp(I
*(d*x+c)))-13/8/a^3/d*ln(exp(I*(d*x+c))+I)+13/8/a^3/d*ln(exp(I*(d*x+c))-I)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{16\,dx\cos{(dx+c)}^4 - 13\,\cos{(dx+c)}^4\log{(\sin{(dx+c)}+1)} + 13\,\cos{(dx+c)}^4\log{(-\sin{(dx+c)}+1)} + 2\,(}{16\,a^3d\cos{(dx+c)}^4}$$

[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/16*(16*d*x*cos(d*x + c)^4 - 13*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 13*
cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(11*cos(d*x + c)^2 - 8*cos(d*x +
c) + 2)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4)

Sympy [F]

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^8(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(tan(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
[Out] Integral(tan(c + d*x)**8/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(93) = 186$.

Time = 0.30 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.60

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{2\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{13\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{21\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^3-\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{16\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{13\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{13\log\left(\frac{\sin}{\cos(}\right.}{}}{8d}$$

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] 1/8*(2*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 13*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 21*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7)/(a^3 - 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 16*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^3 - 13*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 13*l
og(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 5.00 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.24

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{8(dx+c)}{a^3}-\frac{13\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}+\frac{13\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{2\left(21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-13\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4a^3}}{8d}$$

```
[In] integrate(tan(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)/a^3 - 13*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 13*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(21*tan(1/2*d*x + 1/2*c)^7 + 3*tan(1/
2*d*x + 1/2*c)^5 - 13*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/2*c))/((ta
n(1/2*d*x + 1/2*c)^2 - 1)^4*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 14.98 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.49

$$\int \frac{\tan^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3} - \frac{13\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^3\,d} + \frac{\frac{21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}$$

```
[In] int(tan(c + d*x)^8/(a + a/cos(c + d*x))^3,x)
```

```
[Out] x/a^3 - (13*atanh(tan(c/2 + (d*x)/2)))/(4*a^3*d) + ((5*tan(c/2 + (d*x)/2))/
4 - (13*tan(c/2 + (d*x)/2)^3)/4 + (3*tan(c/2 + (d*x)/2)^5)/4 + (21*tan(c/2
+ (d*x)/2)^7)/4)/(d*(6*a^3*tan(c/2 + (d*x)/2)^4 - 4*a^3*tan(c/2 + (d*x)/2)^
2 - 4*a^3*tan(c/2 + (d*x)/2)^6 + a^3*tan(c/2 + (d*x)/2)^8 + a^3))
```

3.98 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3} dx = -\frac{x}{a^3} + \frac{7\text{arctanh}(\sin(c+dx))}{2a^3d} - \frac{5\tan(c+dx)}{2a^3d} - \frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d}$$

```
[Out] -x/a^3+7/2*arctanh(sin(d*x+c))/a^3/d-5/2*tan(d*x+c)/a^3/d-1/2*(1-sec(d*x+c)
)*tan(d*x+c)/a^3/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3973, 3860, 3999, 3852, 8, 3855}

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3} dx = \frac{7\text{arctanh}(\sin(c+dx))}{2a^3d} - \frac{5\tan(c+dx)}{2a^3d} - \frac{\tan(c+dx)(1-\sec(c+dx))}{2a^3d} - \frac{x}{a^3}$$

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) + (7*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (5*Tan[c + d*x])/(2*a^3*d)
 - ((1 - Sec[c + d*x])*Tan[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3999

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (-a+a\sec(c+dx))^3\,dx}{a^6}\\
&= -\frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d} - \frac{\int (-a+a\sec(c+dx))(-2a+5a\sec(c+dx))\,dx}{2a^5}\\
&= -\frac{x}{a^3} - \frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d} - \frac{5\int \sec^2(c+dx)\,dx}{2a^3} + \frac{7\int \sec(c+dx)\,dx}{2a^3}\\
&= -\frac{x}{a^3} + \frac{7\operatorname{arctanh}(\sin(c+dx))}{2a^3d} - \frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d}\\
&\quad + \frac{5\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{2a^3d}
\end{aligned}
$$

$$= -\frac{x}{a^3} + \frac{7\text{arctanh}(\sin(c+dx))}{2a^3d} - \frac{5\tan(c+dx)}{2a^3d} - \frac{(1-\sec(c+dx))\tan(c+dx)}{2a^3d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(66) = 132$.

Time = 1.73 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.65

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-4x - \frac{14\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{14\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{}{d(\text{co}}\right.}{}$$

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-4*x - (14*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]])/d + (14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + 1/(d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2) - (12*Sin[d*x])/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]
)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]))))/(a^3*(1 + Sec[c + d*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{x}{a^3} - \frac{i\left(e^{3i(dx+c)}+6\,e^{2i(dx+c)}-e^{i(dx+c)}+6\right)}{d\,a^3\left(e^{2i(dx+c)}+1\right)^2} - \frac{7\ln\left(e^{i(dx+c)}-i\right)}{2a^3d} + \frac{7\ln\left(e^{i(dx+c)}+i\right)}{2a^3d}$
derivativedivides	$\frac{\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{7}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2}}{a^3d}$
default	$\frac{\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{7}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2}}{a^3d}$

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -x/a^3-I*(exp(3*I*(d*x+c))+6*exp(2*I*(d*x+c))-exp(I*(d*x+c))+6)/d/a^3/(exp(
2*I*(d*x+c))+1)^2-7/2/a^3/d*ln(exp(I*(d*x+c))-I)+7/2/a^3/d*ln(exp(I*(d*x+c)
)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{4\,dx\cos\left(dx+c\right)^2 - 7\,\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) + 7\,\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\,(6\cdots}{4\,a^3d\cos\left(dx+c\right)^2}$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
[Out] -1/4*(4*d*x*cos(d*x + c)^2 - 7*cos(d*x + c)^2*log(sin(d*x + c) + 1) + 7*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*cos(d*x + c) - 1)*sin(d*x + c))/(
a^3*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
[Out] Integral(tan(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(58) = 116.

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.59

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{2\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{2\,d}$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2*(2*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4) + 4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 -
 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 7*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 2.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.47

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{2\,(dx+c)}{a^3} - \frac{7\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} + \frac{7\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} - \frac{2\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^3}}{2\,d}$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)/a^3 - 7*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + 7*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 2*(7*tan(1/2*d*x + 1/2*c)^3 - 5*tan(1/2*
d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d} - \frac{x}{a^3} - \frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}$$

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - x/a^3 - (5*tan(c/2 + (d*x)/2) - 7*t
an(c/2 + (d*x)/2)^3)/(d*(a^3*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2
)^2 + a^3))
```

3.99 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3}+\frac{\operatorname{arctanh}(\sin(c+dx))}{a^3d}-\frac{4\tan(c+dx)}{a^2d(a+a\sec(c+dx))}$$

```
[Out] x/a^3+arctanh(sin(d*x+c))/a^3/d-4*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3973, 3971, 3554, 8, 2686, 3852, 2701, 327, 213}

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^3d}+\frac{4\cot(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}+\frac{x}{a^3}$$

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 + ArcTanh[Sin[c + d*x]]/(a^3*d) + (4*Cot[c + d*x])/(a^3*d) - (4*Csc[c
 + d*x])/(a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \cot^2(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^2(c+dx)+3a^3\cot(c+dx)\csc(c+dx)-3a^3\csc^2(c+dx)+a^3\csc^2(c+dx)\sec(c+dx)\right)}{a^6}$$
$$= -\frac{\int \cot^2(c+dx)\,dx}{a^3}+\frac{\int \csc^2(c+dx)\sec(c+dx)\,dx}{a^3} + \frac{3\int \cot(c+dx)\csc(c+dx)\,dx}{a^3}-\frac{3\int \csc^2(c+dx)\,dx}{a^3}$$
$$= \frac{\cot(c+dx)}{a^3d}+\frac{\int 1\,dx}{a^3}-\frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{a^3d} + \frac{3\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{a^3d}-\frac{3\text{Subst}(\int 1\,dx,x,\csc(c+dx))}{a^3d}$$
$$= \frac{x}{a^3}+\frac{4\cot(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{a^3d}$$
$$= \frac{x}{a^3}+\frac{\text{arctanh}(\sin(c+dx))}{a^3d}+\frac{4\cot(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 117 vs. 2(46) = 92.

Time = 0.85 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.54

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{8\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(dx-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}\right.\right.\right.\right.}{a^3d(1+\sec(c+dx))^3}$$

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (8*Cos[(c + d*x)/2]^5*Sec[c + d*x]^3*(Cos[(c + d*x)/2]*(d*x - Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 4
*Sec[c/2]*Sin[(d*x)/2]))/(a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{3}d}$	61
default	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^{3}d}$	61
risch	$\frac{x}{a^{3}}-\frac{8i}{a^{3}d\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^{3}d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^{3}d}$	70

[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 4/d/a^3*(-tan(1/2*d*x+1/2*c)+1/4*ln(tan(1/2*d*x+1/2*c)+1)-1/4*ln(tan(1/2*d*x+1/2*c)-1)+1/2*arctan(tan(1/2*d*x+1/2*c)))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$=\frac{2\,dx\cos\left(dx+c\right)+2\,dx+\left(\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(\cos\left(dx+c\right)+1\right)\log\left(-\sin\left(dx+c\right)\right.}{2\left(a^3d\cos\left(dx+c\right)+a^3d\right)}$$

[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/2*(2*d*x*cos(d*x + c) + 2*d*x + (cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - (cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 8*sin(d*x + c))/(a^3*d*cos(d*x + c) + a^3*d)

Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx=\frac{\int \frac{\tan^4{(c+dx)}}{\sec^3{(c+dx)}+3\sec^2{(c+dx)}+3\sec{(c+dx)}+1}\,dx}{a^3}$$

[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**3,x)

[Out] Integral(tan(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)/a**3

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(46) = 92$.

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.13

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} + \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3} - \frac{4\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{d}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^3 - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3 - 4*sin(d
*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 1.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{dx+c}{a^3} + \frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3} - \frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3} - \frac{4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}}{d}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^3 + log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^3 - 4*tan(1/2*d*x + 1/2*c)/a^3)/d
```

Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3} + \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right) - 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3\,d}$$

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^3,x)
```

```
[Out] x/a^3 + (2*atanh(tan(c/2 + (d*x)/2)) - 4*tan(c/2 + (d*x)/2))/(a^3*d)
```

3.100 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx=-\frac{x}{a^3}+\frac{2\tan(c+dx)}{a^2d(a+a\sec(c+dx))}-\frac{\tan^3(c+dx)}{3d(a+a\sec(c+dx))^3}$$

```
[Out] -x/a^3+2*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))-1/3*tan(d*x+c)^3/d/(a+a*sec(d*x+
c))^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3973, 3971, 3554, 8, 2686, 2687, 30}

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx=\frac{4\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{x}{a^3}$$

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + (4*Cot[c + d*x]^3)/(3*a^3*d) + (3*Csc[c +
 d*x])/(a^3*d) - (4*Csc[c + d*x]^3)/(3*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \cot^4(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(-a^3\cot^4(c+dx)+3a^3\cot^3(c+dx)\csc(c+dx)-3a^3\cot^2(c+dx)\csc^2(c+dx)+a^3\cot(c+dx\right.}{a^6}$$

$$= -\frac{\int \cot^4(c+dx)\,dx}{a^3}+\frac{\int \cot(c+dx)\csc^3(c+dx)\,dx}{a^3} + \frac{3\int \cot^3(c+dx)\csc(c+dx)\,dx}{a^3}-\frac{3\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a^3}$$

$$= \frac{\cot^3(c+dx)}{3a^3d} + \frac{\int \cot^2(c+dx)\,dx}{a^3} - \frac{\text{Subst}\left(\int x^2\,dx, x, \csc(c+dx)\right)}{a^3d} - \frac{3\text{Subst}\left(\int x^2\,dx, x, -\cot(c+dx)\right)}{a^3d} - \frac{3\text{Subst}\left(\int (-1+x^2)\,dx, x, \csc(c+dx)\right)}{a^3d}$$

$$= -\frac{\cot(c+dx)}{a^3d} + \frac{4\cot^3(c+dx)}{3a^3d} + \frac{3\csc(c+dx)}{a^3d} - \frac{4\csc^3(c+dx)}{3a^3d} - \frac{\int 1\,dx}{a^3}$$

$$= -\frac{x}{a^3} - \frac{\cot(c+dx)}{a^3d} + \frac{4\cot^3(c+dx)}{3a^3d} + \frac{3\csc(c+dx)}{a^3d} - \frac{4\csc^3(c+dx)}{3a^3d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 125 vs. $2(60) = 120$.

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.08

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(180dx\cos\left(\frac{dx}{2}\right) + 180dx\cos\left(c+\frac{dx}{2}\right) + 60dx\cos\left(c+\frac{3dx}{2}\right) + 60dx\cos\left(2c+\frac{3dx}{2}\right)\right.}{480a^3d}$$

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/480*(Sec[c/2]*Sec[(c + d*x)/2]^3*(180*d*x*Cos[(d*x)/2] + 180*d*x*Cos[c +
 (d*x)/2] + 60*d*x*Cos[c + (3*d*x)/2] + 60*d*x*Cos[2*c + (3*d*x)/2] - 471*S
in[(d*x)/2] + 351*Sin[c + (d*x)/2] - 277*Sin[c + (3*d*x)/2] - 3*Sin[2*c + (
3*d*x)/2]))/(a^3*d)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$	45
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$	45
risch	$-\frac{x}{a^3}+\frac{2i\left(9\,e^{2i(dx+c)}+12\,e^{i(dx+c)}+7\right)}{3d\,a^3\left(e^{i(dx+c)}+1\right)^3}$	54

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/3*tan(1/2*d*x+1/2*c)^3+2*tan(1/2*d*x+1/2*c)-2*arctan(tan(1/2*d*
x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{3\,dx\cos(dx+c)^2+6\,dx\cos(dx+c)+3\,dx-(7\cos(dx+c)+5)\sin(dx+c)}{3\left(a^3d\cos(dx+c)^2+2\,a^3d\cos(dx+c)+a^3d\right)}$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (7*cos(d*x + c) +
 5)*sin(d*x + c))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\tan^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{\frac{6\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{6\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3\,d}$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*((6*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/a^3 - 6*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{3\,(dx+c)}{a^3} + \frac{a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 6\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^9}}{3\,d}$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^3 + (a^6*tan(1/2*d*x + 1/2*c)^3 - 6*a^6*tan(1/2*d*x + 1
/2*c))/a^9)/d
```

Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - 6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + 3\,d\,x}{3\,a^3\,d}$$

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(tan(c/2 + (d*x)/2)^3 - 6*tan(c/2 + (d*x)/2) + 3*d*x)/(3*a^3*d)
```

3.101 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\begin{aligned}\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = & -\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d} \\ & +\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{10\csc^3(c+dx)}{3a^3d} \\ & +\frac{11\csc^5(c+dx)}{5a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}\end{aligned}$$

```
[Out] -x/a^3-cot(d*x+c)/a^3/d+1/3*cot(d*x+c)^3/a^3/d-1/5*cot(d*x+c)^5/a^3/d+4/7*c
ot(d*x+c)^7/a^3/d+3*csc(d*x+c)/a^3/d-10/3*csc(d*x+c)^3/a^3/d+11/5*csc(d*x+c
)^5/a^3/d-4/7*csc(d*x+c)^7/a^3/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3973, 3971, 3554, 8, 2686, 200, 2687, 30, 276}

$$\begin{aligned}\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = & \frac{4\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^3(c+dx)}{3a^3d} \\ & -\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}+\frac{11\csc^5(c+dx)}{5a^3d} \\ & -\frac{10\csc^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{x}{a^3}\end{aligned}$$

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + Cot[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]^5
/(5*a^3*d) + (4*Cot[c + d*x]^7)/(7*a^3*d) + (3*Csc[c + d*x])/(a^3*d) - (10*
Csc[c + d*x]^3)/(3*a^3*d) + (11*Csc[c + d*x]^5)/(5*a^3*d) - (4*Csc[c + d*x]
^7)/(7*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \cot^8(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6} \\
&= \frac{\int \left(-a^3\cot^8(c+dx)+3a^3\cot^7(c+dx)\csc(c+dx)-3a^3\cot^6(c+dx)\csc^2(c+dx)+a^3\cot^5(c+d\right.}{a^6} \\
&= -\frac{\int \cot^8(c+dx)\,dx}{a^3}+\frac{\int \cot^5(c+dx)\csc^3(c+dx)\,dx}{a^3} \\
&\quad +\frac{3\int \cot^7(c+dx)\csc(c+dx)\,dx}{a^3}-\frac{3\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a^3} \\
&= \frac{\cot^7(c+dx)}{7a^3d}+\frac{\int \cot^6(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\text{Subst}\left(\int x^6\,dx,x,-\cot(c+dx)\right)}{a^3d}-\frac{3\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&= -\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\int \cot^4(c+dx)\,dx}{a^3} \\
&\quad -\frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&\quad -\frac{3\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&= \frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\csc(c+dx)}{a^3d} \\
&\quad -\frac{10\csc^3(c+dx)}{3a^3d}+\frac{11\csc^5(c+dx)}{5a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}+\frac{\int \cot^2(c+dx)\,dx}{a^3} \\
&= -\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\csc(c+dx)}{a^3d} \\
&\quad -\frac{10\csc^3(c+dx)}{3a^3d}+\frac{11\csc^5(c+dx)}{5a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}-\frac{\int 1\,dx}{a^3}
\end{aligned}
$$

$$= -\frac{x}{a^3} - \frac{\cot(c+dx)}{a^3d} + \frac{\cot^3(c+dx)}{3a^3d} - \frac{\cot^5(c+dx)}{5a^3d} + \frac{4\cot^7(c+dx)}{7a^3d} + \frac{3\csc(c+dx)}{a^3d} - \frac{10\csc^3(c+dx)}{3a^3d} + \frac{11\csc^5(c+dx)}{5a^3d} - \frac{4\csc^7(c+dx)}{7a^3d}$$

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.76

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\csc\left(\frac{c}{2}\right)\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)(-5880dx\cos(dx)+5880dx\cos(2c+dx)-5880dx\cos(c+$$

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c/2]*Csc[(c + d*x)/2]*Sec[c/2]*Sec[(c + d*x)/2]^7*(-5880*d*x*Cos[d*x]
+ 5880*d*x*Cos[2*c + d*x] - 5880*d*x*Cos[c + 2*d*x] + 5880*d*x*Cos[3*c + 2*
d*x] - 2520*d*x*Cos[2*c + 3*d*x] + 2520*d*x*Cos[4*c + 3*d*x] - 420*d*x*Cos[
3*c + 4*d*x] + 420*d*x*Cos[5*c + 4*d*x] + 4200*Sin[c] + 11032*Sin[d*x] - 23
282*Sin[c + d*x] - 23282*Sin[2*(c + d*x)] - 9978*Sin[3*(c + d*x)] - 1663*Si
n[4*(c + d*x)] + 13720*Sin[2*c + d*x] + 15512*Sin[c + 2*d*x] + 9240*Sin[3*c
 + 2*d*x] + 8088*Sin[2*c + 3*d*x] + 2520*Sin[4*c + 3*d*x] + 1768*Sin[3*c +
4*d*x]))/(215040*a^3*d)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d\,a^3}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d\,a^3}$
risch	$-\frac{x}{a^3}+\frac{2i\left(315\,e^{7i(dx+c)}+1155\,e^{6i(dx+c)}+1715\,e^{5i(dx+c)}+525\,e^{4i(dx+c)}-1379\,e^{3i(dx+c)}-1939\,e^{2i(dx+c)}-1011\,e^{i(dx+}\right.}{105d\,a^3\left(e^{i(dx+c)}+1\right)^7\left(e^{i(dx+c)}-1\right)}$

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^3*(-1/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d*x+1/2*c)^5-16/3*tan(1/2
*d*x+1/2*c)^3+26*tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-32*arctan(tan(1/2*
d*x+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{221\cos(dx+c)^4+348\cos(dx+c)^3-25\cos(dx+c)^2+105\left(dx\cos(dx+c)^3+3\,dx\cos(dx+c)^2+3\right.}{105\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)\right.}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/105*(221*cos(d*x + c)^4 + 348*cos(d*x + c)^3 - 25*cos(d*x + c)^2 + 105*(
d*x*cos(d*x + c)^3 + 3*d*x*cos(d*x + c)^2 + 3*d*x*cos(d*x + c) + d*x)*sin(d
*x + c) - 303*cos(d*x + c) - 136)/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cot^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.93

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{\frac{2730\sin(dx+c)}{\cos(dx+c)+1}-\frac{560\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{126\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3}-\frac{3360\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{105(\cos(dx+c)+1)}{a^3\sin(dx+c)}}{1680\,d}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/1680*((2730*sin(d*x + c)/(cos(d*x + c) + 1) - 560*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 126*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/a^3 - 3360*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^
3 - 105*(cos(d*x + c) + 1)/(a^3*sin(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.69

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{1680\,(dx+c)}{a^3}+\frac{105}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{15\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-126\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+560\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2730\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{21}}}{1680\,d}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/1680*(1680*(d*x + c)/a^3 + 105/(a^3*tan(1/2*d*x + 1/2*c)) + (15*a^18*tan
(1/2*d*x + 1/2*c)^7 - 126*a^18*tan(1/2*d*x + 1/2*c)^5 + 560*a^18*tan(1/2*d*
x + 1/2*c)^3 - 2730*a^18*tan(1/2*d*x + 1/2*c))/a^21)/d
```

Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.64

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{x}{a^3}-\frac{\frac{221\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{105}-\frac{268\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{105}+\frac{257\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{420}-\frac{31\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{280}+\frac{1}{112}}{a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

```
[Out] - x/a^3 - ((257*cos(c/2 + (d*x)/2)^4)/420 - (31*cos(c/2 + (d*x)/2)^2)/280 -
 (268*cos(c/2 + (d*x)/2)^6)/105 + (221*cos(c/2 + (d*x)/2)^8)/105 + 1/112)/(
a^3*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2))
```

3.102 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3}+\frac{\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc(c+dx)}{a^3d}+\frac{13\csc^3(c+dx)}{3a^3d}-\frac{21\csc^5(c+dx)}{5a^3d}+\frac{15\csc^7(c+dx)}{7a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}$$

```
[Out] x/a^3+cot(d*x+c)/a^3/d-1/3*cot(d*x+c)^3/a^3/d+1/5*cot(d*x+c)^5/a^3/d-1/7*co
t(d*x+c)^7/a^3/d+4/9*cot(d*x+c)^9/a^3/d-3*csc(d*x+c)/a^3/d+13/3*csc(d*x+c)^
3/a^3/d-21/5*csc(d*x+c)^5/a^3/d+15/7*csc(d*x+c)^7/a^3/d-4/9*csc(d*x+c)^9/a^
3/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3973, 3971, 3554, 8, 2686, 200, 2687, 30, 276}

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{4\cot^9(c+dx)}{9a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot(c+dx)}{a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}+\frac{15\csc^7(c+dx)}{7a^3d}-\frac{21\csc^5(c+dx)}{5a^3d}+\frac{13\csc^3(c+dx)}{3a^3d}-\frac{3\csc(c+dx)}{a^3d}+\frac{x}{a^3}$$

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] x/a^3 + Cot[c + d*x]/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + Cot[c + d*x]^5/(5
*a^3*d) - Cot[c + d*x]^7/(7*a^3*d) + (4*Cot[c + d*x]^9)/(9*a^3*d) - (3*Csc[
c + d*x])/(a^3*d) + (13*Csc[c + d*x]^3)/(3*a^3*d) - (21*Csc[c + d*x]^5)/(5*
a^3*d) + (15*Csc[c + d*x]^7)/(7*a^3*d) - (4*Csc[c + d*x]^9)/(9*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \cot^{10}(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(-a^3\cot^{10}(c+dx)+3a^3\cot^9(c+dx)\csc(c+dx)-3a^3\cot^8(c+dx)\csc^2(c+dx)+a^3\cot^7(c+d\right.}{a^6}$$

$$= -\frac{\int \cot^{10}(c+dx)\,dx}{a^3}+\frac{\int \cot^7(c+dx)\csc^3(c+dx)\,dx}{a^3} + \frac{3\int \cot^9(c+dx)\csc(c+dx)\,dx}{a^3}-\frac{3\int \cot^8(c+dx)\csc^2(c+dx)\,dx}{a^3}$$

$$= \frac{\cot^9(c+dx)}{9a^3d}+\frac{\int \cot^8(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx,x,\csc(c+dx)\right)}{a^3d} - \frac{3\text{Subst}\left(\int x^8\,dx,x,-\cot(c+dx)\right)}{a^3d}-\frac{3\text{Subst}\left(\int \left(-1+x^2\right)^4\,dx,x,\csc(c+dx)\right)}{a^3d}$$

$$= -\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{\int \cot^6(c+dx)\,dx}{a^3} - \frac{\text{Subst}\left(\int \left(-x^2+3x^4-3x^6+x^8\right)\,dx,x,\csc(c+dx)\right)}{a^3d} - \frac{3\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\,dx,x,\csc(c+dx)\right)}{a^3d}$$

$$= \frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc(c+dx)}{a^3d}+\frac{13\csc^3(c+dx)}{3a^3d} - \frac{21\csc^5(c+dx)}{5a^3d}+\frac{15\csc^7(c+dx)}{7a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}+\frac{\int \cot^4(c+dx)\,dx}{a^3}$$

$$= -\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^7(c+dx)}{7a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc(c+dx)}{a^3d} + \frac{13\csc^3(c+dx)}{3a^3d}-\frac{21\csc^5(c+dx)}{5a^3d}+\frac{15\csc^7(c+dx)}{7a^3d}-\frac{4\csc^9(c+dx)}{9a^3d} - \frac{\int \cot^2(c+dx)\,dx}{a^3}$$

$$= \frac{\cot(c+dx)}{a^3d} - \frac{\cot^3(c+dx)}{3a^3d} + \frac{\cot^5(c+dx)}{5a^3d} - \frac{\cot^7(c+dx)}{7a^3d} + \frac{4\cot^9(c+dx)}{9a^3d} - \frac{3\csc(c+dx)}{a^3d} + \frac{13\csc^3(c+dx)}{3a^3d} - \frac{21\csc^5(c+dx)}{5a^3d} + \frac{15\csc^7(c+dx)}{7a^3d} - \frac{4\csc^9(c+dx)}{9a^3d} + \frac{\int 1\,dx}{a^3}$$

$$= \frac{x}{a^3} + \frac{\cot(c+dx)}{a^3d} - \frac{\cot^3(c+dx)}{3a^3d} + \frac{\cot^5(c+dx)}{5a^3d} - \frac{\cot^7(c+dx)}{7a^3d} + \frac{4\cot^9(c+dx)}{9a^3d} - \frac{3\csc(c+dx)}{a^3d} + \frac{13\csc^3(c+dx)}{3a^3d} - \frac{21\csc^5(c+dx)}{5a^3d} + \frac{15\csc^7(c+dx)}{7a^3d} - \frac{4\csc^9(c+dx)}{9a^3d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 366 vs. $2(177) = 354$.

Time = 1.89 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.07

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\csc\left(\frac{c}{2}\right)\csc^3(2(c+dx))\sec\left(\frac{c}{2}\right)(181440dx\cos(dx) - 181440dx\cos(2c+dx) + 136080dx\cos(c+2dx) - 13}{}$$

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c/2]*Csc[2*(c + d*x)]^3*Sec[c/2]*(181440*d*x*Cos[d*x] - 181440*d*x*Cos
[2*c + d*x] + 136080*d*x*Cos[c + 2*d*x] - 136080*d*x*Cos[3*c + 2*d*x] - 100
80*d*x*Cos[2*c + 3*d*x] + 10080*d*x*Cos[4*c + 3*d*x] - 60480*d*x*Cos[3*c +
4*d*x] + 60480*d*x*Cos[5*c + 4*d*x] - 30240*d*x*Cos[4*c + 5*d*x] + 30240*d*
x*Cos[6*c + 5*d*x] - 5040*d*x*Cos[5*c + 6*d*x] + 5040*d*x*Cos[7*c + 6*d*x]
- 169344*Sin[c] - 338112*Sin[d*x] + 675036*Sin[c + d*x] + 506277*Sin[2*(c +
 d*x)] - 37502*Sin[3*(c + d*x)] - 225012*Sin[4*(c + d*x)] - 112506*Sin[5*(c
 + d*x)] - 18751*Sin[6*(c + d*x)] - 431424*Sin[2*c + d*x] - 375552*Sin[c +
2*d*x] - 201600*Sin[3*c + 2*d*x] - 41248*Sin[2*c + 3*d*x] + 84000*Sin[4*c +
 3*d*x] + 155712*Sin[3*c + 4*d*x] + 100800*Sin[5*c + 4*d*x] + 98016*Sin[4*c
 + 5*d*x] + 30240*Sin[6*c + 5*d*x] + 21376*Sin[5*c + 6*d*x]))/(80640*a^3*d*
(1 + Sec[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-99\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+128\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}\right.}}{64d\,a^3}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-99\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+128\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}\right.}}{64d\,a^3}$
risch	$\frac{x}{a^3}-\frac{2i\left(945\,e^{11i(dx+c)}+3150\,e^{10i(dx+c)}+2625\,e^{9i(dx+c)}-6300\,e^{8i(dx+c)}-13482\,e^{7i(dx+c)}-5292\,e^{6i(dx+c)}+10566\,e^{5i(dx}\right.}{315d\,a^3\left(e^{i(dx+c)}+1\right)^9\left(e^{i(dx+c)}-1\right)^3}$

[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/64/d/a^3*(-1/9*tan(1/2*d*x+1/2*c)^9+8/7*tan(1/2*d*x+1/2*c)^7-29/5*tan(1/2*d*x+1/2*c)^5+64/3*tan(1/2*d*x+1/2*c)^3-99*tan(1/2*d*x+1/2*c)+128*arctan(tan(1/2*d*x+1/2*c))-1/3/tan(1/2*d*x+1/2*c)^3+8/tan(1/2*d*x+1/2*c))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.22

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{668\,\cos(dx+c)^6+1059\,\cos(dx+c)^5-573\,\cos(dx+c)^4-1813\,\cos(dx+c)^3-393\,\cos(dx+c)^2+315}{315\left(a^3d\cos(dx+c)^5+3\,a^3d\cos(dx+c)^4+\right.}$$

[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/315*(668*cos(d*x + c)^6 + 1059*cos(d*x + c)^5 - 573*cos(d*x + c)^4 - 1813*cos(d*x + c)^3 - 393*cos(d*x + c)^2 + 315*(d*x*cos(d*x + c)^5 + 3*d*x*cos(d*x + c)^4 + 2*d*x*cos(d*x + c)^3 - 2*d*x*cos(d*x + c)^2 - 3*d*x*cos(d*x + c) - d*x)*sin(d*x + c) + 789*cos(d*x + c) + 368)/((a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))

Sympy [F]

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cot^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**3,x)

[Out] Integral(cot(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)/a**3

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{\frac{\frac{31185\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{6720\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1827\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{360\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^3}-\frac{40320\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{105\left(\frac{24\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right.}{a^3\sin}}{20160\,d}$$

[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] -1/20160*((31185*sin(d*x + c)/(cos(d*x + c) + 1) - 6720*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1827*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 360*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^3 - 40320*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 105*(24*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a^3*sin(d*x + c)^3))/d

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.74

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$=\frac{\frac{20160\,(dx+c)}{a^3}+\frac{105\left(24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-\frac{35\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-360\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1827\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6720\,a^{24}\tan\left(\frac{1}{2}\right.}{a^{27}}}{20160\,d}$$

[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] 1/20160*(20160*(d*x + c)/a^3 + 105*(24*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*tan(1/2*d*x + 1/2*c)^3) - (35*a^24*tan(1/2*d*x + 1/2*c)^9 - 360*a^24*tan(1/2*d*x + 1/2*c)^7 + 1827*a^24*tan(1/2*d*x + 1/2*c)^5 - 6720*a^24*tan(1/2*d*x + 1/2*c)^3 + 31185*a^24*tan(1/2*d*x + 1/2*c))/a^27)/d

Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,(c+d\,x)-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,(c+d\,x)}{a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\right)} - \frac{\frac{668\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{315}-\frac{983\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{210}+\frac{346\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{105}-\frac{2291\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{2520}+\frac{173\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{840}-\frac{19\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{672}+\frac{1}{576}}{a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (cos(c/2 + (d*x)/2)^9*(c + d*x) - cos(c/2 + (d*x)/2)^11*(c + d*x))/(a^3*d*(
cos(c/2 + (d*x)/2)^9 - cos(c/2 + (d*x)/2)^11)) - ((173*cos(c/2 + (d*x)/2)^4
)/840 - (19*cos(c/2 + (d*x)/2)^2)/672 - (2291*cos(c/2 + (d*x)/2)^6)/2520 +
(346*cos(c/2 + (d*x)/2)^8)/105 - (983*cos(c/2 + (d*x)/2)^10)/210 + (668*cos
(c/2 + (d*x)/2)^12)/315 + 1/576)/(a^3*d*(cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*
x)/2) - cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)))
```

3.103 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 215

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx=&-\frac{x}{a^3}-\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}\\&+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}\\&+\frac{3\csc(c+dx)}{a^3d}-\frac{16\csc^3(c+dx)}{3a^3d}+\frac{34\csc^5(c+dx)}{5a^3d}\\&-\frac{36\csc^7(c+dx)}{7a^3d}+\frac{19\csc^9(c+dx)}{9a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}\end{aligned}$$

```
[Out] -x/a^3-cot(d*x+c)/a^3/d+1/3*cot(d*x+c)^3/a^3/d-1/5*cot(d*x+c)^5/a^3/d+1/7*c
ot(d*x+c)^7/a^3/d-1/9*cot(d*x+c)^9/a^3/d+4/11*cot(d*x+c)^11/a^3/d+3*csc(d*x
+c)/a^3/d-16/3*csc(d*x+c)^3/a^3/d+34/5*csc(d*x+c)^5/a^3/d-36/7*csc(d*x+c)^7
/a^3/d+19/9*csc(d*x+c)^9/a^3/d-4/11*csc(d*x+c)^11/a^3/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3973, 3971, 3554, 8, 2686, 200, 2687, 30, 276}

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx=&\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{\cot^7(c+dx)}{7a^3d}\\&-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}\\&-\frac{4\csc^{11}(c+dx)}{11a^3d}+\frac{19\csc^9(c+dx)}{9a^3d}-\frac{36\csc^7(c+dx)}{7a^3d}\\&+\frac{34\csc^5(c+dx)}{5a^3d}-\frac{16\csc^3(c+dx)}{3a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{x}{a^3}\end{aligned}$$

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - Cot[c + d*x]/(a^3*d) + Cot[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]^5
/(5*a^3*d) + Cot[c + d*x]^7/(7*a^3*d) - Cot[c + d*x]^9/(9*a^3*d) + (4*Cot[c
 + d*x]^11)/(11*a^3*d) + (3*Csc[c + d*x])/(a^3*d) - (16*Csc[c + d*x]^3)/(3*
a^3*d) + (34*Csc[c + d*x]^5)/(5*a^3*d) - (36*Csc[c + d*x]^7)/(7*a^3*d) + (1
9*Csc[c + d*x]^9)/(9*a^3*d) - (4*Csc[c + d*x]^11)/(11*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \cot^{12}(c+dx)(-a+a\sec(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(-a^3\cot^{12}(c+dx)+3a^3\cot^{11}(c+dx)\csc(c+dx)-3a^3\cot^{10}(c+dx)\csc^2(c+dx)+a^3\cot^9(c\right.}{a^6}$$

$$= -\frac{\int \cot^{12}(c+dx)\,dx}{a^3}+\frac{\int \cot^9(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^{11}(c+dx)\csc(c+dx)\,dx}{a^3}-\frac{3\int \cot^{10}(c+dx)\csc^2(c+dx)\,dx}{a^3}$$

$$= \frac{\cot^{11}(c+dx)}{11a^3d}+\frac{\int \cot^{10}(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int x^2(-1+x^2)^4\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\text{Subst}\left(\int x^{10}\,dx,x,-\cot(c+dx)\right)}{a^3d}-\frac{3\text{Subst}\left(\int (-1+x^2)^5\,dx,x,\csc(c+dx)\right)}{a^3d}$$

$$= -\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{\int \cot^8(c+dx)\,dx}{a^3}-\frac{\text{Subst}\left(\int (x^2-4x^4+6x^6-4x^8+x^{10})\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\text{Subst}\left(\int (-1+5x^2-10x^4+10x^6-5x^8+x^{10})\,dx,x,\csc(c+dx)\right)}{a^3d}$$

$$= \frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{16\csc^3(c+dx)}{3a^3d}+\frac{34\csc^5(c+dx)}{5a^3d}-\frac{36\csc^7(c+dx)}{7a^3d}+\frac{19\csc^9(c+dx)}{9a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}+\frac{\int \cot^6(c+dx)\,dx}{a^3}$$

$$= -\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{3\csc(c+dx)}{a^3d}-\frac{16\csc^3(c+dx)}{3a^3d}+\frac{34\csc^5(c+dx)}{5a^3d}-\frac{36\csc^7(c+dx)}{7a^3d}+\frac{19\csc^9(c+dx)}{9a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}-\frac{\int \cot^4(c+dx)\,dx}{a^3}$$

$$
\begin{aligned}
&= \frac{\cot^3(c+dx)}{3a^3d} - \frac{\cot^5(c+dx)}{5a^3d} + \frac{\cot^7(c+dx)}{7a^3d} - \frac{\cot^9(c+dx)}{9a^3d} \\
&\quad + \frac{4\cot^{11}(c+dx)}{11a^3d} + \frac{3\csc(c+dx)}{a^3d} - \frac{16\csc^3(c+dx)}{3a^3d} + \frac{34\csc^5(c+dx)}{5a^3d} \\
&\quad - \frac{36\csc^7(c+dx)}{7a^3d} + \frac{19\csc^9(c+dx)}{9a^3d} - \frac{4\csc^{11}(c+dx)}{11a^3d} + \frac{\int \cot^2(c+dx)\,dx}{a^3} \\
&= -\frac{\cot(c+dx)}{a^3d} + \frac{\cot^3(c+dx)}{3a^3d} - \frac{\cot^5(c+dx)}{5a^3d} + \frac{\cot^7(c+dx)}{7a^3d} - \frac{\cot^9(c+dx)}{9a^3d} \\
&\quad + \frac{4\cot^{11}(c+dx)}{11a^3d} + \frac{3\csc(c+dx)}{a^3d} - \frac{16\csc^3(c+dx)}{3a^3d} + \frac{34\csc^5(c+dx)}{5a^3d} \\
&\quad - \frac{36\csc^7(c+dx)}{7a^3d} + \frac{19\csc^9(c+dx)}{9a^3d} - \frac{4\csc^{11}(c+dx)}{11a^3d} - \frac{\int 1\,dx}{a^3} \\
&= -\frac{x}{a^3} - \frac{\cot(c+dx)}{a^3d} + \frac{\cot^3(c+dx)}{3a^3d} - \frac{\cot^5(c+dx)}{5a^3d} + \frac{\cot^7(c+dx)}{7a^3d} \\
&\quad - \frac{\cot^9(c+dx)}{9a^3d} + \frac{4\cot^{11}(c+dx)}{11a^3d} + \frac{3\csc(c+dx)}{a^3d} - \frac{16\csc^3(c+dx)}{3a^3d} \\
&\quad + \frac{34\csc^5(c+dx)}{5a^3d} - \frac{36\csc^7(c+dx)}{7a^3d} + \frac{19\csc^9(c+dx)}{9a^3d} - \frac{4\csc^{11}(c+dx)}{11a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 4.22 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.83

$$
\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(231(-25+28\cos(c+dx))\cot^2\left(\frac{c}{2}\right)\csc^4\left(\frac{1}{2}(c+dx)\right)+561145\sec^2\left(\frac{1}{2}(c+\right.\right.}{}
$$

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/110880*(Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(231*(-25 + 28*Cos[c + d*x])*C
ot[c/2]^2*Csc[(c + d*x)/2]^4 + 561145*Sec[(c + d*x)/2]^2 - 184650*Sec[(c +
d*x)/2]^4 + 41320*Sec[(c + d*x)/2]^6 - 5425*Sec[(c + d*x)/2]^8 + 315*Sec[(c
 + d*x)/2]^10 - 1736335*Csc[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + 561145*Csc
[c/2]*Sec[(c + d*x)/2]^3*Sin[(d*x)/2] - 184650*Csc[c/2]*Sec[(c + d*x)/2]^5*
Sin[(d*x)/2] + 41320*Csc[c/2]*Sec[(c + d*x)/2]^7*Sin[(d*x)/2] - 5425*Csc[c/
2]*Sec[(c + d*x)/2]^9*Sin[(d*x)/2] + 315*Csc[c/2]*Sec[(c + d*x)/2]^11*Sin[(
d*x)/2] + 6468*Csc[c/2]^3*Csc[(c + d*x)/2]^3*Sin[c]*Sin[(d*x)/2] + 231*Cot[
c/2]*(3840*d*x - Csc[c/2]*Csc[(c + d*x)/2]*(743 + 3*Csc[(c + d*x)/2]^4)*Sin
[(d*x)/2]))*Tan[c/2])/(a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}+\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{46\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-\frac{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+382\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-512\arctan(}{256d\,a^{3}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{11}+\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{46\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-\frac{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+382\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-512\arctan(}{256d\,a^{3}}$
risch	$-\frac{x}{a^{3}}+\frac{2i(10395\,e^{15i(dx+c)}+31185\,e^{14i(dx+c)}+1155\,e^{13i(dx+c)}-148995\,e^{12i(dx+c)}-190113\,e^{11i(dx+c)}+117117\,e^{10i(d}}{}$

[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/256/d/a^3*(-1/11*tan(1/2*d*x+1/2*c)^11+10/9*tan(1/2*d*x+1/2*c)^9-46/7*tan(1/2*d*x+1/2*c)^7+26*tan(1/2*d*x+1/2*c)^5-256/3*tan(1/2*d*x+1/2*c)^3+382*tan(1/2*d*x+1/2*c)-512*arctan(tan(1/2*d*x+1/2*c))-1/5/tan(1/2*d*x+1/2*c)^5+10/3/tan(1/2*d*x+1/2*c)^3-46/tan(1/2*d*x+1/2*c))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.31

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{7453\cos(dx+c)^8+11964\cos(dx+c)^7-11866\cos(dx+c)^6-30542\cos(dx+c)^5+90\cos(dx+c}{3465\left(a^3d\cos(d\right.}$$

[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] -1/3465*(7453*cos(d*x + c)^8 + 11964*cos(d*x + c)^7 - 11866*cos(d*x + c)^6 - 30542*cos(d*x + c)^5 + 90*cos(d*x + c)^4 + 26438*cos(d*x + c)^3 + 8539*cos(d*x + c)^2 + 3465*(d*x*cos(d*x + c)^7 + 3*d*x*cos(d*x + c)^6 + d*x*cos(d*x + c)^5 - 5*d*x*cos(d*x + c)^4 - 5*d*x*cos(d*x + c)^3 + d*x*cos(d*x + c)^2 + 3*d*x*cos(d*x + c) + d*x)*sin(d*x + c) - 7671*cos(d*x + c) - 3712)/((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))

Sympy [F]

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cot^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.01

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{5\left(\frac{264726\sin(dx+c)}{\cos(dx+c)+1}-\frac{59136\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{18018\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{4554\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{770\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{63\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a^3}-\frac{1774080\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{887040\,d}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/887040*(5*(264726*sin(d*x + c)/(cos(d*x + c) + 1) - 59136*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 + 18018*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4554*sin
(d*x + c)^7/(cos(d*x + c) + 1)^7 + 770*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
- 63*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^3 - 1774080*arctan(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3 + 231*(50*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
690*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a^3*sin(
d*x + c)^5))/d
```

Giac [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.74

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{887040\,(dx+c)}{a^3}+\frac{231\left(690\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-50\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\right)}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}+\frac{5\left(63a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-770a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+4554a^{30}\tan(\right.}{}}{887040\,d}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/887040*(887040*(d*x + c)/a^3 + 231*(690*tan(1/2*d*x + 1/2*c)^4 - 50*tan(
1/2*d*x + 1/2*c)^2 + 3)/(a^3*tan(1/2*d*x + 1/2*c)^5) + 5*(63*a^30*tan(1/2*d
*x + 1/2*c)^11 - 770*a^30*tan(1/2*d*x + 1/2*c)^9 + 4554*a^30*tan(1/2*d*x +
1/2*c)^7 - 18018*a^30*tan(1/2*d*x + 1/2*c)^5 + 59136*a^30*tan(1/2*d*x + 1/2
*c)^3 - 264726*a^30*tan(1/2*d*x + 1/2*c))/a^33)/d
```

Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.18

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{693\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+315\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-3850\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+22770\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin}{}$$

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(693*cos(c/2 + (d*x)/2)^16 + 315*sin(c/2 + (d*x)/2)^16 - 3850*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 22770*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)
/2)^12 - 90090*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 295680*cos(c/2
+ (d*x)/2)^8*sin(c/2 + (d*x)/2)^8 - 1323630*cos(c/2 + (d*x)/2)^10*sin(c/2 +
 (d*x)/2)^6 + 159390*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 11550*cos
(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2 + 887040*cos(c/2 + (d*x)/2)^11*sin(
c/2 + (d*x)/2)^5*(c + d*x))/(887040*a^3*d*cos(c/2 + (d*x)/2)^11*sin(c/2 + (
d*x)/2)^5)
```

3.104 $\int (a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 310

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx = \frac{ae^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{ae^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{6ae^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} - \frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}$$

```
[Out] 1/2*a*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/2*
a*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/4*a*e^
(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/2)
+1/4*a*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d*2^(1/2)-6/5*a*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x
)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/sin(2*d*x+2*c
)^(1/2)-6/5*a*e*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/d+2/15*e*(5*a+3*a*sec(d*x+c
))*(e*tan(d*x+c))^(3/2)/d
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3966, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx=\frac{ae^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{ae^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}-\frac{ae^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}+\frac{ae^{5/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}+\frac{6ae^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}}-\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d}+\frac{2e(3a\sec(c+dx)+5a)(e\tan(c+dx))^{3/2}}{15d}$$

```
[In] Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d)
- (a*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
d*x]]])/(2*Sqrt[2]*d) + (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (6*a*e^2*Cos[c + d*x]*EllipticE
[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*d*Sqrt[Sin[2*c + 2*d*x]]) - (6
*a*e*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*d) + (2*e*(5*a + 3*a*Sec[c + d
*x])*(e*Tan[c + d*x])^(3/2))/(15*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
```

```
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d} \\
&\quad -\frac{1}{5}\left(2e^2\right)\int\left(\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx \\
&= \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d} \\
&\quad -\frac{1}{5}\left(3ae^2\right)\int\sec(c+dx)\sqrt{e\tan(c+dx)}\,dx-\left(ae^2\right)\int\sqrt{e\tan(c+dx)}\,dx
\end{aligned}
$$

$$= -\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d} + \frac{1}{5}(6ae^2)\int\cos(c+dx)\sqrt{e\tan(c+dx)}\,dx - \frac{(ae^3)\operatorname{Subst}\left(\int\frac{\sqrt{x}}{e^2+x^2}\,dx,x,e\tan(c+dx)\right)}{d}$$

$$= -\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d} - \frac{(2ae^3)\operatorname{Subst}\left(\int\frac{x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{d} + \frac{\left(6ae^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{5\sqrt{\sin(c+dx)}}$$

$$= -\frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d} + \frac{(ae^3)\operatorname{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{d} - \frac{(ae^3)\operatorname{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{d} + \frac{\left(6ae^2\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int\sqrt{\sin(2c+2dx)}\,dx}{5\sqrt{\sin(2c+2dx)}}$$

$$= \frac{6ae^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} - \frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d} - \frac{(ae^{5/2})\operatorname{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{(ae^{5/2})\operatorname{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{(ae^3)\operatorname{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2d} - \frac{(ae^3)\operatorname{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2d}$$

$$= -\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{6ae^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} - \frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d} - \frac{(ae^{5/2})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{(ae^{5/2})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$

$$= \frac{ae^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{ae^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{6ae^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} - \frac{6ae\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2e(5a+3a\sec(c+dx))(e\tan(c+dx))^{3/2}}{15d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.58 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.60

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx = \frac{a(1+\cos(c+dx))\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(12+20\cos(c+dx)-36\cos^2(c+dx)+\frac{24\,\text{H}}{}\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*(12 + 20*Cos[c + d*x]
 - 36*Cos[c + d*x]^2 + (24*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2
```

```
])/Sqrt[Sec[c + d*x]^2] + 15*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Cot[c + d*
x]^2*Sqrt[Sin[2*(c + d*x)]] + 15*Cot[c + d*x]^2*Log[Cos[c + d*x] + Sin[c +
d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]])*(e*Tan[c + d*x])^(5/
2))/(60*d)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(274) = 548$.

Time = 6.00 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.87

method	result
parts	$\frac{2ae\left(\frac{(e\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{e^2\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+\cdots\right)\right)}{8(e^2)^{\frac{1}{4}}}\cdots\right)}{d}$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a/d*e*(1/3*(e*tan(d*x+c))^(3/2)-1/8*e^2/(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*
x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)+(e
^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)
^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))
^(1/2)+1)))-1/5*a/d*2^(1/2)*(-6*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)
-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-co
t(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/
2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF(
(csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3-6*(csc(d*x+c)-cot
(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1
/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+3*(
csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-
csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*co
s(d*x+c)^2+3*cos(d*x+c)^3*2^(1/2)-4*2^(1/2)*cos(d*x+c)^2+2^(1/2))*e^2*(e*ta
n(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*sec(d*x+c)*tan(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))(e\tan(c + dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int (a + a\sec(c + dx))(e\tan(c + dx))^{5/2}\,dx = a\left(\int (e\tan(c + dx))^{\frac{5}{2}}\,dx + \int (e\tan(c + dx))^{\frac{5}{2}}\sec(c + dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(5/2),x)
[Out] a*(Integral((e*tan(c + d*x))**(5/2), x) + Integral((e*tan(c + d*x))**(5/2)*
sec(c + d*x), x))
```

Maxima [F(-2)]

Exception generated.

$$\int (a + a\sec(c + dx))(e\tan(c + dx))^{5/2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx=\int (a\sec{(dx+c)}+a)(e\tan{(dx+c)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx=\int (e\tan(c+d\,x))^{5/2}\left(a+\frac{a}{\cos{(c+d\,x)}}\right)\,dx$$

```
[In] int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x)),x)
[Out] int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x)), x)
```

3.105 $\int (a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 282

$$\begin{aligned}\int (a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx &= \frac{ae^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\ &\quad-\frac{ae^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\ &\quad+\frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d}\\ &\quad-\frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d}\\ &\quad-\frac{ae^2\,\mathrm{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}}\\ &\quad+\frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}\end{aligned}$$

```
[Out] 1/2*a*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/2*
a*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/4*a*e^
(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/2)
-1/4*a*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d*2^(1/2)+1/3*a*e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF
(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*tan(d*x+c)
)^(1/2)+2/3*e*(3*a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3966, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\begin{aligned}\int(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx &= \frac{ae^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\ &-\frac{ae^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}\\ &+\frac{ae^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}\\ &-\frac{ae^{3/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}\\ &-\frac{ae^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\operatorname{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3d\sqrt{e\tan(c+dx)}}\\ &+\frac{2e(a\sec(c+dx)+3a)\sqrt{e\tan(c+dx)}}{3d}\end{aligned}$$

```
[In] Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d)
- (a*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) + (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
d*x]]])/(2*Sqrt[2]*d) - (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a*e^2*EllipticF[c - Pi/4 + d*x
, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[e*Tan[c + d*x]]) + (2*e
*(3*a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(3*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}-\frac{1}{3}\left(2e^2\right)\int\frac{\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx\\
&= \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}\\
&\quad-\frac{1}{3}\left(ae^2\right)\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx-\left(ae^2\right)\int\frac{1}{\sqrt{e\tan(c+dx)}}\,dx\\
&= \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}\\
&\quad-\frac{\left(ae^3\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}(e^2+x^2)}\,dx,x,e\tan(c+dx)\right)}{d}\\
&\quad-\frac{\left(ae^2\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2e(3a + a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d} \\
&\quad - \frac{(2ae^3)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&\quad - \frac{\left(ae^2\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3\sqrt{e\tan(c+dx)}} \\
&= -\frac{ae^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{2e(3a + a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d} \\
&\quad - \frac{(ae^2)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&\quad - \frac{(ae^2)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&= -\frac{ae^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{2e(3a + a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d} \\
&\quad + \frac{(ae^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad + \frac{(ae^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad - \frac{(ae^2)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2d} \\
&\quad - \frac{(ae^2)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2d}
\end{aligned}
$$

$$= \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{ae^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d} - \frac{(ae^{3/2})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{(ae^{3/2})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$

$$= \frac{ae^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{ae^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{ae^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2e(3a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}{3d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.76

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{ae\cos(2(c+dx))\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec^2(c+dx)}\sqrt{e\tan(c+dx)}\left(4\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] -1/12*(a*e*Cos[2*(c + d*x)]*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sqrt[Sec[c +
d*x]^2]*Sqrt[e*Tan[c + d*x]]*(4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*S
```

```
qrt[Tan[c + d*x]]], -1]*Sqrt[Tan[c + d*x]] + Sqrt[Sec[c + d*x]^2]*(12*Sin[c
 + d*x] + 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] - 3*
Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d
*x)]] + 4*Tan[c + d*x])))/(d*(-1 + Tan[c + d*x]^2))
```

Maple [A] (verified)

Time = 5.59 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.32

method	result
parts	$\frac{2ae\left(\sqrt{e\tan(dx+c)}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\tan(c}}{(e^2)^{\frac{1}{4}}}\cdots\right)\right)}{8}\cdots\right)}{d}\cdots$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a/d*e*((e*tan(d*x+c))^(1/2)-1/8*(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)+(e^
2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)-(e^2)^(1/4
)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(
e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1
)))-1/3*a/d*2^(1/2)*(-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c
)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1
)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-
cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)+2^(1/2)*sin(d*x+c))*e*tan(d*x+c
)*(e*tan(d*x+c))^(1/2)/(cos(d*x+c)^2-1)
```

Fricas [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx = a\left(\int (e\tan{(c+dx)})^{\frac{3}{2}}\,dx + \int (e\tan{(c+dx)})^{\frac{3}{2}}\sec{(c+dx)}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(3/2),x)
[Out] a*(Integral((e*tan(c + d*x))**(3/2), x) + Integral((e*tan(c + d*x))**(3/2)*
sec(c + d*x), x))
```

Maxima [F(-2)]

Exception generated.

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx = \int (a\sec{(dx+c)}+a)(e\tan{(dx+c)})^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx=\int (e\tan(c+d\,x))^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

```
[In] int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x)),x)
[Out] int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x)), x)
```

3.106 $\int (a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 272

$$
\begin{aligned}
&\int (a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx \\
&= -\frac{a\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\
&\quad+\frac{a\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad-\frac{a\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad-\frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}+\frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}
\end{aligned}
$$

```
[Out] -1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1/2
*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1/4*a*l
n(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/d*2^(1/2
)-1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)
/d*2^(1/2)+2*a*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ell
ipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/sin(2*d*x+2*c)^(1/
2)+2*a*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/d/e
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\begin{aligned}
&\int (a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx \\
&= -\frac{a\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d} \\
&\quad+\frac{a\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} \\
&\quad-\frac{a\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} \\
&\quad+\frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{2a\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}
\end{aligned}$$

```
[In] Int[(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
)) + (a*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*d) + (a*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*d) - (a*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*a*Cos[c + d*x]*EllipticE[
c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (2*a*
Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*e)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= a\int \sqrt{e\tan(c+dx)}\,dx + a\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx \\
&= \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} \\
&\quad -(2a)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx + \frac{(ae)\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{d} \\
&= \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} + \frac{(2ae)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&\quad - \frac{\left(2a\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{\sqrt{\sin(c+dx)}} \\
&= \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - \frac{(ae)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&\quad + \frac{(ae)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&\quad - \frac{\left(2a\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int \sqrt{\sin(2c+2dx)}\,dx}{\sqrt{\sin(2c+2dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} \\
&\quad+\frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} \\
&\quad+\frac{\left(a\sqrt{e}\right)\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad+\frac{\left(a\sqrt{e}\right)\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad+\frac{(ae)\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2d} \\
&\quad+\frac{(ae)\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad-\frac{a\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad-\frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} \\
&\quad+\frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} \\
&\quad+\frac{\left(a\sqrt{e}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\
&\quad-\frac{\left(a\sqrt{e}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\
&\quad+\frac{a\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad-\frac{a\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad-\frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} \\
&\quad+\frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.67

$$\int (a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx =$$

$$-\frac{a(1+\cos(c+dx))\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}\left(3\sqrt{\sec^2(c+dx)}\left(-4\sin^2(c+dx)+\mathrm{ar}\right.\right.}{}$$

[In] Integrate[(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]],x]

[Out] -1/12*(a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[e*Tan[c + d*x]]*(3*Sqrt[Sec[c + d*x]^2]*(-4*Sin[c + d*x]^2 + ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]) + 8*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^2))/(d*Sqrt[Sec[c + d*x]^2])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(244) = 488.

Time = 5.04 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.86

method	result
parts	$\frac{ae\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)\right)}{4d\left(e^2\right)^{\frac{1}{4}}}-\frac{a\sqrt{}}{}$
default	Expression too large to display

[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4*a/d*e/(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1))-a/d*2^(1/2)*(-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(

```
1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2
),1/2*2^(1/2))+2^(1/2)*cos(d*x+c)-2^(1/2))*(e*tan(d*x+c))^(1/2)*csc(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int (a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx = a\left(\int \sqrt{e\tan{(c+dx)}}\,dx + \int \sqrt{e\tan{(c+dx)}}\sec{(c+dx)}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*tan(c + d*x)), x) + Integral(sqrt(e*tan(c + d*x))*sec(c
+ d*x), x))
```

Maxima [F(-2)]

Exception generated.

$$\int (a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (a + a\sec(c + dx))\sqrt{e\tan(c + dx)}\,dx = \int (a\sec(dx + c) + a)\sqrt{e\tan(dx + c)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))\sqrt{e\tan(c + dx)}\,dx = \int \sqrt{e\tan(c + d\,x)}\left(a + \frac{a}{\cos(c + d\,x)}\right)\,dx$$

```
[In] int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x)),x)
[Out] int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x)), x)
```

3.107 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 244

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = &-\frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}\\&-\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}\\&+\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}\\&+\frac{a\,\mathrm{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}}\end{aligned}$$

```
[Out] -1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+1/2
*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)-1/4*a*l
n(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/2)/e^(1/2
)+1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/
2)/e^(1/2)-a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+
1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = -\frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}} - \frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}} + \frac{a\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}} + \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[e]
)) + (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt
[e]) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]
])/(2*Sqrt[2]*d*Sqrt[e]) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*
Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (a*EllipticF[c - Pi/4 + d*x,
 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx + a\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx\\
&= \frac{(ae)\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{d} + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}\\
&= \frac{(2ae)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d}\\
&\quad + \frac{\left(a\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{\sqrt{e\tan(c+dx)}}\\
&= \frac{a\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}}\\
&\quad + \frac{a\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d}\\
&\quad + \frac{a\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d}
\end{aligned}$$

$$= \frac{a \operatorname{EllipticF}\left(c - \frac{\pi}{4} + dx, 2\right) \sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2d} + \frac{a\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2d} - \frac{a\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{a\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}$$

$$= -\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} + \frac{a \operatorname{EllipticF}\left(c - \frac{\pi}{4} + dx, 2\right) \sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}$$

$$= -\frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} + \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} + \frac{a \operatorname{EllipticF}\left(c - \frac{\pi}{4} + dx, 2\right) \sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 0.87 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.90

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\tan(c + dx)}}\,dx$$
$$= \frac{20a\,\text{AppellF1}\left(\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan^2\left(\frac{1}{2}(c +\right.\right.}{d\left(2\left(2\,\text{AppellF1}\left(\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \tan^2\left(\frac{1}{2}(c + dx)\right), -\tan^2\left(\frac{1}{2}(c + dx)\right)\right) - \text{AppellF1}\left(\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \tan^2\left(\frac{1}{2}(c + dx)\right)\right.\right.\right.}$$

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] (20*a*AppellF1[1/4, 1/2, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*C
os[(c + d*x)/2]^2*(1 + Sec[c + d*x])*Sin[c + d*x])/(d*(2*(2*AppellF1[5/4, 1
/2, 2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[5/4, 3/2, 1
, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 5*Ap
pellF1[1/4, 1/2, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[
c + d*x]))*Sqrt[e*Tan[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.58

method	result
default	$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)a\sqrt{2}\left(i\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)+\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\right)\sqrt{\cot(dx+c)-}}{d\sqrt{e\tan(dx+c)}}$
parts	$\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)\right)}{4de}-$

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2-1/2*I)*a/d*2^(1/2)*(I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1
/2*I,1/2*2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*
2^(1/2)))*(cot(d*x+c)-csc(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cs
c(d*x+c)-cot(d*x+c)+1)^(1/2)/(e*tan(d*x+c))^(1/2)*(1+sec(d*x+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = a\left(\int \frac{1}{\sqrt{e\tan{(c+dx)}}}\,dx + \int \frac{\sec{(c+dx)}}{\sqrt{e\tan{(c+dx)}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*tan(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*tan(c
 + d*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx = \int \frac{a\sec{(dx+c)}+a}{\sqrt{e\tan{(dx+c)}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*tan(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx=\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{e\tan\left(c+d\,x\right)}}\,dx$$

```
[In] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(1/2), x)
```

3.108 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 305

$$
\begin{aligned}
\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx &= \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\
&- \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\
&- \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\
&+ \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\
&- \frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}} \\
&+ \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3}
\end{aligned}
$$

```
[Out] 1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/2*
a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/4*a*ln
(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(3/2)*2^(1/2)
+1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(3/2
)*2^(1/2)-2*(a+a*sec(d*x+c))/d/e/(e*tan(d*x+c))^(1/2)+2*a*cos(d*x+c)*(sin(c
+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2)
)*(e*tan(d*x+c))^(1/2)/d/e^2/sin(2*d*x+2*c)^(1/2)+2*a*cos(d*x+c)*(e*tan(d*x
+c))^(3/2)/d/e^3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx = \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} - \frac{a\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}} - \frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}} + \frac{a\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}} + \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3} - \frac{2a\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}} - \frac{2(a\sec(c+dx)+a)}{de\sqrt{e\tan(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2))
- (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)
) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/
(2*Sqrt[2]*d*e^(3/2)) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - (2*(a + a*Sec[c + d*x]))/(d*e*S
qrt[e*Tan[c + d*x]]) - (2*a*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[
e*Tan[c + d*x]])/(d*e^2*Sqrt[Sin[2*c + 2*d*x]]) + (2*a*Cos[c + d*x]*(e*Tan[
c + d*x])^(3/2))/(d*e^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
```

```
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}+\frac{2\int\left(-\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{e^2}\\
&= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}-\frac{a\int\sqrt{e\tan(c+dx)}\,dx}{e^2}+\frac{a\int\sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{e^2}\\
&= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}}+\frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3}\\
&\quad -\frac{(2a)\int\cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{e^2}-\frac{a\text{Subst}\left(\int\frac{\sqrt{x}}{e^2+x^2}\,dx,x,e\tan(c+dx)\right)}{de}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} + \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3} \\
&\quad - \frac{(2a)\mathrm{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de} \\
&\quad - \frac{\left(2a\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{e^2\sqrt{\sin(c+dx)}} \\
&= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} + \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3} \\
&\quad + \frac{a\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de} \\
&\quad - \frac{a\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de} \\
&\quad - \frac{\left(2a\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int \sqrt{\sin(2c+2dx)}\,dx}{e^2\sqrt{\sin(2c+2dx)}} \\
&= -\frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3} \\
&\quad - \frac{a\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\
&\quad - \frac{a\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\
&\quad - \frac{a\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2de} \\
&\quad - \frac{a\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2de}
\end{aligned}
$$

$$= -\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} - \frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}} + \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3} - \frac{a\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} + \frac{a\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}$$

$$= \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} - \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} - \frac{2(a+a\sec(c+dx))}{de\sqrt{e\tan(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}} + \frac{2a\cos(c+dx)(e\tan(c+dx))^{3/2}}{de^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.63 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.64

$$\int\frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx = -\frac{a(1+\cos(c+dx))\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{e\tan(c+dx)}\left(3\sqrt{\sec^2(c+dx)}\left(2+4\cos(c+dx)+2\cos\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] -1/12*(a*(1 + Cos[c + d*x])*Csc[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[e*Tan[c +
d*x]]*(3*Sqrt[Sec[c + d*x]^2]*(2 + 4*Cos[c + d*x] + 2*Cos[2*(c + d*x)] - Ar
cSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] - Log[Cos[c + d*x]
 + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]) + 8*Hyper
geometric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^2))/(d*e^2*Sqrt[S
ec[c + d*x]^2])
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.68

method	result
parts	$\frac{2ae\left(-\frac{1}{e^{2}\sqrt{e\tan(dx+c)}}-\frac{\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\tan(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}\cdots\right.\right.}{8e^{2}\left(e^{2}\right)^{\frac{1}{4}}}\cdots\right.}{d}$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a/d*e*(-1/e^2/(e*tan(d*x+c))^(1/2)-1/8/e^2/(e^2)^(1/4)*2^(1/2)*(ln((e*tan
(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)
+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e
^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+
c))^(1/2)+1)))-a/d*2^(1/2)/e/(e*tan(d*x+c))^(1/2)*(-2*(csc(d*x+c)-cot(d*x+c
)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ell
ipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*(csc(d*x+c)-cot(d*x+c
)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+(csc(d*x+c)
-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)
)^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+2
^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx = a\left(\int \frac{1}{(e\tan(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\sec(c+dx)}{(e\tan(c+dx))^{\frac{3}{2}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
[Out] a*(Integral((e*tan(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*tan(c +
 d*x))**(3/2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx = \int \frac{a\sec(dx+c)+a}{(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx = \int \frac{a+\frac{a}{\cos(c+d\,x)}}{(e\tan(c+d\,x))^{3/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(3/2), x)
```

3.109 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 282

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx = \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} - \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{a\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}}$$

```
[Out] 1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1/2*
a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/4*a*ln
(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(5/2)*2^(1/2)
-1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(5/2
)*2^(1/2)+1/3*a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos
(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/e^2/(e*tan(d*x+c)
)^(1/2)-2/3*(a+a*sec(d*x+c))/d/e/(e*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3967, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$
\begin{aligned}
\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx &= \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} \\
&\quad-\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{5/2}} \\
&\quad+\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}} \\
&\quad-\frac{a\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}} \\
&\quad-\frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3de^2\sqrt{e\tan(c+dx)}}-\frac{2(a\sec(c+dx)+a)}{3de(e\tan(c+dx))^{3/2}}
\end{aligned}
$$

```
[In] Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2))
- (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)
) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/
(2*Sqrt[2]*d*e^(5/2)) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (2*(a + a*Sec[c + d*x]))/(3*d*e
*(e*Tan[c + d*x])^(3/2)) - (a*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqr
t[Sin[2*c + 2*d*x]])/(3*d*e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}+\frac{2\int\frac{-\frac{3a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3e^2}\\
&= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}-\frac{a\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3e^2}-\frac{a\int\frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{e^2}\\
&= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}-\frac{a\text{Subst}\left(\int\frac{1}{\sqrt{x}(e^2+x^2)}\,dx,x,e\tan(c+dx)\right)}{de}\\
&\quad-\frac{\left(a\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3e^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}\\
&= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}-\frac{(2a)\text{Subst}\left(\int\frac{1}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{de}\\
&\quad-\frac{\left(a\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int\frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3e^2\sqrt{e\tan(c+dx)}}
\end{aligned}
$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{a\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}}$$
$$-\frac{a\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{de^2} - \frac{a\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{de^2}$$

$$= -\frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}} - \frac{a\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}}$$
$$+\frac{a\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}}$$
$$+\frac{a\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}}$$
$$-\frac{a\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2de^2}$$
$$-\frac{a\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2de^2}$$

$$= \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}}$$
$$-\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}$$
$$-\frac{a\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}}$$
$$-\frac{a\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}$$
$$+\frac{a\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}$$

$$= \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} - \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}$$
$$+\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}}$$
$$-\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{2(a+a\sec(c+dx))}{3de(e\tan(c+dx))^{3/2}}$$
$$-\frac{a\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.71

$$\int \frac{a + a\sec(c + dx)}{(e\tan(c + dx))^{5/2}}\,dx =$$

$$-\frac{a\csc(c+dx)\left(\sqrt{\sec^2(c+dx)}\left(2\cot\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{3}{2}(c+dx)\right)\csc\left(\frac{1}{2}(c+dx)\right)-3\arcsin(\cos(c+dx\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/6*(a*Csc[c + d*x]*(Sqrt[Sec[c + d*x]^2]*(2*Cot[(c + d*x)/2] + 2*Cos[(3*(
c + d*x))/2]*Csc[(c + d*x)/2] - 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[
Sin[2*(c + d*x)]] + 3*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x
)]]]*Sqrt[Sin[2*(c + d*x)]]) - 4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*
Sqrt[Tan[c + d*x]]], -1]*Sqrt[Tan[c + d*x]])*Sqrt[e*Tan[c + d*x]])/(d*e^3*S
qrt[Sec[c + d*x]^2])
```

Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.62

method	result
parts	$\frac{2ae\left(-\frac{1}{3e^2(e\tan(dx+c))^{\frac{3}{2}}}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e}}{(}\right.\right.}{8e^4}\right.}{d}$
default	$\frac{a\sqrt{2}(\cos(dx+c)+1)\left(3i\sin(dx+c)\sqrt{\csc(dx+c)-\cot(dx+c)+1}\sqrt{\cot(dx+c)-\csc(dx+c)+1}\sqrt{\cot(dx+c)-\csc(dx+c)}\ \mathrm{EllipticPi}\left(\sqrt{\csc}\right.\right.}{}$

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a/d*e*(-1/3/e^2/(e*tan(d*x+c))^(3/2)-1/8/e^4*(e^2)^(1/4)*2^(1/2)*(ln((e*t
an(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+
c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/
(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*
x+c))^(1/2)+1)))+1/6*a/d*2^(1/2)*(1-cos(d*x+c))^2*(2*(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)
*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*(-cot(d*x+c)+csc(d*
x+c))-(1-cos(d*x+c))^4*csc(d*x+c)^4+1)/((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d
*x+c)-csc(d*x+c))^(1/2)/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*c
```

```
sc(d*x+c))^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2/(-e/((1-cos(d*x+c))^2*
csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d*x+c)))^(5/2)*csc(d*x+c)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx = a\left(\int \frac{1}{(e\tan(c+dx))^{\frac{5}{2}}}\,dx+\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{\frac{5}{2}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
[Out] a*(Integral((e*tan(c + d*x))**(-5/2), x) + Integral(sec(c + d*x)/(e*tan(c +
 d*x))**(5/2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + a\sec(c + dx)}{(e\tan(c + dx))^{5/2}}\,dx = \int \frac{a\sec\left(dx + c\right) + a}{\left(e\tan\left(dx + c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\tan(c + dx))^{5/2}}\,dx = \int \frac{a + \frac{a}{\cos(c+d\,x)}}{\left(e\tan\left(c + d\,x\right)\right)^{5/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(5/2), x)
```

3.110 $\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 346

$$
\begin{aligned}
\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx = & -\frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} \\
& + \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} \\
& - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} - \frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} \\
& + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} + \frac{6a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}} \\
& - \frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5}
\end{aligned}
$$

```
[Out] -1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1/2
*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1/4*a*l
n(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(7/2)*2^(1/2
)-1/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(7/
2)*2^(1/2)+2/5*(5*a+3*a*sec(d*x+c))/d/e^3/(e*tan(d*x+c))^(1/2)-6/5*a*cos(d*
x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d
*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/e^4/sin(2*d*x+2*c)^(1/2)-2/5*(a+a*sec(d
*x+c))/d/e/(e*tan(d*x+c))^(5/2)-6/5*a*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/d/e^5
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$
\begin{aligned}
\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx = & -\frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} \\
& +\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{7/2}}+\frac{a\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{7/2}} \\
& -\frac{a\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{7/2}} \\
& -\frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5} \\
& +\frac{6a\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}} \\
& +\frac{2(3a\sec(c+dx)+5a)}{5de^3\sqrt{e\tan(c+dx)}}-\frac{2(a\sec(c+dx)+a)}{5de(e\tan(c+dx))^{5/2}}
\end{aligned}
$$

```
[In] Int[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] -((a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/2)
)) + (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7
/2)) + (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]
])/(2*Sqrt[2]*d*e^(7/2)) - (a*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*
Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (2*(a + a*Sec[c + d*x]))/(5*
d*e*(e*Tan[c + d*x])^(5/2)) + (2*(5*a + 3*a*Sec[c + d*x]))/(5*d*e^3*Sqrt[e*
Tan[c + d*x]]) + (6*a*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[
c + d*x]])/(5*d*e^4*Sqrt[Sin[2*c + 2*d*x]]) - (6*a*Cos[c + d*x]*(e*Tan[c +
d*x])^(3/2))/(5*d*e^5)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
```

```
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}+\frac{2\int\frac{-\frac{5a}{2}-\frac{3}{2}a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{5e^2}\\
&= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}+\frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}}\\
&\quad+\frac{4\int\left(\frac{5a}{4}-\frac{3}{4}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{5e^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{(3a)\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{5e^4} + \frac{a\int \sqrt{e\tan(c+dx)}\,dx}{e^4} \\
&= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} - \frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5} \\
&\quad + \frac{(6a)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{5e^4} + \frac{a\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{de^3} \\
&= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5} + \frac{(2a)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de^3} \\
&\quad + \frac{\left(6a\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{5e^4\sqrt{\sin(c+dx)}} \\
&= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5} - \frac{a\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de^3} \\
&\quad + \frac{a\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de^3} \\
&\quad + \frac{\left(6a\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int \sqrt{\sin(2c+2dx)}\,dx}{5e^4\sqrt{\sin(2c+2dx)}}
\end{aligned}
$$

$$= -\frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}} + \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}}$$
$$+ \frac{6a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}}$$
$$- \frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5}$$
$$+ \frac{a\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$
$$+ \frac{a\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$
$$+ \frac{a\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2de^3}$$
$$+ \frac{a\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2de^3}$$

$$= \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$
$$- \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} - \frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}$$
$$+ \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} + \frac{6a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}}$$
$$- \frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5} + \frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}$$
$$- \frac{a\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}$$

$$= -\frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} + \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}$$
$$+ \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$
$$- \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} - \frac{2(a+a\sec(c+dx))}{5de(e\tan(c+dx))^{5/2}}$$
$$+ \frac{2(5a+3a\sec(c+dx))}{5de^3\sqrt{e\tan(c+dx)}} + \frac{6a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}}$$
$$- \frac{6a\cos(c+dx)(e\tan(c+dx))^{3/2}}{5de^5}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.75 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.73

$$\int \frac{a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx =$$

$$-\frac{a\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))\left(2\cot\left(\frac{1}{2}(c+dx)\right)-19\sin(c+dx)+12\sin^2(c+dx)\tan\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] -1/20*(a*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(1 + Sec[c + d*x])*(2*Cot[(c + d
*x)/2] - 19*Sin[c + d*x] + 12*Sin[c + d*x]^2*Tan[(c + d*x)/2] - 8*Hypergeom
etric2F1[3/4, 3/2, 7/4, -Tan[c + d*x]^2]*Sqrt[Sec[c + d*x]^2]*Sin[c + d*x]^
2*Tan[(c + d*x)/2] + 5*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*(c +
d*x)]]*Tan[(c + d*x)/2] + 5*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c
 + d*x)]]]*Sqrt[Sin[2*(c + d*x)]]*Tan[(c + d*x)/2] + 5*Sin[c + d*x]*Tan[(c
+ d*x)/2]^2))/(d*e^3*Sqrt[e*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.58

method	result
parts	$\frac{2ae\left(-\frac{1}{5e^2(e\tan(dx+c))^{\frac{5}{2}}}+\frac{1}{e^4\sqrt{e\tan(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\right.}{8e^4(e^2)^{\frac{1}{4}}}\right.}{d}$
default	$-\frac{a\sqrt{2}\left(5i\sin(dx+c)^2\sqrt{\cot(dx+c)-\csc(dx+c)+1}\sqrt{\cot(dx+c)-\csc(dx+c)}\operatorname{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\csc(d\right.}{}$

```
[In] int((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a/d*e*(-1/5/e^2/(e*tan(d*x+c))^(5/2)+1/e^4/(e*tan(d*x+c))^(1/2)+1/8/e^4/(
e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/
2)+(e^2)^(1/2))/(e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2
)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^
(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)))+1/5*a/d*2^(1/2)/(e*tan(d*x+c))^
(1/2)/e^3*(-6*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/
2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2*2^(1/2))+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1
```

```
/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2)
,1/2*2^(1/2))-6*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(
1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2
),1/2*2^(1/2))*sec(d*x+c)+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc
(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*
x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+3*2^(1/2)-2^(1/2)*cot(d*x+c)*csc(d*x+
c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\tan(c + dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{(e\tan(c + dx))^{7/2}}\,dx = a\left(\int \frac{1}{(e\tan{(c + dx)})^{\frac{7}{2}}}\,dx + \int \frac{\sec{(c + dx)}}{(e\tan{(c + dx)})^{\frac{7}{2}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))**(7/2),x)
```

```
[Out] a*(Integral((e*tan(c + d*x))**(-7/2), x) + Integral(sec(c + d*x)/(e*tan(c +
 d*x))**(7/2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + a\sec(c + dx)}{(e\tan(c + dx))^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a + a\sec(c + dx)}{(e\tan(c + dx))^{7/2}}\,dx = \int \frac{a\sec(dx + c) + a}{(e\tan(dx + c))^{\frac{7}{2}}}\,dx$$

[In] integrate((a+a*sec(d*x+c))/(e*tan(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)/(e*tan(d*x + c))^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{(e\tan(c + dx))^{7/2}}\,dx = \int \frac{a + \frac{a}{\cos(c+d\,x)}}{(e\tan(c + d\,x))^{7/2}}\,dx$$

[In] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(7/2),x)

[Out] int((a + a/cos(c + d*x))/(e*tan(c + d*x))^(7/2), x)

3.111 $\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 366

$$\begin{aligned}\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx &= \frac{a^2e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\ &- \frac{a^2e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\ &- \frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\ &+ \frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\ &+ \frac{12a^2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} \\ &+ \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} \\ &+ \frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2a^2(e\tan(c+dx))^{7/2}}{7de}\end{aligned}$$

```
[Out] 1/2*a^2*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/
2*a^2*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/4*
a^2*e^(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2
^(1/2)+1/4*a^2*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/d*2^(1/2)-12/5*a^2*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c
+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/si
n(2*d*x+2*c)^(1/2)+2/3*a^2*e*(e*tan(d*x+c))^(3/2)/d-12/5*a^2*e*cos(d*x+c)*(
e*tan(d*x+c))^(3/2)/d+4/5*a^2*e*sec(d*x+c)*(e*tan(d*x+c))^(3/2)/d+2/7*a^2*(
e*tan(d*x+c))^(7/2)/d/e
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3971, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2691, 2693, 2695, 2652, 2719, 2687, 32}

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx=\frac{a^2e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{a^2e^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d}-\frac{a^2e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}+\frac{a^2e^{5/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d}+\frac{12a^2e^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}}+\frac{2a^2(e\tan(c+dx))^{7/2}}{7de}+\frac{2a^2e(e\tan(c+dx))^{3/2}}{3d}-\frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d}+\frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a^2*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d) - (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]])/(2*Sqrt[2]*d) + (a^2*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (12*a^2*e^2*Cos[c + d*x
]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*d*Sqrt[Sin[2*c + 2*
d*x]]) + (2*a^2*e*(e*Tan[c + d*x])^(3/2))/(3*d) - (12*a^2*e*Cos[c + d*x]*(e
*Tan[c + d*x])^(3/2))/(5*d) + (4*a^2*e*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))
/(5*d) + (2*a^2*(e*Tan[c + d*x])^(7/2))/(7*d*e)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2(e\tan(c+dx))^{5/2} + 2a^2\sec(c+dx)(e\tan(c+dx))^{5/2} + a^2\sec^2(c+dx)(e\tan(c+dx))^{5/2}\right)\,dx \\
&= a^2\int (e\tan(c+dx))^{5/2}\,dx + a^2\int \sec^2(c+dx)(e\tan(c+dx))^{5/2}\,dx + \left(2a^2\right)\int \sec(c+dx)(e\tan(c+dx))^{5/2}\,dx \\
&= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} + \frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{a^2\text{Subst}\left(\int (ex)^{5/2}\,dx, x, \tan(c+dx)\right)}{d} \\
&\quad - \left(a^2e^2\right)\int \sqrt{e\tan(c+dx)}\,dx - \frac{1}{5}\left(6a^2e^2\right)\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx \\
&= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2a^2(e\tan(c+dx))^{7/2}}{7de} \\
&\quad + \frac{1}{5}\left(12a^2e^2\right)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx - \frac{\left(a^2e^3\right)\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{d} \\
&= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2a^2(e\tan(c+dx))^{7/2}}{7de} \\
&\quad - \frac{\left(2a^2e^3\right)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&\quad + \frac{\left(12a^2e^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{5\sqrt{\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} \\
&+ \frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2a^2(e\tan(c+dx))^{7/2}}{7de} \\
&+ \frac{(a^2e^3)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&- \frac{(a^2e^3)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&+ \frac{\left(12a^2e^2\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int \sqrt{\sin(2c+2dx)}\,dx}{5\sqrt{\sin(2c+2dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{12a^2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} + \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} \\
&- \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d} \\
&+ \frac{2a^2(e\tan(c+dx))^{7/2}}{7de} - \frac{(a^2e^{5/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&- \frac{(a^2e^{5/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&- \frac{(a^2e^3)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2d} \\
&- \frac{(a^2e^3)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&+ \frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&+ \frac{12a^2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}} + \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} \\
&- \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d} \\
&+ \frac{2a^2(e\tan(c+dx))^{7/2}}{7de} - \frac{(a^2e^{5/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\
&+ \frac{(a^2e^{5/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}
\end{aligned}
$$

$$= \frac{a^2 e^{5/2} \arctan\left(1 - \frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{a^2 e^{5/2} \arctan\left(1 + \frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$
$$- \frac{a^2 e^{5/2} \log\left(\sqrt{e} + \sqrt{e}\tan(c+dx) - \sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d}$$
$$+ \frac{a^2 e^{5/2} \log\left(\sqrt{e} + \sqrt{e}\tan(c+dx) + \sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d}$$
$$+ \frac{12a^2e^2\cos(c+dx)E\left(c - \frac{\pi}{4} + dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5d\sqrt{\sin(2c+2dx)}}$$
$$+ \frac{2a^2e(e\tan(c+dx))^{3/2}}{3d} - \frac{12a^2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{5d}$$
$$+ \frac{4a^2e\sec(c+dx)(e\tan(c+dx))^{3/2}}{5d} + \frac{2a^2(e\tan(c+dx))^{7/2}}{7de}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.89 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.32

$$\int (a + a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx = \frac{2a^2e\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\arctan(\tan(c+dx))\right)(e\tan(c+dx))^{3/2}\left(35 - 42\,\text{Hypergeometri}\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*e*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(e*Tan[c + d*x])^
(3/2)*(35 - 42*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 35*Hyper
geometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2] + 42*Sqrt[Sec[c + d*x]^2] + 15*T
an[c + d*x]^2))/(105*d)
```

Maple [A] (warning: unable to verify)

Time = 6.60 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.65

method	result
parts	$\frac{2a^{2}e\left(\frac{(e\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{e^{2}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-(e^{2})^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\tan(dx+c)+(e^{2})^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{(e^{2})^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{(e^{2})^{\frac{1}{4}}}\right.}{8(e^{2})^{\frac{1}{4}}}\right.}{d}$
default	Expression too large to display

[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 2*a^2/d*e*(1/3*(e*tan(d*x+c))^(3/2)-1/8*e^2/(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)))+2/7*a^2*(e*tan(d*x+c))^(7/2)/d/e-2/5*a^2/d*2^(1/2)*(-6*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3-6*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+3*cos(d*x+c)^3*2^(1/2)-4*2^(1/2)*cos(d*x+c)^2+2^(1/2))*e^2*(e*tan(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*sec(d*x+c)*tan(d*x+c)

Fricas [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^2(e\tan(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{5/2}\,dx = \int (e\tan(c+d\,x))^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\,dx$$

```
[In] int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2, x)
```

3.112 $\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 335

$$\begin{aligned}\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx &= \frac{a^2e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\ &-\frac{a^2e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\ &+\frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d}\\ &-\frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d}\\ &-\frac{2a^2e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}}\\ &+\frac{2a^2e\sqrt{e\tan(c+dx)}}{d}+\frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d}+\frac{2a^2(e\tan(c+dx))^{5/2}}{5de}\end{aligned}$$

```
[Out] 1/2*a^2*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/
2*a^2*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/4*
a^2*e^(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2
^(1/2)-1/4*a^2*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/d*2^(1/2)+2/3*a^2*e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)
*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*
tan(d*x+c))^(1/2)+2*a^2*e*(e*tan(d*x+c))^(1/2)/d+4/3*a^2*e*sec(d*x+c)*(e*ta
n(d*x+c))^(1/2)/d+2/5*a^2*(e*tan(d*x+c))^(5/2)/d/e
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3971, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2691, 2694, 2653, 2720, 2687, 32}

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx = \frac{a^2e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{a^2e^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d} + \frac{a^2e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} - \frac{a^2e^{3/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} - \frac{2a^2e^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3d\sqrt{e\tan(c+dx)}} + \frac{2a^2(e\tan(c+dx))^{5/2}}{5de} + \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d
) - (a^2*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[
2]*d) + (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*a^2*e^2*EllipticF[c
- Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*Sqrt[e*Tan[c + d
*x]]) + (2*a^2*e*Sqrt[e*Tan[c + d*x]])/d + (4*a^2*e*Sec[c + d*x]*Sqrt[e*Tan
[c + d*x]])/(3*d) + (2*a^2*(e*Tan[c + d*x])^(5/2))/(5*d*e)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2(e\tan(c+dx))^{3/2} + 2a^2\sec(c+dx)(e\tan(c+dx))^{3/2} + a^2\sec^2(c+dx)(e\tan(c+dx))^{3/2}\right) dx \\
&= a^2\int (e\tan(c+dx))^{3/2}\,dx + a^2\int \sec^2(c+dx)(e\tan(c+dx))^{3/2}\,dx + \left(2a^2\right)\int \sec(c+dx)(e\tan(c+dx))^{3/2}\,dx \\
&= \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} + \frac{a^2\text{Subst}\left(\int (ex)^{3/2}\,dx, x, \tan(c+dx)\right)}{d} \\
&\quad - \frac{1}{3}\left(2a^2e^2\right)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx - \left(a^2e^2\right)\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx \\
&= \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} + \frac{2a^2(e\tan(c+dx))^{5/2}}{5de} - \frac{\left(a^2e^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{d} \\
&\quad - \frac{\left(2a^2e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&= \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} + \frac{2a^2(e\tan(c+dx))^{5/2}}{5de} - \frac{\left(2a^2e^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&\quad - \frac{\left(2a^2e^2\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3\sqrt{e\tan(c+dx)}} \\
&= -\frac{2a^2e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} \\
&\quad + \frac{2a^2(e\tan(c+dx))^{5/2}}{5de} - \frac{\left(a^2e^2\right)\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&\quad - \frac{\left(a^2e^2\right)\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d}
\end{aligned}$$

$$= -\frac{2a^2e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} + \frac{2a^2(e\tan(c+dx))^{5/2}}{5de} + \frac{(a^2e^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{(a^2e^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{(a^2e^2)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2d} - \frac{(a^2e^2)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2d}$$

$$= \frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{2a^2e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} + \frac{2a^2(e\tan(c+dx))^{5/2}}{5de} - \frac{(a^2e^{3/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{(a^2e^{3/2})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$

$$= \frac{a^2e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{a^2e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{2a^2e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3d\sqrt{e\tan(c+dx)}} + \frac{2a^2e\sqrt{e\tan(c+dx)}}{d} + \frac{4a^2e\sec(c+dx)\sqrt{e\tan(c+dx)}}{3d} + \frac{2a^2(e\tan(c+dx))^{5/2}}{5de}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 26.79 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.77

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx = \frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\arctan(\tan(c+dx))\right)(e\tan(c+dx))^{3/2}\left(30\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan}\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(e*Tan[c + d*x])^(3/2
)*(30*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 30*Sqrt[2]*ArcTan[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]]] + 15*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]] - 15*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]] + 120*Sqrt[Tan[c + d*x]] - 80*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan
[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 80*Sqrt[Sec[c + d*x]^2]*Sqrt[Tan[c + d*x]
] + 24*Tan[c + d*x]^(5/2)))/(60*d*Tan[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.19

method	result
parts	$\frac{2a^2e\left(\sqrt{e\tan(dx+c)}-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\tan(dx}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8}\right.}{d}$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2/d*e*((e*tan(d*x+c))^(1/2)-1/8*(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)+(
e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)-(e^2)^(1
/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)
*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)
+1)))+2/5*a^2*(e*tan(d*x+c))^(5/2)/d/e-2/3*a^2/d*2^(1/2)*(-(csc(d*x+c)-cot(
d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/
2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-(csc
(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc
(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d
```

```
*x+c)+2^(1/2)*sin(d*x+c))*e*tan(d*x+c)*(e*tan(d*x+c))^(1/2)/(cos(d*x+c)^2-1
)
```

Fricas [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx=a^2\left(\int (e\tan{(c+dx)})^{\frac{3}{2}}\,dx+\int 2(e\tan{(c+dx)})^{\frac{3}{2}}\sec{(c+dx)}\,dx+\int (e\tan{(c+dx)})^{\frac{3}{2}}\sec^2{(c+dx)}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(3/2),x)
[Out] a**2*(Integral((e*tan(c + d*x))**(3/2), x) + Integral(2*(e*tan(c + d*x))**(
3/2)*sec(c + d*x), x) + Integral((e*tan(c + d*x))**(3/2)*sec(c + d*x)**2, x
))
```

Maxima [F(-2)]

Exception generated.

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^{3/2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^{3/2}\, dx = \int (a\sec(dx + c) + a)^2 (e\tan(dx + c))^{\frac{3}{2}}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^{3/2}\, dx = \int (e\tan(c + d\,x))^{3/2} \left(a + \frac{a}{\cos(c + d\,x)}\right)^2 dx$$

```
[In] int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2,x)
[Out] int((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2, x)
```

3.113 $\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 309

$$\begin{aligned}
&\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx\\
&=-\frac{a^2\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a^2\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\
&\quad+\frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d}\\
&\quad-\frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d}\\
&\quad-\frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}\\
&\quad+\frac{2a^2(e\tan(c+dx))^{3/2}}{3de}+\frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}
\end{aligned}$$

```
[Out] -1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1
/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)+1/4
*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/d*
2^(1/2)-1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
*e^(1/2)/d*2^(1/2)+4*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*P
i+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/sin(2*d*
x+2*c)^(1/2)+2/3*a^2*(e*tan(d*x+c))^(3/2)/d/e+4*a^2*cos(d*x+c)*(e*tan(d*x+c
))^(3/2)/d/e
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3971, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719, 2687, 32}

$$\begin{aligned}
&\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx \\
&= -\frac{a^2\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}+\frac{a^2\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d} \\
&\quad+\frac{2a^2(e\tan(c+dx))^{3/2}}{3de}+\frac{a^2\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} \\
&\quad-\frac{a^2\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d} \\
&\quad+\frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}-\frac{4a^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}}
\end{aligned}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a^2*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*d)) + (a^2*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sq
rt[2]*d) + (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e
*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (a^2*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c +
 d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (4*a^2*Cos[c + d*x]*
EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[Sin[2*c + 2*d*x]
]) + (2*a^2*(e*Tan[c + d*x])^(3/2))/(3*d*e) + (4*a^2*Cos[c + d*x]*(e*Tan[c
+ d*x])^(3/2))/(d*e)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \Big(a^2\sqrt{e\tan(c+dx)}+2a^2\sec(c+dx)\sqrt{e\tan(c+dx)} \\
&\qquad + a^2\sec^2(c+dx)\sqrt{e\tan(c+dx)}\Big)\,dx \\
&= a^2\int \sqrt{e\tan(c+dx)}\,dx + a^2\int \sec^2(c+dx)\sqrt{e\tan(c+dx)}\,dx \\
&\quad + \left(2a^2\right)\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} - (4a^2)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx \\
&\quad + \frac{a^2\text{Subst}\left(\int \sqrt{ex}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(a^2e)\,\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{d} \\
&= \frac{2a^2(e\tan(c+dx))^{3/2}}{3de} + \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} \\
&\quad + \frac{(2a^2e)\,\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&\quad - \frac{\left(4a^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{\sqrt{\sin(c+dx)}} \\
&= \frac{2a^2(e\tan(c+dx))^{3/2}}{3de} + \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} \\
&\quad - \frac{(a^2e)\,\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&\quad + \frac{(a^2e)\,\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{d} \\
&\quad - \frac{\left(4a^2\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int \sqrt{\sin(2c+2dx)}\,dx}{\sqrt{\sin(2c+2dx)}} \\
&= -\frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{2a^2(e\tan(c+dx))^{3/2}}{3de} + \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} \\
&\quad + \frac{(a^2\sqrt{e})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad + \frac{(a^2\sqrt{e})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad + \frac{(a^2e)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2d} \\
&\quad + \frac{(a^2e)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2d}
\end{aligned}
$$

$$= \frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} + \frac{2a^2(e\tan(c+dx))^{3/2}}{3de} + \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de} + \frac{(a^2\sqrt{e})\,\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{(a^2\sqrt{e})\,\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}$$

$$= -\frac{a^2\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{d\sqrt{\sin(2c+2dx)}} + \frac{2a^2(e\tan(c+dx))^{3/2}}{3de} + \frac{4a^2\cos(c+dx)(e\tan(c+dx))^{3/2}}{de}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.68 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.34

$$\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx = \frac{4a^2\cos^5\left(\frac{1}{2}(c+dx)\right)\left(1+2\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right)+\mathrm{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\right.\right.}{3d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] (4*a^2*Cos[(c + d*x)/2]^5*(1 + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*Sec[c + d*x]*Sec[ArcTan[Tan[c + d*x]]/2]^4*Sin[(c + d*x)/2]*Sqrt[e*Tan[c + d*x]])/(3*d)
```

Maple [A] (verified)

Time = 5.43 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.72

method	result
parts	$\frac{a^2 e\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)\right)}{4d\left(e^2\right)^{\frac{1}{4}}}+$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2/d*e/(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)
*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)
-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1))+2/3*a^2*(e*tan(d*x+
c))^(3/2)/d/e-2*a^2/d*2^(1/2)*(-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-
cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)
*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((c
sc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)-2*(csc(d*x+c)-cot(d*x
+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+(csc(d*x+c)-cot(d*x+
c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*E
llipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+2^(1/2)*cos(d*x+c)-2^
(1/2))*(e*tan(d*x+c))^(1/2)*csc(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx = a^2\left(\int \sqrt{e\tan{(c+dx)}}\,dx + \int 2\sqrt{e\tan{(c+dx)}}\sec{(c+dx)}\,dx + \int \sqrt{e\tan{(c+dx)}}\sec^2{(c+dx)}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*tan(c + d*x)), x) + Integral(2*sqrt(e*tan(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*tan(c + d*x))*sec(c + d*x)**2, x))
```

Maxima [F(-2)]

Exception generated.

$$\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx = \int (a\sec{(dx+c)}+a)^2\sqrt{e\tan{(dx+c)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\,dx=\int \sqrt{e\tan\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2\,dx$$

```
[In] int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2,x)
[Out] int((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2, x)
```

3.114 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 278

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx &= -\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} \\ &\quad+\frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} \\ &\quad-\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} \\ &\quad+\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} \\ &\quad+\frac{2a^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} \\ &\quad+\frac{2a^2\sqrt{e\tan(c+dx)}}{de}\end{aligned}$$

```
[Out] -1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+1
/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)-1/4
*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d*2^(1/2)/
e^(1/2)+1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
/d*2^(1/2)/e^(1/2)-2*a^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Elli
pticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/(e*tan(d
*x+c))^(1/2)+2*a^2*(e*tan(d*x+c))^(1/2)/d/e
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3971, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720, 2687, 32}

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx = -\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} + \frac{a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}d\sqrt{e}} + \frac{2a^2\sqrt{e\tan(c+dx)}}{de} - \frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}} + \frac{a^2\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}d\sqrt{e}} + \frac{2a^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{d\sqrt{e\tan(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] -((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*Sqrt[
e])) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*
Sqrt[e]) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*Sqrt[e]) + (2*a^2*EllipticF[c -
 Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(d*Sqrt[e*Tan[c + d*x]
]) + (2*a^2*Sqrt[e*Tan[c + d*x]])/(d*e)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{\sqrt{e\tan(c+dx)}}+\frac{2a^2\sec(c+dx)}{\sqrt{e\tan(c+dx)}}+\frac{a^2\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\right)dx\\
&= a^2\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx+a^2\int \frac{\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx+\left(2a^2\right)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx\\
&= \frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{ex}}\,dx,x,\tan(c+dx)\right)}{d}+\frac{\left(a^2e\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx,x,e\tan(c+dx)\right)}{d}\\
&\quad+\frac{\left(2a^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}\\
&= \frac{2a^2\sqrt{e\tan(c+dx)}}{de}+\frac{\left(2a^2e\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{d}\\
&\quad+\frac{\left(2a^2\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{\sqrt{e\tan(c+dx)}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2a^2 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}}+\frac{2a^2\sqrt{e\tan(c+dx)}}{de} \\
&\quad+\frac{a^2\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{d}+\frac{a^2\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{d} \\
&= \frac{2a^2 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} \\
&\quad+\frac{2a^2\sqrt{e\tan(c+dx)}}{de}+\frac{a^2\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2d} \\
&\quad+\frac{a^2\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2d} \\
&\quad-\frac{a^2\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} \\
&\quad-\frac{a^2\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} \\
&= -\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} \\
&\quad+\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} \\
&\quad+\frac{2a^2 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}} \\
&\quad+\frac{2a^2\sqrt{e\tan(c+dx)}}{de}+\frac{a^2\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} \\
&\quad-\frac{a^2\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} \\
&= -\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}}+\frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} \\
&\quad-\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} \\
&\quad+\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} \\
&\quad+\frac{2a^2 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d\sqrt{e\tan(c+dx)}}+\frac{2a^2\sqrt{e\tan(c+dx)}}{de}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.01 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.79

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx$$
$$=\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\arctan(\tan(c+dx))\right)\left(-2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\sqrt{2}\arctan\left(1\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Tan[c + d*x]],x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^4*Sec[ArcTan[Tan[c + d*x]]/2]^4*(-2*Sqrt[2]*ArcTan[1
- Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d
*x]]] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + Sqrt[2
]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 8*Sqrt[Tan[c + d*x]]
 + 16*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]])
*Sqrt[Tan[c + d*x]])/(4*d*Sqrt[e*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 5.46 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.97

method	result
parts	$\frac{a^2(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)\right)}{4de}$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2/d/e*(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)
*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)
-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1))+2*a^2*(e*tan(d*x+c)
)^(1/2)/d/e+2*a^2/d*2^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2
^(1/2))*(cot(d*x+c)-csc(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(csc(
d*x+c)-cot(d*x+c)+1)^(1/2)/(e*tan(d*x+c))^(1/2)*(1+sec(d*x+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx = a^2\left(\int \frac{1}{\sqrt{e\tan{(c+dx)}}}\,dx + \int \frac{2\sec{(c+dx)}}{\sqrt{e\tan{(c+dx)}}}\,dx + \int \frac{\sec^2{(c+dx)}}{\sqrt{e\tan{(c+dx)}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(1/2),x)
[Out] a**2*(Integral(1/sqrt(e*tan(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
tan(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*tan(c + d*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx=\int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\tan(dx+c)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*tan(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{e\tan(c+d\,x)}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(1/2), x)
```

3.115 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 310

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx &= \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\ &- \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\ &- \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\ &+ \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} - \frac{4a^2}{de\sqrt{e\tan(c+dx)}} \\ &- \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}}\end{aligned}$$

```
[Out] 1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/
2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)-1/4*
a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(3/2)*2
^(1/2)+1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d/e^(3/2)*2^(1/2)-4*a^2/d/e/(e*tan(d*x+c))^(1/2)-4*a^2*cos(d*x+c)/d/e/(e*ta
n(d*x+c))^(1/2)+4*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d
*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/e^2/sin(2*d
*x+2*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2688, 2695, 2652, 2719, 2687, 32}

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx = \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} - \frac{a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{3/2}} - \frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}} + \frac{a^2\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{3/2}} - \frac{4a^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}} - \frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(3/2)
) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(
3/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*d*e^(3/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(3/2)) - (4*a^2)/(d*e*Sqrt[e*Tan
[c + d*x]]) - (4*a^2*Cos[c + d*x])/(d*e*Sqrt[e*Tan[c + d*x]]) - (4*a^2*Cos[
c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(d*e^2*Sqrt[Sin
[2*c + 2*d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2}{(e\tan(c+dx))^{3/2}} + \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{3/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\right)dx$$

$$= a^2\int \frac{1}{(e\tan(c+dx))^{3/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx + (2a^2)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} + \frac{a^2\text{Subst}\left(\int \frac{1}{(ex)^{3/2}}\,dx, x, \tan(c+dx)\right)}{d} - \frac{a^2\int \sqrt{e\tan(c+dx)}\,dx}{e^2} - \frac{(4a^2)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{e^2}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{a^2\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{de} - \frac{\left(4a^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{e^2\sqrt{\sin(c+dx)}}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de} - \frac{\left(4a^2\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int \sqrt{\sin(2c+2dx)}\,dx}{e^2\sqrt{\sin(2c+2dx)}}$$

$$= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}} + \frac{a^2\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de} - \frac{a^2\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de}$$

$$
\begin{aligned}
&= -\frac{4a^2}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2de} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2de} \\
&= -\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\
&\quad + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} - \frac{4a^2}{de\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\
&= \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} - \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\
&\quad - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\
&\quad + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{3/2}} - \frac{4a^2}{de\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{4a^2\cos(c+dx)}{de\sqrt{e\tan(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{de^2\sqrt{\sin(2c+2dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.76 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.77

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx = \frac{a^2\left(-24\left(1+e^{i(c+dx)}+e^{2i(c+dx)}+e^{3i(c+dx)}\right)-3\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(-24*(1 + E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x)))
 - 3*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]
+ 6*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sq
rt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]] + 8*E^((3*I)*(c +
 d*x))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4
*I)*(c + d*x))]))/(6*d*e*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.73

method	result
parts	$\frac{2a^2e\left(-\frac{1}{e^2\sqrt{e\tan(dx+c)}}-\frac{\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e^2\left(e^2\right)^{\frac{1}{4}}}\right.}{d}$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2/d*e*(-1/e^2/(e*tan(d*x+c))^(1/2)-1/8/e^2/(e^2)^(1/4)*2^(1/2)*(ln((e*t
an(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*x+
c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/
(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(d*
x+c))^(1/2)+1)))-2*a^2/d/e/(e*tan(d*x+c))^(1/2)-2*a^2/d*2^(1/2)/e/(e*tan(d*
x+c))^(1/2)*(-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(
1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2
),1/2*2^(1/2))+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1
/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2)
,1/2*2^(1/2))-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(
1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2
),1/2*2^(1/2))*sec(d*x+c)+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d
```

```
*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+
c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+2^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx = a^2\left(\int \frac{1}{(e\tan{(c+dx)})^{\frac{3}{2}}}\,dx + \int \frac{2\sec{(c+dx)}}{(e\tan{(c+dx)})^{\frac{3}{2}}}\,dx + \int \frac{\sec^2{(c+dx)}}{(e\tan{(c+dx)})^{\frac{3}{2}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(3/2),x)
[Out] a**2*(Integral((e*tan(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*ta
n(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*tan(c + d*x))**(3/2),
x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx=\int \frac{(a\sec\left(dx+c\right)+a)^2}{(e\tan\left(dx+c\right))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{(e\tan\left(c+d\,x\right))^{3/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(3/2), x)
```

3.116 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 316

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx &= \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} \\
&- \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} \\
&+ \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} \\
&- \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{4a^2}{3de(e\tan(c+dx))^{3/2}} \\
&- \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{2a^2\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}}
\end{aligned}
$$

```
[Out] 1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1/
2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/4*
a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(5/2)*2
^(1/2)-1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/
d/e^(5/2)*2^(1/2)+2/3*a^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ell
ipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/d/e^2/(e*
tan(d*x+c))^(1/2)-4/3*a^2/d/e/(e*tan(d*x+c))^(3/2)-4/3*a^2*sec(d*x+c)/d/e/(
e*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3971, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2689, 2694, 2653, 2720, 2687, 32}

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx = {} & \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} \\ & -\frac{a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{5/2}} \\ & +\frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}} \\ & -\frac{a^2\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{5/2}} \\ & -\frac{2a^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\operatorname{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3de^2\sqrt{e\tan(c+dx)}} \\ & -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}}-\frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}}\end{aligned}$$

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(5/2)
) - (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(
5/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*d*e^(5/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqr
t[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(5/2)) - (4*a^2)/(3*d*e*(e*Tan[c
 + d*x])^(3/2)) - (4*a^2*Sec[c + d*x])/(3*d*e*(e*Tan[c + d*x])^(3/2)) - (2*
a^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*
e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2}{(e\tan(c+dx))^{5/2}} + \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{5/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\right)\,dx$$

$$= a^2\int \frac{1}{(e\tan(c+dx))^{5/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx + (2a^2)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx$$

$$= -\frac{2a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} + \frac{a^2\text{Subst}\left(\int \frac{1}{(ex)^{5/2}}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2a^2)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3e^2} - \frac{a^2\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{e^2}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{de} - \frac{\left(2a^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3e^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de} - \frac{\left(2a^2\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3e^2\sqrt{e\tan(c+dx)}}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{2a^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}} - \frac{a^2\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de^2} - \frac{a^2\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de^2}$$

$$= -\frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{2a^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}} + \frac{a^2\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} + \frac{a^2\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{a^2\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2de^2} - \frac{a^2\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2de^2}$$

$$= \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{2a^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}} - \frac{a^2\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{a^2\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}$$

$$= \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} - \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{5/2}} - \frac{4a^2}{3de(e\tan(c+dx))^{3/2}} - \frac{4a^2\sec(c+dx)}{3de(e\tan(c+dx))^{3/2}} - \frac{2a^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3de^2\sqrt{e\tan(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.82 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.71

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx = -\frac{a^2\cos^2\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}\arctan(\tan(c+dx))\right)\left(16\,\mathrm{Hypergeometric2F1}\left(-\frac{3}{4}\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/24*(a^2*Cos[(c + d*x)/2]^2*Cos[c + d*x]*Cot[(c + d*x)/2]*Sec[ArcTan[Tan[
c + d*x]]/2]^4*(16*Hypergeometric2F1[-3/4, 1/2, 1/4, -Tan[c + d*x]^2] + 16*
Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*(2*ArcTan[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Lo
g[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])*Tan[c + d*x]^(3/2)))/(d*e^2*Sqrt[e*Tan[c + d*x]
])
```

Maple [A] (verified)

Time = 5.55 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.53

method	result
parts	$\frac{2a^2e\left(-\frac{1}{3e^2(e\tan(dx+c))^{\frac{3}{2}}}-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\tan(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{}}{}\right.\right.}{8e^4}\right.}{d}$
default	$\frac{a^2\sqrt{2}\,(\cos(dx+c)+1)\left(3i\sin(dx+c)\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{\cot(dx+c)-\csc(dx+c)+1}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\,\mathrm{EllipticPi}\left(\sqrt{\mathrm{cs}}\right.\right.}{}$

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2/d*e*(-1/3/e^2/(e*tan(d*x+c))^(3/2)-1/8/e^4*(e^2)^(1/4)*2^(1/2)*(ln((e
*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*tan(d*
x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2
)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*tan(
d*x+c))^(1/2)+1)))-2/3*a^2/d/e/(e*tan(d*x+c))^(3/2)+1/3*a^2/d*2^(1/2)*(1-co
s(d*x+c))^2*(2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c)
)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(
1/2),1/2*2^(1/2))*(-cot(d*x+c)+csc(d*x+c))-(1-cos(d*x+c))^4*csc(d*x+c)^4+1)
```

```
/((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)/((1-cos(d*x+c)
)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^2*csc
(d*x+c)^2-1)^2/(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d*x+c
)))^(5/2)*csc(d*x+c)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^2}{(e\tan(c + dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^2}{(e\tan(c + dx))^{5/2}}\,dx = a^2\left(\int \frac{1}{(e\tan{(c + dx)})^{\frac{5}{2}}}\,dx + \int \frac{2\sec{(c + dx)}}{(e\tan{(c + dx)})^{\frac{5}{2}}}\,dx + \int \frac{\sec^2{(c + dx)}}{(e\tan{(c + dx)})^{\frac{5}{2}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(5/2),x)
```

```
[Out] a**2*(Integral((e*tan(c + d*x))**(-5/2), x) + Integral(2*sec(c + d*x)/(e*ta
n(c + d*x))**(5/2), x) + Integral(sec(c + d*x)**2/(e*tan(c + d*x))**(5/2),
x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + a\sec(c + dx))^2}{(e\tan(c + dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{(e\tan(c+d\,x))^{5/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(5/2), x)
```

3.117 $\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 370

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx =& -\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} \\
&+\frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} \\
&+\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} \\
&-\frac{a^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} \\
&-\frac{4a^2}{5de(e\tan(c+dx))^{5/2}}-\frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}}+\frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} \\
&+\frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}}+\frac{12a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}}
\end{aligned}
$$

```
[Out] -1/2*a^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1
/2*a^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)+1/4
*a^2*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/d/e^(7/2)*
2^(1/2)-1/4*a^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
/d/e^(7/2)*2^(1/2)+2*a^2/d/e^3/(e*tan(d*x+c))^(1/2)+12/5*a^2*cos(d*x+c)/d/e
^3/(e*tan(d*x+c))^(1/2)-12/5*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin
(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/d/
e^4/sin(2*d*x+2*c)^(1/2)-4/5*a^2/d/e/(e*tan(d*x+c))^(5/2)-4/5*a^2*sec(d*x+c
)/d/e/(e*tan(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2689, 2688, 2695, 2652, 2719, 2687, 32}

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = -\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} + \frac{a^2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}de^{7/2}} + \frac{a^2\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{7/2}} - \frac{a^2\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}de^{7/2}} + \frac{12a^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}} - \frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}}$$

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] -((a^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*e^(7/
2))) + (a^2*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*d*
e^(7/2)) + (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (a^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*d*e^(7/2)) - (4*a^2)/(5*d*e*(e*Ta
n[c + d*x])^(5/2)) - (4*a^2*Sec[c + d*x])/(5*d*e*(e*Tan[c + d*x])^(5/2)) +
(2*a^2)/(d*e^3*Sqrt[e*Tan[c + d*x]]) + (12*a^2*Cos[c + d*x])/(5*d*e^3*Sqrt[
e*Tan[c + d*x]]) + (12*a^2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e
*Tan[c + d*x]])/(5*d*e^4*Sqrt[Sin[2*c + 2*d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
```

```
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{(e\tan(c+dx))^{7/2}} + \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{7/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\right) dx \\
&= a^2 \int \frac{1}{(e\tan(c+dx))^{7/2}}\, dx + a^2 \int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\, dx + \left(2a^2\right) \int \frac{\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\, dx \\
&= -\frac{2a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} \\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{1}{(ex)^{7/2}}\, dx, x, \tan(c+dx)\right)}{d} \\
&\quad - \frac{a^2 \int \frac{1}{(e\tan(c+dx))^{3/2}}\, dx}{e^2} - \frac{\left(6a^2\right) \int \frac{\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\, dx}{5e^2} \\
&= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} \\
&\quad + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{a^2 \int \sqrt{e\tan(c+dx)}\, dx}{e^4} + \frac{\left(12a^2\right) \int \cos(c+dx)\sqrt{e\tan(c+dx)}\, dx}{5e^4} \\
&= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}} + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\, dx, x, e\tan(c+dx)\right)}{de^3} \\
&\quad + \frac{\left(12a^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right) \int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\, dx}{5e^4\sqrt{\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de^3} \\
&\quad + \frac{\left(12a^2\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int \sqrt{\sin(2c+2dx)}\,dx}{5e^4\sqrt{\sin(2c+2dx)}} \\
&= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{a^2\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de^3} \\
&\quad + \frac{a^2\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{de^3} \\
&= -\frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{a^2\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} \\
&\quad + \frac{a^2\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} \\
&\quad + \frac{a^2\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2de^3} \\
&\quad + \frac{a^2\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2de^3}
\end{aligned}
$$

$$= \frac{a^2 \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} - \frac{a^2 \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} - \frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}} + \frac{a^2\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} - \frac{a^2\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}}$$

$$= -\frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} + \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} + \frac{a^2 \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} - \frac{a^2 \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}de^{7/2}} - \frac{4a^2}{5de(e\tan(c+dx))^{5/2}} - \frac{4a^2\sec(c+dx)}{5de(e\tan(c+dx))^{5/2}} + \frac{2a^2}{de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)}{5de^3\sqrt{e\tan(c+dx)}} + \frac{12a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5de^4\sqrt{\sin(2c+2dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 14.67 (sec) , antiderivative size = 2820, normalized size of antiderivative = 7.62

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Tan[c + d*x])^(7/2),x]
```

```
[Out] (Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((7*Cot[c/2])/(10*d) - (Cot[c/
2]*Csc[c/2 + (d*x)/2]^2)/(20*d) - (3*(4*Cos[c/2] - Cos[(3*c)/2] + Cos[(5*c)
/2])*Cos[d*x]*Sec[2*c]*Sin[c/2])/(10*d*(-1 + 2*Cos[c])) - (7*Csc[c/2]*Csc[c
/2 + (d*x)/2]*Sin[(d*x)/2])/(10*d) + (Csc[c/2]*Csc[c/2 + (d*x)/2]^3*Sin[(d*
```

```
x)/2])/(20*d) - (3*(2 - 5*Cos[c] + 6*Cos[2*c] + Cos[3*c])*Sec[2*c]*Sin[d*x]
)/(20*d*(-1 + 2*Cos[c])))*Sin[c + d*x]^2*Tan[c + d*x]^2)/(e*Tan[c + d*x])^(
7/2) + ((E^((2*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*
I)*(c + d*x))]] - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d
*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*
Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c +
 d*x]^(7/2))/(16*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((
2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x]
)^(7/2)) + ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)
*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x
)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*
Tan[c + d*x]^(7/2))/(16*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x)))
)/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*T
an[c + d*x])^(7/2)) - ((-(E^((6*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan
[Sqrt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c
 + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^((3*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*
(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*C
os[c])*(e*Tan[c + d*x])^(7/2)) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sq
rt[-1 + E^((4*I)*(c + d*x))]] - 2*E^((2*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))
]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1
+ E^((2*I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a +
 a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2
*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 +
 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) - ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTa
n[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d
*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))
/(1 + E^((2*I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*
(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^((2*I)*c)*Sqrt[((-I)*(-1
 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)
))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) + ((Sqrt[-1 + E^((4*I)*(c + d*x)
)]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*E^((6*I)*c)*Sqrt[-1 + E^((2*I
)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c
+ d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c + d*x]^2*Sec[2*c]*Sec[c/2 + (d*
x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(16*d*E^((3*I)*c)*Sqrt[(
(-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(
c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) - (Cos[c + d*x]^2*(3 - 3
*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4
*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[
2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(20*d*
E^(I*(2*c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c +
 d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2))
- (Cos[c + d*x]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(1 + E
```

```
^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4,
E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*
Tan[c + d*x]^(7/2))/(20*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/
(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan
[c + d*x])^(7/2)) + (E^(I*(c - d*x))*Cos[c + d*x]^2*(3 - 3*E^((4*I)*(c + d*
x)) + E^((4*I)*d*x)*(1 + E^((4*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hyperge
ometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c/2 + (d*x)/2]
^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(8*d*Sqrt[((-I)*(-1 + E^((2*I
)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2
*Cos[c])*(e*Tan[c + d*x])^(7/2)) - (Cos[c + d*x]^2*(3 - 3*E^((4*I)*(c + d*x
)) + E^((4*I)*(c + d*x))*(1 + E^((4*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hy
pergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c/2 + (d*
x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(7/2))/(40*d*E^(I*(3*c + d*x))*
Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((
2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])^(7/2)) - (Cos[c + d*x]^2*
(-3*E^((2*I)*c)*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)*d*x)*(1 + E^((6*I)*c)
)*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(
c + d*x))])*Sec[2*c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Tan[c + d*
x]^(7/2))/(10*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2
*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(-1 + 2*Cos[c])*(e*Tan[c + d*x])
^(7/2))
```

Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.54

method	result
parts	$\frac{2a^2 e\left(-\frac{1}{5e^2(e\tan(dx+c))^{\frac{5}{2}}}+\frac{1}{e^4\sqrt{e\tan(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\tan(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\tan(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\tan(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arcta}\right.}{8e^4(e^2)^{\frac{1}{4}}}\right.}{d}$
default	$\frac{a^2\sqrt{2}\left(5i\sin(dx+c)^2\sqrt{\cot(dx+c)-\csc(dx+c)+1}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\,\sqrt{\csc(dx+c)-\cot(dx+c)+1}\ \mathrm{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(d\right.\right.}{}$

```
[In] int((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2/d*e*(-1/5/e^2/(e*tan(d*x+c))^(5/2)+1/e^4/(e*tan(d*x+c))^(1/2)+1/8/e^4
/(e^2)^(1/4)*2^(1/2)*(ln((e*tan(d*x+c)-(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2))/(e*tan(d*x+c)+(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)-2*arctan(-
2^(1/2)/(e^2)^(1/4)*(e*tan(d*x+c))^(1/2)+1)))-2/5*a^2/d/e/(e*tan(d*x+c))^(5
/2)+2/5*a^2/d*2^(1/2)/(e*tan(d*x+c))^(1/2)/e^3*(-6*(csc(d*x+c)-cot(d*x+c)+1
)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellip
ticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*(csc(d*x+c)-cot(d*x+c)+
```

```
1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elli
pticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-6*(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ell
ipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+3*(csc(d*x+c
)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c
))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+
3*2^(1/2)-2^(1/2)*cot(d*x+c)*csc(d*x+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \int \frac{\left(a\sec\left(dx+c\right)+a\right)^2}{\left(e\tan\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/(e*tan(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(e\tan\left(c+d\,x\right)\right)^{7/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(7/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e*tan(c + d*x))^(7/2), x)
```

3.118 $\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 330

$$
\begin{aligned}
\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx &= \frac{e^{11/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&\quad -\frac{e^{11/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&\quad +\frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad -\frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad +\frac{5e^6\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ad\sqrt{e\tan(c+dx)}} \\
&\quad +\frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad}-\frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad}
\end{aligned}
$$

```
[Out] 1/2*e^(11/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/2
*e^(11/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/4*e^
(11/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1
/2)-1/4*e^(11/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c)
)/a/d*2^(1/2)-5/21*e^6*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ellipt
icF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d
*x+c))^(1/2)+2/21*e^5*(21-5*sec(d*x+c))*(e*tan(d*x+c))^(1/2)/a/d-2/35*e^3*(
7-5*sec(d*x+c))*(e*tan(d*x+c))^(5/2)/a/d
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3973, 3966, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \frac{e^{11/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{11/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad} + \frac{e^{11/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad} - \frac{e^{11/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad} + \frac{5e^6\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{21ad\sqrt{e\tan(c+dx)}} + \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad}$$

```
[In] Int[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^(11/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
 - (e^(11/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
*d) + (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + S
qrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (5*e^6*EllipticF[c - Pi/4 +
 d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(21*a*d*Sqrt[e*Tan[c + d*x]])
 + (2*e^5*(21 - 5*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(21*a*d) - (2*e^3*(7
- 5*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2))/(35*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^2 \int (-a + a\sec(c+dx))(e\tan(c+dx))^{7/2}\,dx}{a^2} \\
&= -\frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} \\
&\quad - \frac{(2e^4)\int \left(-\frac{7a}{2}+\frac{5}{2}a\sec(c+dx)\right)(e\tan(c+dx))^{3/2}\,dx}{7a^2} \\
&= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} \\
&\quad - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} + \frac{(4e^6)\int \frac{-\frac{21a}{4}+\frac{5}{4}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{21a^2}
\end{aligned}
$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} + \frac{(5e^6)\int\frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{21a} - \frac{e^6\int\frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a}$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} - \frac{e^7\text{Subst}\left(\int\frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{ad} + \frac{\left(5e^6\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{21a\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$= \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} - \frac{(2e^7)\,\text{Subst}\left(\int\frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} + \frac{\left(5e^6\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int\frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{21a\sqrt{e\tan(c+dx)}}$$

$$= \frac{5e^6\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ad\sqrt{e\tan(c+dx)}} + \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} - \frac{e^6\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} - \frac{e^6\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad}$$

$$= \frac{5e^6 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ad\sqrt{e\tan(c+dx)}} + \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} + \frac{e^{11/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} + \frac{e^{11/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{e^6\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2ad} - \frac{e^6\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2ad}$$

$$= \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} + \frac{5e^6 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ad\sqrt{e\tan(c+dx)}} + \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad} - \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad} - \frac{e^{11/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{e^{11/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}$$

$$
= \frac{e^{11/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{11/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}
+ \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}
- \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}
+ \frac{5e^6\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ad\sqrt{e\tan(c+dx)}}
+ \frac{2e^5(21-5\sec(c+dx))\sqrt{e\tan(c+dx)}}{21ad}
- \frac{2e^3(7-5\sec(c+dx))(e\tan(c+dx))^{5/2}}{35ad}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 16.03 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.01

$$
\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \frac{e^5\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(1+\sqrt{\sec^2(c+dx)}\right)\sqrt{e\tan(c+dx)}\left(70\sqrt{2}\,\mathrm{arct}\right.}{}
$$

```
[In] Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^5*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*Sqrt[e*Tan[
c + d*x]]*(70*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 70*Sqrt[2]*A
rcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 35*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]] - 35*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]] + 280*Sqrt[Tan[c + d*x]] - 320*Hypergeometric2F1[-1/2, 1/4,
 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 280*Hypergeometric2F1[1/4, 1/2,
 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c + d*x]] + 40*Sqrt[Sec[c + d*x]^2]*Sqrt[Ta
n[c + d*x]] - 56*Tan[c + d*x]^(5/2) + 40*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x]^
(5/2)))/(70*a*d*(1 + Sec[c + d*x])^2*Sqrt[Tan[c + d*x]])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.64 (sec) , antiderivative size = 1026, normalized size of antiderivative = 3.11

method	result	size
default	Expression too large to display	1026

```
[In] int((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/210/a/d*2^(1/2)*(105*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*
x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(cot(d*x+c)-csc(d*x+c)+1)^(
1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*cos(d*x+c)^4-105*I*(csc(d*x+c)-cot(d*x+c
)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^
4+105*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-
csc(d*x+c))^(1/2)*cos(d*x+c)^3-105*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d
*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^3+105*(cot(d*x+c)
-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1
)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*c
os(d*x+c)^4-260*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/
2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2
),1/2*2^(1/2))*cos(d*x+c)^4+105*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)
-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2
*2^(1/2))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^4+105*(cot(d*x+c)-csc(
d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/
2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*
x+c)^3-260*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*
(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2
*2^(1/2))*cos(d*x+c)^3+105*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(
d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1
/2))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^3-252*2^(1/2)*cos(d*x+c)^3*
sin(d*x+c)+80*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+42*cos(d*x+c)*sin(d*x+c)*2^(1
/2)-30*2^(1/2)*sin(d*x+c))*(e*tan(d*x+c))^(1/2)*e^5/(cos(d*x+c)-1)^3/(cos(d
*x+c)+1)^3*sin(d*x+c)^2*tan(d*x+c)^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))**(11/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{11}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^(11/2)/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{11}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(11/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{a+a\sec(c+dx)}\,dx=\int \frac{\cos\left(c+d\,x\right)\,\left(e\tan(c+d\,x)\right)^{11/2}}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

```
[In] int((e*tan(c + d*x))^(11/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(11/2))/(a*(cos(c + d*x) + 1)), x)
```

3.119 $\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 326

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx =& -\frac{e^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\ &+ \frac{e^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\ &+ \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\ &- \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\ &+ \frac{6e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5ad\sqrt{\sin(2c+2dx)}} \\ &- \frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad}\end{aligned}$$

```
[Out] -1/2*e^(9/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/2
*e^(9/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/4*e^(
9/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2
)-1/4*e^(9/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a
/d*2^(1/2)-6/5*e^4*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)
*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a/d/sin(2*d*x+2*
c)^(1/2)-6/5*e^3*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/a/d-2/15*e^3*(5-3*sec(d*x+
c))*(e*tan(d*x+c))^(3/2)/a/d
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3973, 3966, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = &-\frac{e^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}\\ &+\frac{e^{9/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}\\ &+\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}\\ &-\frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}\\ &+\frac{6e^4\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5ad\sqrt{\sin(2c+2dx)}}\\ &-\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad}-\frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad}\end{aligned}$$

```
[In] Int[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (e^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (6*e^4*Cos[c + d*x]*Elliptic
E[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a*d*Sqrt[Sin[2*c + 2*d*x]]) -
 (6*e^3*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*a*d) - (2*e^3*(5 - 3*Sec[c
+ d*x])*(e*Tan[c + d*x])^(3/2))/(15*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
```

```
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{e^2 \int (-a + a\sec(c+dx))(e\tan(c+dx))^{5/2}\,dx}{a^2}$$

$$
\begin{aligned}
&= -\frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} \\
&\quad -\frac{(2e^4)\int\left(-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{5a^2} \\
&= -\frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} \\
&\quad -\frac{(3e^4)\int\sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{5a}+\frac{e^4\int\sqrt{e\tan(c+dx)}\,dx}{a} \\
&= -\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad}-\frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} \\
&\quad +\frac{(6e^4)\int\cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{5a}+\frac{e^5\text{Subst}\left(\int\frac{\sqrt{x}}{e^2+x^2}\,dx,x,e\tan(c+dx)\right)}{ad} \\
&= -\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad}-\frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} \\
&\quad +\frac{(2e^5)\,\text{Subst}\left(\int\frac{x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ad} \\
&\quad +\frac{\left(6e^4\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{5a\sqrt{\sin(c+dx)}} \\
&= -\frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad}-\frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} \\
&\quad -\frac{e^5\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ad} \\
&\quad +\frac{e^5\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ad} \\
&\quad +\frac{\left(6e^4\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int\sqrt{\sin(2c+2dx)}\,dx}{5a\sqrt{\sin(2c+2dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5ad\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} \\
&\quad + \frac{e^{9/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad + \frac{e^{9/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad + \frac{e^5\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad} \\
&\quad + \frac{e^5\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad - \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad + \frac{6e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5ad\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad} \\
&\quad + \frac{e^{9/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&\quad - \frac{e^{9/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{e^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&\quad + \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad - \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad + \frac{6e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5ad\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{6e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ad} - \frac{2e^3(5-3\sec(c+dx))(e\tan(c+dx))^{3/2}}{15ad}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 17.80 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.40

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \frac{4e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-1+\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right)-\text{Hy}\right.}{}$$

```
[In] Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (4*e^3*Cos[(c + d*x)/2]^2*(-1 + Hypergeometric2F1[-1/2, 3/4, 7/4, -Tan[c +
d*x]^2] - Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + Hypergeometri
c2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])
*(e*Tan[c + d*x])^(3/2))/(3*a*d*(1 + Sec[c + d*x])^2)
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.86 (sec) , antiderivative size = 1166, normalized size of antiderivative = 3.58

method	result	size
default	Expression too large to display	1166

```
[In] int((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/a/d*2^(1/2)*e^4*(-15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc
(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d
*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-15*I*(cot(d*x+c)-csc(d*x
+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*
EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c
)^3+15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc
(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-
1/2*I,1/2*2^(1/2))*cos(d*x+c)^3+15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d
*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+18*(csc(d*x+c)-
cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))
^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3-
15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c
)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,
1/2*2^(1/2))*cos(d*x+c)^3-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-cs
c(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^
(1/2))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^3-36*(csc(d*x+c)-cot(d*x+
c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*E
llipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+18*(csc(
```

```
d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(
d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*
x+c)^2-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(cs
c(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2
-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*
x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+
c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-36*(csc(d*x+c)-c
ot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^
(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+2
8*cos(d*x+c)^3*2^(1/2)-24*2^(1/2)*cos(d*x+c)^2-10*2^(1/2)*cos(d*x+c)+6*2^(1
/2))*(e*tan(d*x+c))^(1/2)*sin(d*x+c)*tan(d*x+c)^2/(cos(d*x+c)^2-1)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))**(9/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{9}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{9}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{9/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\tan(c+d\,x))^{9/2}}{a\,(\cos(c+d\,x)+1)}\,dx$$

```
[In] int((e*tan(c + d*x))^(9/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(9/2))/(a*(cos(c + d*x) + 1)), x)
```

3.120 $\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 295

$$\begin{aligned}
\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx &= -\frac{e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&+\frac{e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&-\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&+\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&-\frac{e^4\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} \\
&-\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad}
\end{aligned}$$

```
[Out] -1/2*e^(7/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/2
*e^(7/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/4*e^(
7/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2
)+1/4*e^(7/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a
/d*2^(1/2)+1/3*e^4*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(
cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d*x+c
))^(1/2)-2/3*e^3*(3-sec(d*x+c))*(e*tan(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3973, 3966, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx &= -\frac{e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\ &+ \frac{e^{7/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad} \\ &- \frac{e^{7/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad} \\ &+ \frac{e^{7/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad} \\ &- \frac{e^4\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3ad\sqrt{e\tan(c+dx)}} \\ &- \frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad}\end{aligned}$$

```
[In] Int[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) - (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) + (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) - (e^4*EllipticF[c - Pi/4 + d*
x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*a*d*Sqrt[e*Tan[c + d*x]]) - (
2*e^3*(3 - Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])/(3*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^2 \int (-a+a\sec(c+dx))(e\tan(c+dx))^{3/2}\,dx}{a^2} \\
&= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} - \frac{(2e^4)\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a^2} \\
&= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} - \frac{e^4\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a} + \frac{e^4\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} \\
&= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} + \frac{e^5\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{ad} \\
&\quad - \frac{\left(e^4\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3a\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} \\
&\quad + \frac{(2e^5)\,\mathrm{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} \\
&\quad - \frac{\left(e^4\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3a\sqrt{e\tan(c+dx)}} \\
&= -\frac{e^4\,\mathrm{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} + \frac{e^4\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} \\
&\quad + \frac{e^4\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} \\
&= -\frac{e^4\,\mathrm{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} \\
&\quad - \frac{e^{7/2}\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad - \frac{e^{7/2}\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad + \frac{e^4\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad} \\
&\quad + \frac{e^4\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad}
\end{aligned}
$$

$$= -\frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} + \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{e^4\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} - \frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad} + \frac{e^{7/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{7/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}$$

$$= -\frac{e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} + \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{e^4\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} - \frac{2e^3(3-\sec(c+dx))\sqrt{e\tan(c+dx)}}{3ad}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 22.33 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.92

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \frac{e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(1+\sqrt{\sec^2(c+dx)}\right)\left(-2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan\right.\right.}{}$$

```
[In] Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^3*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*(-2*Sqrt[2]
*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]
] + Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 8*Sqrt[Tan
```

```
[c + d*x]] + 8*Hypergeometric2F1[-1/2, 1/4, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[
c + d*x]] - 8*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2]*Sqrt[Tan[c
+ d*x]])*Sqrt[e*Tan[c + d*x]])/(2*a*d*(1 + Sec[c + d*x])^2*Sqrt[Tan[c + d*x
]])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.03 (sec) , antiderivative size = 976, normalized size of antiderivative = 3.31

method	result	size
default	Expression too large to display	976

```
[In] int((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/d*2^(1/2)*(3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c
))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+
1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-3*I*(cot(d*x+c)-csc(d*x+c)+1)^
(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*Ellipti
cPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+3*I
*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-
cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/
2*2^(1/2))*cos(d*x+c)-3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d
*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x
+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)+8*(csc(d*x+c)-cot(d*x+c)+1)^
(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellipti
cF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-3*(cot(d*x+c)-
csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)
^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*co
s(d*x+c)^2-3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+8*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(
d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)-3*(cot(d*x+c)-csc(d*x+c)+1)
^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(
1/2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)-3*(c
ot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot
(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2
^(1/2))*cos(d*x+c)+6*cos(d*x+c)*sin(d*x+c)*2^(1/2)-2*2^(1/2)*sin(d*x+c))*(e
*tan(d*x+c))^(1/2)*e^3/(cos(d*x+c)-1)^2/(cos(d*x+c)+1)^2*sin(d*x+c)^2*tan(d
*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(e\tan(c+d\,x)\right)^{7/2}}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

```
[In] int((e*tan(c + d*x))^(7/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(7/2))/(a*(cos(c + d*x) + 1)), x)
```

3.121 $\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 285

$$
\begin{aligned}
\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx &= \frac{e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&- \frac{e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&- \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&+ \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&- \frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}} \\
&+ \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad}
\end{aligned}
$$

```
[Out] 1/2*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/2*
e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/4*e^(5
/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)
+1/4*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/
d*2^(1/2)+2*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*El
lipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a/d/sin(2*d*x+2*c)^
(1/2)+2*e*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/a/d
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3973, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$
\begin{aligned}
\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx &= \frac{e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&- \frac{e^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad} \\
&- \frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad} \\
&+ \frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad} \\
&- \frac{2e^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}} \\
&+ \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad}
\end{aligned}
$$

```
[In] Int[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
- (e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
) - (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*a*d) + (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[
2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) - (2*e^2*Cos[c + d*x]*EllipticE[c
 - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a*d*Sqrt[Sin[2*c + 2*d*x]]) + (2*e
*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
```

```
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^2 \int (-a + a\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx}{a^2} \\
&= -\frac{e^2 \int \sqrt{e\tan(c+dx)}\,dx}{a} + \frac{e^2 \int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a} \\
&= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} - \frac{(2e^2)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a} \\
&\quad - \frac{e^3\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{ad}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} - \frac{(2e^3)\,\mathrm{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} \\
&\quad - \frac{\left(2e^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{a\sqrt{\sin(c+dx)}} \\
&= \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} + \frac{e^3\mathrm{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} \\
&\quad - \frac{e^3\mathrm{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} \\
&\quad - \frac{\left(2e^2\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int \sqrt{\sin(2c+2dx)}\,dx}{a\sqrt{\sin(2c+2dx)}} \\
&= -\frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} \\
&\quad - \frac{e^{5/2}\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad - \frac{e^{5/2}\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad - \frac{e^3\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad} \\
&\quad - \frac{e^3\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad} \\
&= -\frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad + \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad - \frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad} - \frac{e^{5/2}\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&\quad + \frac{e^{5/2}\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}
\end{aligned}
$$

$$= \frac{e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}$$
$$- \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}$$
$$+ \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}$$
$$- \frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}}$$
$$+ \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{ad}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 16.52 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.37

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left(\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right)-\text{Hy}\right.}{3ad(1+\text{se}}$$

```
[In] Integrate[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Csc[c + d*x]*(Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c
 + d*x]^2] - Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*(1 + Sqrt[Sec
[c + d*x]^2])*(e*Tan[c + d*x])^(5/2))/(3*a*d*(1 + Sec[c + d*x])^2)
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.64 (sec) , antiderivative size = 1100, normalized size of antiderivative = 3.86

method	result	size
default	Expression too large to display	1100

```
[In] int((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/d*2^(1/2)*(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d*x
+c)))^(5/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^3*(I*(csc(d*x+c)-cot(d*x+c)+1
)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*E
llipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*
x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2
)-I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(co
```

```
t(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1
/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-co
t(d*x+c)+1)*csc(d*x+c))^(1/2)-4*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*
x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c
)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-
1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)+2*(csc(d*x+c)-cot(d*x+c)+1)^
(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ell
ipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d
*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)-(csc(d*x+c)
-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*
x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2
))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc
(d*x+c))^(1/2)-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c)
)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^
(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(cs
c(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)-4*(1-cos(d*x+c))^2*((1-cos(d*x+c))
^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)*csc(d*x+c)^2)/(1-cos(d*x+c))^2
*sin(d*x+c)^2/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))
^(1/2)/((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)
*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1
/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\tan(c+dx))^{\frac{5}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate((e*tan(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**(5/2)/(sec(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,\left(e\tan(c+d\,x)\right)^{5/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

```
[In] int((e*tan(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(5/2))/(a*(cos(c + d*x) + 1)), x)
```

3.122 $\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 257

$$
\begin{aligned}
\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx &= \frac{e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&- \frac{e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&+ \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&- \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&+ \frac{e^2\,\mathrm{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{ad\sqrt{e\tan(c+dx)}}
\end{aligned}
$$

```
[Out] 1/2*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)-1/2*
e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)+1/4*e^(3
/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)
-1/4*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/
d*2^(1/2)-e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c
+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d*x+c))^(1
/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3973, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx &= \frac{e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\ &- \frac{e^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad} \\ &+ \frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad} \\ &- \frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad} \\ &+ \frac{e^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{ad\sqrt{e\tan(c+dx)}}\end{aligned}$$

```
[In] Int[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d)
- (e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
) + (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*
x]]])/(2*Sqrt[2]*a*d) - (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[
2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (e^2*EllipticF[c - Pi/4 + d*x,
2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(a*d*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^2 \int \frac{-a+a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a^2}\\
&= -\frac{e^2 \int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} + \frac{e^2 \int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a}\\
&= -\frac{e^3\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{ad} + \frac{\left(e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}\\
&= -\frac{(2e^3)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad}\\
&\quad + \frac{\left(e^2\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{a\sqrt{e\tan(c+dx)}}\\
&= \frac{e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{ad\sqrt{e\tan(c+dx)}}\\
&\quad - \frac{e^2\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad}\\
&\quad - \frac{e^2\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{e^2 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{ad\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{e^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad + \frac{e^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad - \frac{e^{2}\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2ad} \\
&\quad - \frac{e^{2}\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2ad} \\
&= \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad + \frac{e^2 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{ad\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{e^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&\quad + \frac{e^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&= \frac{e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&\quad + \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad + \frac{e^2 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{ad\sqrt{e\tan(c+dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.61 (sec) , antiderivative size = 1211, normalized size of antiderivative = 4.71

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\csc(c+dx)\left(\frac{8\cos(c)\cos(dx)\sec(2c)\sin^2\left(\frac{c}{2}\right)}{d}-\frac{16\cos\left(\frac{c}{2}\right)\sec(2c)\sin^3\left(\frac{c}{2}\right)\sin(dx)}{d}\right)(e\tan}{a+a\sec(c+dx)}$$

$$-\frac{2e^{-i(c+dx)}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(2c)\sec(c+dx)(e\tan(c+dx))^{3/2}}{d(a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)}$$

$$-\frac{e^{-2ic}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(e^{4ic}\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\mathrm{arct}}{2d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\tan^{\frac{3}{2}}}$$

$$-\frac{e^{-2ic}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2e^{4ic}\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\mathrm{arct}}{2d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\tan^{\frac{3}{2}}}$$

$$+\frac{e^{-i(2c+dx)}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3\left(-1+e^{4i(c+dx)}\right)+e^{4i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1-e^{4i(c+dx)}}\,\mathrm{Hypergeom}}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+}$$

$$-\frac{e^{-idx}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3-3e^{4i(c+dx)}+e^{2i(c+2dx)}\left(-1+e^{2ic}\right)\sqrt{1-e^{4i(c+dx)}}\,\mathrm{Hypergeometric2F}}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx}$$

$$-\frac{4\sqrt[4]{-1}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\mathrm{EllipticF}\left(i\mathrm{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right),-1\right)\sec^4(c+dx)(e\tan(c+dx))^{3/2}}{d(a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)\left(1+\tan^2(c+dx)\right)^{3/2}}$$

```
[In] Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^2*Csc[c + d*x]*((8*Cos[c]*Cos[d*x]*Sec[2*c]*Sin[c/2]^2)
/d - (16*Cos[c/2]*Sec[2*c]*Sin[c/2]^3*Sin[d*x])/d)*(e*Tan[c + d*x])^(3/2))/
(a + a*Sec[c + d*x]) - (2*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2
*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Se
c[c + d*x]*(e*Tan[c + d*x])^(3/2))/(d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*
Tan[c + d*x]^(3/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)
*(c + d*x)))]*(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
E^((4*I)*(c + d*x))]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*
(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)
))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(2
*d*E^((2*I)*c)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]
^(3/2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))
]*(Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] +
2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*
ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/
2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(2*d*E^((2*I)*
```

```
c)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]^(3/2)) + (S
qrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 +
(d*x)/2]^2*(3*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4
*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(3*d*E^(I*(2
*c + d*x))*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]^(3/
2)) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Co
s[c/2 + (d*x)/2]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 +
 E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4
, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*(e*Tan[c + d*x])^(3/2))/(3*d*
E^(I*d*x)*(-1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*Tan[c + d*x]^(3/2
)) - (4*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^2*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt
[Tan[c + d*x]]], -1]*Sec[c + d*x]^4*(e*Tan[c + d*x])^(3/2))/(d*(a + a*Sec[c
 + d*x])*Tan[c + d*x]^(3/2)*(1 + Tan[c + d*x]^2)^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.75 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.80

method	result
default	$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{2}\left(i\,\mathrm{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)+\mathrm{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)-2\,\mathrm{EllipticF}\left(\sqrt{\right.}\right.}{}$

```
[In] int((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2-1/2*I)/a/d*2^(1/2)*(I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1
/2*I,1/2*2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*
2^(1/2))-2*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*I*Ellip
ticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2)))*(e*tan(d*x+c))^(1/2)*(cs
c(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-cs
c(d*x+c))^(1/2)*e/(cos(d*x+c)-1)*sin(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\tan(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate((e*tan(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,\left(e\tan(c+d\,x)\right)^{3/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

```
[In] int((e*tan(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

3.123 $\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 315

$$\begin{aligned}\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx &= -\frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}\\
&\quad+\frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\
&\quad-\frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\
&\quad+\frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}}\\
&\quad-\frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}}\\
&\quad+\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade}\end{aligned}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/4*ln(
e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a/d*2^(1/2
)-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a
/d*2^(1/2)+2*e*(1-sec(d*x+c))/a/d/(e*tan(d*x+c))^(1/2)+2*cos(d*x+c)*(sin(c+
1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))
*(e*tan(d*x+c))^(1/2)/a/d/sin(2*d*x+2*c)^(1/2)+2*cos(d*x+c)*(e*tan(d*x+c))^
(3/2)/a/d/e
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3973, 3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = -\frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}-\frac{\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}+\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade}+\frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}}-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}}$$

```
[In] Int[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (2*e*(1 - Sec[c + d*x]))/(a*
d*Sqrt[e*Tan[c + d*x]]) - (2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt
[e*Tan[c + d*x]])/(a*d*Sqrt[Sin[2*c + 2*d*x]]) + (2*Cos[c + d*x]*(e*Tan[c +
 d*x])^(3/2))/(a*d*e)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^2 \int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2} \\
&= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\int \left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{\int \sqrt{e\tan(c+dx)}\,dx}{a} + \frac{\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a} \\
&= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} \\
&\quad - \frac{2\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a} + \frac{e\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{ad} \\
&= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} \\
&\quad + \frac{(2e)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} \\
&\quad - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{a\sqrt{\sin(c+dx)}} \\
&= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} \\
&\quad - \frac{e\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} \\
&\quad + \frac{e\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} \\
&\quad - \frac{\left(2\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int \sqrt{\sin(2c+2dx)}\,dx}{a\sqrt{\sin(2c+2dx)}} \\
&= \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} - \frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} \\
&\quad + \frac{\sqrt{e}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad + \frac{\sqrt{e}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad + \frac{e\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad} \\
&\quad + \frac{e\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad + \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} - \frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade} + \frac{\sqrt{e}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&\quad - \frac{\sqrt{e}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&= -\frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} \\
&\quad + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} \\
&\quad - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} + \frac{2e(1-\sec(c+dx))}{ad\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{ad\sqrt{\sin(2c+2dx)}} + \frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}}{ade}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.58 (sec) , antiderivative size = 2715, normalized size of antiderivative = 8.62

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^2*Sec[c + d*x]*((-2*Cos[c/2]*Cos[d*x]*Sec[2*c]*(4*Sin[c
/2] + Sin[(3*c)/2] + Sin[(5*c)/2]))/(d*(1 + 2*Cos[c])) - (4*Sec[c/2]*Sec[c/
2 + (d*x)/2]*Sin[(d*x)/2])/d - ((-2 - 5*Cos[c] - 6*Cos[2*c] + Cos[3*c])*Sec
[2*c]*Sin[d*x])/(d*(1 + 2*Cos[c])) - (4*Tan[c/2])/d)*Sqrt[e*Tan[c + d*x]])/
(a + a*Sec[c + d*x]) + ((E^((2*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[
Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 +
 E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)
*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d
```

```
*x]])/(2*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c
+ d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqr
t[Tan[c + d*x]]) - ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sq
rt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 +
E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*
(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*
x]])/(2*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*
(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*
Sqrt[Tan[c + d*x]]) - ((-(E^((6*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan
[Sqrt[-1 + E^((4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c +
 d*x]])/(2*d*E^((3*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*
I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x
])*Sqrt[Tan[c + d*x]]) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 +
E^((4*I)*(c + d*x))]] - 2*E^((2*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1
 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c +
 d*x]])/(2*d*E^(I*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*S
qrt[Tan[c + d*x]]) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((
4*I)*(c + d*x))]] - 2*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(
c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x
]])/(2*d*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*S
qrt[Tan[c + d*x]]) + ((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((
4*I)*(c + d*x))]] - 2*E^((6*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(
c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x
]])/(2*d*E^((3*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*S
qrt[Tan[c + d*x]]) + (Cos[c/2 + (d*x)/2]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^(
(4*I)*(c + d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeome
tric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*T
an[c + d*x]])/(3*d*E^(I*(2*c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))
/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a
*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) + (Cos[c/2 + (d*x)/2]^2*(3 - 3*E^((4*I)*
(c + d*x)) + E^((2*I)*(c + 2*d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c +
 d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec
[c + d*x]*Sqrt[e*Tan[c + d*x]])/(3*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c
 + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[
c])*(a + a*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) + (5*E^(I*(c - d*x))*Cos[c/2 +
 (d*x)/2]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*d*x)*(1 + E^((4*I)*c))*Sq
rt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c +
```

```
d*x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x]])/(6*d*Sqrt[((-I)*(-1 + E
^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(
1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) - (Cos[c/2 + (d*x)/2
]^2*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*(1 + E^((4*I)*c))*Sqrt
[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*
x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x]])/(6*d*E^(I*(3*c + d*x))*Sq
rt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*
I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqrt[Tan[c + d*x]]) + (2
*Cos[c/2 + (d*x)/2]^2*(-3*E^((2*I)*c)*(-1 + E^((4*I)*(c + d*x))) + E^((4*I)
*d*x)*(1 + E^((6*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2
, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]*Sqrt[e*Tan[c + d*x]
])/(3*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c +
 d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])*Sqrt
[Tan[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.81

method	result
default	$\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{2}\left(2i\,\mathrm{EllipticE}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticF}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticPi}\left(\sqrt{\csc(dx+c)-\cdots}\right.\right.$

```
[In] int((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2-1/2*I)/a/d*2^(1/2)*(2*I*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*
2^(1/2))-I*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-I*Ellipti
cPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi((csc(
d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+2*EllipticE((csc(d*x+c)-c
ot(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1
/2*2^(1/2)))*(e*tan(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(cot(d*x+c)+csc(d*x+c)
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{e\tan(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate((e*tan(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
[Out] Integral(sqrt(e*tan(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\tan(dx+c)}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\tan(dx+c)}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,\sqrt{e\tan(c+d\,x)}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

```
[In] int((e*tan(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

3.124 $\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 290

$$\begin{aligned}\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx &= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}}\\ &\quad-\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}}\\ &\quad+\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}}+\frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}}\\ &\quad-\frac{\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}}\end{aligned}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)+1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)-1/4*ln(
e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)/e^(1/2
)+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/
2)/e^(1/2)+1/3*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(
c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/(e*tan(d*x+c))^(
1/2)+2/3*e*(1-sec(d*x+c))/a/d/(e*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3973, 3967, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\begin{aligned}
&\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx \\
&= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad\sqrt{e}} \\
&\quad -\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad\sqrt{e}} \\
&\quad +\frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad\sqrt{e}}+\frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} \\
&\quad -\frac{\sqrt{\sin(2c+2dx)}\sec(c+dx)\operatorname{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3ad\sqrt{e\tan(c+dx)}}
\end{aligned}$$

```
[In] Int[1/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e]))
 + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e])
 - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sq
rt[2]*a*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]]/(2*Sqrt[2]*a*d*Sqrt[e]) + (2*e*(1 - Sec[c + d*x]))/(3*a*d*(e*T
an[c + d*x])^(3/2)) - (EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2
*c + 2*d*x]])/(3*a*d*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^2 \int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2}\\
&= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a^2}\\
&= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} - \frac{\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a} + \frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a}\\
&= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} + \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{ad}\\
&\quad - \frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3a\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}
\end{aligned}
$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} + \frac{(2e)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} - \frac{\left(\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3a\sqrt{e\tan(c+dx)}}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} - \frac{\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} + \frac{\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad}$$

$$= \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} - \frac{\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad} + \frac{\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}}$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} + \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}} - \frac{\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}}$$
$$- \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}}$$
$$+ \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} + \frac{2e(1-\sec(c+dx))}{3ad(e\tan(c+dx))^{3/2}}$$
$$- \frac{\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad\sqrt{e\tan(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 1253, normalized size of antiderivative = 4.32

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$$
$$= \frac{2e^{-i(c+dx)}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(2c)\sec(c+dx)\sqrt{\tan(c+dx)}}{3d(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}$$
$$+ \frac{e^{-2ic}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(e^{4ic}\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\right.}{2d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\sqrt{e}}$$
$$+ \frac{e^{-2ic}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2e^{4ic}\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\right.}{2d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\sqrt{e}}$$
$$- \frac{e^{-i(2c+dx)}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3\left(-1+e^{4i(c+dx)}\right)+e^{4i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1-e^{4i(c+dx)}}\,\text{Hyperg}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\sqrt{e\tan}}$$
$$+ \frac{e^{-idx}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3-3e^{4i(c+dx)}+e^{2i(c+2dx)}\left(-1+e^{2ic}\right)\sqrt{1-e^{4i(c+dx)}}\,\text{Hypergeometr}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))\sqrt{e\tan(c+}}$$
$$+ \frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\left(-\frac{4}{3d}+\frac{2(3-2\cos(c)+3\cos(2c))\cos(dx)\sec(2c)}{3d}+\frac{2\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}-\frac{2\sec(2c)(-2\sin(c)+3\sin(2c))}{3d}\right.}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}$$
$$+ \frac{4\sqrt[4]{-1}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right),-1\right)\sec^4(c+dx)\sqrt{\tan(c+dx)}}{3d(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}\left(1+\tan^2(c+dx)\right)^{3/2}}$$

```
[In] Integrate[1/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] (2*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E
^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[Tan[c +
 d*x]])/(3*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (
Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(E^((4*I)
*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] +
 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqr
t[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2
]^2*Sec[2*c]*Sec[c + d*x]*Sqrt[Tan[c + d*x]])/(2*d*E^((2*I)*c)*(-1 + E^((2*
I)*(c + d*x)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (Sqrt[((-I)*(-1
 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[-1 + E^((4*I)*(c
+ d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4*I)*c)*Sqrt[-1 + E
^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*
I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*S
ec[c + d*x]*Sqrt[Tan[c + d*x]])/(2*d*E^((2*I)*c)*(-1 + E^((2*I)*(c + d*x)))
*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) - (Sqrt[((-I)*(-1 + E^((2*I)*(c
 + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*(3*(-1 + E^((4*I
)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c
 + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*S
ec[c + d*x]*Sqrt[Tan[c + d*x]])/(3*d*E^(I*(2*c + d*x))*(-1 + E^((2*I)*(c +
d*x)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (Sqrt[((-I)*(-1 + E^((2
*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*(3 - 3*E^(
(4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^((4*
I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2
*c]*Sec[c + d*x]*Sqrt[Tan[c + d*x]])/(3*d*E^(I*d*x)*(-1 + E^((2*I)*(c + d*x
)))*(a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) + (Cos[c/2 + (d*x)/2]^2*Sec[
c + d*x]*(-4/(3*d) + (2*(3 - 2*Cos[c] + 3*Cos[2*c])*Cos[d*x]*Sec[2*c])/(3*d
) + (2*Sec[c/2 + (d*x)/2]^2)/(3*d) - (2*Sec[2*c]*(-2*Sin[c] + 3*Sin[2*c])*S
in[d*x])/(3*d))*Tan[c + d*x])/((a + a*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]) +
 (4*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^2*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan
[c + d*x]]], -1]*Sec[c + d*x]^4*Sqrt[Tan[c + d*x]])/(3*d*(a + a*Sec[c + d*x
])*Sqrt[e*Tan[c + d*x]]*(1 + Tan[c + d*x]^2)^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.75 (sec) , antiderivative size = 611, normalized size of antiderivative = 2.11

method	result
default	$-\frac{\sqrt{2}\left(3i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\,\operatorname{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\right.\right.}{}$

```
[In] int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/d*2^(1/2)*(3*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot
```

```
(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x
+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-
2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((
csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*(csc(d*x+c)-cot(d*x
+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1
/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*(csc(d*x+c)-co
t(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c
))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+
3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(
d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2
*I,1/2*2^(1/2))+2*(1-cos(d*x+c))^3*csc(d*x+c)^3-2*csc(d*x+c)+2*cot(d*x+c))*
(1-cos(d*x+c))/((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)/
((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/(-e/((1
-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d*x+c)))^(1/2)*csc(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx=\frac{\int \frac{1}{\sqrt{e\tan{(c+dx)}}\sec{(c+dx)}+\sqrt{e\tan{(c+dx)}}}\,dx}{a}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*tan(c + d*x))*sec(c + d*x) + sqrt(e*tan(c + d*x))), x)/a
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{(a\sec{(dx+c)}+a)\sqrt{e\tan{(dx+c)}}}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{(a\sec{(dx+c)}+a)\sqrt{e\tan{(dx+c)}}}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{\cos{(c+d\,x)}}{a\,\sqrt{e\tan{(c+d\,x)}}\,\left(\cos{(c+d\,x)}+1\right)}\,dx$$

```
[In] int(1/((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*tan(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

3.125 $\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 359

$$
\begin{aligned}
\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx &= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}} \\
&\quad - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}} - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{3/2}} \\
&\quad + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{3/2}} \\
&\quad + \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5ade^2\sqrt{\sin(2c+2dx)}} - \frac{6\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ade^3}
\end{aligned}
$$

```
[Out] 1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(3/2)*2^(1/2)-1/2*
arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(3/2)*2^(1/2)-1/4*ln(e
^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(3/2)*2^(1/2)
+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(3/2
)*2^(1/2)-2/5*(5-3*sec(d*x+c))/a/d/e/(e*tan(d*x+c))^(1/2)-6/5*cos(d*x+c)*(s
in(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(
1/2))*(e*tan(d*x+c))^(1/2)/a/d/e^2/sin(2*d*x+2*c)^(1/2)+2/5*e*(1-sec(d*x+c)
)/a/d/(e*tan(d*x+c))^(5/2)-6/5*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/a/d/e^3
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3973, 3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ade^{3/2}} - \frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ade^{3/2}} + \frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ade^{3/2}} - \frac{6\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ade^3} + \frac{6\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5ade^2\sqrt{\sin(2c+2dx)}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} + \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}}$$

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(3/2)) -
ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(3/2)) -
Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sqrt[
2]*a*d*e^(3/2)) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]]/(2*Sqrt[2]*a*d*e^(3/2)) + (2*e*(1 - Sec[c + d*x]))/(5*a*d*(e*Tan[
c + d*x])^(5/2)) - (2*(5 - 3*Sec[c + d*x]))/(5*a*d*e*Sqrt[e*Tan[c + d*x]])
+ (6*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a*d
*e^2*Sqrt[Sin[2*c + 2*d*x]]) - (6*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(5*a
*d*e^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
```

```
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{e^2 \int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx}{a^2}$$

$$
\begin{aligned}
&=\frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}}+\frac{2\int\frac{\frac{5a}{2}-\frac{3}{2}a\sec(c+dx)}{(e\tan(c+dx))^{3/2}}dx}{5a^2}\\
&=\frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}}-\frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}}\\
&\quad+\frac{4\int\left(-\frac{5a}{4}-\frac{3}{4}a\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{5a^2e^2}\\
&=\frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}}-\frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}}\\
&\quad-\frac{3\int\sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{5ae^2}-\frac{\int\sqrt{e\tan(c+dx)}\,dx}{ae^2}\\
&=\frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}}-\frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}}-\frac{6\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ade^3}\\
&\quad+\frac{6\int\cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{5ae^2}-\frac{\text{Subst}\left(\int\frac{\sqrt{x}}{e^2+x^2}dx,x,e\tan(c+dx)\right)}{ade}\\
&=\frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}}-\frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}}\\
&\quad-\frac{6\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ade^3}-\frac{2\text{Subst}\left(\int\frac{x^2}{e^2+x^4}dx,x,\sqrt{e\tan(c+dx)}\right)}{ade}\\
&\quad+\frac{\left(6\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{5ae^2\sqrt{\sin(c+dx)}}\\
&=\frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}}-\frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}}-\frac{6\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ade^3}\\
&\quad+\frac{\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}dx,x,\sqrt{e\tan(c+dx)}\right)}{ade}-\frac{\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}dx,x,\sqrt{e\tan(c+dx)}\right)}{ade}\\
&\quad+\frac{\left(6\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int\sqrt{\sin(2c+2dx)}\,dx}{5ae^2\sqrt{\sin(2c+2dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} \\
&+ \frac{6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5ade^2\sqrt{\sin(2c+2dx)}} \\
&- \frac{6\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ade^3} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{3/2}} \\
&- \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{3/2}} \\
&- \frac{\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ade} \\
&- \frac{\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ade}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{3/2}} \\
&+ \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{3/2}} + \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} \\
&- \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} + \frac{6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5ade^2\sqrt{\sin(2c+2dx)}} \\
&- \frac{6\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ade^3} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}} \\
&+ \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{3/2}} \\
&- \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{3/2}} \\
&+ \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{3/2}} \\
&+ \frac{2e(1-\sec(c+dx))}{5ad(e\tan(c+dx))^{5/2}} - \frac{2(5-3\sec(c+dx))}{5ade\sqrt{e\tan(c+dx)}} \\
&+ \frac{6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5ade^2\sqrt{\sin(2c+2dx)}} - \frac{6\cos(c+dx)(e\tan(c+dx))^{3/2}}{5ade^3}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.99 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = -\frac{4\csc(c+dx)\left(15\cot^2(c+dx)-3\cot^4(c+dx)+3\cot^4(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\tan^2\right.\right.}{}$$

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (-4*Csc[c + d*x]*(15*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 3*Cot[c + d*x]^4*H
ypergeometric2F1[-5/4, -1/2, -1/4, -Tan[c + d*x]^2] - 15*Cot[c + d*x]^2*Hyp
ergeometric2F1[-1/2, -1/4, 3/4, -Tan[c + d*x]^2] - 5*Hypergeometric2F1[1/2,
 3/4, 7/4, -Tan[c + d*x]^2] + 5*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x
]^2])*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2*Sqrt[e*Tan[c + d*x]])/(
15*a*d*e^2)
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.99 (sec) , antiderivative size = 1116, normalized size of antiderivative = 3.11

method	result	size
default	Expression too large to display	1116

```
[In] int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/a/d*2^(1/2)*(5*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*co
t(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*
x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(
d*x+c)^2-1)*csc(d*x+c))^(1/2)-5*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(
d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*
x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*((1-cos(d*x
+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)+12*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)
*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elliptic
E((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*((1-cos(d*x+
c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-6*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(
2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF(
(csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*((1-cos(d*x+c)
)^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-5*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-
2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((
csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*((1-c
```

```
os(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-5*(csc(d*x+c)-cot(d*x+c)+1)^
(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ell
ipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+
c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)+((1-cos(d*x+c))*((1
-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(1-cos(d*x+c))^4*csc(d*x+c
)^4-((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(1-
cos(d*x+c))^2*csc(d*x+c)^2-5*(1-cos(d*x+c))^2*((1-cos(d*x+c))^3*csc(d*x+c)^
3+cot(d*x+c)-csc(d*x+c))^(1/2)*csc(d*x+c)^2+5*((1-cos(d*x+c))^3*csc(d*x+c)^
3+cot(d*x+c)-csc(d*x+c))^(1/2))*(1-cos(d*x+c))/((1-cos(d*x+c))^3*csc(d*x+c)
^3+cot(d*x+c)-csc(d*x+c))^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2/(-e/((1
-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d*x+c)))^(3/2)*csc(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \frac{\int \frac{1}{(e\tan{(c+dx)})^{\frac{3}{2}}\sec{(c+dx)}+(e\tan{(c+dx)})^{\frac{3}{2}}}\,dx}{a}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
[Out] Integral(1/((e*tan(c + d*x))**(3/2)*sec(c + d*x) + (e*tan(c + d*x))**(3/2))
, x)/a
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec{(dx+c)}+a)\,(e\tan{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)}{a\,(e\tan(c+d\,x))^{3/2}\,(\cos(c+d\,x)+1)}\,dx$$

```
[In] int(1/((e*tan(c + d*x))^(3/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*tan(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

3.126 $\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 328

$$\begin{aligned}\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx &= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}} \\ &- \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{5/2}} \\ &- \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{5/2}} + \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} \\ &- \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ade^2\sqrt{e\tan(c+dx)}}\end{aligned}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(5/2)*2^(1/2)-1/2*
arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d/e^(5/2)*2^(1/2)+1/4*ln(e
^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(5/2)*2^(1/2)
-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d/e^(5/2
)*2^(1/2)-5/21*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(
c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a/d/e^2/(e*tan(d*x+c
))^(1/2)+2/7*e*(1-sec(d*x+c))/a/d/(e*tan(d*x+c))^(7/2)-2/21*(7-5*sec(d*x+c)
)/a/d/e/(e*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3973, 3967, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\begin{aligned}\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx &= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}} \\ &- \frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ade^{5/2}} + \frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ade^{5/2}} \\ &- \frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ade^{5/2}} \\ &+ \frac{5\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{21ade^2\sqrt{e\tan(c+dx)}} \\ &- \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}}\end{aligned}$$

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(5/2)) -
ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*e^(5/2)) +
Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sqrt[
2]*a*d*e^(5/2)) - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]]/(2*Sqrt[2]*a*d*e^(5/2)) + (2*e*(1 - Sec[c + d*x]))/(7*a*d*(e*Tan[
c + d*x])^(7/2)) - (2*(7 - 5*Sec[c + d*x]))/(21*a*d*e*(e*Tan[c + d*x])^(3/2
)) + (5*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(
21*a*d*e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^2 \int \frac{-a+a\sec(c+dx)}{(e\tan(c+dx))^{9/2}}\,dx}{a^2} \\
&= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} + \frac{2\int \frac{\frac{7a}{2}-\frac{5}{2}a\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{7a^2} \\
&= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{4\int \frac{-\frac{21a}{4}+\frac{5}{4}a\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{21a^2e^2} \\
&= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{21ae^2} - \frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{ae^2}
\end{aligned}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{ade} + \frac{\left(5\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{21ae^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ade} + \frac{\left(5\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{21ae^2\sqrt{e\tan(c+dx)}}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ade^2\sqrt{e\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ade^2} - \frac{\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ade^2}$$

$$= \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} + \frac{5\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ade^2\sqrt{e\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{5/2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{5/2}} - \frac{\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ade^2} - \frac{\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ade^2}$$

$$
\begin{aligned}
&= \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{5/2}} \\
&\quad - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{5/2}} \\
&\quad + \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} \\
&\quad + \frac{5\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ade^2\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}} \\
&\quad + \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ade^{5/2}} \\
&\quad + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{5/2}} \\
&\quad - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ade^{5/2}} \\
&\quad + \frac{2e(1-\sec(c+dx))}{7ad(e\tan(c+dx))^{7/2}} - \frac{2(7-5\sec(c+dx))}{21ade(e\tan(c+dx))^{3/2}} \\
&\quad + \frac{5\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21ade^2\sqrt{e\tan(c+dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.62 (sec) , antiderivative size = 1299, normalized size of antiderivative = 3.96

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{10e^{-i(c+dx)}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(2c)\sec(c+dx)\tan^{\frac{5}{2}}(c+dx)}{21d(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}$$

$$-\frac{e^{-2ic}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(e^{4ic}\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\text{arct}\right.}{2d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))(e\tan(c+}$$

$$-\frac{e^{-2ic}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2e^{4ic}\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\text{arct}\right.}{2d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))(e\tan(c+}$$

$$+\frac{e^{-i(2c+dx)}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3\left(-1+e^{4i(c+dx)}\right)+e^{4i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1-e^{4i(c+dx)}}\,\text{Hypergeom}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))(e\tan(c+dx}$$

$$-\frac{e^{-idx}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3-3e^{4i(c+dx)}+e^{2i(c+2dx)}\left(-1+e^{2ic}\right)\sqrt{1-e^{4i(c+dx)}}\,\text{Hypergeometric2F}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))(e\tan(c+dx))^{5/}}$$

$$+\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\left(\frac{40}{21d}-\frac{\csc^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6d}-\frac{2(21-10\cos(c)+21\cos(2c))\cos(dx)\sec(2c)}{21d}-\frac{13\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{14d}+\frac{\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{14d}\right.}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}$$

$$-\frac{20\sqrt[4]{-1}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right),-1\right)\sec^4(c+dx)\tan^{\frac{5}{2}}(c+dx)}{21d(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}\left(1+\tan^2(c+dx)\right)^{3/2}}$$

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (-10*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 +
 E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Tan[c + d*
x]^(5/2))/(21*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)
) - (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(E^(
(4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x)
)]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTan
h[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d
*x)/2]^2*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(2*d*E^((2*I)*c)*(-1 + E
^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) - (Sqrt[((
-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[-1 + E^((4
*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4*I)*c)*Sqrt
[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1 +
 E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^2*Sec
[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(2*d*E^((2*I)*c)*(-1 + E^((2*I)*(c +
```

```
 d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) + (Sqrt[((-I)*(-1 + E^
((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2*(3*(-1
+ E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 - E^
((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*S
ec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(3*d*E^(I*(2*c + d*x))*(-1 + E^((2
*I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) - (Sqrt[((-I)*
(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^2
*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 + E^((2*I)*c))*Sqrt
[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*
x))])*Sec[2*c]*Sec[c + d*x]*Tan[c + d*x]^(5/2))/(3*d*E^(I*d*x)*(-1 + E^((2*
I)*(c + d*x)))*(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)) + (Cos[c/2 + (d
*x)/2]^2*Sec[c + d*x]*(40/(21*d) - Csc[c/2 + (d*x)/2]^2/(6*d) - (2*(21 - 10
*Cos[c] + 21*Cos[2*c])*Cos[d*x]*Sec[2*c])/(21*d) - (13*Sec[c/2 + (d*x)/2]^2
)/(14*d) + Sec[c/2 + (d*x)/2]^4/(14*d) + (2*Sec[2*c]*(-10*Sin[c] + 21*Sin[2
*c])*Sin[d*x])/(21*d))*Tan[c + d*x]^3)/((a + a*Sec[c + d*x])*(e*Tan[c + d*x
])^(5/2)) - (20*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^2*EllipticF[I*ArcSinh[(-1)^(1
/4)*Sqrt[Tan[c + d*x]]], -1]*Sec[c + d*x]^4*Tan[c + d*x]^(5/2))/(21*d*(a +
a*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)*(1 + Tan[c + d*x]^2)^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 744, normalized size of antiderivative = 2.27

method	result
default	$\frac{\sqrt{2}\,(1-\cos(dx+c))^2 \left(42i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\ \mathrm{EllipticPi}\left(\sqrt{\csc(dx+c}\right.\right.}{}$

```
[In] int(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/84/a/d*2^(1/2)*(1-cos(d*x+c))^2*(42*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-
2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((
csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-cot(d*x+c)+csc(d*x+
c))-42*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2
)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2+1/2*I,1/2*2^(1/2))*(-cot(d*x+c)+csc(d*x+c))-3*(1-cos(d*x+c))^6*csc(d*x+
c)^6-104*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2
)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1
/2*2^(1/2))*(-cot(d*x+c)+csc(d*x+c))+42*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-
2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((
csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-cot(d*x+c)+csc(d*x+
c))+42*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*
(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/
2+1/2*I,1/2*2^(1/2))*(-cot(d*x+c)+csc(d*x+c))+33*(1-cos(d*x+c))^4*csc(d*x+c
```

```
)^4-37*(1-cos(d*x+c))^2*csc(d*x+c)^2+7)/((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(
d*x+c)-csc(d*x+c))^(1/2)/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*
csc(d*x+c))^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2/(-e/((1-cos(d*x+c))^2
*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d*x+c)))^(5/2)*csc(d*x+c)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \frac{\int \frac{1}{(e\tan(c+dx))^{\frac{5}{2}}\sec(c+dx)+(e\tan(c+dx))^{\frac{5}{2}}}\,dx}{a}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
[Out] Integral(1/((e*tan(c + d*x))**(5/2)*sec(c + d*x) + (e*tan(c + d*x))**(5/2))
, x)/a
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)}{a\,(e\tan(c+d\,x))^{5/2}\,(\cos(c+d\,x)+1)}\,dx$$

```
[In] int(1/((e*tan(c + d*x))^(5/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*tan(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

3.127 $\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 372

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^{13/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{13/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} + \frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{12e^6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}} + \frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d} + \frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d} - \frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d} + \frac{2e^3(e\tan(c+dx))^{7/2}}{7a^2d}$$

```
[Out] 1/2*e^(13/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1
/2*e^(13/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/
4*e^(13/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/
d*2^(1/2)+1/4*e^(13/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/a^2/d*2^(1/2)+12/5*e^6*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c
+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2/
d/sin(2*d*x+2*c)^(1/2)+2/3*e^5*(e*tan(d*x+c))^(3/2)/a^2/d+12/5*e^5*cos(d*x+
c)*(e*tan(d*x+c))^(3/2)/a^2/d-4/5*e^5*sec(d*x+c)*(e*tan(d*x+c))^(3/2)/a^2/d
+2/7*e^3*(e*tan(d*x+c))^(7/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3973, 3971, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2691, 2693, 2695, 2652, 2719, 2687, 32}

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}dx = &\frac{e^{13/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}\\ &-\frac{e^{13/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}\\ &-\frac{e^{13/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d}\\ &+\frac{e^{13/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d}\\ &-\frac{12e^6\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}}\\ &+\frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}\\ &-\frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}+\frac{2e^3(e\tan(c+dx))^{7/2}}{7a^2d}\end{aligned}$$

```
[In] Int[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (e^(13/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*
d) - (e^(13/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*a^2*d) - (e^(13/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(13/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (12*e^6*Cos[c + d*x
]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(5*a^2*d*Sqrt[Sin[2*c
+ 2*d*x]]) + (2*e^5*(e*Tan[c + d*x])^(3/2))/(3*a^2*d) + (12*e^5*Cos[c + d*x
]*(e*Tan[c + d*x])^(3/2))/(5*a^2*d) - (4*e^5*Sec[c + d*x]*(e*Tan[c + d*x])^
(3/2))/(5*a^2*d) + (2*e^3*(e*Tan[c + d*x])^(7/2))/(7*a^2*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^4 \int (-a+a\sec(c+dx))^2 (e\tan(c+dx))^{5/2}\,dx}{a^4}\\
&= \frac{e^4 \int \left(a^2 (e\tan(c+dx))^{5/2} - 2a^2 \sec(c+dx)(e\tan(c+dx))^{5/2} + a^2\sec^2(c+dx)(e\tan(c+dx))^{5/2}\right)}{a^4}\\
&= \frac{e^4 \int (e\tan(c+dx))^{5/2}\,dx}{a^2} + \frac{e^4 \int \sec^2(c+dx)(e\tan(c+dx))^{5/2}\,dx}{a^2}\\
&\quad - \frac{(2e^4)\int \sec(c+dx)(e\tan(c+dx))^{5/2}\,dx}{a^2}\\
&= \frac{2e^5 (e\tan(c+dx))^{3/2}}{3a^2 d} - \frac{4e^5 \sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2 d}\\
&\quad + \frac{e^4 \mathrm{Subst}\left(\int (ex)^{5/2}\,dx, x, \tan(c+dx)\right)}{a^2 d} - \frac{e^6 \int \sqrt{e\tan(c+dx)}\,dx}{a^2}\\
&\quad + \frac{(6e^6)\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{5a^2}\\
&= \frac{2e^5 (e\tan(c+dx))^{3/2}}{3a^2 d} + \frac{12e^5 \cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2 d}\\
&\quad - \frac{4e^5 \sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2 d} + \frac{2e^3 (e\tan(c+dx))^{7/2}}{7a^2 d}\\
&\quad - \frac{(12e^6)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{5a^2} - \frac{e^7 \mathrm{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{a^2 d}
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}\\
&\quad-\frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}+\frac{2e^3(e\tan(c+dx))^{7/2}}{7a^2d}\\
&\quad-\frac{(2e^7)\,\text{Subst}\left(\int\frac{x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{a^2d}\\
&\quad-\frac{\left(12e^6\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{5a^2\sqrt{\sin(c+dx)}}\\
&=\frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}\\
&\quad-\frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}+\frac{2e^3(e\tan(c+dx))^{7/2}}{7a^2d}\\
&\quad+\frac{e^7\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{a^2d}\\
&\quad-\frac{e^7\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{a^2d}\\
&\quad-\frac{\left(12e^6\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int\sqrt{\sin(2c+2dx)}\,dx}{5a^2\sqrt{\sin(2c+2dx)}}\\
&=-\frac{12e^6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}}+\frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}\\
&\quad+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}-\frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}\\
&\quad+\frac{2e^3(e\tan(c+dx))^{7/2}}{7a^2d}-\frac{e^{13/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}\\
&\quad-\frac{e^{13/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d}\\
&\quad-\frac{e^7\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2a^2d}\\
&\quad-\frac{e^7\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2a^2d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad+\frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad-\frac{12e^6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}}+\frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d} \\
&\quad+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}-\frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d} \\
&\quad+\frac{2e^3(e\tan(c+dx))^{7/2}}{7a^2d}-\frac{e^{13/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&\quad+\frac{e^{13/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&= \frac{e^{13/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}-\frac{e^{13/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&\quad-\frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad+\frac{e^{13/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad-\frac{12e^6\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}} \\
&\quad+\frac{2e^5(e\tan(c+dx))^{3/2}}{3a^2d}+\frac{12e^5\cos(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d} \\
&\quad-\frac{4e^5\sec(c+dx)(e\tan(c+dx))^{3/2}}{5a^2d}+\frac{2e^3(e\tan(c+dx))^{7/2}}{7a^2d}
\end{aligned}
$$

Mathematica [F]

$$
\int\frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx=\int\frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx
$$

```
[In] Integrate[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2,x]
[Out] Integrate[(e*Tan[c + d*x])^(13/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.19 (sec) , antiderivative size = 1181, normalized size of antiderivative = 3.17

method	result	size
default	Expression too large to display	1181

```
[In] int((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/210/a^2/d*2^(1/2)*e^6*(105*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)
-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2
*2^(1/2))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^3-105*I*(cot(d*x+c)-cs
c(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(
1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(
d*x+c)^3+105*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2
)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*
x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^4-105*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)
*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((
csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^4+105*(cot
(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d
*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*x+c)-cot(d*x+c)+1)^
(1/2)*cos(d*x+c)^4+504*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+
c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+
1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^4-252*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot
(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticF((csc(d*
x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^4+105*(cot(d*x+c)-csc(d*x+
c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*E
llipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)
^4+105*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellipt
icPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*x+c)-cot
(d*x+c)+1)^(1/2)*cos(d*x+c)^3+504*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-
cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3-252*(csc(d*x+c)-cot(d*x+c)+1)
^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellipt
icF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+105*(cot(d*x+
c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))
*cos(d*x+c)^3-292*cos(d*x+c)^4*2^(1/2)+336*cos(d*x+c)^3*2^(1/2)+10*2^(1/2)*
cos(d*x+c)^2-84*2^(1/2)*cos(d*x+c)+30*2^(1/2))*(e*tan(d*x+c))^(1/2)*sin(d*x
+c)^2*tan(d*x+c)^3/(cos(d*x+c)^2-1)^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))**(13/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^{\frac{13}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(13/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(13/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{13/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos{(c+d\,x)}^2\,(e\tan(c+d\,x))^{13/2}}{a^2\,{(\cos{(c+d\,x)}+1)}^2}\,dx$$

```
[In] int((e*tan(c + d*x))^(13/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(13/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.128 $\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 339

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^{11/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{11/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} + \frac{2e^6\,\mathrm{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}} + \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d} - \frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d} + \frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d}$$

```
[Out] 1/2*e^(11/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1
/2*e^(11/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1/
4*e^(11/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/
d*2^(1/2)-1/4*e^(11/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(
d*x+c))/a^2/d*2^(1/2)-2/3*e^6*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)
*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/d
/(e*tan(d*x+c))^(1/2)+2*e^5*(e*tan(d*x+c))^(1/2)/a^2/d-4/3*e^5*sec(d*x+c)*(
e*tan(d*x+c))^(1/2)/a^2/d+2/5*e^3*(e*tan(d*x+c))^(5/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3973, 3971, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2691, 2694, 2653, 2720, 2687, 32}

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^{11/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{11/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d} + \frac{e^{11/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d} - \frac{e^{11/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d} + \frac{2e^6\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3a^2d\sqrt{e\tan(c+dx)}} + \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d} - \frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d} + \frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d}$$

```
[In] Int[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (e^(11/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*
d) - (e^(11/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]
*a^2*d) + (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(11/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*
x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (2*e^6*EllipticF[c
- Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*Sqrt[e*Tan[c
 + d*x]]) + (2*e^5*Sqrt[e*Tan[c + d*x]])/(a^2*d) - (4*e^5*Sec[c + d*x]*Sqrt
[e*Tan[c + d*x]])/(3*a^2*d) + (2*e^3*(e*Tan[c + d*x])^(5/2))/(5*a^2*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
```

```
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^4 \int (-a + a\sec(c+dx))^2 (e\tan(c+dx))^{3/2}\,dx}{a^4} \\
&= \frac{e^4 \int \left(a^2 (e\tan(c+dx))^{3/2} - 2a^2 \sec(c+dx)(e\tan(c+dx))^{3/2} + a^2 \sec^2(c+dx)(e\tan(c+dx))^{3/2}\right)}{a^4} \\
&= \frac{e^4 \int (e\tan(c+dx))^{3/2}\,dx}{a^2} + \frac{e^4 \int \sec^2(c+dx)(e\tan(c+dx))^{3/2}\,dx}{a^2} \\
&\quad - \frac{(2e^4) \int \sec(c+dx)(e\tan(c+dx))^{3/2}\,dx}{a^2} \\
&= \frac{2e^5 \sqrt{e\tan(c+dx)}}{a^2 d} - \frac{4e^5 \sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2 d} \\
&\quad + \frac{e^4 \text{Subst}\left(\int (ex)^{3/2}\,dx, x, \tan(c+dx)\right)}{a^2 d} \\
&\quad + \frac{(2e^6) \int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a^2} - \frac{e^6 \int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a^2} \\
&= \frac{2e^5 \sqrt{e\tan(c+dx)}}{a^2 d} - \frac{4e^5 \sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2 d} + \frac{2e^3 (e\tan(c+dx))^{5/2}}{5a^2 d} \\
&\quad - \frac{e^7 \text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{a^2 d} + \frac{\left(2e^6 \sqrt{\sin(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3a^2 \sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&= \frac{2e^5 \sqrt{e\tan(c+dx)}}{a^2 d} - \frac{4e^5 \sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2 d} \\
&\quad + \frac{2e^3 (e\tan(c+dx))^{5/2}}{5a^2 d} - \frac{(2e^7)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2 d} \\
&\quad + \frac{\left(2e^6 \sec(c+dx)\sqrt{\sin(2c+2dx)}\right) \int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3a^2 \sqrt{e\tan(c+dx)}} \\
&= \frac{2e^6 \,\text{EllipticF}\left(c - \frac{\pi}{4} + dx, 2\right) \sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2 d \sqrt{e\tan(c+dx)}} \\
&\quad + \frac{2e^5 \sqrt{e\tan(c+dx)}}{a^2 d} - \frac{4e^5 \sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2 d} \\
&\quad + \frac{2e^3 (e\tan(c+dx))^{5/2}}{5a^2 d} - \frac{e^6 \text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2 d} \\
&\quad - \frac{e^6 \text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2 d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2e^6 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d} - \frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d} \\
&\quad + \frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d} + \frac{e^{11/2}\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad + \frac{e^{11/2}\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad - \frac{e^6\operatorname{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2a^2d} \\
&\quad - \frac{e^6\operatorname{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2a^2d} \\
&= \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad - \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad + \frac{2e^6 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d} - \frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d} \\
&\quad + \frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d} - \frac{e^{11/2}\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&\quad + \frac{e^{11/2}\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&= \frac{e^{11/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{11/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&\quad + \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad - \frac{e^{11/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad + \frac{2e^6 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{2e^5\sqrt{e\tan(c+dx)}}{a^2d} - \frac{4e^5\sec(c+dx)\sqrt{e\tan(c+dx)}}{3a^2d} + \frac{2e^3(e\tan(c+dx))^{5/2}}{5a^2d}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx$$

```
[In] Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2,x]
[Out] Integrate[(e*Tan[c + d*x])^(11/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.51 (sec) , antiderivative size = 1007, normalized size of antiderivative = 2.97

method	result	size
default	Expression too large to display	1007

```
[In] int((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/30/a^2/d*2^(1/2)*(15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d
*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/
2))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^3-15*I*(cot(d*x+c)-csc(d*x+c
)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*El
lipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^
3+15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellipt
icPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*x+c)-cot
(d*x+c)+1)^(1/2)*cos(d*x+c)^2-15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x
+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c
)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-15*(cot(d*x+c)-cs
c(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(
d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(
d*x+c)^3-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(
csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1
/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^3+50*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(
d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3-15*(cot(d*x+c)-csc(d*x+c)
+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*Ell
ipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2
-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+
c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I
,1/2*2^(1/2))*cos(d*x+c)^2+50*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-c
sc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(
d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+24*cos(d*x+c)^2*2^(1/2)*sin(d*x+c
```

```
)-20*cos(d*x+c)*sin(d*x+c)*2^(1/2)+6*2^(1/2)*sin(d*x+c))*(e*tan(d*x+c))^(1/
2)*e^5/(cos(d*x+c)-1)^3/(cos(d*x+c)+1)^3*sin(d*x+c)^3*tan(d*x+c)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))**(11/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^{\frac{11}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(11/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{11/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(e\tan(c+d\,x)\right)^{11/2}}{a^2\,\left(\cos\left(c+d\,x\right)+1\right)^2}\,dx$$

```
[In] int((e*tan(c + d*x))^(11/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(11/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.129 $\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 312

$$\begin{aligned}
\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx &= -\frac{e^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&+ \frac{e^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&+ \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&- \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&+ \frac{4e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}} \\
&+ \frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d} - \frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}
\end{aligned}$$

```
[Out] -1/2*e^(9/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1
/2*e^(9/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1/4
*e^(9/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*
2^(1/2)-1/4*e^(9/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x
+c))/a^2/d*2^(1/2)-4*e^4*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*P
i+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2/d/sin(
2*d*x+2*c)^(1/2)+2/3*e^3*(e*tan(d*x+c))^(3/2)/a^2/d-4*e^3*cos(d*x+c)*(e*tan
(d*x+c))^(3/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3973, 3971, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719, 2687, 32}

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = -\frac{e^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{e^{9/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d} + \frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d} - \frac{e^{9/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d} + \frac{4e^4\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}} + \frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d} - \frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}$$

```
[In] Int[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (e^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) + (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(9/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (4*e^4*Cos[c + d*x]*
EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a^2*d*Sqrt[Sin[2*c + 2*
d*x]]) + (2*e^3*(e*Tan[c + d*x])^(3/2))/(3*a^2*d) - (4*e^3*Cos[c + d*x]*(e*
Tan[c + d*x])^(3/2))/(a^2*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{e^4 \int (-a + a\sec(c+dx))^2 \sqrt{e\tan(c+dx)}\,dx}{a^4}$$

$$= \frac{e^4 \int \left(a^2\sqrt{e\tan(c+dx)} - 2a^2\sec(c+dx)\sqrt{e\tan(c+dx)} + a^2\sec^2(c+dx)\sqrt{e\tan(c+dx)}\right)\,dx}{a^4}$$

$$= \frac{e^4 \int \sqrt{e\tan(c+dx)}\,dx}{a^2} + \frac{e^4 \int \sec^2(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a^2} - \frac{(2e^4) \int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a^2}$$

$$= -\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2 d} + \frac{(4e^4) \int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a^2} + \frac{e^4\text{Subst}\left(\int \sqrt{ex}\,dx, x, \tan(c+dx)\right)}{a^2 d} + \frac{e^5\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{a^2 d}$$

$$= \frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2 d} - \frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2 d} + \frac{(2e^5)\,\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2 d} + \frac{\left(4e^4\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right) \int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{a^2\sqrt{\sin(c+dx)}}$$

$$= \frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2 d} - \frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2 d} - \frac{e^5\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2 d} + \frac{e^5\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2 d} + \frac{\left(4e^4\cos(c+dx)\sqrt{e\tan(c+dx)}\right) \int \sqrt{\sin(2c+2dx)}\,dx}{a^2\sqrt{\sin(2c+2dx)}}$$

$$
\begin{aligned}
&= \frac{4e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}} \\
&\quad+\frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d} \\
&\quad+\frac{e^{9/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad+\frac{e^{9/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad+\frac{e^5\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2a^2d} \\
&\quad+\frac{e^5\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2a^2d} \\
&= \frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad-\frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad+\frac{4e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}}+\frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d} \\
&\quad-\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}+\frac{e^{9/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&\quad-\frac{e^{9/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&= -\frac{e^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}+\frac{e^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&\quad+\frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad-\frac{e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad+\frac{4e^4\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}} \\
&\quad+\frac{2e^3(e\tan(c+dx))^{3/2}}{3a^2d}-\frac{4e^3\cos(c+dx)(e\tan(c+dx))^{3/2}}{a^2d}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx=\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx$$

```
[In] Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Integrate[(e*Tan[c + d*x])^(9/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.92 (sec) , antiderivative size = 1141, normalized size of antiderivative = 3.66

method	result	size
default	Expression too large to display	1141

```
[In] int((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a^2/d*2^(1/2)*e^4*(3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc
(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d
*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-3*I*(cot(d*x+c)-csc(d*x+
c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*E
llipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)
^2+3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d
*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/
2*I,1/2*2^(1/2))*cos(d*x+c)-3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)
-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-c
ot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)+24*(csc(d*x+c)-cot(d*x
+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-12*(csc
(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc
(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d
*x+c)^2+3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(cs
c(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2
+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x
+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c
)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+24*(csc(d*x+c)-co
t(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(
1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)-12*(
csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-
csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*co
s(d*x+c)+3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*x+c)
```

```
-cot(d*x+c)+1)^(1/2)*cos(d*x+c)+3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)
-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)-10*2^(1/2)*cos(d*x+c
)^2+12*2^(1/2)*cos(d*x+c)-2*2^(1/2))*(e*tan(d*x+c))^(1/2)*sin(d*x+c)^2*tan(
d*x+c)/(cos(d*x+c)^2-1)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))**(9/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^{\frac{9}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^{\frac{9}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(9/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{9/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2\,(e\tan(c+d\,x))^{9/2}}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int((e*tan(c + d*x))^(9/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(9/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.130 $\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 281

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx &= -\frac{e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\ &+ \frac{e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\ &- \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\ &+ \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\ &- \frac{2e^4\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{a^2d\sqrt{e\tan(c+dx)}}+\frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d}\end{aligned}$$

```
[Out] -1/2*e^(7/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1
/2*e^(7/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/4
*e^(7/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*
2^(1/2)+1/4*e^(7/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x
+c))/a^2/d*2^(1/2)+2*e^4*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Elli
pticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*t
an(d*x+c))^(1/2)+2*e^3*(e*tan(d*x+c))^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3973, 3971, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720, 2687, 32}

$$\begin{aligned}
&\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = -\frac{e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}\\
&+\frac{e^{7/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}\\
&-\frac{e^{7/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d}\\
&+\frac{e^{7/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d}\\
&-\frac{2e^4\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{a^2d\sqrt{e\tan(c+dx)}}+\frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d}
\end{aligned}$$

```
[In] Int[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) - (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(7/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (2*e^4*EllipticF[c -
 Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(a^2*d*Sqrt[e*Tan[c +
d*x]]) + (2*e^3*Sqrt[e*Tan[c + d*x]])/(a^2*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{e^4 \int \frac{(-a+a\sec(c+dx))^2}{\sqrt{e\tan(c+dx)}}\,dx}{a^4}\\
&= \frac{e^4 \int \left(\frac{a^2}{\sqrt{e\tan(c+dx)}}-\frac{2a^2\sec(c+dx)}{\sqrt{e\tan(c+dx)}}+\frac{a^2\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\right)\,dx}{a^4}\\
&= \frac{e^4 \int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a^2}+\frac{e^4 \int \frac{\sec^2(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a^2}-\frac{(2e^4) \int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{a^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^4 \text{Subst}\left(\int \frac{1}{\sqrt{ex}} \, dx, x, \tan(c+dx)\right)}{a^2 d} + \frac{e^5 \text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)} \, dx, x, e\tan(c+dx)\right)}{a^2 d} \\
&\quad - \frac{\left(2e^4 \sqrt{\sin(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}} \, dx}{a^2 \sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&= \frac{2e^3 \sqrt{e\tan(c+dx)}}{a^2 d} + \frac{(2e^5) \text{Subst}\left(\int \frac{1}{e^2+x^4} \, dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2 d} \\
&\quad - \frac{\left(2e^4 \sec(c+dx)\sqrt{\sin(2c+2dx)}\right) \int \frac{1}{\sqrt{\sin(2c+2dx)}} \, dx}{a^2 \sqrt{e\tan(c+dx)}} \\
&= -\frac{2e^4 \text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right) \sec(c+dx)\sqrt{\sin(2c+2dx)}}{a^2 d\sqrt{e\tan(c+dx)}} + \frac{2e^3 \sqrt{e\tan(c+dx)}}{a^2 d} \\
&\quad + \frac{e^4 \text{Subst}\left(\int \frac{e-x^2}{e^2+x^4} \, dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2 d} + \frac{e^4 \text{Subst}\left(\int \frac{e+x^2}{e^2+x^4} \, dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2 d} \\
&= -\frac{2e^4 \text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right) \sec(c+dx)\sqrt{\sin(2c+2dx)}}{a^2 d\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{2e^3 \sqrt{e\tan(c+dx)}}{a^2 d} - \frac{e^{7/2} \text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2} \, dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2 d} \\
&\quad - \frac{e^{7/2} \text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2} \, dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2 d} \\
&\quad + \frac{e^4 \text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2} \, dx, x, \sqrt{e\tan(c+dx)}\right)}{2a^2 d} \\
&\quad + \frac{e^4 \text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2} \, dx, x, \sqrt{e\tan(c+dx)}\right)}{2a^2 d} \\
&= -\frac{e^{7/2} \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2 d} \\
&\quad + \frac{e^{7/2} \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2 d} \\
&\quad - \frac{2e^4 \text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right) \sec(c+dx)\sqrt{\sin(2c+2dx)}}{a^2 d\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{2e^3 \sqrt{e\tan(c+dx)}}{a^2 d} + \frac{e^{7/2} \text{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, 1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2 d} \\
&\quad - \frac{e^{7/2} \text{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, 1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2 d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&\quad - \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad + \frac{e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad - \frac{2e^4\,\mathrm{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{a^2d\sqrt{e\tan(c+dx)}} + \frac{2e^3\sqrt{e\tan(c+dx)}}{a^2d}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx
$$

```
[In] Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
[Out] Integrate[(e*Tan[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.34 (sec) , antiderivative size = 961, normalized size of antiderivative = 3.42

method	result	size
default	Expression too large to display	961

```
[In] int((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/2/a^2/d*2^(1/2)*(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d
*x+c)))^(7/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^4*(I*(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)
*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(
d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1
/2)-I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(
cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2
+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-
cot(d*x+c)+1)*csc(d*x+c))^(1/2)-6*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(
d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c
)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)+(csc(d*x+c)-cot(d*x+c)+1)^
```

```
(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ell
ipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+
c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)+
(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*
x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I
,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*
x+c)+1)*csc(d*x+c))^(1/2)+4*((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d
*x+c))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))/(1-cos(d*x+c))^3*sin(d*x+c)^3/((1-co
s(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c
))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)/(
(1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx=\int \frac{(e\tan(dx+c))^{\frac{7}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx=\int \frac{\cos(c+d\,x)^2\,(e\tan(c+d\,x))^{7/2}}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int((e*tan(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(7/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.131 $\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 310

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx &= \frac{e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\ &- \frac{e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\ &- \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\ &+ \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} \\ &+ \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}}\end{aligned}$$

```
[Out] 1/2*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/
2*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/4*
e^(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*2
^(1/2)+1/4*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+
c))/a^2/d*2^(1/2)-4*e^3/a^2/d/(e*tan(d*x+c))^(1/2)+4*e^3*cos(d*x+c)/a^2/d/(
e*tan(d*x+c))^(1/2)-4*e^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*
Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2/d/sin
(2*d*x+2*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3973, 3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2688, 2695, 2652, 2719, 2687, 32}

$$\begin{aligned}\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx &= \frac{e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\ &-\frac{e^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d} \\ &-\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d} \\ &+\frac{e^{5/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d}-\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} \\ &+\frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}}+\frac{4e^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}}\end{aligned}$$

```
[In] Int[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*d
) - (e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
^2*d) - (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
+ d*x]]])/(2*Sqrt[2]*a^2*d) + (e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
 Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(a^2*d*Sqrt[e*T
an[c + d*x]]) + (4*e^3*Cos[c + d*x])/(a^2*d*Sqrt[e*Tan[c + d*x]]) + (4*e^2*
Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(a^2*d*Sqrt
[Sin[2*c + 2*d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
```

```
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^4\int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{3/2}}\,dx}{a^4}\\
&= \frac{e^4\int \left(\frac{a^2}{(e\tan(c+dx))^{3/2}}-\frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{3/2}}+\frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\right)dx}{a^4}\\
&= \frac{e^4\int \frac{1}{(e\tan(c+dx))^{3/2}}\,dx}{a^2}+\frac{e^4\int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2}-\frac{(2e^4)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2}\\
&= -\frac{2e^3}{a^2d\sqrt{e\tan(c+dx)}}+\frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}}-\frac{e^2\int \sqrt{e\tan(c+dx)}\,dx}{a^2}\\
&\quad+\frac{(4e^2)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{a^2}+\frac{e^4\text{Subst}\left(\int \frac{1}{(ex)^{3/2}}\,dx,x,\tan(c+dx)\right)}{a^2d}\\
&= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}}+\frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}}-\frac{e^3\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx,x,e\tan(c+dx)\right)}{a^2d}\\
&\quad+\frac{\left(4e^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{a^2\sqrt{\sin(c+dx)}}\\
&= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}}+\frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}}\\
&\quad-\frac{(2e^3)\,\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{a^2d}\\
&\quad+\frac{\left(4e^2\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int \sqrt{\sin(2c+2dx)}\,dx}{a^2\sqrt{\sin(2c+2dx)}}\\
&= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}}+\frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}}\\
&\quad+\frac{4e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}}\\
&\quad+\frac{e^3\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{a^2d}\\
&\quad-\frac{e^3\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{a^2d}
\end{aligned}
$$

$$= -\frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}} - \frac{e^{5/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{e^{5/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{e^3\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2a^2d} - \frac{e^3\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2a^2d}$$

$$= -\frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} + \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}} - \frac{e^{5/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{e^{5/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}$$

$$= \frac{e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} + \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{4e^3}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^3\cos(c+dx)}{a^2d\sqrt{e\tan(c+dx)}} + \frac{4e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{a^2d\sqrt{\sin(2c+2dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.36 (sec) , antiderivative size = 812, normalized size of antiderivative = 2.62

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^2(c+dx)\left(\frac{32\cos\left(\frac{c}{2}\right)\cos(dx)\sec(2c)\sin\left(\frac{c}{2}\right)}{d}+\frac{16\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{d} - \cdots\right.}{(a+a\sec(c+dx))^2}$$
$$+\frac{e^{-2ic}\left(-e^{4ic}\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{\frac{-1+e^{2i(c+\cdots}}{1+e^{2i(c+\cdots}}}\right.\right.}{d\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\tan^{\frac{5}{2}}(\cdots}$$
$$-\frac{e^{-2ic}\left(\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)-2e^{4ic}\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{\frac{-1+e^{2i(c+dx}}{1+e^{2i(c+dx)}}}\right.\right.}{d\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\tan^{\frac{5}{2}}(\cdots}$$
$$-\frac{8e^{i(c-dx)}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3-3e^{4i(c+dx)}+e^{4idx}(1+e^{4ic})\sqrt{1-e^{4i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e^{4i(c+dx)}\right)\right.}{3d\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\tan^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[(e*Tan[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^4*Csc[c + d*x]^2*((32*Cos[c/2]*Cos[d*x]*Sec[2*c]*Sin[c/
2])/d + (16*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (16*Cos[c]*Sec[2*
c]*Sin[d*x])/d + (16*Tan[c/2])/d)*(e*Tan[c + d*x])^(5/2))/(a + a*Sec[c + d*
x])^2 + ((-(E^((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^(
(4*I)*(c + d*x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))
]])*Cos[c/2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5/2))/(d
*E^((2*I)*c)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)
))]*(1 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(5/2)) -
((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2
*E^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*A
rcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2
 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5/2))/(d*E^((2*I)*c
)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^
((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(5/2)) - (8*E^(I*(c
- d*x))*Cos[c/2 + (d*x)/2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*d*x)*(1
+ E^((4*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/
4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*(e*Tan[c + d*x])^(5/2))/(3
*d*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E
^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Tan[c + d*x]^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.49 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\sqrt{2}\, e^{2} \left(i \,\mathrm{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1}, \frac{1}{2}+\frac{i}{2}, \frac{\sqrt{2}}{2}\right)-i \,\mathrm{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1}, \frac{1}{2}-\frac{i}{2}, \frac{\sqrt{2}}{2}\right)+\mathrm{EllipticPi}\left(\sqrt{\csc\right.\right.}}{}$

[In] int((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -1/2/a^2/d*2^(1/2)*e^2*(I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-8*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+4*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2)))*(e*tan(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*sin(d*x+c)/(cos(d*x+c)-1)

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{(e\tan(c+dx))^{\frac{5}{2}}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

[In] integrate((e*tan(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)

[Out] Integral((e*tan(c + d*x))**(5/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2

Maxima [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan\left(dx+c\right))^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan\left(dx+c\right))^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,(e\tan(c+d\,x))^{5/2}}{a^2\,\left(\cos\left(c+d\,x\right)+1\right)^2}\,dx$$

```
[In] int((e*tan(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(5/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.132 $\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 316

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{2e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}}$$

```
[Out] 1/2*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/
2*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1/4*
e^(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*2
^(1/2)-1/4*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+
c))/a^2/d*2^(1/2)-2/3*e^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ell
ipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*
tan(d*x+c))^(1/2)-4/3*e^3/a^2/d/(e*tan(d*x+c))^(3/2)+4/3*e^3*sec(d*x+c)/a^2
/d/(e*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3973, 3971, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2689, 2694, 2653, 2720, 2687, 32}

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d} + \frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d} - \frac{e^{3/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d} - \frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{2e^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3a^2d\sqrt{e\tan(c+dx)}}$$

```
[In] Int[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2*d
) - (e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a
^2*d) + (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
+ d*x]]])/(2*Sqrt[2]*a^2*d) - (e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] +
 Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(3*a^2*d*(e*Tan
[c + d*x])^(3/2)) + (4*e^3*Sec[c + d*x])/(3*a^2*d*(e*Tan[c + d*x])^(3/2)) +
 (2*e^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(
3*a^2*d*Sqrt[e*Tan[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
```

```
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^4 \int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{5/2}}\,dx}{a^4} \\
&= \frac{e^4 \int \left(\frac{a^2}{(e\tan(c+dx))^{5/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{5/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\right)dx}{a^4} \\
&= \frac{e^4 \int \frac{1}{(e\tan(c+dx))^{5/2}}\,dx}{a^2} + \frac{e^4 \int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2} \\
&= -\frac{2e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{(2e^2)\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3a^2} \\
&\quad - \frac{e^2 \int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a^2} + \frac{e^4\text{Subst}\left(\int \frac{1}{(ex)^{5/2}}\,dx, x, \tan(c+dx)\right)}{a^2d} \\
&= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} \\
&\quad - \frac{e^3\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{a^2d} \\
&\quad + \frac{\left(2e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3a^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} \\
&\quad - \frac{(2e^3)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2d} \\
&\quad + \frac{\left(2e^2\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3a^2\sqrt{e\tan(c+dx)}} \\
&= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} \\
&\quad + \frac{2e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{e^2\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2d} \\
&\quad - \frac{e^2\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2d}
\end{aligned}
$$

$$= -\frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{2e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}} + \frac{e^{3/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} + \frac{e^{3/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{e^2\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2a^2d} - \frac{e^2\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2a^2d}$$

$$= \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{2e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}} - \frac{e^{3/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{e^{3/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}$$

$$= \frac{e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} - \frac{e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} - \frac{4e^3}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{4e^3\sec(c+dx)}{3a^2d(e\tan(c+dx))^{3/2}} + \frac{2e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3a^2d\sqrt{e\tan(c+dx)}}$$

Mathematica [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$$

```
[In] Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
[Out] Integrate[(e*Tan[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2, x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.27 (sec) , antiderivative size = 638, normalized size of antiderivative = 2.02

method	result
default	$-\frac{\sqrt{2}\left(-\frac{e(-\cot(dx+c)+\csc(dx+c))}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\right)^{\frac{3}{2}}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^2\left(-3i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{2-2\csc(dx+c)+2\,\mathrm{co}}\right.}{}$

```
[In] int((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/6/a^2/d*2^(1/2)*(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d
*x+c)))^(3/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2*(-3*I*(csc(d*x+c)-cot(d*x
+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1
/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*I*(
csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x
+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,
1/2*2^(1/2))+10*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c
))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^
(1/2),1/2*2^(1/2))-3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(
d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+
c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*c
sc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc
(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*(1-cos(d*x+c))^3*csc(d
*x+c)^3+4*csc(d*x+c)-4*cot(d*x+c))/(1-cos(d*x+c))*sin(d*x+c)/((1-cos(d*x+c)
)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^3*csc
(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{(e\tan(c+dx))^{\frac{3}{2}}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate((e*tan(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
[Out] Integral((e*tan(c + d*x))**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)
/a**2
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(e\tan(c+d\,x)\right)^{3/2}}{a^2\,\left(\cos\left(c+d\,x\right)+1\right)^2}\,dx$$

```
[In] int((e*tan(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(3/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.133 $\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 363

$$\begin{aligned}\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\ &\quad+\frac{\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\ &\quad+\frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\ &\quad-\frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\ &\quad-\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}}+\frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} \\ &\quad+\frac{2e}{a^2d\sqrt{e\tan(c+dx)}}-\frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+dx)}} \\ &\quad-\frac{12\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}}\end{aligned}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)+1
/2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)+1/4
*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a^2/d*
2^(1/2)-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(
1/2)/a^2/d*2^(1/2)+2*e/a^2/d/(e*tan(d*x+c))^(1/2)-12/5*e*cos(d*x+c)/a^2/d/(
e*tan(d*x+c))^(1/2)+12/5*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*P
i+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/a^2/d/sin(
```

```
2*d*x+2*c)^(1/2)-4/5*e^3/a^2/d/(e*tan(d*x+c))^(5/2)+4/5*e^3*sec(d*x+c)/a^2/
d/(e*tan(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3973, 3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2689, 2688, 2695, 2652, 2719, 2687, 32}

$$\begin{aligned}\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = &-\frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d}\\ &+\frac{\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d}-\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}}\\ &+\frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}}+\frac{2e}{a^2d\sqrt{e\tan(c+dx)}}\\ &+\frac{\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d}\\ &-\frac{\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d}\\ &-\frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+dx)}}\\ &-\frac{12\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}}\end{aligned}$$

```
[In] Int[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a^2
*d)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2
]*a^2*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan
[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x
] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a^2*d) - (4*e^3)/(5*a^2*d*(e*
Tan[c + d*x])^(5/2)) + (4*e^3*Sec[c + d*x])/(5*a^2*d*(e*Tan[c + d*x])^(5/2)
) + (2*e)/(a^2*d*Sqrt[e*Tan[c + d*x]]) - (12*e*Cos[c + d*x])/(5*a^2*d*Sqrt[
e*Tan[c + d*x]]) - (12*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan
[c + d*x]])/(5*a^2*d*Sqrt[Sin[2*c + 2*d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^4 \int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{7/2}}\,dx}{a^4} \\
&= \frac{e^4 \int \left(\frac{a^2}{(e\tan(c+dx))^{7/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{7/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\right)dx}{a^4} \\
&= \frac{e^4 \int \frac{1}{(e\tan(c+dx))^{7/2}}\,dx}{a^2} + \frac{e^4 \int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{7/2}}\,dx}{a^2} \\
&= -\frac{2e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} - \frac{e^2\int \frac{1}{(e\tan(c+dx))^{3/2}}\,dx}{a^2} \\
&\quad + \frac{(6e^2)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{5a^2} + \frac{e^4\text{Subst}\left(\int \frac{1}{(ex)^{7/2}}\,dx, x, \tan(c+dx)\right)}{a^2d} \\
&= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} \\
&\quad + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{\int \sqrt{e\tan(c+dx)}\,dx}{a^2} - \frac{12\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{5a^2}
\end{aligned}
$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+dx)}} + \frac{e\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{a^2d} - \frac{\left(12\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{5a^2\sqrt{\sin(c+dx)}}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+dx)}} + \frac{(2e)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2d} - \frac{\left(12\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int \sqrt{\sin(2c+2dx)}\,dx}{5a^2\sqrt{\sin(2c+2dx)}}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+dx)}} - \frac{12\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}} - \frac{e\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2d} + \frac{e\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2d}$$

$$= -\frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} - \frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+dx)}} - \frac{12\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} + \frac{e\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2a^2d} + \frac{e\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2a^2d}$$

$$
\begin{aligned}
&= \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad - \frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+dx)}} - \frac{12\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{\sqrt{e}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&\quad - \frac{\sqrt{e}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&= -\frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} + \frac{\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d} \\
&\quad + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d} \\
&\quad - \frac{4e^3}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{4e^3\sec(c+dx)}{5a^2d(e\tan(c+dx))^{5/2}} + \frac{2e}{a^2d\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{12e\cos(c+dx)}{5a^2d\sqrt{e\tan(c+dx)}} - \frac{12\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{5a^2d\sqrt{\sin(2c+2dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 2792, normalized size of antiderivative = 7.69

$$
\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[Sqrt[e*Tan[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*((-24*Cos[c/2]*Cos[d*x]*Sec[2*c]*(4*Si
n[c/2] + Sin[(3*c)/2] + Sin[(5*c)/2]))/(5*d*(1 + 2*Cos[c])) - (56*Sec[c/2]*
Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*S
in[(d*x)/2])/(5*d) - (12*(-2 - 5*Cos[c] - 6*Cos[2*c] + Cos[3*c])*Sec[2*c]*S
```

```
in[d*x])/(5*d*(1 + 2*Cos[c])) - (56*Tan[c/2])/(5*d) + (4*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(5*d))*Sqrt[e*Tan[c + d*x]])/(a + a*Sec[c + d*x])^2 + ((E^((2*
I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]
 - 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[S
qrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)
/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^(I*c)*Sqrt[((-I)*(
-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*
x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) - ((-(E^((4*
I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]]
) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[
Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x
)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((2*I)*c)*Sqrt[((
-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c
 + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) - ((-(E
^((6*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*
x))]]) + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Arc
Tanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 +
 (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((3*I)*c)*Sq
rt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*
I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) +
((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2
*E^((2*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*A
rcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2
 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^(I*c)*Sqrt
[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)
*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) + ((
Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2*E
^((4*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Arc
Tanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 +
 (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((2*I)*c)*Sq
rt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*
I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) +
((Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] - 2
*E^((6*I)*c)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*A
rcTanh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2
 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*E^((3*I)*c)*
Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((
2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]])
+ (4*Cos[c/2 + (d*x)/2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*
(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4,
 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(
5*d*E^(I*(2*c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*
(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^
2*Sqrt[Tan[c + d*x]]) + (4*Cos[c/2 + (d*x)/2]^4*(3 - 3*E^((4*I)*(c + d*x))
+ E^((2*I)*(c + 2*d*x))*(1 + E^((2*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hyp
```

```
ergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2
*Sqrt[e*Tan[c + d*x]])/(5*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x)))
)/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a +
a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) + (2*E^(I*(c - d*x))*Cos[c/2 + (d*x)/
2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*d*x)*(1 + E^((4*I)*c))*Sqrt[1 -
E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))])
*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(d*Sqrt[((-I)*(-1 + E^((2*I)
*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*C
os[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) - (2*Cos[c/2 + (d*x)/2]^4
*(3 - 3*E^((4*I)*(c + d*x)) + E^((4*I)*(c + d*x))*(1 + E^((4*I)*c))*Sqrt[1
- E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))
])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d*x]])/(5*d*E^(I*(3*c + d*x))*Sqr
t[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I
)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2*Sqrt[Tan[c + d*x]]) + (
8*Cos[c/2 + (d*x)/2]^4*(-3*E^((2*I)*c)*(-1 + E^((4*I)*(c + d*x))) + E^((4*I
)*d*x)*(1 + E^((6*I)*c))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/
2, 3/4, 7/4, E^((4*I)*(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[e*Tan[c + d
*x]])/(5*d*E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(
c + d*x)))]*(1 + E^((2*I)*(c + d*x)))*(1 + 2*Cos[c])*(a + a*Sec[c + d*x])^2
*Sqrt[Tan[c + d*x]])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.23 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.94

method	result
default	$\frac{\sqrt{2}\,\sqrt{-\frac{e(-\cot(dx+c)+\csc(dx+c))}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)\left(5i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\right.}{}$

```
[In] int((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/a^2/d*2^(1/2)*(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d
*x+c)))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(5*I*(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)
*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*I*(csc
(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)
-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2
*2^(1/2))+24*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^
(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/
2),1/2*2^(1/2))-12*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*
x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+
1)^(1/2),1/2*2^(1/2))-5*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*c
ot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d
```

```
*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-
2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((
csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*(1-cos(d*x+c))^4*cs
c(d*x+c)^4-2*(1-cos(d*x+c))^2*csc(d*x+c)^2)/((1-cos(d*x+c))*((1-cos(d*x+c))
^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x
+c)-csc(d*x+c))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{e\tan{(c+dx)}}}{\sec^2{(c+dx)}+2\sec{(c+dx)}+1}\,dx}{a^2}$$

```
[In] integrate((e*tan(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)
[Out] Integral(sqrt(e*tan(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

Maxima [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e\tan(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{e\tan\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sqrt(e*tan(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\sqrt{e\tan\left(c+d\,x\right)}}{a^2\,\left(\cos\left(c+d\,x\right)+1\right)^2}\,dx$$

```
[In] int((e*tan(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^(1/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.134 $\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 365

$$
\begin{aligned}
&\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx \\
&= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}} \\
&\quad - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d\sqrt{e}} \\
&\quad + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d\sqrt{e}} - \frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} \\
&\quad + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} - \frac{20e\sec(c+dx)}{21a^2d(e\tan(c+dx))^{3/2}} \\
&\quad - \frac{10\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21a^2d\sqrt{e\tan(c+dx)}}
\end{aligned}
$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)+1
/2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)-1/4
*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2/d*2^(1/2)/
e^(1/2)+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a^2
/d*2^(1/2)/e^(1/2)+10/21*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Elli
pticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*t
an(d*x+c))^(1/2)-4/7*e^3/a^2/d/(e*tan(d*x+c))^(7/2)+4/7*e^3*sec(d*x+c)/a^2/
d/(e*tan(d*x+c))^(7/2)+2/3*e/a^2/d/(e*tan(d*x+c))^(3/2)-20/21*e*sec(d*x+c)/
a^2/d/(e*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3973, 3971, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2689, 2694, 2653, 2720, 2687, 32}

$$\begin{aligned}
&\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx\\
&= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}a^2d\sqrt{e}}\\
&\quad-\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}}+\frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}}+\frac{2e}{3a^2d(e\tan(c+dx))^{3/2}}\\
&\quad-\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d\sqrt{e}}\\
&\quad+\frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}a^2d\sqrt{e}}-\frac{20e\sec(c+dx)}{21a^2d(e\tan(c+dx))^{3/2}}\\
&\quad-\frac{10\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{21a^2d\sqrt{e\tan(c+dx)}}
\end{aligned}$$

```
[In] Int[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a^2*d*Sqrt[e]
)) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a^2*d*Sqrt
[e]) - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(
2*Sqrt[2]*a^2*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqr
t[e*Tan[c + d*x]]]/(2*Sqrt[2]*a^2*d*Sqrt[e]) - (4*e^3)/(7*a^2*d*(e*Tan[c +
d*x])^(7/2)) + (4*e^3*Sec[c + d*x])/(7*a^2*d*(e*Tan[c + d*x])^(7/2)) + (2*e
)/(3*a^2*d*(e*Tan[c + d*x])^(3/2)) - (20*e*Sec[c + d*x])/(21*a^2*d*(e*Tan[c
 + d*x])^(3/2)) - (10*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*
c + 2*d*x]])/(21*a^2*d*Sqrt[e*Tan[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
```

```
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{e^4 \int \frac{(-a+a\sec(c+dx))^2}{(e\tan(c+dx))^{9/2}}\,dx}{a^4} \\
&= \frac{e^4 \int \left(\frac{a^2}{(e\tan(c+dx))^{9/2}} - \frac{2a^2\sec(c+dx)}{(e\tan(c+dx))^{9/2}} + \frac{a^2\sec^2(c+dx)}{(e\tan(c+dx))^{9/2}}\right)dx}{a^4} \\
&= \frac{e^4 \int \frac{1}{(e\tan(c+dx))^{9/2}}\,dx}{a^2} + \frac{e^4 \int \frac{\sec^2(c+dx)}{(e\tan(c+dx))^{9/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{9/2}}\,dx}{a^2} \\
&= -\frac{2e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} - \frac{e^2\int \frac{1}{(e\tan(c+dx))^{5/2}}\,dx}{a^2} \\
&\quad + \frac{(10e^2)\int \frac{\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{7a^2} + \frac{e^4\text{Subst}\left(\int \frac{1}{(ex)^{9/2}}\,dx, x, \tan(c+dx)\right)}{a^2d} \\
&= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} \\
&\quad - \frac{20e\sec(c+dx)}{21a^2d(e\tan(c+dx))^{3/2}} - \frac{10\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{21a^2} + \frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a^2} \\
&= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} \\
&\quad - \frac{20e\sec(c+dx)}{21a^2d(e\tan(c+dx))^{3/2}} + \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx, x, e\tan(c+dx)\right)}{a^2d} \\
&\quad - \frac{\left(10\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{21a^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} \\
&\quad - \frac{20e\sec(c+dx)}{21a^2d(e\tan(c+dx))^{3/2}} + \frac{(2e)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a^2d} \\
&\quad - \frac{\left(10\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{21a^2\sqrt{e\tan(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} \\
&\quad - \frac{20e\sec(c+dx)}{21a^2d(e\tan(c+dx))^{3/2}} - \frac{10\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21a^2d\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{a^2d} + \frac{\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{a^2d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} \\
&\quad + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} - \frac{20e\sec(c+dx)}{21a^2d(e\tan(c+dx))^{3/2}} \\
&\quad - \frac{10\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21a^2d\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2a^2d} \\
&\quad + \frac{\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2a^2d} \\
&\quad - \frac{\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d\sqrt{e}} \\
&\quad - \frac{\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d\sqrt{e}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d\sqrt{e}} \\
&\quad + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d\sqrt{e}} - \frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}} \\
&\quad + \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} - \frac{20e\sec(c+dx)}{21a^2d(e\tan(c+dx))^{3/2}} \\
&\quad - \frac{10\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21a^2d\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}} \\
&\quad - \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}}
\end{aligned}
$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a^2d\sqrt{e}}$$
$$- \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d\sqrt{e}}$$
$$+ \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a^2d\sqrt{e}} - \frac{4e^3}{7a^2d(e\tan(c+dx))^{7/2}}$$
$$+ \frac{4e^3\sec(c+dx)}{7a^2d(e\tan(c+dx))^{7/2}} + \frac{2e}{3a^2d(e\tan(c+dx))^{3/2}} - \frac{20e\sec(c+dx)}{21a^2d(e\tan(c+dx))^{3/2}}$$
$$- \frac{10\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{21a^2d\sqrt{e\tan(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.51 (sec) , antiderivative size = 1281, normalized size of antiderivative = 3.51

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx$$
$$= \frac{40e^{-i(c+dx)}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(2c)\sec^2(c+dx)\sqrt{\tan(c+dx)}}{21d(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}$$
$$+ \frac{e^{-2ic}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(e^{4ic}\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\right.}{d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\sqrt{e}}$$
$$+ \frac{e^{-2ic}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(\sqrt{-1+e^{4i(c+dx)}}\arctan\left(\sqrt{-1+e^{4i(c+dx)}}\right)+2e^{4ic}\sqrt{-1+e^{2i(c+dx)}}\sqrt{1+e^{2i(c+dx)}}\right.}{d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\sqrt{e}}$$
$$- \frac{2e^{-i(2c+dx)}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3\left(-1+e^{4i(c+dx)}\right)+e^{4i(c+dx)}\left(-1+e^{2ic}\right)\sqrt{1-e^{4i(c+dx)}}\,\text{Hyper}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\sqrt{e\,\text{ta}}}$$
$$+ \frac{2e^{-idx}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3-3e^{4i(c+dx)}+e^{2i(c+2dx)}\left(-1+e^{2ic}\right)\sqrt{1-e^{4i(c+dx)}}\,\text{Hypergeomet}\right.}{3d\left(-1+e^{2i(c+dx)}\right)(a+a\sec(c+dx))^2\sqrt{e\tan(c}}$$
$$+ \frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\left(-\frac{104}{21d}+\frac{4(21-20\cos(c)+21\cos(2c))\cos(dx)\sec(2c)}{21d}+\frac{64\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{21d}-\frac{2\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{7d}-\frac{4\,\text{se}}{}\right.}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}$$
$$+ \frac{80\sqrt[4]{-1}\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right),-1\right)\sec^5(c+dx)\sqrt{\tan(c+dx)}}{21d(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}\left(1+\tan^2(c+dx)\right)^{3/2}}$$

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] (40*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(1 +
E^((2*I)*(c + d*x)))*Cos[c/2 + (d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[
c + d*x]])/(21*d*E^(I*(c + d*x))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]
]) + (Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(E^
((4*I)*c)*Sqrt[-1 + E^((4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x
))]] + 2*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTa
nh[Sqrt[(-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (
d*x)/2]^4*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(d*E^((2*I)*c)*(-1 +
E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (Sqrt[(
(-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*(Sqrt[-1 + E^((
4*I)*(c + d*x))]*ArcTan[Sqrt[-1 + E^((4*I)*(c + d*x))]] + 2*E^((4*I)*c)*Sqr
t[-1 + E^((2*I)*(c + d*x))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[(-1
+ E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]])*Cos[c/2 + (d*x)/2]^4*Se
c[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(d*E^((2*I)*c)*(-1 + E^((2*I)*(c
+ d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) - (2*Sqrt[((-I)*(-1 +
 E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*x)/2]^4*(3*(
-1 + E^((4*I)*(c + d*x))) + E^((4*I)*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 -
 E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*(c + d*x))]
)*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(3*d*E^(I*(2*c + d*x))*(-1 +
E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt
[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]*Cos[c/2 + (d*
x)/2]^4*(3 - 3*E^((4*I)*(c + d*x)) + E^((2*I)*(c + 2*d*x))*(-1 + E^((2*I)*c
))*Sqrt[1 - E^((4*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((4*I)*
(c + d*x))])*Sec[2*c]*Sec[c + d*x]^2*Sqrt[Tan[c + d*x]])/(3*d*E^(I*d*x)*(-1
 + E^((2*I)*(c + d*x)))*(a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (Cos
[c/2 + (d*x)/2]^4*Sec[c + d*x]^2*(-104/(21*d) + (4*(21 - 20*Cos[c] + 21*Cos
[2*c])*Cos[d*x]*Sec[2*c])/(21*d) + (64*Sec[c/2 + (d*x)/2]^2)/(21*d) - (2*Se
c[c/2 + (d*x)/2]^4)/(7*d) - (4*Sec[2*c]*(-20*Sin[c] + 21*Sin[2*c])*Sin[d*x]
)/(21*d))*Tan[c + d*x])/((a + a*Sec[c + d*x])^2*Sqrt[e*Tan[c + d*x]]) + (80
*(-1)^(1/4)*Cos[c/2 + (d*x)/2]^4*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c
+ d*x]]], -1]*Sec[c + d*x]^5*Sqrt[Tan[c + d*x]])/(21*d*(a + a*Sec[c + d*x])
^2*Sqrt[e*Tan[c + d*x]]*(1 + Tan[c + d*x]^2)^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.60 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\sqrt{2}\left(21 i \sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\,\operatorname{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1}\right.\right.}{}$

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/42/a^2/d*2^(1/2)*(21*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2
*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot
(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-21*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2
)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellipti
cPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*(1-cos(d*x+c))
^5*csc(d*x+c)^5-62*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*
x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+
1)^(1/2),1/2*2^(1/2))+21*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*
cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(
d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+21*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(
2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi
((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+26*(1-cos(d*x+c))^3
*csc(d*x+c)^3-23*csc(d*x+c)+23*cot(d*x+c))*(1-cos(d*x+c))/((1-cos(d*x+c))^3
*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)/((1-cos(d*x+c))*((1-cos(d*x+c))^
2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(
-cot(d*x+c)+csc(d*x+c)))^(1/2)*csc(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx = \frac{\int \frac{1}{\sqrt{e\tan{(c+dx)}}\sec^2{(c+dx)}+2\sqrt{e\tan{(c+dx)}}\sec{(c+dx)}+\sqrt{e\tan{(c+dx)}}}\,dx}{a^2}$$

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*tan(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*tan(c + d*x))*s
ec(c + d*x) + sqrt(e*tan(c + d*x))), x)/a**2
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{e\tan(dx+c)}}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{e\tan(dx+c)}}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*tan(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\tan(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^2}{a^2\,\sqrt{e\tan(c+d\,x)}\,(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int(1/((e*tan(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*tan(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

3.135 $\int \sqrt{a+a\sec(c+dx)}\tan^5(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \sqrt{a+a\sec(c+dx)}\tan^5(c+dx)\,dx = -\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^4d}$$

```
[Out] 2/3*(a+a*sec(d*x+c))^(3/2)/a/d+2/5*(a+a*sec(d*x+c))^(5/2)/a^2/d-6/7*(a+a*se
c(d*x+c))^(7/2)/a^3/d+2/9*(a+a*sec(d*x+c))^(9/2)/a^4/d-2*arctanh((a+a*sec(d
*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3965, 90, 52, 65, 213}

$$\int \sqrt{a+a\sec(c+dx)}\tan^5(c+dx)\,dx = \frac{2(a\sec(c+dx)+a)^{9/2}}{9a^4d} - \frac{6(a\sec(c+dx)+a)^{7/2}}{7a^3d} + \frac{2(a\sec(c+dx)+a)^{5/2}}{5a^2d} - \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} + \frac{2(a\sec(c+dx)+a)^{3/2}}{3ad} + \frac{2\sqrt{a\sec(c+dx)+a}}{d}$$

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Se
c[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a*d) + (2*(a + a*Sec[c +
 d*x])^(5/2))/(5*a^2*d) - (6*(a + a*Sec[c + d*x])^(7/2))/(7*a^3*d) + (2*(a
+ a*Sec[c + d*x])^(9/2))/(9*a^4*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4 d}$$

$$= \frac{\mathrm{Subst}\left(\int \left(-3a^2(a+ax)^{5/2}+\frac{a^2(a+ax)^{5/2}}{x}+a(a+ax)^{7/2}\right)\,dx,x,\sec(c+dx)\right)}{a^4d}$$

$$= -\frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d}+\frac{2(a+a\sec(c+dx))^{9/2}}{9a^4d}+\frac{\mathrm{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx,x,\sec(c+dx)\right)}{a^2d}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}-\frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^4d}+\frac{\mathrm{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx,x,\sec(c+dx)\right)}{ad}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3ad}+\frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}-\frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^4d}+\frac{\mathrm{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx,x,\sec(c+dx)\right)}{d}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}-\frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^4d}+\frac{a\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx,x,\sec(c+dx)\right)}{d}$$

$$= \frac{2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3ad}+\frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d}+\frac{2(a+a\sec(c+dx))^{9/2}}{9a^4d} + \frac{2\mathrm{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a+a\sec(c+dx)}\right)}{d}$$

$$= -\frac{2\sqrt{a}\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3ad} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}-\frac{6(a+a\sec(c+dx))^{7/2}}{7a^3d}+\frac{2(a+a\sec(c+dx))^{9/2}}{9a^4d}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.69

$$\int \sqrt{a+a\sec(c+dx)}\tan^5(c+dx)\,dx$$
$$= \frac{2\sqrt{a(1+\sec(c+dx))}\left(-315\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}(383-34\sec(c+dx)-132}{315d\sqrt{1+\sec(c+dx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*(-315*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt[
1 + Sec[c + d*x]]*(383 - 34*Sec[c + d*x] - 132*Sec[c + d*x]^2 + 5*Sec[c + d
*x]^3 + 35*Sec[c + d*x]^4)))/(315*d*Sqrt[1 + Sec[c + d*x]])
```

Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

method	result
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(315\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+383-34\sec(dx+c)-132\sec(dx+c)^2+5\sec(dx+c)^3+35\sec(dx+c}{315d}$

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315/d*(a*(1+sec(d*x+c)))^(1/2)*(315*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+383-34*sec(d*x+c)-132*sec(d*x+c)^2
+5*sec(d*x+c)^3+35*sec(d*x+c)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.03

$$\int \sqrt{a+a\sec(c+dx)}\tan^5(c+dx)\,dx$$
$$= \left[\frac{315\sqrt{a}\cos(dx+c)^4\log\left(-8a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8a\,\mathrm{co}\right.}{630}\right.$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/630*(315*sqrt(a)*cos(d*x + c)^4*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) + 4*(383*cos(d*x + c)^4 - 34*cos(d*x + c)^3 - 132*cos(d*
x + c)^2 + 5*cos(d*x + c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d
*cos(d*x + c)^4), 1/315*(315*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^4 +
2*(383*cos(d*x + c)^4 - 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 + 5*cos(d*x
+ c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4)]
```

Sympy [F]

$$\int \sqrt{a + a\sec(c + dx)}\tan^5(c + dx)\,dx = \int \sqrt{a\left(\sec\left(c + dx\right) + 1\right)}\tan^5\left(c + dx\right)dx$$

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**5,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \sqrt{a + a\sec(c + dx)}\tan^5(c + dx)\,dx$$
$$= \frac{315\sqrt{a}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right) + 630\sqrt{a+\frac{a}{\cos(dx+c)}} + \frac{70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{9}{2}}}{a^4} - \frac{270\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^3} + \frac{126\left(a+\frac{a}{\cos(dx+c)}\right)}{a^2}}{315\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/315*(315*sqrt(a)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos
(d*x + c)) + sqrt(a))) + 630*sqrt(a + a/cos(d*x + c)) + 70*(a + a/cos(d*x +
 c))^(9/2)/a^4 - 270*(a + a/cos(d*x + c))^(7/2)/a^3 + 126*(a + a/cos(d*x +
c))^(5/2)/a^2 + 210*(a + a/cos(d*x + c))^(3/2)/a)/d
```

Giac [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^5(c+dx)\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\tan\left(dx+c\right)^5\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(c+dx)}\tan^5(c+dx)\,dx = \int \tan(c+d\,x)^5\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(1/2), x)
```

3.136 $\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 99

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx = \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+a\sec(c+dx)}}{d}-\frac{2(a+a\sec(c+dx))^{3/2}}{3ad}+\frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}$$

```
[Out] -2/3*(a+a*sec(d*x+c))^(3/2)/a/d+2/5*(a+a*sec(d*x+c))^(5/2)/a^2/d+2*arctanh(
(a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3965, 81, 52, 65, 213}

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx = \frac{2(a\sec(c+dx)+a)^{5/2}}{5a^2d}+\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}-\frac{2(a\sec(c+dx)+a)^{3/2}}{3ad}-\frac{2\sqrt{a\sec(c+dx)+a}}{d}$$

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + a*Sec
[c + d*x]])/d - (2*(a + a*Sec[c + d*x])^(3/2))/(3*a*d) + (2*(a + a*Sec[c +
d*x])^(5/2))/(5*a^2*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2d}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$
\begin{aligned}
&= -\frac{2(a+a\sec(c+dx))^{3/2}}{3ad}+\frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}-\frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx,x,\sec(c+dx)\right)}{d}\\
&= -\frac{2\sqrt{a+a\sec(c+dx)}}{d}-\frac{2(a+a\sec(c+dx))^{3/2}}{3ad}\\
&\quad+\frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}-\frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx,x,\sec(c+dx)\right)}{d}\\
&= -\frac{2\sqrt{a+a\sec(c+dx)}}{d}-\frac{2(a+a\sec(c+dx))^{3/2}}{3ad}\\
&\quad+\frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}-\frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a+a\sec(c+dx)}\right)}{d}\\
&= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+a\sec(c+dx)}}{d}\\
&\quad-\frac{2(a+a\sec(c+dx))^{3/2}}{3ad}+\frac{2(a+a\sec(c+dx))^{5/2}}{5a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.81

$$
\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx
$$
$$
=\frac{2\sqrt{a(1+\sec(c+dx))}\left(15\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}(-17+\sec(c+dx)+3\sec^2(c
$$
$$
15d\sqrt{1+\sec(c+dx)}
$$

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*(15*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt[1
+ Sec[c + d*x]]*(-17 + Sec[c + d*x] + 3*Sec[c + d*x]^2)))/(15*d*Sqrt[1 + Se
c[c + d*x]])
```

Maple [A] (verified)

Time = 4.74 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\left(15\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+17-\sec(dx+c)-3\sec(dx+c)^2\right)}{15d}$	81

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*(a*(1+sec(d*x+c)))^(1/2)*(15*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+17-sec(d*x+c)-3*sec(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.62

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx$$

$$= \left[\frac{15\sqrt{a}\cos(dx+c)^2\log\left(-8\,a\cos(dx+c)^2 - 4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} - 8\,a\cos\right.}{30\,d\cos(dx+c)^2}\right.$$

$$-\frac{15\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos(dx+c)^2+2\left(17\cos(dx+c)^2-\cos(dx+c)-3\right)\sqrt{\frac{a\cos}{\cos}}}{15\,d\cos(dx+c)^2}$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
os(d*x + c) - a) - 4*(17*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2), -1/15*(15*sqrt(-a)*arctan(2*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
 c) + a))*cos(d*x + c)^2 + 2*(17*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx = \int \sqrt{a\left(\sec(c+dx)+1\right)}\tan^3(c+dx)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx$$
$$= -\frac{15\sqrt{a}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+30\sqrt{a+\frac{a}{\cos(dx+c)}}-\frac{6\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^2}+\frac{10\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a}}{15\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="maxima")
[Out] -1/15*(15*sqrt(a)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(
d*x + c)) + sqrt(a))) + 30*sqrt(a + a/cos(d*x + c)) - 6*(a + a/cos(d*x + c)
)^(5/2)/a^2 + 10*(a + a/cos(d*x + c))^(3/2)/a)/d
```

Giac [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx = \int \sqrt{a\sec(dx+c)+a}\tan(dx+c)^3\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(c+dx)}\tan^3(c+dx)\,dx = \int \tan(c+d\,x)^3\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(1/2), x)
```

3.137 $\int \sqrt{a+a\sec(c+dx)}\tan(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \sqrt{a+a\sec(c+dx)}\tan(c+dx)\,dx=-\frac{2\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{a+a\sec(c+dx)}}{d}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+a*sec(d*x+c))^(1/
2)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3965, 52, 65, 213}

$$\int \sqrt{a+a\sec(c+dx)}\tan(c+dx)\,dx=\frac{2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Se
c[c + d*x]])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{d} \\
&= \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} \\
&= \frac{2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} \\
&= -\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+a\sec(c+dx)}}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \sqrt{a+a\sec(c+dx)}\tan(c+dx)\,dx = \frac{\sqrt{a(1+\sec(c+dx))}\left(-2\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)+2\sqrt{1+\sec(c+dx)}\right)}{d\sqrt{1+\sec(c+dx)}}$$

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x],x]
[Out] (Sqrt[a*(1 + Sec[c + d*x])]*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + 2*Sqrt[1
 + Sec[c + d*x]]))/(d*Sqrt[1 + Sec[c + d*x]])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2\sqrt{a+a\sec(dx+c)}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	42
default	$\frac{2\sqrt{a+a\sec(dx+c)}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	42

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(2*(a+a*sec(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1
/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.61

$$\int \sqrt{a+a\sec(c+dx)}\tan(c+dx)\,dx = \left[\frac{\sqrt{a}\log\left(-8\,a\cos(dx+c)^2+4\left(2\,\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos(dx+c)-a\right)}{2\,d}\right.$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) +
4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d, (sqrt(-a)*arctan(2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))
 + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d]
```

Sympy [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan(c+dx)\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\tan\left(c+dx\right)dx$$

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \sqrt{a+a\sec(c+dx)}\tan(c+dx)\,dx = \frac{\sqrt{a}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+2\sqrt{a+\frac{a}{\cos(dx+c)}}}{d}$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="maxima")
[Out] (sqrt(a)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
 + sqrt(a))) + 2*sqrt(a + a/cos(d*x + c)))/d
```

Giac [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan(c+dx)\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\tan\left(dx+c\right)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \sqrt{a + a\sec(c + dx)}\tan(c + dx)\,dx = \frac{2\sqrt{a + \frac{a}{\cos(c+d\,x)}}}{d} - \frac{2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{d}$$

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^(1/2),x)
[Out] (2*(a + a/cos(c + d*x))^(1/2))/d - (2*a^(1/2)*atanh((a + a/cos(c + d*x))^(1
/2)/a^(1/2)))/d
```

3.138 $\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-arctanh(1/2*(a+a*sec(d*
x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3965, 88, 65, 213}

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

```
[In] Int[Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (Sqrt[2]*Sqrt[a]*
ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^2\text{Subst}\left(\int \frac{1}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= -\frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= -\frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}\\
&\quad + \frac{(2a)\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}\\
&= \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{\left(2\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)\right)\sqrt{a(1+\sec(c+dx))}}{d\sqrt{1+\sec(c+dx)}}$$

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] - Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*x]
]/Sqrt[2]])*Sqrt[a*(1 + Sec[c + d*x])])/(d*Sqrt[1 + Sec[c + d*x]])
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(2\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\right)}{d}$	92

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(1+sec(d*x+c)))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(2*arctan(
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.32

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+2\sqrt{a}\log\left(-2a\cos(dx+c)-2\sqrt{a}\sqrt{\frac{a\cos(dx+c}{\cos(dx+c}}\right.}{2d}\right.$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 2*sqr
t(a)*log(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c) - a))/d, (sqrt(2)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 2*sqr
t(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(a*cos(d*x + c) + a)))/d]
```

Sympy [F]

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cot\left(c+dx\right)dx$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x), x)
```

Maxima [F]

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\cot\left(dx+c\right)\,dx$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c), x)
```

Giac [F]

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\cot\left(dx+c\right)\,dx$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cot(c+d\,x)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)*(a + a/cos(c + d*x))^(1/2), x)
```

3.139 $\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{7\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d} + \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+7/8*arctanh(1/2*(a+a*s
ec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d+1/4*a/d/(a+a*sec(d*x+c)
)^(1/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

$= \{3965, 105, 157, 162, 65, 213\}$

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} + \frac{7\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d} + \frac{a}{4d\sqrt{a\sec(c+dx)+a}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Cot[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (7*Sqrt[a]*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(4*Sqrt[2]*d) + a/(4*d*Sqrt
[a + a*Sec[c + d*x]]) + a/(2*d*(1 - Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a^4\text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} - \frac{a\text{Subst}\left(\int \frac{2a^2+\frac{3a^2x}{2}}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{2d}\\
&= \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{-2a^4+\frac{a^4x}{4}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{2a^2d}\\
&= \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}}\\
&\quad + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}\\
&\quad - \frac{(7a^2)\,\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{8d}
\end{aligned}
$$

$$= \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} - \frac{(7a)\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{4d}$$

$$= -\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{7\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d} + \frac{a}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a}{2d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.55 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.66

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{\cot^2(c+dx)\left(-2-7\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))+8\,\text{Hypergeo}\right.}{4d}$$

```
[In] Integrate[Cot[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
[Out] (Cot[c + d*x]^2*(-2 - 7*Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/
2]*(-1 + Sec[c + d*x]) + 8*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]
]*(-1 + Sec[c + d*x]))*Sqrt[a*(1 + Sec[c + d*x])])/(4*d)
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(-7\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)-16\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+6\cot(dx\right.}{8d}$

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/8/d*(a*(1+sec(d*x+c)))^(1/2)*(-7*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-16*arctan((-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+6*cot(d*x+
c)^2-2*cot(d*x+c)*csc(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.25

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \left[\frac{8\left(\cos\left(dx+c\right)^2-1\right)\sqrt{a}\log\left(-8\,a\cos\left(dx+c\right)^2+4\left(2\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\right.}{}\right.$$
$$-\frac{7\left(\sqrt{2}\cos\left(dx+c\right)^2-\sqrt{2}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-8\left(\cos\left(dx+c\right)^2-1\right)\sqrt{-a}\arctan}{8\left(d\cos\left(dx+c\right)^2-d\right)}$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*(cos(d*x + c)^2 - 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*
x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) -
8*a*cos(d*x + c) - a) + 7*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(a)*log((2
*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 3*a
*cos(d*x + c) + a)/(cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^2 - cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - d), -1/8*(7*
(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 8*(co
s(d*x + c)^2 - 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(3*cos(d*x + c)^2 - cos(
d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - d)]
```

Sympy [F]

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cot^3\left(c+dx\right)dx$$

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**3, x)
```

Maxima [F]

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\cot\left(dx+c\right)^3\,dx$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^3, x)
```

Giac [F]

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\cot\left(dx+c\right)^3\,dx$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cot(c+d\,x)^3\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(1/2), x)
```

3.140 $\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 193

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{107\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} + \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} - \frac{15a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} - \frac{21a}{64d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 43/96*a^2/d/(a+a*sec(d*x+c))^(3/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+
c))^(3/2)-15/16*a^2/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)+2*arctanh((a+a*
sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-107/128*arctanh(1/2*(a+a*sec(d*x+c))^(
1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d-21/64*a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3965, 105, 156, 157, 162, 65, 213}

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{43a^2}{96d(a\sec(c+dx)+a)^{3/2}} - \frac{15a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{3/2}} + \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} - \frac{107\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} - \frac{21a}{64d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Cot[c + d*x]^5*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (107*Sqrt[a]*ArcT
anh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(64*Sqrt[2]*d) + (43*a^2)/
(96*d*(a + a*Sec[c + d*x])^(3/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a + a*Se
c[c + d*x])^(3/2)) - (15*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])
^(3/2)) - (21*a)/(64*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{a^6 \text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} - \frac{a^3\text{Subst}\left(\int \frac{4a^2+\frac{7a^2x}{2}}{x(-a+ax)^2(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} - \frac{15a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{8a^4+\frac{75a^4x}{4}}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{8d}$$

$$= \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} - \frac{15a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{-24a^6-\frac{129a^6x}{8}}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{24a^3d}$$

$$= \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} - \frac{15a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} - \frac{21a}{64d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{24a^8-\frac{63a^8x}{16}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{24a^6d}$$

$$= \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} - \frac{15a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} - \frac{21a}{64d\sqrt{a+a\sec(c+dx)}} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{(107a^2)\,\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{128d}$$

$$= \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} - \frac{15a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} - \frac{21a}{64d\sqrt{a+a\sec(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} + \frac{(107a)\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{64d}$$

$$= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{107\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} + \frac{43a^2}{96d(a+a\sec(c+dx))^{3/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{3/2}} - \frac{15a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} - \frac{21a}{64d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.59 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.53

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{\cot^4(c+dx)\left(-2\left(57+32\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1+\sec(c+dx)\right)(-1+\sec(c+dx))^2-45\sec(c\right.\right.}{}$$

```
[In] Integrate[Cot[c + d*x]^5*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]^4*(-2*(57 + 32*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d
*x]]*(-1 + Sec[c + d*x])^2 - 45*Sec[c + d*x]) + 107*Hypergeometric2F1[-3/2,
 1, -1/2, (1 + Sec[c + d*x])/2]*(-1 + Sec[c + d*x])^2)*Sqrt[a*(1 + Sec[c +
d*x])])/(96*d)
```

Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.90

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(321\sqrt{2}\,\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+768\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+410\cot\right.}{384d}$

```
[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/384/d*(a*(1+sec(d*x+c)))^(1/2)*(321*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+768*arctan((-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+410*co
t(d*x+c)^4-142*cot(d*x+c)^3*csc(d*x+c)-298*cot(d*x+c)^2*csc(d*x+c)^2+126*cs
c(d*x+c)^3*cot(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.74

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\left[\frac{384\left(\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2+1\right)\sqrt{a}\log\left(-8\,a\cos\left(dx+c\right)^2-4\left(2\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\right.}{}\right.$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/768*(384*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sqrt(a)*log(-8*a*cos(d*
x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 321*(sqrt(2)*cos(d*x + c)^4 -
 2*sqrt(2)*cos(d*x + c)^2 + sqrt(2))*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos
(d*x + c) - 1)) - 4*(205*cos(d*x + c)^4 - 71*cos(d*x + c)^3 - 149*cos(d*x +
 c)^2 + 63*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d), 1/384*(321*(sqrt(2)*cos(d*x + c)^4 - 2*
sqrt(2)*cos(d*x + c)^2 + sqrt(2))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 384*(c
os(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(205
*cos(d*x + c)^4 - 71*cos(d*x + c)^3 - 149*cos(d*x + c)^2 + 63*cos(d*x + c))
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4 - 2*d*cos(d*x +
 c)^2 + d)]
```

Sympy [F]

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\,(\sec{(c+dx)}+1)}\cot^5{(c+dx)}\,dx$$

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**5, x)
```

Maxima [F(-1)]

Timed out.

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec{(dx+c)}+a}\cot{(dx+c)}^5\,dx$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^5(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cot(c+d\,x)^5\,\sqrt{a+\frac{a}{\cos{(c+d\,x)}}}\,dx$$

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(1/2), x)
```

3.141 $\int \sqrt{a+a\sec(c+dx)}\tan^6(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 222

$$\int \sqrt{a+a\sec(c+dx)}\tan^6(c+dx)\,dx = -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^3\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{2a^4\tan^7(c+dx)}{d(a+a\sec(c+dx))^{7/2}} + \frac{10a^5\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}} + \frac{2a^6\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+2*a*tan(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)-2/3*a^2*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2
/5*a^3*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2*a^4*tan(d*x+c)^7/d/(a+a*sec(
d*x+c))^(7/2)+10/9*a^5*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+2/11*a^6*tan(d
*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 472, 209}

$$\int \sqrt{a+a\sec(c+dx)}\tan^6(c+dx)\,dx = \frac{2a^6\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}} + \frac{10a^5\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}} + \frac{2a^4\tan^7(c+dx)}{d(a\sec(c+dx)+a)^{7/2}} + \frac{2a^3\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}} - \frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}} - \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^6,x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^2*Tan[c + d*x]^3)/(3*d
*(a + a*Sec[c + d*x])^(3/2)) + (2*a^3*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d
*x])^(5/2)) + (2*a^4*Tan[c + d*x]^7)/(d*(a + a*Sec[c + d*x])^(7/2)) + (10*a
^5*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (2*a^6*Tan[c + d*x]^1
1)/(11*d*(a + a*Sec[c + d*x])^(11/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(2a^4)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)^3}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{(2a^4)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+7x^6+5ax^8+a^2x^{10}-\frac{1}{a^3(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^3\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} \\
&\quad +\frac{2a^4\tan^7(c+dx)}{d(a+a\sec(c+dx))^{7/2}}+\frac{10a^5\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}} \\
&\quad +\frac{2a^6\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}}+\frac{(2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} \\
&\quad -\frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^3\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}+\frac{2a^4\tan^7(c+dx)}{d(a+a\sec(c+dx))^{7/2}} \\
&\quad +\frac{10a^5\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}}+\frac{2a^6\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 7.81 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.60

$$\int \sqrt{a+a\sec(c+dx)}\tan^6(c+dx)\,dx =$$

$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(1+\sec(c+dx))}\Big(3960\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{11}{2}}(c+dx)+}{}$$

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^6,x]
```

```
[Out] -1/3960*(Sec[(c + d*x)/2]*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c + d*x])]*(3960*S
qrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(11/2) + 792*Sin[(c +
d*x)/2] - 1386*Sin[(3*(c + d*x))/2] + 495*Sin[(5*(c + d*x))/2] - 616*Sin[(7
*(c + d*x))/2] - 247*Sin[(11*(c + d*x))/2]))/d
```

Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.05

method	result
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(-495\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)-495\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{}{(\cos}\right.\right.}{}$

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2/495/d*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(-495*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*cos(d*x+c)-495*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+
c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+494*sin(d*x+c)+247*ta
n(d*x+c)-186*sec(d*x+c)*tan(d*x+c)-155*tan(d*x+c)*sec(d*x+c)^2+50*tan(d*x+c
)*sec(d*x+c)^3+45*sec(d*x+c)^4*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.67

$$\int \sqrt{a+a\sec(c+dx)}\tan^6(c+dx)\,dx$$
$$=\left[\frac{495\left(\cos(dx+c)^6+\cos(dx+c)^5\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{495\,(}\right.$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] [1/495*(495*(cos(d*x + c)^6 + cos(d*x + c)^5)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(494*cos(d*x + c)^5 +
 247*cos(d*x + c)^4 - 186*cos(d*x + c)^3 - 155*cos(d*x + c)^2 + 50*cos(d*x
+ c) + 45)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^6 + d*cos(d*x + c)^5), 2/495*(495*(cos(d*x + c)^6 + cos(d*x + c)^5)*s
qrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)
*sin(d*x + c))) + (494*cos(d*x + c)^5 + 247*cos(d*x + c)^4 - 186*cos(d*x +
c)^3 - 155*cos(d*x + c)^2 + 50*cos(d*x + c) + 45)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)]
```

Sympy [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^6(c+dx)\,dx=\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\tan^6\left(c+dx\right)dx$$

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**6,x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**6, x)
```

Maxima [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^6(c+dx)\,dx=\int \sqrt{a\sec\left(dx+c\right)+a}\tan\left(dx+c\right)^6\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="maxima")
[Out] 1/990*(495*((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + 4*cos(2*d*x + 2*c)^3
 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 6*c
os(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + 4*cos(2*d*x
+ 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)
^2 + 6*cos(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) - 1) - 2*(d*cos(2*d*x + 2*c)^4 + d*sin(2*d*x + 2*c)^4
+ 4*d*cos(2*d*x + 2*c)^3 + 6*d*cos(2*d*x + 2*c)^2 + 2*(d*cos(2*d*x + 2*c)^2
 + 2*d*cos(2*d*x + 2*c) + d)*sin(2*d*x + 2*c)^2 + 4*d*cos(2*d*x + 2*c) + d)
*integrate((((cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*co
s(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*
c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x +
4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x +
 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2
*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*si
n(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*
cos(13/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*s
in(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x +
2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d
*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d
*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*co
```

s(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) -
15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))
*sin(13/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(14*d*x + 14*
c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x
 + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6
*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(
2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c
)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x +
6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(13/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(14*d*x + 14*c)*cos(2*d*x + 2
*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d
*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(
2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + s
in(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c)
+ 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x +
2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + sin(2*d*x + 2*c)^2)*sin(13/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(6*
cos(12*d*x + 12*c) + 15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6
*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + c
os(14*d*x + 14*c)^2 + 12*(15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*
cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(12*d*x + 12*c
) + 36*cos(12*d*x + 12*c)^2 + 30*(20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c)
 + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 225*cos(10*d
*x + 10*c)^2 + 40*(15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2
*c))*cos(8*d*x + 8*c) + 400*cos(8*d*x + 8*c)^2 + 30*(6*cos(4*d*x + 4*c) + c
os(2*d*x + 2*c))*cos(6*d*x + 6*c) + 225*cos(6*d*x + 6*c)^2 + 36*cos(4*d*x +
 4*c)^2 + 12*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(6*
sin(12*d*x + 12*c) + 15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6
*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + s
in(14*d*x + 14*c)^2 + 12*(15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*
sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(12*d*x + 12*c
) + 36*sin(12*d*x + 12*c)^2 + 30*(20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c)
 + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 225*sin(10*d
*x + 10*c)^2 + 40*(15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(8*d*x + 8*c) + 400*sin(8*d*x + 8*c)^2 + 30*(6*sin(4*d*x + 4*c) + s
in(2*d*x + 2*c))*sin(6*d*x + 6*c) + 225*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x +
 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(6*cos(12*d*x + 1
2*c) + 15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + cos(14*d*x + 14
c)^2 + 12(15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*
c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 36*cos(12*
d*x + 12*c)^2 + 30*(20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x

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 + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 225*cos(10*d*x + 10*c)^2 +
 40*(15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x
 + 8*c) + 400*cos(8*d*x + 8*c)^2 + 30*(6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c
))*cos(6*d*x + 6*c) + 225*cos(6*d*x + 6*c)^2 + 36*cos(4*d*x + 4*c)^2 + 12*c
os(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(6*sin(12*d*x + 1
2*c) + 15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) +
6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + sin(14*d*x + 14
*c)^2 + 12*(15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*
c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 36*sin(12*
d*x + 12*c)^2 + 30*(20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x
 + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 225*sin(10*d*x + 10*c)^2 +
 40*(15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x
 + 8*c) + 400*sin(8*d*x + 8*c)^2 + 30*(6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 225*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 12*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x) + 12*(d*cos(2*d*x + 2*c)^4 + d*s
in(2*d*x + 2*c)^4 + 4*d*cos(2*d*x + 2*c)^3 + 6*d*cos(2*d*x + 2*c)^2 + 2*(d*
cos(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*c) + d)*sin(2*d*x + 2*c)^2 + 4*d*cos
(2*d*x + 2*c) + d)*integrate((((cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos
(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) +
 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c
) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x +
 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10
*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*s
in(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + si
n(2*d*x + 2*c)^2)*cos(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (
cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c)
 + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x +
 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x
 + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*
d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*s
in(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)
*sin(2*d*x + 2*c))*sin(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c
)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x
 + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(
2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(1
4*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15
*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c)
 - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*
c))*cos(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(14*d*x + 1
4*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d
*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos
(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(
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2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)
*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x +
 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(11/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)))/(((2*(6*cos(12*d*x + 12*c) + 15*cos(10*d*x + 10*c) + 20*cos(8*d*x
 + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(
14*d*x + 14*c) + cos(14*d*x + 14*c)^2 + 12*(15*cos(10*d*x + 10*c) + 20*cos(
8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))
*cos(12*d*x + 12*c) + 36*cos(12*d*x + 12*c)^2 + 30*(20*cos(8*d*x + 8*c) + 1
5*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10
*c) + 225*cos(10*d*x + 10*c)^2 + 40*(15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*
c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 400*cos(8*d*x + 8*c)^2 + 30*(6*co
s(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 225*cos(6*d*x + 6*c)^
2 + 36*cos(4*d*x + 4*c)^2 + 12*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*
x + 2*c)^2 + 2*(6*sin(12*d*x + 12*c) + 15*sin(10*d*x + 10*c) + 20*sin(8*d*x
 + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(
14*d*x + 14*c) + sin(14*d*x + 14*c)^2 + 12*(15*sin(10*d*x + 10*c) + 20*sin(
8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))
*sin(12*d*x + 12*c) + 36*sin(12*d*x + 12*c)^2 + 30*(20*sin(8*d*x + 8*c) + 1
5*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10
*c) + 225*sin(10*d*x + 10*c)^2 + 40*(15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*
c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 400*sin(8*d*x + 8*c)^2 + 30*(6*si
n(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 225*sin(6*d*x + 6*c)^
2 + 36*sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*
x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2
*(6*cos(12*d*x + 12*c) + 15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*c
os(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(14*d*x + 14*c)
 + cos(14*d*x + 14*c)^2 + 12*(15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) +
 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(12*d*x +
12*c) + 36*cos(12*d*x + 12*c)^2 + 30*(20*cos(8*d*x + 8*c) + 15*cos(6*d*x +
6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 225*cos(
10*d*x + 10*c)^2 + 40*(15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x
 + 2*c))*cos(8*d*x + 8*c) + 400*cos(8*d*x + 8*c)^2 + 30*(6*cos(4*d*x + 4*c)
 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 225*cos(6*d*x + 6*c)^2 + 36*cos(4*d
*x + 4*c)^2 + 12*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2
*(6*sin(12*d*x + 12*c) + 15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*s
in(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(14*d*x + 14*c)
 + sin(14*d*x + 14*c)^2 + 12*(15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) +
 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(12*d*x +
12*c) + 36*sin(12*d*x + 12*c)^2 + 30*(20*sin(8*d*x + 8*c) + 15*sin(6*d*x +
6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 225*sin(
10*d*x + 10*c)^2 + 40*(15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x
 + 2*c))*sin(8*d*x + 8*c) + 400*sin(8*d*x + 8*c)^2 + 30*(6*sin(4*d*x + 4*c)
 + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 225*sin(6*d*x + 6*c)^2 + 36*sin(4*d

```
*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x) - 30*(d*cos(2*
d*x + 2*c)^4 + d*sin(2*d*x + 2*c)^4 + 4*d*cos(2*d*x + 2*c)^3 + 6*d*cos(2*d*
x + 2*c)^2 + 2*(d*cos(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*c) + d)*sin(2*d*x
 + 2*c)^2 + 4*d*cos(2*d*x + 2*c) + d)*integrate((((cos(14*d*x + 14*c)*cos(2*
d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*
cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*
c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c
)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x
 + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2
*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*s
in(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x +
 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x
 + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d
*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*c
os(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6
*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(9/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (
(cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c
) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x
 + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*
x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2
*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*
sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
(cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c
) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x
 + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*
x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin
(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) +
 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c
) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(9/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)))/(((2*(6*cos(12*d*x + 12*c) + 15*cos(10*d*x + 10*c)
+ 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*
x + 2*c))*cos(14*d*x + 14*c) + cos(14*d*x + 14*c)^2 + 12*(15*cos(10*d*x + 1
0*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos
(2*d*x + 2*c))*cos(12*d*x + 12*c) + 36*cos(12*d*x + 12*c)^2 + 30*(20*cos(8*
d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*c
os(10*d*x + 10*c) + 225*cos(10*d*x + 10*c)^2 + 40*(15*cos(6*d*x + 6*c) + 6*
cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 400*cos(8*d*x + 8*c
)^2 + 30*(6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 225*cos
```

(6*d*x + 6*c)^2 + 36*cos(4*d*x + 4*c)^2 + 12*cos(4*d*x + 4*c)*cos(2*d*x + 2
*c) + cos(2*d*x + 2*c)^2 + 2*(6*sin(12*d*x + 12*c) + 15*sin(10*d*x + 10*c)
+ 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*
x + 2*c))*sin(14*d*x + 14*c) + sin(14*d*x + 14*c)^2 + 12*(15*sin(10*d*x + 1
0*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin
(2*d*x + 2*c))*sin(12*d*x + 12*c) + 36*sin(12*d*x + 12*c)^2 + 30*(20*sin(8*
d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*s
in(10*d*x + 10*c) + 225*sin(10*d*x + 10*c)^2 + 40*(15*sin(6*d*x + 6*c) + 6*
sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 400*sin(8*d*x + 8*c
)^2 + 30*(6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 225*sin
(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))^2 + (2*(6*cos(12*d*x + 12*c) + 15*cos(10*d*x + 10*c) + 20*cos(8*d*x
 + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(
14*d*x + 14*c) + cos(14*d*x + 14*c)^2 + 12*(15*cos(10*d*x + 10*c) + 20*cos(
8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))
*cos(12*d*x + 12*c) + 36*cos(12*d*x + 12*c)^2 + 30*(20*cos(8*d*x + 8*c) + 1
5*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10
*c) + 225*cos(10*d*x + 10*c)^2 + 40*(15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*
c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 400*cos(8*d*x + 8*c)^2 + 30*(6*co
s(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 225*cos(6*d*x + 6*c)^
2 + 36*cos(4*d*x + 4*c)^2 + 12*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*
x + 2*c)^2 + 2*(6*sin(12*d*x + 12*c) + 15*sin(10*d*x + 10*c) + 20*sin(8*d*x
 + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(
14*d*x + 14*c) + sin(14*d*x + 14*c)^2 + 12*(15*sin(10*d*x + 10*c) + 20*sin(
8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))
*sin(12*d*x + 12*c) + 36*sin(12*d*x + 12*c)^2 + 30*(20*sin(8*d*x + 8*c) + 1
5*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10
*c) + 225*sin(10*d*x + 10*c)^2 + 40*(15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*
c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 400*sin(8*d*x + 8*c)^2 + 30*(6*si
n(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 225*sin(6*d*x + 6*c)^
2 + 36*sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*
x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x)
+ 40*(d*cos(2*d*x + 2*c)^4 + d*sin(2*d*x + 2*c)^4 + 4*d*cos(2*d*x + 2*c)^3
+ 6*d*cos(2*d*x + 2*c)^2 + 2*(d*cos(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*c) +
 d)*sin(2*d*x + 2*c)^2 + 4*d*cos(2*d*x + 2*c) + d)*integrate((((cos(14*d*x
+ 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(1
0*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*
cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + c
os(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12
*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*
x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(7/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(

```
2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) +
20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c)
 + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c
) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x
 + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*
d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin
(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2
*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x +
 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x
 + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos
(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*c
os(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - (cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*c
os(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8
*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x +
 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x
+ 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(
2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*s
in(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)
*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(6*cos(12*d*x + 12*c) + 15*cos(1
0*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + cos(14*d*x + 14*c)^2 + 12*(15*
cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*
x + 4*c) + cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 36*cos(12*d*x + 12*c)^2 +
 30*(20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2
*d*x + 2*c))*cos(10*d*x + 10*c) + 225*cos(10*d*x + 10*c)^2 + 40*(15*cos(6*d
*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 400*c
os(8*d*x + 8*c)^2 + 30*(6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x +
6*c) + 225*cos(6*d*x + 6*c)^2 + 36*cos(4*d*x + 4*c)^2 + 12*cos(4*d*x + 4*c)
*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(6*sin(12*d*x + 12*c) + 15*sin(1
0*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4
*c) + sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + sin(14*d*x + 14*c)^2 + 12*(15*
sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*
x + 4*c) + sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 36*sin(12*d*x + 12*c)^2 +
 30*(20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2
*d*x + 2*c))*sin(10*d*x + 10*c) + 225*sin(10*d*x + 10*c)^2 + 40*(15*sin(6*d
*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 400*s
in(8*d*x + 8*c)^2 + 30*(6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x +
6*c) + 225*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))^2 + (2*(6*cos(12*d*x + 12*c) + 15*cos(10*d*x + 10*c)
+ 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*
```

x + 2*c))*cos(14*d*x + 14*c) + cos(14*d*x + 14*c)^2 + 12*(15*cos(10*d*x + 1
0*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos
(2*d*x + 2*c))*cos(12*d*x + 12*c) + 36*cos(12*d*x + 12*c)^2 + 30*(20*cos(8*
d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*c
os(10*d*x + 10*c) + 225*cos(10*d*x + 10*c)^2 + 40*(15*cos(6*d*x + 6*c) + 6*
cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 400*cos(8*d*x + 8*c
)^2 + 30*(6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 225*cos
(6*d*x + 6*c)^2 + 36*cos(4*d*x + 4*c)^2 + 12*cos(4*d*x + 4*c)*cos(2*d*x + 2
*c) + cos(2*d*x + 2*c)^2 + 2*(6*sin(12*d*x + 12*c) + 15*sin(10*d*x + 10*c)
+ 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*
x + 2*c))*sin(14*d*x + 14*c) + sin(14*d*x + 14*c)^2 + 12*(15*sin(10*d*x + 1
0*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin
(2*d*x + 2*c))*sin(12*d*x + 12*c) + 36*sin(12*d*x + 12*c)^2 + 30*(20*sin(8*
d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*s
in(10*d*x + 10*c) + 225*sin(10*d*x + 10*c)^2 + 40*(15*sin(6*d*x + 6*c) + 6*
sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 400*sin(8*d*x + 8*c
)^2 + 30*(6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 225*sin
(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)), x) - 30*(d*cos(2*d*x + 2*c)^4 + d*sin(2*d*x + 2*c)^4 + 4*d*cos(2
*d*x + 2*c)^3 + 6*d*cos(2*d*x + 2*c)^2 + 2*(d*cos(2*d*x + 2*c)^2 + 2*d*cos(
2*d*x + 2*c) + d)*sin(2*d*x + 2*c)^2 + 4*d*cos(2*d*x + 2*c) + d)*integrate(
(((cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2
*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*
x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*
d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*s
in(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c)
 + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2
*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x +
14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*
d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*si
n(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*s
in(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 1
0*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x
 + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*
d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20
*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) +
 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c)
- 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x +
 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*
x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x +

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 2*c), cos(2*d*x + 2*c))) - (cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12
*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20
*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) +
 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14
*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*
x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(
6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2
*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(6*cos(12*d*x + 12
*c) + 15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6
*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + cos(14*d*x + 14*
c)^2 + 12*(15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c
) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 36*cos(12*d
*x + 12*c)^2 + 30*(20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 225*cos(10*d*x + 10*c)^2 +
40*(15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x
+ 8*c) + 400*cos(8*d*x + 8*c)^2 + 30*(6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)
)*cos(6*d*x + 6*c) + 225*cos(6*d*x + 6*c)^2 + 36*cos(4*d*x + 4*c)^2 + 12*co
s(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(6*sin(12*d*x + 12
*c) + 15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6
*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + sin(14*d*x + 14*
c)^2 + 12*(15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c
) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 36*sin(12*d
*x + 12*c)^2 + 30*(20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x
+ 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 225*sin(10*d*x + 10*c)^2 +
40*(15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x
+ 8*c) + 400*sin(8*d*x + 8*c)^2 + 30*(6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c)
)*sin(6*d*x + 6*c) + 225*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 12*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(6*cos(12*d*x + 12*c) + 15*cos(1
0*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + cos(14*d*x + 14*c)^2 + 12*(15*
cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*
x + 4*c) + cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 36*cos(12*d*x + 12*c)^2 +
 30*(20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2
*d*x + 2*c))*cos(10*d*x + 10*c) + 225*cos(10*d*x + 10*c)^2 + 40*(15*cos(6*d
*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 400*c
os(8*d*x + 8*c)^2 + 30*(6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x +
6*c) + 225*cos(6*d*x + 6*c)^2 + 36*cos(4*d*x + 4*c)^2 + 12*cos(4*d*x + 4*c)
*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(6*sin(12*d*x + 12*c) + 15*sin(1
0*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4
*c) + sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + sin(14*d*x + 14*c)^2 + 12*(15*
sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*
x + 4*c) + sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 36*sin(12*d*x + 12*c)^2 +
 30*(20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2
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*d*x + 2*c))*sin(10*d*x + 10*c) + 225*sin(10*d*x + 10*c)^2 + 40*(15*sin(6*d
*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 400*s
in(8*d*x + 8*c)^2 + 30*(6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x +
6*c) + 225*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)), x) + 12*(d*cos(2*d*x + 2*c)^4 + d*sin(2*d*x + 2*c)
^4 + 4*d*cos(2*d*x + 2*c)^3 + 6*d*cos(2*d*x + 2*c)^2 + 2*(d*cos(2*d*x + 2*c
)^2 + 2*d*cos(2*d*x + 2*c) + d)*sin(2*d*x + 2*c)^2 + 4*d*cos(2*d*x + 2*c) +
 d)*integrate((((cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)
*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x +
 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x
 + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*
x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*si
n(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)
*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^
2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)
*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x
+ 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2
*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14
*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*
cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c)
- 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c
))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(14*d*x + 14
*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*
x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(
6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin
(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*
c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x +
 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(14*d*x + 14*c)*cos(2*d*x + 2
*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d
*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(
2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + s
in(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c)
+ 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x +
2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(6*c
os(12*d*x + 12*c) + 15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*
d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + co
s(14*d*x + 14*c)^2 + 12*(15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*c
os(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(12*d*x + 12*c)
 + 36*cos(12*d*x + 12*c)^2 + 30*(20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c)
```

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+ 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 225*cos(10*d*
x + 10*c)^2 + 40*(15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*
c))*cos(8*d*x + 8*c) + 400*cos(8*d*x + 8*c)^2 + 30*(6*cos(4*d*x + 4*c) + co
s(2*d*x + 2*c))*cos(6*d*x + 6*c) + 225*cos(6*d*x + 6*c)^2 + 36*cos(4*d*x +
4*c)^2 + 12*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(6*s
in(12*d*x + 12*c) + 15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*
d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + si
n(14*d*x + 14*c)^2 + 12*(15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*s
in(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(12*d*x + 12*c)
 + 36*sin(12*d*x + 12*c)^2 + 30*(20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c)
+ 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 225*sin(10*d*
x + 10*c)^2 + 40*(15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*
c))*sin(8*d*x + 8*c) + 400*sin(8*d*x + 8*c)^2 + 30*(6*sin(4*d*x + 4*c) + si
n(2*d*x + 2*c))*sin(6*d*x + 6*c) + 225*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x +
4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(6*cos(12*d*x + 12
*c) + 15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6
*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + cos(14*d*x + 14*
c)^2 + 12*(15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c
) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 36*cos(12*d
*x + 12*c)^2 + 30*(20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 225*cos(10*d*x + 10*c)^2 +
40*(15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x
+ 8*c) + 400*cos(8*d*x + 8*c)^2 + 30*(6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)
)*cos(6*d*x + 6*c) + 225*cos(6*d*x + 6*c)^2 + 36*cos(4*d*x + 4*c)^2 + 12*co
s(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(6*sin(12*d*x + 12
*c) + 15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6
*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + sin(14*d*x + 14*
c)^2 + 12*(15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c
) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 36*sin(12*d
*x + 12*c)^2 + 30*(20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x
+ 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 225*sin(10*d*x + 10*c)^2 +
40*(15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x
+ 8*c) + 400*sin(8*d*x + 8*c)^2 + 30*(6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c)
)*sin(6*d*x + 6*c) + 225*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 12*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x) - 2*(d*cos(2*d*x + 2*c)^4 + d*sin
(2*d*x + 2*c)^4 + 4*d*cos(2*d*x + 2*c)^3 + 6*d*cos(2*d*x + 2*c)^2 + 2*(d*co
s(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*c) + d)*sin(2*d*x + 2*c)^2 + 4*d*cos(2
*d*x + 2*c) + d)*integrate((((cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(1
2*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 2
0*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c)
+ 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 1
4*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d
```

```
*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin
(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(
2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos
(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) +
15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*
c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x +
4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x
 + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(
2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*si
n(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*si
n(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2
*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*
x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*
x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos
(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 1
5*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(14*d*x + 14*c)*
cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x +
10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*
x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x
 + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(
2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)
*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)))/(((2*(6*cos(12*d*x + 12*c) + 15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c
) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(14*d*x
 + 14*c) + cos(14*d*x + 14*c)^2 + 12*(15*cos(10*d*x + 10*c) + 20*cos(8*d*x
+ 8*c) + 15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(1
2*d*x + 12*c) + 36*cos(12*d*x + 12*c)^2 + 30*(20*cos(8*d*x + 8*c) + 15*cos(
6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) +
225*cos(10*d*x + 10*c)^2 + 40*(15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + c
os(2*d*x + 2*c))*cos(8*d*x + 8*c) + 400*cos(8*d*x + 8*c)^2 + 30*(6*cos(4*d*
x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 225*cos(6*d*x + 6*c)^2 + 36
*cos(4*d*x + 4*c)^2 + 12*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*
c)^2 + 2*(6*sin(12*d*x + 12*c) + 15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c
) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(14*d*x
 + 14*c) + sin(14*d*x + 14*c)^2 + 12*(15*sin(10*d*x + 10*c) + 20*sin(8*d*x
+ 8*c) + 15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(1
2*d*x + 12*c) + 36*sin(12*d*x + 12*c)^2 + 30*(20*sin(8*d*x + 8*c) + 15*sin(
6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) +
225*sin(10*d*x + 10*c)^2 + 40*(15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + s
in(2*d*x + 2*c))*sin(8*d*x + 8*c) + 400*sin(8*d*x + 8*c)^2 + 30*(6*sin(4*d*
x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 225*sin(6*d*x + 6*c)^2 + 36
```

```
*sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*
c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(6*co
s(12*d*x + 12*c) + 15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*cos(6*d
*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + cos
(14*d*x + 14*c)^2 + 12*(15*cos(10*d*x + 10*c) + 20*cos(8*d*x + 8*c) + 15*co
s(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(12*d*x + 12*c)
+ 36*cos(12*d*x + 12*c)^2 + 30*(20*cos(8*d*x + 8*c) + 15*cos(6*d*x + 6*c) +
 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 225*cos(10*d*x
 + 10*c)^2 + 40*(15*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + cos(2*d*x + 2*c
))*cos(8*d*x + 8*c) + 400*cos(8*d*x + 8*c)^2 + 30*(6*cos(4*d*x + 4*c) + cos
(2*d*x + 2*c))*cos(6*d*x + 6*c) + 225*cos(6*d*x + 6*c)^2 + 36*cos(4*d*x + 4
*c)^2 + 12*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(6*si
n(12*d*x + 12*c) + 15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*sin(6*d
*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + sin
(14*d*x + 14*c)^2 + 12*(15*sin(10*d*x + 10*c) + 20*sin(8*d*x + 8*c) + 15*si
n(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(12*d*x + 12*c)
+ 36*sin(12*d*x + 12*c)^2 + 30*(20*sin(8*d*x + 8*c) + 15*sin(6*d*x + 6*c) +
 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 225*sin(10*d*x
 + 10*c)^2 + 40*(15*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + sin(2*d*x + 2*c
))*sin(8*d*x + 8*c) + 400*sin(8*d*x + 8*c)^2 + 30*(6*sin(4*d*x + 4*c) + sin
(2*d*x + 2*c))*sin(6*d*x + 6*c) + 225*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4
*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x))*(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*sqrt(a) - 8*(11*(45*s
in(8*d*x + 8*c) + 72*sin(6*d*x + 6*c) + 126*sin(4*d*x + 4*c) + 56*sin(2*d*x
 + 2*c))*cos(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (495*c
os(8*d*x + 8*c) + 792*cos(6*d*x + 6*c) + 1386*cos(4*d*x + 4*c) + 616*cos(2*
d*x + 2*c) + 247)*sin(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
)*sqrt(a))/((d*cos(2*d*x + 2*c)^4 + d*sin(2*d*x + 2*c)^4 + 4*d*cos(2*d*x +
2*c)^3 + 6*d*cos(2*d*x + 2*c)^2 + 2*(d*cos(2*d*x + 2*c)^2 + 2*d*cos(2*d*x +
 2*c) + d)*sin(2*d*x + 2*c)^2 + 4*d*cos(2*d*x + 2*c) + d)*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4))
```

Giac [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^6(c+dx)\,dx=\int \sqrt{a\sec\left(dx+c\right)+a}\tan\left(dx+c\right)^6\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^6,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(c+dx)}\tan^6(c+dx)\,dx = \int \tan(c+d\,x)^6\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(1/2), x)
```

3.142 $\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{6a^3\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}+\frac{2a^4\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d-2*a*tan(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+6/
5*a^3*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2/7*a^4*tan(d*x+c)^7/d/(a+a*sec
(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used

$= \{3972,\, 472,\, 209\}$

$$\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx = \frac{2a^4\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{6a^3\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^4,x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Tan[c + d*x]^3)/(3*d*
(a + a*Sec[c + d*x])^(3/2)) + (6*a^3*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*
x])^(5/2)) + (2*a^4*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(2a^3)\operatorname{Subst}\left(\int \frac{x^4(2+ax^2)^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^3)\operatorname{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+3x^4+ax^6+\frac{1}{a^2(1+ax^2)}\right)\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{6a^3\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{2a^4\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} - \frac{(2a)\text{Subst}\left(\int\frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{6a^3\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{2a^4\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}$$

Mathematica [A] (verified)

Time = 6.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.69

$$\int\sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx = \frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(1+\sec(c+dx))}\left(105\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{7}{2}}(c+dx)+35\sin\ldots\right)}{105d}$$

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^4,x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(105*Sqrt[2]*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) + 35*Sin[(c + d*x)/2] - 2
8*Sin[(3*(c + d*x))/2] - 23*Sin[(7*(c + d*x))/2]))/(105*d)
```

Maple [A] (verified)

Time = 5.18 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.26

method	result
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(105\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+105\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{arctanh}\left(\ldots\right.\right.}{105d(\cos(dx+c)+1)\ldots}$

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/105/d*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(105*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*cos(d*x+c)+105*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c
)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-92*sin(d*x+c)-46*tan(d
*x+c)+18*sec(d*x+c)*tan(d*x+c)+15*tan(d*x+c)*sec(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.07

$$\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx$$

$$= \left[\frac{105\left(\cos\left(dx+c\right)^4+\cos\left(dx+c\right)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{105\left(d\cos\left(dx+c\right)^4+d\cos\right.}\right.$$

$$-\frac{2\left(105\left(\cos\left(dx+c\right)^4+\cos\left(dx+c\right)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\left(92\cos\left(dx+c\right)^3+46\cos\left(\right.\right.\right.}{105\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] [1/105*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(92*cos(d*x + c)^3 +
46*cos(d*x + c)^2 - 18*cos(d*x + c) - 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), -2/105*(105*(co
s(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (92*cos(d*x + c)^3 + 46*c
os(d*x + c)^2 - 18*cos(d*x + c) - 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\tan^4\left(c+dx\right)dx$$

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**4,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**4, x)
```

Maxima [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx=\int \sqrt{a\sec(dx+c)+a}\tan(dx+c)^4\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/210*(105*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - 1) - 2*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*
d*cos(2*d*x + 2*c) + d)*integrate((((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) +
4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) +
 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10
*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x +
6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2
*c)^2)*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x +
2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x
 + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x
 + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d
*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(9/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(
2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos
(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*co
s(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*c
os(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos
(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*co
s(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) +
 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c)
+ 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(9/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)))/(((2*(4*cos(8*d*x + 8*c) + 6*cos(6*d*x + 6*c) + 4*cos
(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2
 + 8*(6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x
 + 8*c) + 16*cos(8*d*x + 8*c)^2 + 12*(4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)
)*cos(6*d*x + 6*c) + 36*cos(6*d*x + 6*c)^2 + 16*cos(4*d*x + 4*c)^2 + 8*cos(
```

```
4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x + 8*c)
+ 6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x +
10*c) + sin(10*d*x + 10*c)^2 + 8*(6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) +
 sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*sin(8*d*x + 8*c)^2 + 12*(4*sin(4*d
*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 36*sin(6*d*x + 6*c)^2 + 16
*sin(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c
)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(4*cos
(8*d*x + 8*c) + 6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))
*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 + 8*(6*cos(6*d*x + 6*c) + 4*cos(
4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 16*cos(8*d*x + 8*c)^2 +
 12*(4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 36*cos(6*d*x
 + 6*c)^2 + 16*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + c
os(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x + 8*c) + 6*sin(6*d*x + 6*c) + 4*sin(4*d*
x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 8*
(6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*
c) + 16*sin(8*d*x + 8*c)^2 + 12*(4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 36*sin(6*d*x + 6*c)^2 + 16*sin(4*d*x + 4*c)^2 + 8*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)), x) + 8*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x
+ 2*c)^2 + 2*d*cos(2*d*x + 2*c) + d)*integrate((((cos(10*d*x + 10*c)*cos(2*
d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2
*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + si
n(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6
*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
sin(2*d*x + 2*c)^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
(cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c)
+ 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c)
 - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c
) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*
c))*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 1
0*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x +
 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x
+ 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x
 + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d
*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*
d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2
*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(
2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*si
n(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(4*cos(8*d*x + 8*c) + 6*cos(6*d*x +
 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + cos(10*
d*x + 10*c)^2 + 8*(6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*
```

c))*cos(8*d*x + 8*c) + 16*cos(8*d*x + 8*c)^2 + 12*(4*cos(4*d*x + 4*c) + cos (2*d*x + 2*c))*cos(6*d*x + 6*c) + 36*cos(6*d*x + 6*c)^2 + 16*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x + 8*c) + 6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 8*(6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*sin(8*d*x + 8*c)^2 + 12*(4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 36*sin(6*d*x + 6*c)^2 + 16*sin(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(4*cos(8*d*x + 8*c) + 6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 + 8*(6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 16*cos(8*d*x + 8*c)^2 + 12*(4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 36*cos(6*d*x + 6*c)^2 + 16*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x + 8*c) + 6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 8*(6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*sin(8*d*x + 8*c)^2 + 12*(4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 36*sin(6*d*x + 6*c)^2 + 16*sin(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x) - 12*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*c) + d)*integrate((((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(4*cos(8*d*x + 8*c) +

6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 + 8*(6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 16*cos(8*d*x + 8*c)^2 + 12*(4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 36*cos(6*d*x + 6*c)^2 + 16*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x + 8*c) + 6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 8*(6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*sin(8*d*x + 8*c)^2 + 12*(4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 36*sin(6*d*x + 6*c)^2 + 16*sin(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(4*cos(8*d*x + 8*c) + 6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 + 8*(6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 16*cos(8*d*x + 8*c)^2 + 12*(4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 36*cos(6*d*x + 6*c)^2 + 16*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x + 8*c) + 6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 8*(6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*sin(8*d*x + 8*c)^2 + 12*(4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 36*sin(6*d*x + 6*c)^2 + 16*sin(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x) + 8*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*c) + d)*integrate((((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*

c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(4*cos(8
*d*x + 8*c) + 6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*c
os(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 + 8*(6*cos(6*d*x + 6*c) + 4*cos(4*
d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 16*cos(8*d*x + 8*c)^2 + 1
2*(4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 36*cos(6*d*x +
 6*c)^2 + 16*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos
(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x + 8*c) + 6*sin(6*d*x + 6*c) + 4*sin(4*d*x
+ 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 8*(6
*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c)
 + 16*sin(8*d*x + 8*c)^2 + 12*(4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6
*d*x + 6*c) + 36*sin(6*d*x + 6*c)^2 + 16*sin(4*d*x + 4*c)^2 + 8*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))^2 + (2*(4*cos(8*d*x + 8*c) + 6*cos(6*d*x + 6*c)
+ 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + cos(10*d*x +
10*c)^2 + 8*(6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*co
s(8*d*x + 8*c) + 16*cos(8*d*x + 8*c)^2 + 12*(4*cos(4*d*x + 4*c) + cos(2*d*x
 + 2*c))*cos(6*d*x + 6*c) + 36*cos(6*d*x + 6*c)^2 + 16*cos(4*d*x + 4*c)^2 +
 8*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x
+ 8*c) + 6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10
*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 8*(6*sin(6*d*x + 6*c) + 4*sin(4*d*x +
 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*sin(8*d*x + 8*c)^2 + 12*(4*
sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 36*sin(6*d*x + 6*c)
^2 + 16*sin(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*
x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x)
- 2*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*c) + d
)*integrate((((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos
(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*co
s(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) +
 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c)
+ 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(10*d*x + 10*c
) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*
c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2
*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x +
2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x
 + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*
x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d
*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*
d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(10*
d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(
6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2

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*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*si
n(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/
(((2*(4*cos(8*d*x + 8*c) + 6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*
d*x + 2*c))*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 + 8*(6*cos(6*d*x + 6*
c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 16*cos(8*d*x
 + 8*c)^2 + 12*(4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 3
6*cos(6*d*x + 6*c)^2 + 16*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x + 4*c)*cos(2*d*x
 + 2*c) + cos(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x + 8*c) + 6*sin(6*d*x + 6*c) +
 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 1
0*c)^2 + 8*(6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin
(8*d*x + 8*c) + 16*sin(8*d*x + 8*c)^2 + 12*(4*sin(4*d*x + 4*c) + sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c) + 36*sin(6*d*x + 6*c)^2 + 16*sin(4*d*x + 4*c)^2 +
8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(4*cos(8*d*x + 8*c) + 6*cos(
6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) +
cos(10*d*x + 10*c)^2 + 8*(6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d
*x + 2*c))*cos(8*d*x + 8*c) + 16*cos(8*d*x + 8*c)^2 + 12*(4*cos(4*d*x + 4*c
) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 36*cos(6*d*x + 6*c)^2 + 16*cos(4*d
*x + 4*c)^2 + 8*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*
(4*sin(8*d*x + 8*c) + 6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x +
 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 8*(6*sin(6*d*x + 6*c) +
4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*sin(8*d*x + 8*
c)^2 + 12*(4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 36*sin
(6*d*x + 6*c)^2 + 16*sin(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)), x))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(3/4)*sqrt(a) - 16*(7*(5*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(7
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (35*cos(4*d*x + 4*c)
+ 28*cos(2*d*x + 2*c) + 23)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)))*sqrt(a))/((d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos
(2*d*x + 2*c) + d)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(3/4))
```

Giac [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx=\int \sqrt{a\sec{(dx+c)}+a}\tan{(dx+c)}^4\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^4,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(c+dx)}\tan^4(c+dx)\,dx=\int \tan(c+d\,x)^4\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(1/2), x)
```

3.143 $\int \sqrt{a+a\sec(c+dx)}\tan^2(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \sqrt{a+a\sec(c+dx)}\tan^2(c+dx)\,dx = -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+2*a*tan(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3972, 470, 327, 209}

$$\int \sqrt{a+a\sec(c+dx)}\tan^2(c+dx)\,dx = \frac{2a^2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}} - \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Tan[c + d*x]^3)/(3*d
*(a + a*Sec[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(2a^2)\,\text{Subst}\left(\int \frac{x^2(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{(2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.48 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.35

$$\int \sqrt{a+a\sec(c+dx)}\tan^2(c+dx)\,dx$$
$$= \frac{8\sqrt{2}\left(\frac{1}{1+\sec(c+dx)}\right)^{7/2}\sqrt{a(1+\sec(c+dx))}\left(-\frac{\cos(c+dx)(7+3\cos(c+dx))\csc^4\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-3\text{arctanh}\left(\sqrt{1-\sec(\right.\right.}{24\sqrt{1-\sec(c+dx)}}\right.}{}$$

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (8*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(7/2)*Sqrt[a*(1 + Sec[c + d*x])]*(-1/2
4*(Cos[c + d*x]*(7 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^4*Sec[(c + d*x)/2]^2*
(-3*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + (-1 + 4*Cos[c + d*x])*Sq
rt[1 - Sec[c + d*x]]))/Sqrt[1 - Sec[c + d*x]] - (4*Hypergeometric2F1[2, 7/2
, 9/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x)/2]^2)
/7)*Tan[c + d*x]^3)/(3*d*(1 - Tan[(c + d*x)/2]^2)^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(84) = 168.

Time = 4.73 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\left(3\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{3}{2}}-2(1-\cos(dx+c))^3\csc(dx+c)^3+6\csc(dx+c)-\right.}{3d(-\cot(dx+c)+\csc(dx+c)-1)(\csc(dx+c)-\cot(dx+c)+1)}$

```
[In] int((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*(3*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(
-cot(d*x+c)+csc(d*x+c)))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)-2*(1-cos(d
*x+c))^3*csc(d*x+c)^3+6*csc(d*x+c)-6*cot(d*x+c))*(-2*a/((1-cos(d*x+c))^2*cs
c(d*x+c)^2-1))^(1/2)/(-cot(d*x+c)+csc(d*x+c)-1)/(csc(d*x+c)-cot(d*x+c)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.95

$$\int \sqrt{a+a\sec(c+dx)}\tan^2(c+dx)\,dx$$
$$= \left[\frac{3\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+}{3\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}\right.$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^2 +
2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c
) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d
*x + c)), 2/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x + c))/(d*cos(
d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^2(c+dx)\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\tan^2\left(c+dx\right)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*tan(c + d*x)**2, x)
```

Maxima [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^2(c+dx)\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\tan\left(dx+c\right)^2\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*(2*d*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4
*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*
```

```
c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*
c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c
) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*
c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c
))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x + 6*c)
*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^
2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(2*cos(4*d
*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 4*cos
(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2
+ 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x +
6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2
*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 +
 (2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*x +
6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2
*d*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
+ sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)), x) - 4*d*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*c
os(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*si
n(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*si
n(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin
(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*si
n(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin
(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(
6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2
*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((
2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*x + 6*
c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d
*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))^2 + (2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) +
cos(6*d*x + 6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x +
2*c) + cos(2*d*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6
```

```
*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c
)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)), x) + 2*d*integrate((((cos(6*d*x + 6*c)*cos(2*d*x
+ 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d
*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*
x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d
*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*
x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d
*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x +
2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))/(((2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + co
s(6*d*x + 6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*
c) + cos(2*d*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d
*x + 6*c) + sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))^2 + (2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d
*x + 6*c) + cos(6*d*x + 6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*
cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x +
 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x) - arctan2((cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a
) - 8*sqrt(a)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/((c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*d)
```

Giac [F]

$$\int \sqrt{a+a\sec(c+dx)}\tan^2(c+dx)\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\tan\left(dx+c\right)^2\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(c+dx)}\tan^2(c+dx)\,dx = \int \tan(c+d\,x)^2\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(1/2), x)
```

3.144 $\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{2}d} - \frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+1/2*arctan(1
/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)-cot
(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3972, 491, 536, 209}

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}d} - \frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

```
[In] Int[Cot[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (S
qrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
Sqrt[2]*d) - (Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{\text{Subst}\left(\int \frac{-3a-a^2x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{a\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&\quad + \frac{(2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{2}d} \\
&\quad - \frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$
\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx
$$

$$
= \frac{\sqrt{a(1+\sec(c+dx))}\left(-\frac{4\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\sec(c+dx)}}}\right)\sqrt{\frac{\cos(c+dx)}{(1+\cos(c+dx))^2}}}{\sqrt{\frac{1}{1+\cos(c+dx)}}} - 2\cot(c+dx) + \sqrt{2}\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right.}{2d}
$$

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sec[c + d*x])]*((-4*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c +
d*x])^(-1)]]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2])/Sqrt[(1 + Cos[c + d*x
])^(-1)] - 2*Cot[c + d*x] + Sqrt[2]*ArcSin[Tan[(c + d*x)/2]]*Sqrt[(1 + Sec[
c + d*x])^(-1)]))/(2*d)
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.48

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-4\sqrt{-\right.}{2d}$

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(a*(1+sec(d*x+c)))^(1/2)*(ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*co
t(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)-4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x
+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2*cot(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.87

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{2}\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3\,a\cos(dx+c)^2-2\,a\cos(dx+c)+a}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)\sin(dx+c)+2\sqrt{-a}\log\left(\right.}{4}\right.$$
$$-\frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)+2\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2\,a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin}{2\,d\sin(dx+c)}$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x +
 c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 2*sqrt(-a)*l
og(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(c
os(d*x + c) + 1))*sin(d*x + c) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c))/(d*sin(d*x + c)), -1/2*(sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(
d*x + c) + 2*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin
(d*x + c) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(
d*x + c))]
```

Sympy [F]

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a(\sec(c+dx)+1)}\cot^2(c+dx)\,dx$$

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**2, x)
```

Maxima [F]

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cot(dx+c)^2\,dx$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

Giac [F]

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cot(dx+c)^2\,dx$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cot(c+d\,x)^2\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(1/2), x)
```

3.145 $\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 196

$$
\begin{aligned}
&\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx \\
&= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{9\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{2}d} \\
&\quad + \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8d} + \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12ad} \\
&\quad - \frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{4ad}
\end{aligned}
$$

```
[Out] 1/12*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a/d-1/4*cos(d*x+c)*cot(d*x+c)^3*se
c(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a/d+2*arctan(a^(1/2)*tan(d*x+c)/(
a+a*sec(d*x+c))^(1/2))*a^(1/2)/d-9/16*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)
/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)+7/8*cot(d*x+c)*(a+a*sec(d*x+c))^
(1/2)/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{3972, 483, 597, 536, 209\}$

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} - \frac{9\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{8\sqrt{2}d}$$
$$+ \frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{12ad} + \frac{7\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8d}$$
$$- \frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{4ad}$$

```
[In] Int[Cot[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (9*
Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/
(8*Sqrt[2]*d) + (7*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(8*d) + (Cot[c +
d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(12*a*d) - (Cos[c + d*x]*Cot[c + d*x]^3*
Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(4*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
```

```
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} \\
&= -\frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{4ad} \\
&\quad -\frac{\text{Subst}\left(\int \frac{-a-5a^2x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2a^2d} \\
&= \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12ad} \\
&\quad -\frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{4ad} \\
&\quad +\frac{\text{Subst}\left(\int \frac{21a^2-3a^3x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{12a^2d} \\
&= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8d} + \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12ad} \\
&\quad -\frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{4ad} \\
&\quad -\frac{\text{Subst}\left(\int \frac{69a^3+21a^4x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{24a^2d}
\end{aligned}$$

$$= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8d} + \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12ad} - \frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{4ad} + \frac{(9a)\mathrm{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} - \frac{(2a)\mathrm{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{9\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{2}d} + \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8d} + \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12ad} - \frac{\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{4ad}$$

Mathematica [A] (warning: unable to verify)

Time = 7.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.18

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{\left(-9\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right) + 16\sqrt{2}\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}\right)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{1+\sec(c}{16d\sqrt{\sec(c+dx)}} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\frac{2}{3}\csc\left(\frac{1}{2}(c+dx)\right) - \frac{1}{24}\csc^3\left(\frac{1}{2}(c+dx)\right) - \frac{31}{24}\sin\left(\frac{1}{2}(c+dx)\right) + \frac{1}{16}\right.}{d}$$

```
[In] Integrate[Cot[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((-9*ArcSin[Tan[(c + d*x)/2]] + 16*Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]])*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
Sec[(c + d*x)/2]^2]*Sqrt[1 + Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])/(16*
d*Sqrt[Sec[c + d*x]]) + (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*((2*Cs
c[(c + d*x)/2])/3 - Csc[(c + d*x)/2]^3/24 - (31*Sin[(c + d*x)/2])/24 + (Sec
[(c + d*x)/2]*Tan[(c + d*x)/2])/16))/d
```

Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(-27\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+96}{48d}$

[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/48/d*(a*(1+sec(d*x+c)))^(1/2)*(-27*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2 -2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d *x+c)+1))^(1/2)+96*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(c os(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-62*cot(d*x+c)^3+4*cot(d*x+ c)^2*csc(d*x+c)+42*cot(d*x+c)*csc(d*x+c)^2)

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.79

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \left[\frac{27\left(\sqrt{2}\cos\left(dx+c\right)^2-\sqrt{2}\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin\left(\right.}{}\right.$$

[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/96*(27*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(-a)*log((2*sqrt(2)*sqrt(- a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a* cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 48*(cos(d*x + c)^2 - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c os(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d *x + c) + 4*(31*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 21*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2 - d)*sin(d*x + c)), 1 /48*(48*(cos(d*x + c)^2 - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d* x + c) - a))*sin(d*x + c) + 27*(sqrt(2)*cos(d*x + c)^2 - sqrt(2))*sqrt(a)*a rctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a) *sin(d*x + c)))*sin(d*x + c) + 2*(31*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 21 *cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))]

Sympy [F]

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cot^4\left(c+dx\right)dx$$

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**4, x)
```

Maxima [F]

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\cot\left(dx+c\right)^4\,dx$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cot(d*x + c)^4, x)
```

Giac [F]

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\cot\left(dx+c\right)^4\,dx$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cot(c+d\,x)^4\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(1/2), x)
```

3.146 $\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 280

$$\begin{aligned}
&\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx\\
&=-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{151\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{128\sqrt{2}d}\\
&\quad-\frac{105\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128d}-\frac{23\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{192ad}\\
&\quad+\frac{87\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{160a^2d}\\
&\quad-\frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{32a^2d}\\
&\quad-\frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{16a^2d}
\end{aligned}$$

```
[Out] -23/192*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a/d+87/160*cot(d*x+c)^5*(a+a*se
c(d*x+c))^(5/2)/a^2/d-17/32*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2*(a
+a*sec(d*x+c))^(5/2)/a^2/d-1/16*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2*d*x+1/2*c
)^4*(a+a*sec(d*x+c))^(5/2)/a^2/d-2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c
))^(1/2))*a^(1/2)/d+151/256*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(
d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)-105/128*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/
d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3972, 483, 593, 597, 536, 209}

$$\begin{aligned}
&\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx \\
&= \frac{87\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{160a^2d} \\
&\quad -\frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{16a^2d} \\
&\quad -\frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{32a^2d} \\
&\quad -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{151\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{128\sqrt{2}d} \\
&\quad -\frac{23\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{192ad}-\frac{105\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{128d}
\end{aligned}$$

```
[In] Int[Cot[c + d*x]^6*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
51*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(128*Sqrt[2]*d) - (105*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(128*d) -
(23*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(192*a*d) + (87*Cot[c + d*x]
^5*(a + a*Sec[c + d*x])^(5/2))/(160*a^2*d) - (17*Cos[c + d*x]*Cot[c + d*x]^
5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(32*a^2*d) - (Cos[c + d*x]
^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(5/2))/(16*a^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} \\
&= -\frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{16a^2d} \\
&\quad -\frac{\text{Subst}\left(\int \frac{-a-9a^2x^2}{x^6(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^3d}
\end{aligned}$$

$$= -\frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{32a^2d} - \frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{16a^2d} - \frac{\text{Subst}\left(\int \frac{-87a^2-119a^3x^2}{x^6(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{16a^4d}$$

$$= \frac{87\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{160a^2d} - \frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{32a^2d} - \frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{16a^2d} + \frac{\text{Subst}\left(\int \frac{-115a^3-435a^4x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{160a^4d}$$

$$= -\frac{23\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{192ad} + \frac{87\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{160a^2d} - \frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{32a^2d} - \frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{16a^2d} - \frac{\text{Subst}\left(\int \frac{1575a^4-345a^5x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{960a^4d}$$

$$= -\frac{105\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128d} - \frac{23\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{192ad} + \frac{87\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{160a^2d} - \frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{32a^2d} - \frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{16a^2d} + \frac{\text{Subst}\left(\int \frac{5415a^5+1575a^6x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{1920a^4d}$$

$$= -\frac{105\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128d} - \frac{23\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{192ad} + \frac{87\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{160a^2d} - \frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{32a^2d} - \frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{16a^2d} - \frac{(151a)\mathrm{Subst}\left(\int\frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128d} + \frac{(2a)\mathrm{Subst}\left(\int\frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{151\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{128\sqrt{2}d} - \frac{105\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128d} - \frac{23\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{192ad} + \frac{87\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{160a^2d} - \frac{17\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{32a^2d} - \frac{\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{16a^2d}$$

Mathematica [A] (verified)

Time = 4.66 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.75

$$\int\cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{\sqrt{a(1+\sec(c+dx))}\left(-\frac{(5207+172\cos(c+dx)-4572\cos(2(c+dx))-556\cos(3(c+dx))+2821\cos(4(c+dx)))\csc^5(c+dx)}{4\sqrt{\sec(c+dx)}} - 7680\,\mathrm{arcta}\right.}{}$$

```
[In] Integrate[Cot[c + d*x]^6*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sec[c + d*x])]*(-1/4*((5207 + 172*Cos[c + d*x] - 4572*Cos[2*(c
 + d*x)] - 556*Cos[3*(c + d*x)] + 2821*Cos[4*(c + d*x)])*Csc[c + d*x]^5)/Sq
rt[Sec[c + d*x]] - 7680*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1
)]]*Sqrt[Sec[c + d*x]/(1 + Sec[c + d*x])^2]*Sqrt[1 + Sec[c + d*x]] + 2265*A
rcSin[Tan[(c + d*x)/2]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[(1 + Sec[c + d*x])^(-
1)]*Sqrt[1 + Sec[c + d*x]]))/(3840*d*Sqrt[Sec[c + d*x]])
```

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.81

method	result
default	$\frac{\left(2265 \sin(dx+c)^5 \sqrt{2}\, \ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-7680 \sin\right.}{}$

[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3840/d*(2265*sin(d*x+c)^5*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-7680*sin(d*x+c)^5*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-5642*cos(d*x+c)^5+556*cos(d*x+c)^4+7928*cos(d*x+c)^3-460*cos(d*x+c)^2-3150*cos(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)*csc(d*x+c)^5

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 646, normalized size of antiderivative = 2.31

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \left[\frac{2265\left(\sqrt{2}\cos(dx+c)^4-2\sqrt{2}\cos(dx+c)^2+\sqrt{2}\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{}\right.$$

$$-\frac{3840\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin(dx+c)+}{}$$

[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/7680*(2265*(sqrt(2)*cos(d*x + c)^4 - 2*sqrt(2)*cos(d*x + c)^2 + sqrt(2))*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 3840*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*(2821*cos(d*x + c)^5 - 278*cos(d*x + c)^4 - 3964*cos(d*x + c)^3 + 230*cos(d*x + c)^2

```
+ 1575*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x +
 c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c)), -1/3840*(3840*(cos(d*x + c)^
4 - 2*cos(d*x + c)^2 + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x +
 c) - a))*sin(d*x + c) + 2265*(sqrt(2)*cos(d*x + c)^4 - 2*sqrt(2)*cos(d*x +
 c)^2 + sqrt(2))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(2821*cos(d*x +
c)^5 - 278*cos(d*x + c)^4 - 3964*cos(d*x + c)^3 + 230*cos(d*x + c)^2 + 1575
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^4
- 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))]
```

Sympy [F]

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cot^6\left(c+dx\right)dx$$

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cot(c + d*x)**6, x)
```

Maxima [F(-1)]

Timed out.

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\cot\left(dx+c\right)^6\,dx$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^6(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\int \cot(c+d\,x)^6\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(1/2), x)
```

3.147 $\int (a+a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 169

$$\int (a+a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx = -\frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d} - \frac{2(a+a\sec(c+dx))^{9/2}}{3a^3d} + \frac{2(a+a\sec(c+dx))^{11/2}}{11a^4d}$$

```
[Out] -2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*(a+a*sec(d*x+c))^(
3/2)/d+2/5*(a+a*sec(d*x+c))^(5/2)/a/d+2/7*(a+a*sec(d*x+c))^(7/2)/a^2/d-2/3*
(a+a*sec(d*x+c))^(9/2)/a^3/d+2/11*(a+a*sec(d*x+c))^(11/2)/a^4/d+2*a*(a+a*se
c(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3965, 90, 52, 65, 213}

$$\int (a+a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx = -\frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} + \frac{2(a\sec(c+dx)+a)^{11/2}}{11a^4d} - \frac{2(a\sec(c+dx)+a)^{9/2}}{3a^3d} + \frac{2(a\sec(c+dx)+a)^{7/2}}{7a^2d} + \frac{2(a\sec(c+dx)+a)^{5/2}}{5ad} + \frac{2(a\sec(c+dx)+a)^{3/2}}{3d} + \frac{2a\sqrt{a\sec(c+dx)+a}}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^5,x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a*Sqrt[a + a*
Sec[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec[c +
 d*x])^(5/2))/(5*a*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^2*d) - (2*(a +
a*Sec[c + d*x])^(9/2))/(3*a^3*d) + (2*(a + a*Sec[c + d*x])^(11/2))/(11*a^4*
d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{7/2} + \frac{a^2(a+ax)^{7/2}}{x} + a(a+ax)^{9/2}\right)\,dx, x, \sec(c+dx)\right)}{a^4 d}$$

$$= -\frac{2(a+a\sec(c+dx))^{9/2}}{3a^3 d} + \frac{2(a+a\sec(c+dx))^{11/2}}{11a^4 d} + \frac{\text{Subst}\left(\int \frac{(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2 d}$$

$$= \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2 d} - \frac{2(a+a\sec(c+dx))^{9/2}}{3a^3 d} + \frac{2(a+a\sec(c+dx))^{11/2}}{11a^4 d} + \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2 d} - \frac{2(a+a\sec(c+dx))^{9/2}}{3a^3 d} + \frac{2(a+a\sec(c+dx))^{11/2}}{11a^4 d} + \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2 d} - \frac{2(a+a\sec(c+dx))^{9/2}}{3a^3 d} + \frac{2(a+a\sec(c+dx))^{11/2}}{11a^4 d} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2 d} - \frac{2(a+a\sec(c+dx))^{9/2}}{3a^3 d} + \frac{2(a+a\sec(c+dx))^{11/2}}{11a^4 d} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2 d} - \frac{2(a+a\sec(c+dx))^{9/2}}{3a^3 d} + \frac{2(a+a\sec(c+dx))^{11/2}}{11a^4 d} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}$$

$$= -\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2 d} - \frac{2(a+a\sec(c+dx))^{9/2}}{3a^3 d} + \frac{2(a+a\sec(c+dx))^{11/2}}{11a^4 d}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.66

$$\int (a+a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx = \frac{2(a(1+\sec(c+dx)))^{3/2}\left(-1155\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}(1656+327\sec\right.}{1155d(1+\sec(c}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^5,x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(3/2)*(-1155*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sq
rt[1 + Sec[c + d*x]]*(1656 + 327*Sec[c + d*x] - 534*Sec[c + d*x]^2 - 325*Se
c[c + d*x]^3 + 140*Sec[c + d*x]^4 + 105*Sec[c + d*x]^5)))/(1155*d*(1 + Sec[
c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.66

method	result
default	$\frac{2a\sqrt{a(1+\sec(dx+c))}\left(1155\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+1656+327\sec(dx+c)-534\sec(dx+c)^2-325\sec(dx+c)^3+140\right.}{1155d}$

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155/d*a*(a*(1+sec(d*x+c)))^(1/2)*(1155*arctan((-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1656+327*sec(d*x+c)-534*sec(d*
x+c)^2-325*sec(d*x+c)^3+140*sec(d*x+c)^4+105*sec(d*x+c)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.98

$$\int (a+a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx = \left[\frac{1155\,a^{\frac{3}{2}}\cos(dx+c)^5\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)}{\cos(dx+c}}\right.}{}\right.$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/2310*(1155*a^(3/2)*cos(d*x + c)^5*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x
 + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8
*a*cos(d*x + c) - a) + 4*(1656*a*cos(d*x + c)^5 + 327*a*cos(d*x + c)^4 - 53
4*a*cos(d*x + c)^3 - 325*a*cos(d*x + c)^2 + 140*a*cos(d*x + c) + 105*a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^5), 1/1155*(1155*sqrt
(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^5 + 2*(1656*a*cos(d*x + c)^5 + 327*
a*cos(d*x + c)^4 - 534*a*cos(d*x + c)^3 - 325*a*cos(d*x + c)^2 + 140*a*cos(
d*x + c) + 105*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^
5)]
```

Sympy [F]

$$\int (a+a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx=\int \left(a(\sec{(c+dx)}+1)\right)^{\frac{3}{2}}\tan^5{(c+dx)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**5,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.96

$$\int (a+a\sec(c+dx))^{3/2}\tan^5(c+dx)\,dx=\frac{1155\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+770\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}+\frac{210\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{11}{2}}}{a^4}-\frac{770\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{9}{2}}}{a^3}+\frac{330}{}}{1155\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/1155*(1155*a^(3/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/c
os(d*x + c)) + sqrt(a))) + 770*(a + a/cos(d*x + c))^(3/2) + 210*(a + a/cos(
d*x + c))^(11/2)/a^4 - 770*(a + a/cos(d*x + c))^(9/2)/a^3 + 330*(a + a/cos(
d*x + c))^(7/2)/a^2 + 462*(a + a/cos(d*x + c))^(5/2)/a + 2310*sqrt(a + a/co
s(d*x + c))*a)/d
```

Giac [F]

$$\int (a + a\sec(c + dx))^{3/2} \tan^5(c + dx)\, dx = \int (a\sec(dx + c) + a)^{\frac{3}{2}} \tan(dx + c)^5\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^5,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2} \tan^5(c + dx)\, dx = \int \tan(c + d\,x)^5 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{3/2} dx$$

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(3/2), x)
```

3.148 $\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx = \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2a\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d}$$

```
[Out] 2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-2/3*(a+a*sec(d*x+c))^(3
/2)/d-2/5*(a+a*sec(d*x+c))^(5/2)/a/d+2/7*(a+a*sec(d*x+c))^(7/2)/a^2/d-2*a*(
a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3965, 81, 52, 65, 213}

$$\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx = \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} + \frac{2(a\sec(c+dx)+a)^{7/2}}{7a^2d} - \frac{2(a\sec(c+dx)+a)^{5/2}}{5ad} - \frac{2(a\sec(c+dx)+a)^{3/2}}{3d} - \frac{2a\sqrt{a\sec(c+dx)+a}}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*a*Sqrt[a + a*S
ec[c + d*x]])/d - (2*(a + a*Sec[c + d*x])^(3/2))/(3*d) - (2*(a + a*Sec[c +
d*x])^(5/2))/(5*a*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^2*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2 d}$$

$$
\begin{aligned}
&= \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{ad} \\
&= -\frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{d} \\
&= -\frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} \\
&\quad + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d} - \frac{a\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{d} \\
&= -\frac{2a\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} \\
&\quad + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d} - \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} \\
&= -\frac{2a\sqrt{a+a\sec(c+dx)}}{d} - \frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} \\
&\quad + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d} - \frac{(2a)\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} \\
&= \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2a\sqrt{a+a\sec(c+dx)}}{d} \\
&\quad - \frac{2(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5ad} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$
\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx = \frac{2(a(1+\sec(c+dx)))^{3/2}\left(105\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}(-146-32\sec(c+
$$

$$
\frac{}{105d(1+\sec(c+dx))^{3/2}}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(3/2)*(105*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(-146 - 32*Sec[c + d*x] + 24*Sec[c + d*x]^2 + 15*Sec[c +
 d*x]^3)))/(105*d*(1 + Sec[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

method	result	siz
default	$-\frac{2a\sqrt{a(1+\sec(dx+c))}\left(105\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+146+32\sec(dx+c)-24\sec(dx+c)^2-15\sec(dx+c)^3\right)}{105d}$	92

[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] -2/105/d*a*(a*(1+sec(d*x+c)))^(1/2)*(105*arctan((-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+146+32*sec(d*x+c)-24*sec(d*x+c)
^2-15*sec(d*x+c)^3)

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.40

$$\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx = \left[\frac{105\,a^{\frac{3}{2}}\cos(dx+c)^3\log\left(-8\,a\cos(dx+c)^2-4\left(2\,\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+}{\cos(dx+c)}}\right.}{21}\right.$$

$$-\frac{105\sqrt{-a}a\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos(dx+c)^3+2\left(146\,a\cos(dx+c)^3+32\,a\cos(dx+c)^2-\right.}{105\,d\cos(dx+c)^3}$$

[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="fricas")

[Out] [1/210*(105*a^(3/2)*cos(d*x + c)^3*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) - 4*(146*a*cos(d*x + c)^3 + 32*a*cos(d*x + c)^2 - 24*a*c
os(d*x + c) - 15*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c
)^3), -1/105*(105*sqrt(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^3 + 2*(146*a*
cos(d*x + c)^3 + 32*a*cos(d*x + c)^2 - 24*a*cos(d*x + c) - 15*a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^3)]

Sympy [F]

$$\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\tan^3(c+dx)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**3,x)
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx =$$
$$-\frac{105\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}-\frac{30\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^2}+\frac{42\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a}+210\sqrt{a+\frac{a}{\cos(dx+}}}{105\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="maxima")
[Out] -1/105*(105*a^(3/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/co
s(d*x + c)) + sqrt(a))) + 70*(a + a/cos(d*x + c))^(3/2) - 30*(a + a/cos(d*x
 + c))^(7/2)/a^2 + 42*(a + a/cos(d*x + c))^(5/2)/a + 210*sqrt(a + a/cos(d*x
 + c))*a)/d
```

Giac [F]

$$\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^3\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^{3/2}\tan^3(c+dx)\,dx=\int \tan(c+d\,x)^3\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(3/2), x)
```

3.149 $\int (a+a\sec(c+dx))^{3/2}\tan(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (a+a\sec(c+dx))^{3/2}\tan(c+dx)\,dx = -\frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d}$$

```
[Out] -2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*(a+a*sec(d*x+c))^(
3/2)/d+2*a*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3965, 52, 65, 213}

$$\int (a+a\sec(c+dx))^{3/2}\tan(c+dx)\,dx = -\frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} + \frac{2a\sqrt{a\sec(c+dx)+a}}{d} + \frac{2(a\sec(c+dx)+a)^{3/2}}{3d}$$

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x],x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a*Sqrt[a + a*
Sec[c + d*x]])/d + (2*(a + a*Sec[c + d*x])^(3/2))/(3*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d}\\
&\quad + \frac{(2a)\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}\\
&= -\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a\sqrt{a+a\sec(c+dx)}}{d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int (a+a\sec(c+dx))^{3/2}\tan(c+dx)\,dx = \frac{2(a(1+\sec(c+dx)))^{3/2}\left(-3\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}(4+\sec(c+dx))\right)}{3d(1+\sec(c+dx))^{3/2}}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x],x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(3/2)*(-3*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt[
1 + Sec[c + d*x]]*(4 + Sec[c + d*x])))/(3*d*(1 + Sec[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{2(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2\sqrt{a+a\sec(dx+c)}\,a-2a^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	57
default	$\frac{\frac{2(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2\sqrt{a+a\sec(dx+c)}\,a-2a^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	57

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/3*(a+a*sec(d*x+c))^(3/2)+2*(a+a*sec(d*x+c))^(1/2)*a-2*a^(3/2)*arctan
h((a+a*sec(d*x+c))^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.26

$$\int (a+a\sec(c+dx))^{3/2}\tan(c+dx)\,dx = \left[\frac{3\,a^{\frac{3}{2}}\cos(dx+c)\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\right.}{6\,d\cos(dx+c)}\right.$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^(3/2)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d
*x + c) - a) + 4*(4*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c)))/(d*cos(d*x + c)), 1/3*(3*sqrt(-a)*a*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)
+ 2*(4*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(
d*x + c))]
```

Sympy [F]

$$\int (a+a\sec(c+dx))^{3/2}\tan(c+dx)\,dx=\int (a(\sec(c+dx)+1))^{\frac{3}{2}}\tan(c+dx)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c),x)
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18

$$\int (a+a\sec(c+dx))^{3/2}\tan(c+dx)\,dx=\frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{a}{\cos(dx+c)}}\,a}{3\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="maxima")
[Out] 1/3*(3*a^(3/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x
 + c)) + sqrt(a))) + 2*(a + a/cos(d*x + c))^(3/2) + 6*sqrt(a + a/cos(d*x +
c))*a)/d
```

Giac [F]

$$\int (a+a\sec(c+dx))^{3/2}\tan(c+dx)\,dx=\int (a\sec(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 14.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int (a+a\sec(c+dx))^{3/2}\tan(c+dx)\,dx = \frac{2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{3\,d} - \frac{2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{d} + \frac{2\,a\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{d}$$

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] (2*(a + a/cos(c + d*x))^(3/2))/(3*d) - (2*a^(3/2)*atanh((a + a/cos(c + d*x)
)^(1/2)/a^(1/2)))/d + (2*a*(a + a/cos(c + d*x))^(1/2))/d
```

3.150 $\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{2}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

```
[Out] 2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-2*a^(3/2)*arctanh(1/2*(
a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3965, 85, 65, 213}

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{2}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[2]*a^(3/2
)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a^2\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x(-a+ax)}\,dx, x, \sec(c+dx)\right)}{d}\\
&= -\frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}+\frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= -\frac{(2a)\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}\\
&\quad +\frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}\\
&= \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{\left(2\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)-2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)\right)(a(1+\sec(c+dx)))^{3/2}}{d(1+\sec(c+dx))^{3/2}}$$

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
[Out] ((2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] - 2*Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*
x]]/Sqrt[2]])*(a*(1 + Sec[c + d*x]))^(3/2))/(d*(1 + Sec[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

method	result	size
default	$-\frac{2a\sqrt{a(1+\sec(dx+c))}\,\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\right)}{d}$	91

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/d*a*(a*(1+sec(d*x+c)))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(arctan(
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(58) = 116$.

Time = 0.33 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.33

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{\sqrt{2}a^{\frac{3}{2}}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3\,a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+a^{\frac{3}{2}}\log\left(-2\,a\cos\left(dx+c\right)-2\,\sqrt{a}\sqrt{}\right.}{d}\right.$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a^(3/2)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + a^(3/2)*l
og(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c) - a))/d, 2*(sqrt(2)*sqrt(-a)*a*arctan(sqrt(2)*sqrt(-a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - sqrt(-a
)*a*arctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a
*cos(d*x + c) + a)))/d]
```

Sympy [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\cot(c+dx)\,dx$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*cot(c + d*x), x)
```

Maxima [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)\,dx$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c), x)
```

Giac [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)\,dx$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\int \cot(c+d\,x)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)*(a + a/cos(c + d*x))^(3/2), x)
```

3.151 $\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{5a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d} + \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))}$$

```
[Out] -2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+5/4*a^(3/2)*arctanh(1/
2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+1/2*a*(a+a*sec(d*x+c))^
(1/2)/d/(1-sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3965, 105, 162, 65, 213}

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} + \frac{5a^{3/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d} + \frac{a\sqrt{a\sec(c+dx)+a}}{2d(1-\sec(c+dx))}$$

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (5*a^(3/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*d) + (a*Sqrt[a +
 a*Sec[c + d*x]])/(2*d*(1 - Sec[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^4\text{Subst}\left(\int \frac{1}{x(-a+ax)^2\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} \\
&= \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))} - \frac{a\text{Subst}\left(\int \frac{2a^2+\frac{a^2x}{2}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{2d}
\end{aligned}$$

$$= \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} - \frac{(5a^3)\,\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{4d}$$

$$= \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} - \frac{(5a^2)\,\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{2d}$$

$$= -\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{5a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d} + \frac{a\sqrt{a+a\sec(c+dx)}}{2d(1-\sec(c+dx))}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{(a(1+\sec(c+dx)))^{3/2}\left(-2\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) + \frac{5\text{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\sqrt{1+\sec(c+}}{2(-1+\sec(c+}$$

$$d(1+\sec(c+dx))^{3/2}$$

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(3/2)*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + (5*ArcT
anh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]])/(2*Sqrt[2]) - Sqrt[1 + Sec[c + d*x]]/(
2*(-1 + Sec[c + d*x]))))/(d*(1 + Sec[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

method	result
default	$\frac{a\sqrt{a(1+\sec(dx+c))}\left(5\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+8\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-2\cot(dx+c)^2-\right.}{4d}$

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*a*(a*(1+sec(d*x+c)))^(1/2)*(5*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+8*arctan((-cos(d
```

```
*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2*cot(d*x+c
)^2-2*cot(d*x+c)*csc(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(86) = 172$.

Time = 0.33 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.47

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)+8\,(a\cos(dx+c)-a)\sqrt{a}\log\left(-2\,a\cos(dx+c)+2\,\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}} \cdots\right.}{8\,(\cdots}\right.$$

$$-\frac{5\,(\sqrt{2}a\cos(dx+c)-\sqrt{2}a)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-8\,(a\cos(dx+c)-a)\sqrt{-a}\arctan\cdots}{4\,(d\cos(dx+c)-d)}$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 8*(a*cos(d
*x + c) - a)*sqrt(a)*log(-2*a*cos(d*x + c) + 2*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c) - a) + 5*(sqrt(2)*a*cos(d*x + c) - sqrt(2)
*a)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)))/(d*cos(d*x + c) -
 d), -1/4*(5*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-a)*arctan(sqrt(2)*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c
) + a)) - 8*(a*cos(d*x + c) - a)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 2*a*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*cos(d*x + c) - d)]
```

Sympy [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\cot^3(c+dx)\,dx$$

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*cot(c + d*x)**3, x)
```

Maxima [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^3\,dx$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c)^3, x)
```

Giac [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^3\,dx$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cot(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(3/2), x)
```

3.152 $\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 171

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{71a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d} + \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-71/64*a^(3/2)*arctanh(1
/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+7/32*a^2/d/(a+a*sec(d*
x+c))^(1/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+c))^(1/2)-13/16*a^2/d/(
1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3965, 105, 156, 157, 162, 65, 213}

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} - \frac{71a^{3/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d} + \frac{7a^2}{32d\sqrt{a\sec(c+dx)+a}} - \frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a\sec(c+dx)+a}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (71*a^(3/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*d) + (7*a^2)/(3
2*d*Sqrt[a + a*Sec[c + d*x]]) - a^2/(4*d*(1 - Sec[c + d*x])^2*Sqrt[a + a*Se
c[c + d*x]]) - (13*a^2)/(16*d*(1 - Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{a^6 \text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{a^3 \text{Subst}\left(\int \frac{4a^2+\frac{5a^2x}{2}}{x(-a+ax)^2(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{8a^4+\frac{39a^4x}{4}}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{8d}$$

$$= \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{-8a^6-\frac{7a^6x}{8}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{8a^3d}$$

$$
\begin{aligned}
&= \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} \\
&\quad - \frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} \\
&\quad + \frac{(71a^3)\,\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{64d} \\
&= \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} \\
&\quad - \frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}} \\
&\quad - \frac{(2a)\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} \\
&\quad + \frac{(71a^2)\,\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{32d} \\
&= \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{71a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d} + \frac{7a^2}{32d\sqrt{a+a\sec(c+dx)}} \\
&\quad - \frac{a^2}{4d(1-\sec(c+dx))^2\sqrt{a+a\sec(c+dx)}} - \frac{13a^2}{16d(1-\sec(c+dx))\sqrt{a+a\sec(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.72 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.61

$$
\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\left(-34+71\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))^2-64\,\text{Hype}\right.}{32d(-1+\sec(c+dx))^2\sqrt{a(1}}
$$

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(-34 + 71*Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/2]*(-1 +
Sec[c + d*x])^2 - 64*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]]*(-1
+ Sec[c + d*x])^2 + 26*Sec[c + d*x]))/(32*d*(-1 + Sec[c + d*x])^2*Sqrt[a*(1
 + Sec[c + d*x])])
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.02

method	result
default	$-\frac{a\sqrt{a(1+\sec(dx+c))}\left(71\sqrt{2}\,\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+128\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+54\cot}{64d}$

[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/64/d*a*(a*(1+sec(d*x+c)))^(1/2)*(71*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+128*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+54*cot(d*x+c)^4+30*cot(d*x+c)^3*csc(d*x+c)-38*cot(d*x+c)^2*csc(d*x+c)^2-14*csc(d*x+c)^3*cot(d*x+c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(138) = 276$.

Time = 0.42 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.44

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{64\left(a\cos\left(dx+c\right)^3 - a\cos\left(dx+c\right)^2 - a\cos\left(dx+c\right) + a\right)\sqrt{a}\log\left(-8\,a\cos\left(dx+c\right)^2 - 4\,(2\right.}{}\right.$$

[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/128*(64*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 71*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) - 4*(27*a*cos(d*x + c)^3 - 12*a*cos(d*x + c)^2 - 7*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d), 1/64*(71*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 64*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(27*a*cos(d*x + c)^3 - 12*a*cos(d*x + c)^2 - 7*a*cos(d*x + c))*sq

```
rt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2
 - d*cos(d*x + c) + d)]
```

Sympy [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^5\,dx$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cot(c+d\,x)^5\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(3/2), x)
```

3.153 $\int (a+a\sec(c+dx))^{3/2}\tan^6(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 258

$$\begin{aligned}\int (a+a\sec(c+dx))^{3/2}\tan^6(c+dx)\,dx &= -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &\quad+\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}\\ &\quad+\frac{30a^5\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}+\frac{34a^6\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}}\\ &\quad+\frac{14a^7\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}}+\frac{2a^8\tan^{13}(c+dx)}{13d(a+a\sec(c+dx))^{13/2}}\end{aligned}$$

```
[Out] -2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)-2/3*a^3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
+2/5*a^4*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+30/7*a^5*tan(d*x+c)^7/d/(a+a
*sec(d*x+c))^(7/2)+34/9*a^6*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+14/11*a^7
*tan(d*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)+2/13*a^8*tan(d*x+c)^13/d/(a+a*sec(
d*x+c))^(13/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used

$= \{3972, 472, 209\}$

$$\int (a+a\sec(c+dx))^{3/2}\tan^6(c+dx)\,dx = -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{2a^8\tan^{13}(c+dx)}{13d(a\sec(c+dx)+a)^{13/2}} + \frac{14a^7\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}} + \frac{34a^6\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}} + \frac{30a^5\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}} + \frac{2a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}} - \frac{2a^3\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}} + \frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^6,x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^3*Tan[c + d*x]^3)/(3
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (30*a^5*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(34*a^6*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (14*a^7*Tan[c +
d*x]^11)/(11*d*(a + a*Sec[c + d*x])^(11/2)) + (2*a^8*Tan[c + d*x]^13)/(13*d
*(a + a*Sec[c + d*x])^(13/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(2a^5)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{(2a^5)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+15x^6+17ax^8+7a^2x^{10}+a^3x^{12}-\frac{1}{a^3(1+ax^2)}\right)\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} \\
&\quad+\frac{30a^5\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}+\frac{34a^6\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}}+\frac{14a^7\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}} \\
&\quad+\frac{2a^8\tan^{13}(c+dx)}{13d(a+a\sec(c+dx))^{13/2}}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} \\
&\quad+\frac{2a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}+\frac{30a^5\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}+\frac{34a^6\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}} \\
&\quad+\frac{14a^7\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}}+\frac{2a^8\tan^{13}(c+dx)}{13d(a+a\sec(c+dx))^{13/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 9.38 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.57

$$\int (a+a\sec(c+dx))^{3/2}\tan^6(c+dx)\,dx = \frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\sqrt{a(1+\sec(c+dx))}\left(-1441440\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos$$

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^6,x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^6*Sqrt[a*(1 + Sec[c + d*x])]*(-1441440*Sqr
t[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(13/2) + 164736*Sin[(c +
 d*x)/2] + 81081*Sin[(3*(c + d*x))/2] + 134849*Sin[(5*(c + d*x))/2] + 98176
*Sin[(9*(c + d*x))/2] + 45045*Sin[(11*(c + d*x))/2] + 32429*Sin[(13*(c + d*
x))/2]))/(1441440*d)
```

Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.97

method	result
default	$-\frac{2a\sqrt{a(1+\sec(dx+c))}\left(45045\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+45045\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}}{}$

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -2/45045/d*a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(45045*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))*cos(d*x+c)+45045*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(
sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-32429*sin(d*x
+c)-38737*tan(d*x+c)+4731*sec(d*x+c)*tan(d*x+c)+26465*tan(d*x+c)*sec(d*x+c)
^2+6265*tan(d*x+c)*sec(d*x+c)^3-7875*sec(d*x+c)^4*tan(d*x+c)-3465*tan(d*x+c
)*sec(d*x+c)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.61

$$\int (a+a\sec(c+dx))^{3/2}\tan^6(c+dx)\,dx = \left[\frac{45045\left(a\cos(dx+c)^7+a\cos(dx+c)^6\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+}{\cos(dx+c)+1}\right.}{}\right.$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] [1/45045*(45045*(a*cos(d*x + c)^7 + a*cos(d*x + c)^6)*sqrt(-a)*log((2*a*cos
(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(32429*a*cos(
d*x + c)^6 + 38737*a*cos(d*x + c)^5 - 4731*a*cos(d*x + c)^4 - 26465*a*cos(d
*x + c)^3 - 6265*a*cos(d*x + c)^2 + 7875*a*cos(d*x + c) + 3465*a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^7 + d*cos(d*x
+ c)^6), 2/45045*(45045*(a*cos(d*x + c)^7 + a*cos(d*x + c)^6)*sqrt(a)*arcta
n(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c
))) + (32429*a*cos(d*x + c)^6 + 38737*a*cos(d*x + c)^5 - 4731*a*cos(d*x + c
)^4 - 26465*a*cos(d*x + c)^3 - 6265*a*cos(d*x + c)^2 + 7875*a*cos(d*x + c)
+ 3465*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
+ c)^7 + d*cos(d*x + c)^6)]
```

Sympy [F]

$$\int (a + a\sec(c + dx))^{3/2}\tan^6(c + dx)\,dx = \int (a(\sec{(c + dx)} + 1))^{\frac{3}{2}}\tan^6{(c + dx)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**6,x)
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**6, x)
```

Maxima [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2}\tan^6(c + dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int (a + a\sec(c + dx))^{3/2}\tan^6(c + dx)\,dx = \int (a\sec{(dx + c)} + a)^{\frac{3}{2}}\tan{(dx + c)}^6\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^6,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2}\tan^6(c + dx)\,dx = \int \tan(c + d\,x)^6\left(a + \frac{a}{\cos{(c + d\,x)}}\right)^{3/2}\,dx$$

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(3/2), x)
```

3.154 $\int (a+a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 194

$$\int (a+a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{14a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{10a^5\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} + \frac{2a^6\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2*a^2*tan(d*x
+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+
14/5*a^4*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+10/7*a^5*tan(d*x+c)^7/d/(a+a
*sec(d*x+c))^(7/2)+2/9*a^6*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 472, 209}

$$\int (a+a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{2a^6\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}} + \frac{10a^5\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}} + \frac{14a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}} + \frac{2a^3\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}} - \frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^4,x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*Tan[c + d*x]^3)/(3*
d*(a + a*Sec[c + d*x])^(3/2)) + (14*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (10*a^5*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(2*a^6*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(2a^4)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)^3}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{(2a^4)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+7x^4+5ax^6+a^2x^8+\frac{1}{a^2(1+ax^2)}\right)\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} \\
&\quad +\frac{14a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}+\frac{10a^5\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} \\
&\quad +\frac{2a^6\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}}-\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}
\end{aligned}$$

$$= \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{14a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{10a^5\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} + \frac{2a^6\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}}$$

Mathematica [A] (verified)

Time = 7.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.63

$$\int (a+a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx = \frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(1+\sec(c+dx))}\Big(2520\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{9}{2}}(c+dx)$$

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^4,x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^4*Sqrt[a*(1 + Sec[c + d*x])]*(2520*Sqrt[2]
*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(9/2) + 126*Sin[(c + d*x)/2]
 - 288*Sin[(5*(c + d*x))/2] - 315*Sin[(7*(c + d*x))/2] - 169*Sin[(9*(c + d*
x))/2]))/(2520*d)
```

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12

method	result
default	$\frac{2a\sqrt{a(1+\sec(dx+c))}\left(315\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+315\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{arctanh}\left(\frac{}{(\cos(}\right.\right.}{315d(c}$

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315/d*a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(315*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)+315*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x
+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-169*sin(d*x+c)-242*t
an(d*x+c)-24*sec(d*x+c)*tan(d*x+c)+85*tan(d*x+c)*sec(d*x+c)^2+35*tan(d*x+c)
*sec(d*x+c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.91

$$\int (a+a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx = \left[\frac{315\left(a\cos(dx+c)^5+a\cos(dx+c)^4\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a}{\cos(dx+c)+1}\right.}{3}\right.$$

$$-\frac{2\left(315\left(a\cos(dx+c)^5+a\cos(dx+c)^4\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\left(169\,a\cos(dx+c)^4+242\,a\right.\right.}{315\left(d\cos(dx+c)^5+d\cos\right.}$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] [1/315*(315*(a*cos(d*x + c)^5 + a*cos(d*x + c)^4)*sqrt(-a)*log((2*a*cos(d*x
 + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*s
in(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(169*a*cos(d*x +
c)^4 + 242*a*cos(d*x + c)^3 + 24*a*cos(d*x + c)^2 - 85*a*cos(d*x + c) - 35*
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5
+ d*cos(d*x + c)^4), -2/315*(315*(a*cos(d*x + c)^5 + a*cos(d*x + c)^4)*sqrt
(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*si
n(d*x + c))) + (169*a*cos(d*x + c)^4 + 242*a*cos(d*x + c)^3 + 24*a*cos(d*x
+ c)^2 - 85*a*cos(d*x + c) - 35*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F]

$$\int (a+a\sec(c+dx))^{3/2}\tan^4(c+dx)\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\tan^4(c+dx)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**4,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**4, x)
```

Maxima [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2}\tan^4(c + dx)\, dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int (a + a\sec(c + dx))^{3/2}\tan^4(c + dx)\, dx = \int (a\sec(dx + c) + a)^{\frac{3}{2}}\tan(dx + c)^4\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2}\tan^4(c + dx)\, dx = \int \tan(c + d\,x)^4 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{3/2} dx$$

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(3/2), x)
```

3.155 $\int (a+a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 128

$$\int (a+a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx=-\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^3\tan^3(c+dx)}{d(a+a\sec(c+dx))^{3/2}}+\frac{2a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}$$

```
[Out] -2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)+2*a^3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2
/5*a^4*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 472, 209}

$$\int (a+a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx=-\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^4\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2a^3\tan^3(c+dx)}{d(a\sec(c+dx)+a)^{3/2}}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^2,x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*Tan[c + d*x]^3)/(d
*(a + a*Sec[c + d*x])^(3/2)) + (2*a^4*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d
*x])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(2a^3)\,\text{Subst}\left(\int \frac{x^2(2+ax^2)^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{(2a^3)\,\text{Subst}\left(\int \left(\frac{1}{a}+3x^2+ax^4-\frac{1}{a(1+ax^2)}\right)\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3\tan^3(c+dx)}{d(a+a\sec(c+dx))^{3/2}} \\
&\quad + \frac{2a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} \\
&\quad + \frac{2a^3\tan^3(c+dx)}{d(a+a\sec(c+dx))^{3/2}} + \frac{2a^4\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.79 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

$$\int (a+a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx = \frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(1+\sec(c+dx))}\left(-10\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{5}{2}}(c+\right.}{10d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]^2,x]
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*(-10*Sqrt[2]*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 5*Sin[(3*(c + d*x))/2
] + Sin[(5*(c + d*x))/2]))/(10*d)
```

Maple [A] (warning: unable to verify)

Time = 3.48 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.46

method	result
default	$-\frac{a\left(5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^{2}\csc(dx+c)^{2}-1}}\right)\left((1-\cos(dx+c))^{2}\csc(dx+c)^{2}-1\right)^{\frac{5}{2}}+2(1-\cos(dx+c))^{5}\csc(dx+c)^{5}-10\csc(dx+\right.}{5d(\csc(dx+c)-\cot(dx+c)+1)^{2}(-\cot(dx+c)+\csc(dx+c)-1)^{2}}$

```
[In] int((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] -1/5/d*a*(5*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)
*(-cot(d*x+c)+csc(d*x+c)))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(5/2)+2*(1-cos
(d*x+c))^5*csc(d*x+c)^5-10*csc(d*x+c)+10*cot(d*x+c))*(-2*a/((1-cos(d*x+c))^
2*csc(d*x+c)^2-1))^(1/2)/(csc(d*x+c)-cot(d*x+c)+1)^2/(-cot(d*x+c)+csc(d*x+c
)-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.51

$$\int (a+a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx = \left[\frac{5\left(a\cos(dx+c)^{3}+a\cos(dx+c)^{2}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos\right.}{\cos(dx+c)+1}}{5\left(d\cos(dx+c)^{3}+d\cos(\right.}\right.$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [1/5*(5*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c
)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(a*cos(d*x + c)^2 + 3
*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 2/5*(5*(a*cos(d*x + c)^3 + a*cos(d*x
 + c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)/(sqrt(a)*sin(d*x + c))) + (a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(
d*x + c)^2)]
```

Sympy [F]

$$\int (a + a\sec(c + dx))^{3/2}\tan^2(c + dx)\,dx = \int \left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}\tan^2\left(c + dx\right)dx$$

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*tan(c + d*x)**2, x)
```

Maxima [F]

$$\int (a + a\sec(c + dx))^{3/2}\tan^2(c + dx)\,dx = \int \left(a\sec\left(dx + c\right) + a\right)^{\frac{3}{2}}\tan\left(dx + c\right)^2\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/10*(5*((a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c
) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (a*cos(2*d*x + 2*
c)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) - 1) - 2*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x +
 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*integrate((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d
*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(
6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2
*d*x + 2*c)^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(
```

```
2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6
*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(
2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4
*d*x + 4*c)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x
 + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(7/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))/((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3
 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2
+ cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x +
2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*
x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
+ 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin
(2*d*x + 2*c))*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))^2 + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 +
4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x +
2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2
*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*
d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*co
s(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) +
(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))^2), x) - 2*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2
*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d*x
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 + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*
d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d
*x + 2*c)^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*
d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d
*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*
d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d
*x + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x +
 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)))/((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*
c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 +
 cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 +
cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*
c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x
+ 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2
*d*x + 2*c))*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))^2 + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*
x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*
c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c
)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*
x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(
4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (c
os(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6
*c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(
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2*d*x + 2*c) + 1))^2), x) + 10*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*
c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d*x
+ 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d
*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*
x + 2*c)^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d
*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*
x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d
*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x +
2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))/((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 +
cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + c
os(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c
)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x +
 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4
*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*
d*x + 2*c))*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))^2 + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x
 + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c
)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)
^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x
 + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4
*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (co
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s(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*
c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))^2), x) - 6*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)
^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d*x +
2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x
 + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x
+ 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x
 + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x
+ 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x
 + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*
c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)))/((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^
2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + co
s(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos
(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4
*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4*(
sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*
x + 2*c))*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1))^2 + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^
3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2
 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x +
```

```
 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d
*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
 + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*si
n(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))^2), x))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(5*a*cos(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(4*d*x + 4*c) - (5*a*cos(4*d*x + 4*c) - a)*sin(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))/((d*cos(2*d*x +
2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*c) + d)*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4))
```

Giac [F]

$$\int (a+a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx=\int (a\sec{(dx+c)}+a)^{\frac{3}{2}}\tan{(dx+c)}^2\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*tan(d*x+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^{3/2}\tan^2(c+dx)\,dx=\int \tan(c+d\,x)^2\left(a+\frac{a}{\cos{(c+d\,x)}}\right)^{3/2}dx$$

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(3/2), x)
```

3.156 $\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}$$

```
[Out] -2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2*a*cot(d*x+
c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 331, 209}

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} - \frac{2a\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2
*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(2a)\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= -\frac{2a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.59

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx =$$

$$-\frac{2\cot(c+dx)\sqrt{\frac{1}{1+\sec(c+dx)}}(a(1+\sec(c+dx)))^{3/2}\left(\sqrt{\cos(c+dx)}\sqrt{\frac{1}{1+\cos(c+dx)}}+\arcsin\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\cos(c+dx)}}}\right)\tan\right.}{d}$$

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cot[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*(a*(1 + Sec[c + d*x]))^(3/2)
*(Sqrt[Cos[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)] + ArcSin[Tan[(c + d*x)/2
]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Tan[(c + d*x)/2]))/d
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

method	result	size
default	$-\frac{2a\sqrt{a(1+\sec(dx+c))}\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\cot(dx+c)\right)}{d}$	85

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a*(a*(1+sec(d*x+c)))^(1/2)*((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh
(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+cot(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(56) = 112$.

Time = 0.34 (sec) , antiderivative size = 264, normalized size of antiderivative = 4.12

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{\sqrt{-a}a\log\left(-\frac{8\,a\cos(dx+c)^3+4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7\,a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(dx\cdots}{2\,d\sin(dx+c)}\right.$$
$$\left. -\frac{a^{\frac{3}{2}}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2\,a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin(dx+c)+2\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{d\sin(dx+c)}\right]$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x +
 c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*co
s(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*a*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c)), -(a^(3/2)*arctan(2*sqrt
(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*
cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*a*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c))]
```

Sympy [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\cot^2(c+dx)\,dx$$

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*cot(c + d*x)**2, x)
```

Maxima [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^2\,dx$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cot(d*x + c)^2, x)
```

Giac [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^2\,dx$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cot(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(3/2), x)
```

3.157 $\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}d} + \frac{3a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{2d} - \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d}$$

```
[Out] 2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-1/3*cot(d*x+c
)^3*(a+a*sec(d*x+c))^(3/2)/d-1/4*a^(3/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1
/2)/(a+a*sec(d*x+c))^(1/2))/d*2^(1/2)+3/2*a*cot(d*x+c)*(a+a*sec(d*x+c))^(1/
2)/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3972, 491, 597, 536, 209}

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} - \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}d} - \frac{\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d} + \frac{3a\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (a^
(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2
*Sqrt[2]*d) + (3*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(2*d) - (Cot[c +
d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= -\frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d} - \frac{\text{Subst}\left(\int \frac{-9a-3a^2x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{3d}\\
&= \frac{3a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{2d} - \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d}\\
&\quad + \frac{\text{Subst}\left(\int \frac{-21a^2-9a^3x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{6d}\\
&= \frac{3a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{2d} - \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d}\\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2d}\\
&\quad - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}d}\\
&\quad + \frac{3a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{2d} - \frac{\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 6.69 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.57

$$
\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx =
$$

$$
-\frac{\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right) - 4\sqrt{2}\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}\right)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{1+\sec(c+dx)}(a}{8d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sec^{\frac{3}{2}}(c+dx)}
$$

$$
+\frac{\cos(c+dx)\sec^3\left(\frac{1}{2}(c+dx)\right)(a(1+\sec(c+dx)))^{3/2}\left(\frac{13}{24}\csc\left(\frac{1}{2}(c+dx)\right) - \frac{1}{24}\csc^3\left(\frac{1}{2}(c+dx)\right) - \frac{11}{12}\sin\left(\frac{1}{2}(c\right.\right.}{d}
$$

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -1/8*((ArcSin[Tan[(c + d*x)/2]] - 4*Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]])*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[
(c + d*x)/2]^4*Sqrt[1 + Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))/(d*Sqrt
[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2)) + (Cos[c + d*x]*Sec[(c + d*x)/2]^3
*(a*(1 + Sec[c + d*x]))^(3/2)*((13*Csc[(c + d*x)/2])/24 - Csc[(c + d*x)/2]^
3/24 - (11*Sin[(c + d*x)/2])/12))/d
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.37

method	result
default	$-\frac{a\sqrt{a(1+\sec(dx+c))}\left(3\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-}{}$

[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/12/d*a*(a*(1+sec(d*x+c)))^(1/2)*(3*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-24*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+22*cot(d*x+c)^3+4*cot(d*x+c)^2*csc(d*x+c)-18*cot(d*x+c)*csc(d*x+c)^2)

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 531, normalized size of antiderivative = 3.69

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{3\left(\sqrt{2}a\cos(dx+c)-\sqrt{2}a\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{}\right.$$

[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/24*(3*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 12*(a*cos(d*x + c) - a)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4*(11*a*cos(d*x + c)^2 - 9*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c)), 1/12*(12*(a*cos(d*x + c) - a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 3*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(11*a*cos(d*x + c)^2 - 9*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c))]

Sympy [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^4\,dx$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cot(c+d\,x)^4\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(3/2), x)
```

3.158 $\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 226

$$
\begin{aligned}
\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx &= -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&+\frac{11a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}d}-\frac{21a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{16d} \\
&+\frac{5\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{24d}+\frac{3\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{20ad} \\
&-\frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{4ad}
\end{aligned}
$$

```
[Out] -2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+5/24*cot(d*x
+c)^3*(a+a*sec(d*x+c))^(3/2)/d+3/20*cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)/a/d
-1/4*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(5/2)/a/
d+11/32*a^(3/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2
))/d*2^(1/2)-21/16*a*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{3972, 483, 597, 536, 209\}$

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{11a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}d} + \frac{3\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{20ad} + \frac{5\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{24d} - \frac{21a\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{16d} - \frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{4ad}$$

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
1*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(16*Sqrt[2]*d) - (21*a*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(16*d) + (5
*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(24*d) + (3*Cot[c + d*x]^5*(a +
 a*Sec[c + d*x])^(5/2))/(20*a*d) - (Cos[c + d*x]*Cot[c + d*x]^5*Sec[(c + d*
x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(4*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}\\
&= -\frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{4ad}\\
&\quad-\frac{\text{Subst}\left(\int \frac{-3a-7a^2x^2}{x^6(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2a^2d}\\
&= \frac{3\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{20ad}\\
&\quad-\frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{4ad}\\
&\quad+\frac{\text{Subst}\left(\int \frac{25a^2-15a^3x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{20a^2d}\\
&= \frac{5\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{24d}+\frac{3\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{20ad}\\
&\quad-\frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{4ad}\\
&\quad-\frac{\text{Subst}\left(\int \frac{315a^3+75a^4x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{120a^2d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{21a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{16d} + \frac{5\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{24d} \\
&\quad + \frac{3\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{20ad} \\
&\quad - \frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{4ad} \\
&\quad + \frac{\text{Subst}\left(\int \frac{795a^4+315a^5x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{240a^2d} \\
&= -\frac{21a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{16d} + \frac{5\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{24d} \\
&\quad + \frac{3\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{20ad} \\
&\quad - \frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{4ad} \\
&\quad - \frac{(11a^2)\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{16d} \\
&\quad + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{11a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}d} \\
&\quad - \frac{21a\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{16d} + \frac{5\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{24d} \\
&\quad + \frac{3\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{20ad} \\
&\quad - \frac{\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{4ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 7.46 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.04

$$
\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{(a(1+\sec(c+dx)))^{3/2}\left(165\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{1+\sec(c
$$

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(3/2)*(165*ArcSin[Tan[(c + d*x)/2]]*Sec[(c + d*x)/2
]^4*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] + 2*Sqrt[2]*(-240*
```

```
ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Sec[(c + d*x)/2]^4*S
qrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] + (Sqrt[(1 + Cos[c + d*
x])^(-1)]*Csc[c + d*x]^5*(281 - 279*Sec[c + d*x] + Cos[2*(c + d*x)]*(-449 +
 351*Sec[c + d*x])))/Sec[c + d*x]^(3/2))))/(960*d*Sqrt[Sec[(c + d*x)/2]^2]*
Sec[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00

method	result
default	$-\frac{a\left(-165\sin(dx+c)^5\sqrt{2}\,\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+96\right.}{}$

```
[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/480/d*a*(-165*sin(d*x+c)^5*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^
2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)+960*sin(d*x+c)^5*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+898*cos(d*x+c)^5+19
6*cos(d*x+c)^4-1432*cos(d*x+c)^3-100*cos(d*x+c)^2+630*cos(d*x+c))*(a*(1+sec
(d*x+c)))^(1/2)*csc(d*x+c)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 708, normalized size of antiderivative = 3.13

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{165\left(\sqrt{2}a\cos(dx+c)^3-\sqrt{2}a\cos(dx+c)^2-\sqrt{2}a\cos(dx+c)+\sqrt{2}a\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}}{}\right.}{}\right.$$

$$-\frac{480\left(a\cos(dx+c)^3-a\cos(dx+c)^2-a\cos(dx+c)+a\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\mathrm{s}}{}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/960*(165*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*
a*cos(d*x + c) + sqrt(2)*a)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d
```

```
*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 -
 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c)
+ 480*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*sqrt(-a)*l
og(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(c
os(d*x + c) + 1))*sin(d*x + c) - 4*(449*a*cos(d*x + c)^4 - 351*a*cos(d*x +
c)^3 - 365*a*cos(d*x + c)^2 + 315*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c)))/((d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)
*sin(d*x + c)), -1/480*(480*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*
x + c) + a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(
d*x + c) + 165*(sqrt(2)*a*cos(d*x + c)^3 - sqrt(2)*a*cos(d*x + c)^2 - sqrt(
2)*a*cos(d*x + c) + sqrt(2)*a)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(4
49*a*cos(d*x + c)^4 - 351*a*cos(d*x + c)^3 - 365*a*cos(d*x + c)^2 + 315*a*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^3 -
d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \cot^6(c + dx)(a + a\sec(c + dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cot^6(c + dx)(a + a\sec(c + dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\int (a\sec(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^6\,dx$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\int \cot(c+d\,x)^6\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(3/2), x)
```

3.159 $\int (a+a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 193

$$\begin{aligned}\int (a+a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx = &-\frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}\\&+\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}+\frac{2a(a+a\sec(c+dx))^{3/2}}{3d}\\&+\frac{2(a+a\sec(c+dx))^{5/2}}{5d}+\frac{2(a+a\sec(c+dx))^{7/2}}{7ad}+\frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d}\\&-\frac{6(a+a\sec(c+dx))^{11/2}}{11a^3d}+\frac{2(a+a\sec(c+dx))^{13/2}}{13a^4d}\end{aligned}$$

```
[Out] -2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*a*(a+a*sec(d*x+c))
^(3/2)/d+2/5*(a+a*sec(d*x+c))^(5/2)/d+2/7*(a+a*sec(d*x+c))^(7/2)/a/d+2/9*(a
+a*sec(d*x+c))^(9/2)/a^2/d-6/11*(a+a*sec(d*x+c))^(11/2)/a^3/d+2/13*(a+a*sec
(d*x+c))^(13/2)/a^4/d+2*a^2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{3965, 90, 52, 65, 213\}$

$$\int (a + a\sec(c + dx))^{5/2}\tan^5(c + dx)\,dx = -\frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} + \frac{2(a\sec(c + dx) + a)^{13/2}}{13a^4d} - \frac{6(a\sec(c + dx) + a)^{11/2}}{11a^3d} + \frac{2(a\sec(c + dx) + a)^{9/2}}{9a^2d} + \frac{2a^2\sqrt{a\sec(c + dx) + a}}{d} + \frac{2(a\sec(c + dx) + a)^{7/2}}{7ad} + \frac{2(a\sec(c + dx) + a)^{5/2}}{5d} + \frac{2a(a\sec(c + dx) + a)^{3/2}}{3d}$$

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^5,x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d + (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec
[c + d*x])^(5/2))/(5*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a*d) + (2*(a +
a*Sec[c + d*x])^(9/2))/(9*a^2*d) - (6*(a + a*Sec[c + d*x])^(11/2))/(11*a^3*
d) + (2*(a + a*Sec[c + d*x])^(13/2))/(13*a^4*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{9/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4 d}\\
&= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{9/2}+\frac{a^2(a+ax)^{9/2}}{x}+a(a+ax)^{11/2}\right)\,dx, x, \sec(c+dx)\right)}{a^4 d}\\
&= -\frac{6(a+a\sec(c+dx))^{11/2}}{11a^3 d}+\frac{2(a+a\sec(c+dx))^{13/2}}{13a^4 d}+\frac{\text{Subst}\left(\int \frac{(a+ax)^{9/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2 d}\\
&= \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2 d}-\frac{6(a+a\sec(c+dx))^{11/2}}{11a^3 d}\\
&\quad+\frac{2(a+a\sec(c+dx))^{13/2}}{13a^4 d}+\frac{\text{Subst}\left(\int \frac{(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{ad}\\
&= \frac{2(a+a\sec(c+dx))^{7/2}}{7ad}+\frac{2(a+a\sec(c+dx))^{9/2}}{9a^2 d}-\frac{6(a+a\sec(c+dx))^{11/2}}{11a^3 d}\\
&\quad+\frac{2(a+a\sec(c+dx))^{13/2}}{13a^4 d}+\frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{2(a+a\sec(c+dx))^{5/2}}{5d}+\frac{2(a+a\sec(c+dx))^{7/2}}{7ad}\\
&\quad+\frac{2(a+a\sec(c+dx))^{9/2}}{9a^2 d}-\frac{6(a+a\sec(c+dx))^{11/2}}{11a^3 d}\\
&\quad+\frac{2(a+a\sec(c+dx))^{13/2}}{13a^4 d}+\frac{a\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{2a(a+a\sec(c+dx))^{3/2}}{3d}+\frac{2(a+a\sec(c+dx))^{5/2}}{5d}+\frac{2(a+a\sec(c+dx))^{7/2}}{7ad}\\
&\quad+\frac{2(a+a\sec(c+dx))^{9/2}}{9a^2 d}-\frac{6(a+a\sec(c+dx))^{11/2}}{11a^3 d}\\
&\quad+\frac{2(a+a\sec(c+dx))^{13/2}}{13a^4 d}+\frac{a^2\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d} \\
&\quad + \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{6(a+a\sec(c+dx))^{11/2}}{11a^3d} \\
&\quad + \frac{2(a+a\sec(c+dx))^{13/2}}{13a^4d} + \frac{a^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} \\
&= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d} \\
&\quad + \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{6(a+a\sec(c+dx))^{11/2}}{11a^3d} \\
&\quad + \frac{2(a+a\sec(c+dx))^{13/2}}{13a^4d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} \\
&= -\frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} \\
&\quad + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} \\
&\quad + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{6(a+a\sec(c+dx))^{11/2}}{11a^3d} + \frac{2(a+a\sec(c+dx))^{13/2}}{13a^4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.81

$$
\int (a+a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx = \frac{(a(1+\sec(c+dx)))^{5/2}\left(-2\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)+2\sqrt{1+\sec(c+dx)}+\frac{2}{3}(1+\sec(c+d
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^5,x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(5/2)*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + 2*Sqrt[
1 + Sec[c + d*x]] + (2*(1 + Sec[c + d*x])^(3/2))/3 + (2*(1 + Sec[c + d*x])^
(5/2))/5 + (2*(1 + Sec[c + d*x])^(7/2))/7 + (2*(1 + Sec[c + d*x])^(9/2))/9
- (6*(1 + Sec[c + d*x])^(11/2))/11 + (2*(1 + Sec[c + d*x])^(13/2))/13))/(d*
(1 + Sec[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 262.51 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.64

method	result
default	$\frac{2a^2\sqrt{a(1+\sec(dx+c))}\left(45045\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+71689+31723\sec(dx+c)-12531\sec(dx+c)^2-27095\sec(dx+}{45045d}$

[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x,method=_RETURNVERBOSE)

[Out] 2/45045/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*(45045*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+71689+31723*sec(d*x+c)-12531*sec(d*x+c)^2-27095*sec(d*x+c)^3-4445*sec(d*x+c)^4+8505*sec(d*x+c)^5+3465*sec(d*x+c)^6)

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.00

$$\int(a+a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx = \left[\frac{45045\,a^{\frac{5}{2}}\cos\left(dx+c\right)^6\log\left(-8\,a\cos\left(dx+c\right)^2+4\left(2\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)}{\cos(dx+c)}}\right.}{}\right.$$

[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="fricas")

[Out] [1/90090*(45045*a^(5/2)*cos(d*x + c)^6*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(71689*a^2*cos(d*x + c)^6 + 31723*a^2*cos(d*x + c)^5 - 12531*a^2*cos(d*x + c)^4 - 27095*a^2*cos(d*x + c)^3 - 4445*a^2*cos(d*x + c)^2 + 8505*a^2*cos(d*x + c) + 3465*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^6), 1/45045*(45045*sqrt(-a)*a^2*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^6 + 2*(71689*a^2*cos(d*x + c)^6 + 31723*a^2*cos(d*x + c)^5 - 12531*a^2*cos(d*x + c)^4 - 27095*a^2*cos(d*x + c)^3 - 4445*a^2*cos(d*x + c)^2 + 8505*a^2*cos(d*x + c) + 3465*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^6)]

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx = \text{Timed out}$$

[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**5,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.94

$$\int (a+a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx = \frac{45045\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+18018\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}+\frac{6930\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{13}{2}}}{a^4}-\frac{24570\left(a+\frac{a}{\cos(dx+c)}\right)}{a^3}\cdots}{4504\cdots}$$

[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="maxima")

[Out] 1/45045*(45045*a^(5/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqrt(a))) + 18018*(a + a/cos(d*x + c))^(5/2) + 6930*(a + a/cos(d*x + c))^(13/2)/a^4 - 24570*(a + a/cos(d*x + c))^(11/2)/a^3 + 10010*(a + a/cos(d*x + c))^(9/2)/a^2 + 12870*(a + a/cos(d*x + c))^(7/2)/a + 30030*(a + a/cos(d*x + c))^(3/2)*a + 90090*sqrt(a + a/cos(d*x + c))*a^2)/d

Giac [F]

$$\int (a+a\sec(c+dx))^{5/2}\tan^5(c+dx)\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^5\,dx$$

[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^5,x, algorithm="giac")

[Out] sage0*x

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^5(c + dx)\, dx = \int \tan(c + d\,x)^5 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{5/2} dx$$

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^(5/2), x)
```

3.160 $\int (a+a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 145

$$\int (a+a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx=\frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}-\frac{2a(a+a\sec(c+dx))^{3/2}}{3d}-\frac{2(a+a\sec(c+dx))^{5/2}}{5d}-\frac{2(a+a\sec(c+dx))^{7/2}}{7ad}+\frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d}$$

```
[Out] 2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-2/3*a*(a+a*sec(d*x+c))^
(3/2)/d-2/5*(a+a*sec(d*x+c))^(5/2)/d-2/7*(a+a*sec(d*x+c))^(7/2)/a/d+2/9*(a+
a*sec(d*x+c))^(9/2)/a^2/d-2*a^2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3965, 81, 52, 65, 213}

$$\int (a+a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx=\frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d}+\frac{2(a\sec(c+dx)+a)^{9/2}}{9a^2d}-\frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}-\frac{2(a\sec(c+dx)+a)^{7/2}}{7ad}-\frac{2(a\sec(c+dx)+a)^{5/2}}{5d}-\frac{2a(a\sec(c+dx)+a)^{3/2}}{3d}$$

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (2*a^2*Sqrt[a + a
*Sec[c + d*x]])/d - (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) - (2*(a + a*Sec[
c + d*x])^(5/2))/(5*d) - (2*(a + a*Sec[c + d*x])^(7/2))/(7*a*d) + (2*(a + a
*Sec[c + d*x])^(9/2))/(9*a^2*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^2d}$$

$$= \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{7/2}}{x}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= -\frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{2(a+a\sec(c+dx))^{5/2}}{5d} - \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{a\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5d} - \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{a^2\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5d} - \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{a^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5d} - \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}$$

$$= \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} - \frac{2(a+a\sec(c+dx))^{5/2}}{5d} - \frac{2(a+a\sec(c+dx))^{7/2}}{7ad} + \frac{2(a+a\sec(c+dx))^{9/2}}{9a^2d}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int (a+a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx=\frac{2(a(1+\sec(c+dx)))^{5/2}\left(315\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}(-493-226\sec(c+\cdots\right.}{315d(1+\sec(c+dx))^{5/2}}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^3,x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(5/2)*(315*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(-493 - 226*Sec[c + d*x] + 12*Sec[c + d*x]^2 + 95*Sec[c
+ d*x]^3 + 35*Sec[c + d*x]^4)))/(315*d*(1 + Sec[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 55.97 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.72

method	result
default	$-\frac{2a^2\sqrt{a(1+\sec(dx+c))}\left(315\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+493+226\sec(dx+c)-12\sec(dx+c)^2-95\sec(dx+c)^3-35\sec\cdots\right.}{315d}$

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*(315*arctan((-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+493+226*sec(d*x+c)-12*sec(d*x
+c)^2-95*sec(d*x+c)^3-35*sec(d*x+c)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.30

$$\int (a+a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx=\left[\frac{315\,a^{\frac{5}{2}}\cos(dx+c)^4\log\left(-8\,a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cdots\right.}{\cdots}\right.$$

$$-\frac{315\sqrt{-a}a^2\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos(dx+c)^4+2\left(493\,a^2\cos(dx+c)^4+226\,a^2\cos(dx+c)^{\cdots}\right.}{315\,d\cos(dx+c)^4}$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/630*(315*a^(5/2)*cos(d*x + c)^4*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) - 4*(493*a^2*cos(d*x + c)^4 + 226*a^2*cos(d*x + c)^3 - 1
2*a^2*cos(d*x + c)^2 - 95*a^2*cos(d*x + c) - 35*a^2)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c)))/(d*cos(d*x + c)^4), -1/315*(315*sqrt(-a)*a^2*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
c) + a))*cos(d*x + c)^4 + 2*(493*a^2*cos(d*x + c)^4 + 226*a^2*cos(d*x + c)^
3 - 12*a^2*cos(d*x + c)^2 - 95*a^2*cos(d*x + c) - 35*a^2)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^4)]
```

Sympy [F]

$$\int (a+a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx=\int \left(a(\sec{(c+dx)}+1)\right)^{\frac{5}{2}}\tan^3{(c+dx)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**3,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*tan(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99

$$\int (a+a\sec(c+dx))^{5/2}\tan^3(c+dx)\,dx=$$

$$-\frac{315\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+126\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}-\frac{70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{9}{2}}}{a^2}+\frac{90\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a}+210\left(a+\frac{a}{\cos(dx\ldots}\right.}{315\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/315*(315*a^(5/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/co
s(d*x + c)) + sqrt(a))) + 126*(a + a/cos(d*x + c))^(5/2) - 70*(a + a/cos(d*
x + c))^(9/2)/a^2 + 90*(a + a/cos(d*x + c))^(7/2)/a + 210*(a + a/cos(d*x +
c))^(3/2)*a + 630*sqrt(a + a/cos(d*x + c))*a^2)/d
```

Giac [F]

$$\int (a + a\sec(c + dx))^{5/2}\tan^3(c + dx)\, dx = \int (a\sec(dx + c) + a)^{\frac{5}{2}}\tan(dx + c)^3\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2}\tan^3(c + dx)\, dx = \int \tan(c + d\,x)^3\left(a + \frac{a}{\cos(c + d\,x)}\right)^{5/2} dx$$

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^(5/2), x)
```

3.161 $\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = -\frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d}$$

```
[Out] -2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+2/3*a*(a+a*sec(d*x+c))
^(3/2)/d+2/5*(a+a*sec(d*x+c))^(5/2)/d+2*a^2*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3965, 52, 65, 213}

$$\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = -\frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a\sec(c+dx)+a}}{d} + \frac{2a(a\sec(c+dx)+a)^{3/2}}{3d} + \frac{2(a\sec(c+dx)+a)^{5/2}}{5d}$$

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x],x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d + (2*a*(a + a*Sec[c + d*x])^(3/2))/(3*d) + (2*(a + a*Sec
[c + d*x])^(5/2))/(5*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{a\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d}\\
&\quad + \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{a^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}
\end{aligned}
$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}$$

$$= -\frac{2a^{5/2}\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{2a(a+a\sec(c+dx))^{3/2}}{3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5d}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = \frac{2(a(1+\sec(c+dx)))^{5/2}\left(-15\mathrm{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)+\sqrt{1+\sec(c+dx)}(23+11\sec(c+dx)+3\sec^2(c+dx))\right)}{15d(1+\sec(c+dx))^{5/2}}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x],x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(5/2)*(-15*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + Sqrt
[1 + Sec[c + d*x]]*(23 + 11*Sec[c + d*x] + 3*Sec[c + d*x]^2)))/(15*d*(1 + S
ec[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 7.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\frac{2(a+a\sec(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2\sqrt{a+a\sec(dx+c)}\,a^2-2a^{\frac{5}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	74
default	$\frac{\frac{2(a+a\sec(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+a\sec(dx+c))^{\frac{3}{2}}}{3}+2\sqrt{a+a\sec(dx+c)}\,a^2-2a^{\frac{5}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	74

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/5*(a+a*sec(d*x+c))^(5/2)+2/3*a*(a+a*sec(d*x+c))^(3/2)+2*(a+a*sec(d*x
+c))^(1/2)*a^2-2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.91

$$\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = \left[\frac{15\,a^{\frac{5}{2}}\cos\left(dx+c\right)^{2}\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\,\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\right.}{30\,d\cos\left(dx+\right.}\right.$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^(5/2)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
os(d*x + c) - a) + 4*(23*a^2*cos(d*x + c)^2 + 11*a^2*cos(d*x + c) + 3*a^2)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2), 1/15*(15*sqrt(
-a)*a^2*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^2 + 2*(23*a^2*cos(d*x + c)^2 + 11*
a^2*cos(d*x + c) + 3*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(d*cos(d
*x + c)^2)]
```

Sympy [F]

$$\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = \int \left(a(\sec\left(c+dx\right)+1)\right)^{\frac{5}{2}}\tan\left(c+dx\right)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*tan(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int (a+a\sec(c+dx))^{5/2}\tan(c+dx)\,dx = \frac{15\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)+6\,\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}+10\,\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a+30\,\sqrt{a+\frac{a}{\cos(dx+c)}}a^{2}}{15\,d}$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/15*(15*a^(5/2)*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d
*x + c)) + sqrt(a))) + 6*(a + a/cos(d*x + c))^(5/2) + 10*(a + a/cos(d*x + c
))^(3/2)*a + 30*sqrt(a + a/cos(d*x + c))*a^2)/d
```

Giac [F]

$$\int (a + a\sec(c + dx))^{5/2}\tan(c + dx)\,dx = \int (a\sec(dx + c) + a)^{\frac{5}{2}}\tan(dx + c)\ dx$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int (a + a\sec(c + dx))^{5/2}\tan(c + dx)\,dx = \frac{2\left(a + \frac{a}{\cos(c+d\,x)}\right)^{5/2}}{5\,d} + \frac{2\,a\left(a + \frac{a}{\cos(c+d\,x)}\right)^{3/2}}{3\,d} + \frac{2\,a^2\,\sqrt{a + \frac{a}{\cos(c+d\,x)}}}{d} + \frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}}{d}$$

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] (2*(a + a/cos(c + d*x))^(5/2))/(5*d) + (a^(5/2)*atan(((a + a/cos(c + d*x))^
(1/2)*1i)/a^(1/2))*2i)/d + (2*a*(a + a/cos(c + d*x))^(3/2))/(3*d) + (2*a^2*
(a + a/cos(c + d*x))^(1/2))/d
```

3.162 $\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{4\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}$$

```
[Out] 2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-4*a^(5/2)*arctanh(1/2*(
a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+2*a^2*(a+a*sec(d*x+c))^(1/
2)/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3965, 86, 162, 65, 213}

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} - \frac{4\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a\sec(c+dx)+a}}{d}$$

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (4*Sqrt[2]*a^(5/2
)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d + (2*a^2*Sqrt[a +
a*Sec[c + d*x]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a^2\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{x(-a+ax)}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} + \frac{a\text{Subst}\left(\int \frac{a^3+3a^3x}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{a^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}\\
&\quad + \frac{(4a^4)\,\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}
\end{aligned}
$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d} - \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} + \frac{(8a^3)\,\mathrm{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}$$

$$= \frac{2a^{5/2}\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{4\sqrt{2}a^{5/2}\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2a^2\sqrt{a+a\sec(c+dx)}}{d}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2(a(1+\sec(c+dx)))^{5/2}\left(\mathrm{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) - 2\sqrt{2}\mathrm{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right) + \sqrt{1+\mathrm{s}\cdots}\right.}{d(1+\sec(c+dx))^{5/2}}$$

```
[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a*(1 + Sec[c + d*x]))^(5/2)*(ArcTanh[Sqrt[1 + Sec[c + d*x]]] - 2*Sqrt[2
]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]] + Sqrt[1 + Sec[c + d*x]]))/(d*(1
+ Sec[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 5.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.22

method	result	siz
default	$-\frac{2a^2\sqrt{a(1+\sec(dx+c))}\left(2\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-1\right)}{d}$	11

```
[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*(2*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+arctan((-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.16

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{2\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)-3\,a\cos(dx+c)-a}{\cos(dx+c)-1}\right)+a^{\frac{5}{2}}\log\left(-2\,a\cos\left(dx+c\right)-2\sqrt{a}\right.}{d}\right.$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] [(2*sqrt(2)*a^(5/2)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + a^(5/2)
*log(-2*a*cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c) - a) + 2*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d, 2*(2*
sqrt(2)*sqrt(-a)*a^2*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - sqrt(-a)*a^2*arctan(sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a))
 + a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cot\left(dx+c\right)\,dx$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c), x)
```

Giac [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec{(dx+c)}+a)^{\frac{5}{2}}\cot{(dx+c)}\,dx$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cot(c+d\,x)\,\left(a+\frac{a}{\cos{(c+d\,x)}}\right)^{5/2}\,dx$$

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)*(a + a/cos(c + d*x))^(5/2), x)
```

3.163 $\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{3a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))}$$

```
[Out] -2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d+3/2*a^(5/2)*arctanh(1/
2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+a^2*(a+a*sec(d*x+c))^(1
/2)/d/(1-sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3965, 101, 162, 65, 213}

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} + \frac{3a^{5/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{a\sec(c+dx)+a}}{d(1-\sec(c+dx))}$$

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d + (3*a^(5/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*d) + (a^2*Sqrt[a +
 a*Sec[c + d*x]])/(d*(1 - Sec[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{a^4\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x(-a+ax)^2}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))} + \frac{a^3\text{Subst}\left(\int \frac{-a-\frac{ax}{2}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))} + \frac{a^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} - \frac{(3a^4)\,\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{2d}$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} - \frac{(3a^3)\,\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}$$

$$= -\frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{3a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{a+a\sec(c+dx)}}{d(1-\sec(c+dx))}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{(a(1+\sec(c+dx)))^{5/2}\left(-2\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right) + \frac{3\text{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\sqrt{1+\sec(c+dx)}}{-1+\sec(c+dx)}\right)}{d(1+\sec(c+dx))^{5/2}}$$

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(5/2)*(-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]] + (3*ArcT
anh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]])/Sqrt[2] - Sqrt[1 + Sec[c + d*x]]/(-1 +
 Sec[c + d*x])))/(d*(1 + Sec[c + d*x])^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(89) = 178$.

Time = 15.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.26

method	result
default	$\frac{a^2\sqrt{a(1+\sec(dx+c))}\left(3\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+4\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\cos(dx+c)\sqrt{-\frac{\cos}{\cos}}\right.}{2d(\cos(dx+c)}$

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*(3*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4*a
rctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)-3*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-4*arctan((-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*cos(d*x+c))/(cos(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(88) = 176$.

Time = 0.34 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.75

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{4\,a^2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)+4\left(a^2\cos(dx+c)-a^2\right)\sqrt{a}\log\left(-2\,a\cos(dx+c)+2\,\sqrt{a}\sqrt{\frac{a}{}}\right.}{4}\right.$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 4*(a^2*c
os(d*x + c) - a^2)*sqrt(a)*log(-2*a*cos(d*x + c) + 2*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c) - a) + 3*(sqrt(2)*a^2*cos(d*x + c) -
 sqrt(2)*a^2)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)))/(d*cos(
d*x + c) - d), 1/2*(2*a^2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c) - 3*(sqrt(2)*a^2*cos(d*x + c) - sqrt(2)*a^2)*sqrt(-a)*arctan(sqrt(2)*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c)
 + a)) + 4*(a^2*cos(d*x + c) - a^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)))/(d*cos(d*x +
c) - d)]
```

Sympy [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^3\,dx$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c)^3, x)
```

Giac [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^3\,dx$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cot(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^(5/2), x)
```

3.164 $\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{43a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d} - \frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))}$$

```
[Out] 2*a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d-43/32*a^(5/2)*arctanh(1
/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-1/4*a^2*(a+a*sec(d*x+c
))^(1/2)/d/(1-sec(d*x+c))^2-11/16*a^2*(a+a*sec(d*x+c))^(1/2)/d/(1-sec(d*x+c
))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3965, 105, 156, 162, 65, 213}

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{d} - \frac{43a^{5/2}\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d} - \frac{11a^2\sqrt{a\sec(c+dx)+a}}{16d(1-\sec(c+dx))} - \frac{a^2\sqrt{a\sec(c+dx)+a}}{4d(1-\sec(c+dx))^2}$$

```
[In] Int[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/d - (43*a^(5/2)*ArcTa
nh[Sqrt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*d) - (a^2*Sqrt[
```

```
a + a*Sec[c + d*x]])/(4*d*(1 - Sec[c + d*x])^2) - (11*a^2*Sqrt[a + a*Sec[c
+ d*x]])/(16*d*(1 - Sec[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
```

```
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a^6\text{Subst}\left(\int \frac{1}{x(-a+ax)^3\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{a^3\text{Subst}\left(\int \frac{4a^2+\frac{3a^2x}{2}}{x(-a+ax)^2\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{4d}\\
&= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))}\\
&\quad + \frac{\text{Subst}\left(\int \frac{8a^4+\frac{11a^4x}{4}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{8d}\\
&= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))}\\
&\quad - \frac{a^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}\\
&\quad + \frac{(43a^4)\,\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{32d}\\
&= -\frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))}\\
&\quad - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d}\\
&\quad + \frac{(43a^3)\,\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{16d}\\
&= \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{43a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d}\\
&\quad - \frac{a^2\sqrt{a+a\sec(c+dx)}}{4d(1-\sec(c+dx))^2} - \frac{11a^2\sqrt{a+a\sec(c+dx)}}{16d(1-\sec(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{(a(1+\sec(c+dx)))^{5/2}\left(64\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)-43\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)+\frac{2\sqrt{1}}{}\right.}{32d(1+\sec(c+dx))^{5/2}}$$

[In] Integrate[Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2),x]

[Out] ((a*(1 + Sec[c + d*x]))^(5/2)*(64*ArcTanh[Sqrt[1 + Sec[c + d*x]]] - 43*Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]] + (2*Sqrt[1 + Sec[c + d*x]]*(-15 + 11*Sec[c + d*x]))/(-1 + Sec[c + d*x])^2))/(32*d*(1 + Sec[c + d*x])^(5/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(122) = 244.

Time = 88.52 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.85

method	result
default	$\frac{a^2\sqrt{a(1+\sec(dx+c))}\left(-43\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-64\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\cos(dx+c)\sqrt{-}\right.}{}$

[In] int(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/32/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)-1)*(-43*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-64*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+43*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+64*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+30*cos(d*x+c)*cot(d*x+c)^2+8*cot(d*x+c)^2-22*cot(d*x+c)*csc(d*x+c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(118) = 236$.

Time = 0.33 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.42

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{64\left(a^2\cos(dx+c)^2-2a^2\cos(dx+c)+a^2\right)\sqrt{a}\log\left(-2a\cos(dx+c)-2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\right.}{}\right.$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(64*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(a)*log(-2*a*
cos(d*x + c) - 2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c) - a) + 43*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqr
t(2)*a^2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) - 4*(15*a^2
*cos(d*x + c)^2 - 11*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c)))/(d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d), 1/32*(43*(sqrt(2)*a^2*cos(d
*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x
 + c) + a)) - 64*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(-a)*a
rctan(sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(
d*x + c) + a)) - 2*(15*a^2*cos(d*x + c)^2 - 11*a^2*cos(d*x + c))*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c)))/(d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)]
```

Sympy [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**5*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^5\,dx$$

```
[In] integrate(cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cot(c+d\,x)^5\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

```
[In] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^5*(a + a/cos(c + d*x))^(5/2), x)
```

3.165 $\int (a+a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 290

$$\begin{aligned}\int (a+a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx &= -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &+\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^5\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}\\ &+\frac{62a^6\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}+\frac{98a^7\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}}+\frac{62a^8\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}}\\ &+\frac{18a^9\tan^{13}(c+dx)}{13d(a+a\sec(c+dx))^{13/2}}+\frac{2a^{10}\tan^{15}(c+dx)}{15d(a+a\sec(c+dx))^{15/2}}\end{aligned}$$

```
[Out] -2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^3*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)-2/3*a^4*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
+2/5*a^5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+62/7*a^6*tan(d*x+c)^7/d/(a+a
*sec(d*x+c))^(7/2)+98/9*a^7*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+62/11*a^8
*tan(d*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)+18/13*a^9*tan(d*x+c)^13/d/(a+a*sec
(d*x+c))^(13/2)+2/15*a^10*tan(d*x+c)^15/d/(a+a*sec(d*x+c))^(15/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used

$= \{3972, 472, 209\}$

$$\int (a+a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx = -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{2a^{10}\tan^{15}(c+dx)}{15d(a\sec(c+dx)+a)^{15/2}} + \frac{18a^9\tan^{13}(c+dx)}{13d(a\sec(c+dx)+a)^{13/2}} + \frac{62a^8\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}} + \frac{98a^7\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}} + \frac{62a^6\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}} + \frac{2a^5\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}} - \frac{2a^4\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}} + \frac{2a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^6,x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a^4*Tan[c + d*x]^3)/(3
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^5*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c +
 d*x])^(5/2)) + (62*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) +
(98*a^7*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (62*a^8*Tan[c +
d*x]^11)/(11*d*(a + a*Sec[c + d*x])^(11/2)) + (18*a^9*Tan[c + d*x]^13)/(13*
d*(a + a*Sec[c + d*x])^(13/2)) + (2*a^10*Tan[c + d*x]^15)/(15*d*(a + a*Sec[
c + d*x])^(15/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{(2a^6)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)^5}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2a^6)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+31x^6+49ax^8+31a^2x^{10}+9a^3x^{12}+a^4x^{14}-\frac{1}{a^3(1+ax^2)}\right) dx, x, -\frac{}{\sqrt{}}\right.}{d}$$

$$= \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^5\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{62a^6\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} + \frac{98a^7\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}} + \frac{62a^8\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}} + \frac{18a^9\tan^{13}(c+dx)}{13d(a+a\sec(c+dx))^{13/2}} + \frac{2a^{10}\tan^{15}(c+dx)}{15d(a+a\sec(c+dx))^{15/2}} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^5\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{62a^6\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} + \frac{98a^7\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}} + \frac{62a^8\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}} + \frac{18a^9\tan^{13}(c+dx)}{13d(a+a\sec(c+dx))^{13/2}} + \frac{2a^{10}\tan^{15}(c+dx)}{15d(a+a\sec(c+dx))^{15/2}}$$

Mathematica [A] (verified)

Time = 8.76 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.60

$$\int (a+a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx = \frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\sqrt{a(1+\sec(c+dx))}\left(-2882880\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^6,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^7*Sqrt[a*(1 + Sec[c + d*x])]*(-2882880*S
qrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(15/2) + 604890*Sin[(c
 + d*x)/2] - 87230*Sin[(3*(c + d*x))/2] + 450450*Sin[(5*(c + d*x))/2] - 137
670*Sin[(7*(c + d*x))/2] + 210210*Sin[(9*(c + d*x))/2] + 75450*Sin[(11*(c +
 d*x))/2] + 90090*Sin[(13*(c + d*x))/2] + 16066*Sin[(15*(c + d*x))/2]))/(28
82880*d)
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.92

$$-\frac{2a^2\sqrt{a\left(1+\sec\left(dx+c\right)\right)}\left(45045\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos\left(dx+c\right)+45045\cdots\right.}{}$$

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x)
```

```
[Out] -2/45045/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(45045*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*cos(d*x+c)+45045*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
h(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-16066*sin(d
*x+c)-53078*tan(d*x+c)-17286*sec(d*x+c)*tan(d*x+c)+30640*tan(d*x+c)*sec(d*x
+c)^2+26810*tan(d*x+c)*sec(d*x+c)^3-2898*sec(d*x+c)^4*tan(d*x+c)-10164*tan(
d*x+c)*sec(d*x+c)^5-3003*tan(d*x+c)*sec(d*x+c)^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.64

$$\int (a+a\sec(c+dx))^{5/2}\tan^6(c+dx)\,dx=\left[\frac{45045\left(a^2\cos\left(dx+c\right)^8+a^2\cos\left(dx+c\right)^7\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx\cdots}{\cos(dx+c)+1}\right.}{}\right.$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] [1/45045*(45045*(a^2*cos(d*x + c)^8 + a^2*cos(d*x + c)^7)*sqrt(-a)*log((2*a
*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(16066*a^
2*cos(d*x + c)^7 + 53078*a^2*cos(d*x + c)^6 + 17286*a^2*cos(d*x + c)^5 - 30
640*a^2*cos(d*x + c)^4 - 26810*a^2*cos(d*x + c)^3 + 2898*a^2*cos(d*x + c)^2
 + 10164*a^2*cos(d*x + c) + 3003*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^8 + d*cos(d*x + c)^7), 2/45045*(45045*(a^2
*cos(d*x + c)^8 + a^2*cos(d*x + c)^7)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (16066*a^2*cos(d*x
 + c)^7 + 53078*a^2*cos(d*x + c)^6 + 17286*a^2*cos(d*x + c)^5 - 30640*a^2*c
os(d*x + c)^4 - 26810*a^2*cos(d*x + c)^3 + 2898*a^2*cos(d*x + c)^2 + 10164*
a^2*cos(d*x + c) + 3003*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^8 + d*cos(d*x + c)^7)]
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^6(c + dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**6,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^6(c + dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int (a + a\sec(c + dx))^{5/2} \tan^6(c + dx)\,dx = \int (a\sec(dx + c) + a)^{\frac{5}{2}} \tan(dx + c)^6\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^6,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2} \tan^6(c + dx)\,dx = \int \tan(c + d\,x)^6 \left(a + \frac{a}{\cos(c + d\,x)}\right)^{5/2} dx$$

```
[In] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^6*(a + a/cos(c + d*x))^(5/2), x)
```

3.166 $\int (a + a\sec(c + dx))^{5/2} \tan^4(c + dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 224

$$\int (a + a\sec(c + dx))^{5/2} \tan^4(c + dx)\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{6a^5\tan^5(c+dx)}{d(a+a\sec(c+dx))^{5/2}} + \frac{34a^6\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} + \frac{14a^7\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}} + \frac{2a^8\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2*a^3*tan(d*x
+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^4*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+
6*a^5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+34/7*a^6*tan(d*x+c)^7/d/(a+a*se
c(d*x+c))^(7/2)+14/9*a^7*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)+2/11*a^8*tan
(d*x+c)^11/d/(a+a*sec(d*x+c))^(11/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 472, 209}

$$\int (a + a\sec(c + dx))^{5/2} \tan^4(c + dx)\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{2a^8\tan^{11}(c+dx)}{11d(a\sec(c+dx)+a)^{11/2}} + \frac{14a^7\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}} + \frac{34a^6\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}} + \frac{6a^5\tan^5(c+dx)}{d(a\sec(c+dx)+a)^{5/2}} + \frac{2a^4\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}} - \frac{2a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^4,x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (2*
a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^4*Tan[c + d*x]^3)/(3*
d*(a + a*Sec[c + d*x])^(3/2)) + (6*a^5*Tan[c + d*x]^5)/(d*(a + a*Sec[c + d*
x])^(5/2)) + (34*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) + (14
*a^7*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2)) + (2*a^8*Tan[c + d*x]
^11)/(11*d*(a + a*Sec[c + d*x])^(11/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(2a^5)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{(2a^5)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+15x^4+17ax^6+7a^2x^8+a^3x^{10}+\frac{1}{a^2(1+ax^2)}\right)\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{6a^5\tan^5(c+dx)}{d(a+a\sec(c+dx))^{5/2}} \\
&\quad+\frac{34a^6\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}+\frac{14a^7\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}} \\
&\quad+\frac{2a^8\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}}-\frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}
\end{aligned}
$$

$$= \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{6a^5\tan^5(c+dx)}{d(a+a\sec(c+dx))^{5/2}} + \frac{34a^6\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} + \frac{14a^7\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}} + \frac{2a^8\tan^{11}(c+dx)}{11d(a+a\sec(c+dx))^{11/2}}$$

Mathematica [A] (verified)

Time = 7.81 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.67

$$\int (a+a\sec(c+dx))^{5/2}\tan^4(c+dx)\,dx = \frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(1+\sec(c+dx))}\left(5544\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{11}{2}}(c\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^4,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^5*Sqrt[a*(1 + Sec[c + d*x])]*(5544*Sqrt[
2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(11/2) - 1386*Sin[(c + d*x
)/2] + 1584*Sin[(3*(c + d*x))/2] - 1386*Sin[(5*(c + d*x))/2] - 143*Sin[(7*(
c + d*x))/2] - 693*Sin[(9*(c + d*x))/2] - 26*Sin[(11*(c + d*x))/2]))/(5544*
d)
```

Maple [A] (verified)

Time = 129.41 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.05

method	result
default	$\frac{2a^2\sqrt{a(1+\sec(dx+c))}\left(693\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+693\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{arctanh}\left(\right.\right.}{}$

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/693/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(693*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))*cos(d*x+c)+693*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d
*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-52*sin(d*x+c)-719*
tan(d*x+c)-366*sec(d*x+c)*tan(d*x+c)+157*tan(d*x+c)*sec(d*x+c)^2+224*tan(d*
x+c)*sec(d*x+c)^3+63*sec(d*x+c)^4*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.90

$$\int (a + a\sec(c + dx))^{5/2}\tan^4(c + dx)\,dx = \left[\frac{693\left(a^2\cos(dx+c)^6 + a^2\cos(dx+c)^5\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+ \cdots}{\cos(dx+c)+1}\right) \cdots}{\cdots}\right.$$

$$-\frac{2\left(693\left(a^2\cos(dx+c)^6 + a^2\cos(dx+c)^5\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right) + \left(52\,a^2\cos(dx+c)^5 + 719 \cdots\right.\right.}{693\left(d\cos(dx \cdots\right.}$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] [1/693*(693*(a^2*cos(d*x + c)^6 + a^2*cos(d*x + c)^5)*sqrt(-a)*log((2*a*cos
(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(52*a^2*cos(d
*x + c)^5 + 719*a^2*cos(d*x + c)^4 + 366*a^2*cos(d*x + c)^3 - 157*a^2*cos(d
*x + c)^2 - 224*a^2*cos(d*x + c) - 63*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5), -2/693*(693*(a
^2*cos(d*x + c)^6 + a^2*cos(d*x + c)^5)*sqrt(a)*arctan(sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (52*a^2*cos(d*x
+ c)^5 + 719*a^2*cos(d*x + c)^4 + 366*a^2*cos(d*x + c)^3 - 157*a^2*cos(d*x
+ c)^2 - 224*a^2*cos(d*x + c) - 63*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2}\tan^4(c + dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^{5/2}\tan^4(c+dx)\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int (a+a\sec(c+dx))^{5/2}\tan^4(c+dx)\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^4\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^4,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^{5/2}\tan^4(c+dx)\,dx = \int \tan(c+d\,x)^4\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^(5/2), x)
```

3.167 $\int (a+a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int (a+a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx=-\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{14a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^5\tan^5(c+dx)}{d(a+a\sec(c+dx))^{5/2}}+\frac{2a^6\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}$$

```
[Out] -2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^3*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)+14/3*a^4*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2
)+2*a^5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2/7*a^6*tan(d*x+c)^7/d/(a+a*s
ec(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 472, 209}

$$\int (a+a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx=-\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^6\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}}+\frac{2a^5\tan^5(c+dx)}{d(a\sec(c+dx)+a)^{5/2}}+\frac{14a^4\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^2,x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^3*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (14*a^4*Tan[c + d*x]^3)/(
3*d*(a + a*Sec[c + d*x])^(3/2)) + (2*a^5*Tan[c + d*x]^5)/(d*(a + a*Sec[c +
d*x])^(5/2)) + (2*a^6*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(2a^4)\operatorname{Subst}\left(\int \frac{x^2(2+ax^2)^3}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{(2a^4)\operatorname{Subst}\left(\int \left(\frac{1}{a}+7x^2+5ax^4+a^2x^6-\frac{1}{a(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{14a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^5\tan^5(c+dx)}{d(a+a\sec(c+dx))^{5/2}} \\
&\quad +\frac{2a^6\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}+\frac{(2a^3)\operatorname{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^3\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} \\
&\quad +\frac{14a^4\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^5\tan^5(c+dx)}{d(a+a\sec(c+dx))^{5/2}}+\frac{2a^6\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int (a + a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx =$$

$$-\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(1+\sec(c+dx))}\left(42\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{7}{2}}(c+dx) - 35\sin\right.}{42d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x]^2,x]
```

```
[Out] -1/42*(a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^3*Sqrt[a*(1 + Sec[c + d*x])]*(42*S
qrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(7/2) - 35*Sin[(c + d*
x)/2] + 7*Sin[(3*(c + d*x))/2] - 21*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*
x))/2]))/d
```

Maple [A] (warning: unable to verify)

Time = 13.54 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.46

method	result
default	$-\frac{a^2\left(21\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{7}{2}}+34(1-\cos(dx+c))^7\csc(dx+c)^7-98(1-\cos\right.}{21d(\csc(dx+c)-\cot(dx+c)+1)^3(-\cot}$

```
[In] int((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/d*a^2*(21*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(
1/2)*(-cot(d*x+c)+csc(d*x+c)))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(7/2)+34*(
1-cos(d*x+c))^7*csc(d*x+c)^7-98*(1-cos(d*x+c))^5*csc(d*x+c)^5+70*(1-cos(d*x
+c))^3*csc(d*x+c)^3+42*csc(d*x+c)-42*cot(d*x+c))*(-2*a/((1-cos(d*x+c))^2*cs
c(d*x+c)^2-1))^(1/2)/(csc(d*x+c)-cot(d*x+c)+1)^3/(-cot(d*x+c)+csc(d*x+c)-1)
^3
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.34

$$\int (a + a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx = \left[\frac{21\left(a^2\cos(dx+c)^4 + a^2\cos(dx+c)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2 + 2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) + a\,c}{\cos(dx+c)+1}\right.}{21\left(d\cos(d\right.}\right.$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [1/21*(21*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d
*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(10*a^2*cos(d*x
 + c)^3 - 16*a^2*cos(d*x + c)^2 - 12*a^2*cos(d*x + c) - 3*a^2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c
)^3), 2/21*(21*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*arctan(sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) -
 (10*a^2*cos(d*x + c)^3 - 16*a^2*cos(d*x + c)^2 - 12*a^2*cos(d*x + c) - 3*a
^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
 + d*cos(d*x + c)^3)]
```

Sympy [F]

$$\int (a+a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx = \int (a(\sec(c+dx)+1))^{\frac{5}{2}}\tan^2(c+dx)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*tan(d*x+c)**2,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*tan(c + d*x)**2, x)
```

Maxima [F]

$$\int (a+a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^2\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/42*(21*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*
x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (a^2*cos
(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - 2*(a^2*d*cos(2*d*x + 2*c)^
2 + a^2*d*sin(2*d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*integrate(
(((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c)
+ cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(9/2*arctan2(sin(2*d*x + 2*c),
```

```
 cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*
c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))*sin(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*c
os(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c
)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)
*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(9/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2
*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*
sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2
)*sin(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((cos(2*d*x + 2*c)^4 + sin(2*d*x +
2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*
x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*
c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2
*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2
+ cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x +
2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*cos(5/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(2*d*x + 2*c)^4 + sin(2*
d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^
2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*si
n(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*
x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 +
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*
(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*
d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*s
in(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x) - 16*(a^2*d*cos(2*d*
x + 2*c)^2 + a^2*d*sin(2*d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*i
```

```
ntegrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*
x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(7/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2
*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*
d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*
d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d
*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(7/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x + 6*c)*cos(2*d*x +
 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*
x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x
 + 2*c)^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((cos(2*d*x + 2*c)^4 + sin
(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*
c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2
*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2
 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos
(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) +
 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin
(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*cos(5
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(2*d*x + 2*c)^4
 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 +
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*
x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x +
2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x +
6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 +
2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))
```

```
*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x) + 28*(a^2*d
*cos(2*d*x + 2*c)^2 + a^2*d*sin(2*d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) +
 a^2*d)*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)
*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c)
+ 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c)
+ 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) -
 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
 ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c)
- cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*
cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x + 6*c)*co
s(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 +
 sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*s
in(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((cos(2*d*x + 2*c
)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3
 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6
*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x
 + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x
 + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2
 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x + 2*c)^3 + (co
s(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*
c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(2*d*x
 + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x +
2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin
(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos
(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x +
2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
```

```
 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x + 2*c)^3
 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*
x + 4*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x) -
10*(a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(2*d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x
 + 2*c) + a^2*d)*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d
*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*
x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*
x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x
 + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*
x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x
 + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x
+ 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x +
 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((cos(2*
d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x
 + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*
sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*
cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x
 + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x + 2*c
)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4
*d*x + 4*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (
cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos
(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x +
 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x +
```

```
2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos
(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x
 + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))
*sin(4*d*x + 4*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)), x))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*sqrt(a) + 8*(7*(3*a^2*sin(6*d*x + 6*c) + 5*a^2*sin(4*d*x + 4*c) + a^2
*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 - (21*a^2*cos(6*d*x + 6*c) + 35*a^2*cos(4*d*x + 4*c) + 7*a^2*cos(2*d*x + 2
*c) + 5*a^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt
(a))/((d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*c) +
 d)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4
))
```

Giac [F]

$$\int (a+a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^2\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*tan(d*x+c)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^{5/2}\tan^2(c+dx)\,dx = \int \tan(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^(5/2), x)
```

3.168 $\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{4a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}$$

```
[Out] -2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-4*a^2*cot(d*
x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 464, 209}

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{4a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (4
*a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(2a^2)\,\text{Subst}\left(\int \frac{2+ax^2}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{4a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{4a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.88

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{\sqrt{2}\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2\cos(c+dx) - \frac{\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)(-1+\cos(c+dx))}{\sqrt{1-\sec(c+dx)}}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{3/2}(a(1}{d\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}$$

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -((Sqrt[2]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*(2*Cos[c + d*x] - (ArcTanh[Sqrt[
1 - Sec[c + d*x]]]*(-1 + Cos[c + d*x]))/Sqrt[1 - Sec[c + d*x]])*((1 + Sec[c
 + d*x])^(-1))^(3/2)*(a*(1 + Sec[c + d*x]))^(5/2))/(d*Sqrt[1 - Tan[(c + d*x
)/2]^2]))
```

Maple [A] (verified)

Time = 6.62 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{2a^{2}\sqrt{a(1+\sec(dx+c))}\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+2\cot(dx+c)\right)}{d}$	89

```
[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcta
nh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2*cot(d*x+
c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(58) = 116.

Time = 0.35 (sec) , antiderivative size = 270, normalized size of antiderivative = 4.09

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{\sqrt{-a}a^2\log\left(-\frac{8\,a\cos(dx+c)^3+4\left(2\,\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-7\,a\cos(dx+c)+a}{\cos(dx+c)+1}\right)\sin(}{2\,d\sin(dx+c)}\right.$$

$$\left.-\frac{a^{\frac{5}{2}}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2\,a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin(dx+c)+4\,a^2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{d\sin(dx+c)}\right]$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*a^2*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x
 + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*
cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 8*a^2*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c)), -(a^(5/2)*arctan(2*
sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(
2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 4*a^2*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(d*sin(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^2\,dx$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c)^2, x)
```

Giac [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^2\,dx$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cot(c+d\,x)^2\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^(5/2), x)
```

3.169 $\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d} - \frac{2a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d}$$

```
[Out] 2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-2/3*a*cot(d*x
+c)^3*(a+a*sec(d*x+c))^(3/2)/d+2*a^2*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 331, 209}

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{2a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{d} - \frac{2a\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}$$

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*
a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/d - (2*a*Cot[c + d*x]^3*(a + a*S
ec[c + d*x])^(3/2))/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(2a)\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= -\frac{2a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{2a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}-\frac{2a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d}\\
&\quad -\frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{d}\\
&\quad -\frac{2a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{3d}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{2\left(\frac{1}{1+\cos(c+dx)}\right)^{3/2}\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},2\sin^2\left(\frac{1}{2}(c+dx)\right)\right)(a(1+\sec(c+dx)))^{5}}{3d\sqrt{\frac{1}{1+\sec(c+dx)}}}$$

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*((1 + Cos[c + d*x])^(-1))^(3/2)*Cot[c + d*x]^3*Hypergeometric2F1[-3/2,
-3/2, -1/2, 2*Sin[(c + d*x)/2]^2]*(a*(1 + Sec[c + d*x]))^(5/2))/(3*d*Sqrt[(
1 + Sec[c + d*x])^(-1)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(84) = 168.

Time = 40.39 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.88

method	result
default	$\frac{2a^2\sqrt{a(1+\sec(dx+c))}\left(3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)-3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{arctanh}\left(\frac{}{(\cos(d}\right.\right.}{3d(\cos(dx+c)-1)}$

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)-1)*(3*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*cos(d*x+c)-3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/
(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4*cot(d*x+c)*cos(d*x+c)-
3*cot(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.70

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{3\left(a^2\cos(dx+c)-a^2\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(dx+c)^3-4\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\cos(dx+c)+1}\right.}{6\left(d\cos(dx+c)-d\right)}\right.$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*cos(d*x + c) - a^2)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*c
os(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) +
 4*(4*a^2*cos(d*x + c)^2 - 3*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c)), 1/3*(3*(a^2*cos(d*x + c)
- a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x +
 c) + 2*(4*a^2*cos(d*x + c)^2 - 3*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c)))/((d*cos(d*x + c) - d)*sin(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^4\,dx$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cot(d*x + c)^4, x)
```

Giac [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^4\,dx$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cot(c+d\,x)^4\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^(5/2), x)
```

3.170 $\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{2}d} - \frac{7a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4d} + \frac{a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{2d} - \frac{\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{5d}$$

```
[Out] -2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+1/2*a*cot(d*
x+c)^3*(a+a*sec(d*x+c))^(3/2)/d-1/5*cot(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)/d+1
/8*a^(5/2)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/d*
2^(1/2)-7/4*a^2*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3972, 491, 597, 536, 209}

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d} + \frac{a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{2}d} - \frac{7a^2\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4d} - \frac{\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{5d} + \frac{a\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{2d}$$

```
[In] Int[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
4*Sqrt[2]*d) - (7*a^2*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4*d) + (a*Cot
[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(2*d) - (Cot[c + d*x]^5*(a + a*Sec[
c + d*x])^(5/2))/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
```

`tegerQ[n - 1/2]`

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{5d} - \frac{\text{Subst}\left(\int \frac{-15a-5a^2x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{5d}$$

$$= \frac{a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{2d} - \frac{\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{5d} + \frac{\text{Subst}\left(\int \frac{-105a^2-45a^3x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{30d}$$

$$= -\frac{7a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4d} + \frac{a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{2d} - \frac{\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{5d} - \frac{\text{Subst}\left(\int \frac{-225a^3-105a^4x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{60d}$$

$$= -\frac{7a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4d} + \frac{a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{2d} - \frac{\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{5d} - \frac{a^3\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{2}d} - \frac{7a^2\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4d} + \frac{a\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{2d} - \frac{\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{5d}$$

Mathematica [A] (verified)

Time = 6.40 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.15

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{\sec^5\left(\frac{1}{2}(c+dx)\right)(a(1+\sec(c+dx)))^{5/2}\left(\frac{1}{40}\cos^3(c+dx)(160\cos(c+dx)-7(17+7\cos(2(c\right.}{}$$

[In] Integrate[Cot[c + d*x]^6*(a + a*Sec[c + d*x])^(5/2),x]

[Out] (Sec[(c + d*x)/2]^5*(a*(1 + Sec[c + d*x]))^(5/2)*((Cos[c + d*x]^3*(160*Cos[c + d*x] - 7*(17 + 7*Cos[2*(c + d*x)]))*Csc[(c + d*x)/2]^5)/40 + ((ArcSin[Tan[(c + d*x)/2]] - 8*Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]])*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2))))/(32*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(147) = 294$.

Time = 194.44 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.11

method	result
default	$\frac{a^2\sqrt{a(1+\sec(dx+c))}\left(5\sqrt{2}\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{co}\right.}{}$

[In] int(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/40/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)-1)*(5*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-80*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-5*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+80*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+98*cos(d*x+c)*cot(d*x+c)^3-62*cot(d*x+c)^3-90*cot(d*x+c)^2*csc(d*x+c)+70*cot(d*x+c)*csc(d*x+c)^2)

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 650, normalized size of antiderivative = 3.69

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{5\left(\sqrt{2}a^2\cos(dx+c)^2-2\sqrt{2}a^2\cos(dx+c)+\sqrt{2}a^2\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\,\mathrm{s}}{\cos(dx+c)^2}\right.}{}\right.$$

$$-\frac{40\left(a^2\cos(dx+c)^2-2\,a^2\cos(dx+c)+a^2\right)\sqrt{a}\arctan\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{2\,a\cos(dx+c)^2+a\cos(dx+c)-a}\right)\sin(dx+c)+5\,(}{}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/80*(5*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)
*a^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 40*(a^2*cos(d*x + c)
^2 - 2*a^2*cos(d*x + c) + a^2)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos
(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4
*(49*a^2*cos(d*x + c)^3 - 80*a^2*cos(d*x + c)^2 + 35*a^2*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c)))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c)
+ d)*sin(d*x + c)), -1/40*(40*(a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c) + a^
2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c)
+ 5*(sqrt(2)*a^2*cos(d*x + c)^2 - 2*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2)
*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(49*a^2*cos(d*x + c)^3 - 80*a^2*
cos(d*x + c)^2 + 35*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
)))/((d*cos(d*x + c)^2 - 2*d*cos(d*x + c) + d)*sin(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**6*(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^6\,dx$$

```
[In] integrate(cot(d*x+c)^6*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \cot^6(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cot(c+d\,x)^6\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

```
[In] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^6*(a + a/cos(c + d*x))^(5/2), x)
```

3.171 $\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d} - \frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d} + \frac{2(a+a\sec(c+dx))^{7/2}}{7a^4d}$$

```
[Out] 2/3*(a+a*sec(d*x+c))^(3/2)/a^2/d-6/5*(a+a*sec(d*x+c))^(5/2)/a^3/d+2/7*(a+a*
sec(d*x+c))^(7/2)/a^4/d-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
+2*(a+a*sec(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3965, 90, 52, 65, 213}

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2(a\sec(c+dx)+a)^{7/2}}{7a^4d} - \frac{6(a\sec(c+dx)+a)^{5/2}}{5a^3d} + \frac{2(a\sec(c+dx)+a)^{3/2}}{3a^2d} - \frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{2\sqrt{a\sec(c+dx)+a}}{ad}$$

```
[In] Int[Tan[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (2*Sqrt[a + a*
Sec[c + d*x]])/(a*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^2*d) - (6*(a + a
*Sec[c + d*x])^(5/2))/(5*a^3*d) + (2*(a + a*Sec[c + d*x])^(7/2))/(7*a^4*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{3/2}}{x}\,dx, x, \sec(c+dx)\right)}{a^4 d}$$

$$=\frac{\text{Subst}\left(\int\left(-3a^2(a+ax)^{3/2}+\frac{a^2(a+ax)^{3/2}}{x}+a(a+ax)^{5/2}\right)\,dx,x,\sec(c+dx)\right)}{a^4d}$$

$$=-\frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d}+\frac{2(a+a\sec(c+dx))^{7/2}}{7a^4d}+\frac{\text{Subst}\left(\int\frac{(a+ax)^{3/2}}{x}\,dx,x,\sec(c+dx)\right)}{a^2d}$$

$$=\frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}-\frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d}+\frac{2(a+a\sec(c+dx))^{7/2}}{7a^4d}+\frac{\text{Subst}\left(\int\frac{\sqrt{a+ax}}{x}\,dx,x,\sec(c+dx)\right)}{ad}$$

$$=\frac{2\sqrt{a+a\sec(c+dx)}}{ad}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}-\frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d}+\frac{2(a+a\sec(c+dx))^{7/2}}{7a^4d}+\frac{\text{Subst}\left(\int\frac{1}{x\sqrt{a+ax}}\,dx,x,\sec(c+dx)\right)}{d}$$

$$=\frac{2\sqrt{a+a\sec(c+dx)}}{ad}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}-\frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d}+\frac{2(a+a\sec(c+dx))^{7/2}}{7a^4d}+\frac{2\text{Subst}\left(\int\frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a+a\sec(c+dx)}\right)}{ad}$$

$$=-\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+\frac{2\sqrt{a+a\sec(c+dx)}}{ad}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}-\frac{6(a+a\sec(c+dx))^{5/2}}{5a^3d}+\frac{2(a+a\sec(c+dx))^{7/2}}{7a^4d}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int\frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$=\frac{2\left(92+46\sec(c+dx)-64\sec^2(c+dx)-3\sec^3(c+dx)+15\sec^4(c+dx)-105\text{arctanh}\left(\sqrt{1+\sec(c+d\right.\right.}{105d\sqrt{a(1+\sec(c+dx))}}$$

```
[In] Integrate[Tan[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*(92 + 46*Sec[c + d*x] - 64*Sec[c + d*x]^2 - 3*Sec[c + d*x]^3 + 15*Sec[c
+ d*x]^4 - 105*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(10
5*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(105\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+92-46\sec(dx+c)-18\sec(dx+c)^2+15\sec(dx+c)^3\right)}{105da}$	94

[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/105/d/a*(a*(1+sec(d*x+c)))^(1/2)*(105*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+92-46*sec(d*x+c)-18*sec(d*x+c)^2+15*sec(d*x+c)^3)

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.26

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{105\sqrt{a}\cos(dx+c)^3\log\left(-8a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8a\right.}{210\,ad\cos(dx}\right.$$

[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/210*(105*sqrt(a)*cos(d*x + c)^3*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(92*cos(d*x + c)^3 - 46*cos(d*x + c)^2 - 18*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3), 1/105*(105*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^3 + 2*(92*cos(d*x + c)^3 - 46*cos(d*x + c)^2 - 18*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3)]

Sympy [F]

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan^5{(c+dx)}}{\sqrt{a(\sec{(c+dx)}+1)}}\,dx$$

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**5/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\frac{105\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}} + \frac{30\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{7}{2}}}{a^4} - \frac{126\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^3} + \frac{70\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^2} + \frac{210\sqrt{a+\frac{a}{\cos(dx+c)}}}{a}}{105\,d}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 1/105*(105*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c
)) + sqrt(a)))/sqrt(a) + 30*(a + a/cos(d*x + c))^(7/2)/a^4 - 126*(a + a/cos
(d*x + c))^(5/2)/a^3 + 70*(a + a/cos(d*x + c))^(3/2)/a^2 + 210*sqrt(a + a/c
os(d*x + c))/a)/d
```

Giac [A] (verification not implemented)

none

Time = 2.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.33

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{105\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(105\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^3-70\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2a-252\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right.}{105\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*sqrt(2)*(105*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)/sqrt(-a))/sqrt(-a) + 2*(105*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3 - 70*(a
*tan(1/2*d*x + 1/2*c)^2 - a)^2*a - 252*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a^2 -
 120*a^3)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{{\tan\left(c+d\,x\right)}^5}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(1/2), x)
```

3.172 $\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}$$

```
[Out] 2/3*(a+a*sec(d*x+c))^(3/2)/a^2/d+2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/
d/a^(1/2)-2*(a+a*sec(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3965, 81, 52, 65, 213}

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2(a\sec(c+dx)+a)^{3/2}}{3a^2d} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2\sqrt{a\sec(c+dx)+a}}{ad}$$

```
[In] Int[Tan[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*Sqrt[a + a*S
ec[c + d*x]])/(a*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^2*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-a+ax)\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{a^2 d}\\
&= \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2 d} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{ad}\\
&= -\frac{2\sqrt{a+a\sec(c+dx)}}{ad} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^2 d} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}
\end{aligned}
$$

$$= -\frac{2\sqrt{a+a\sec(c+dx)}}{ad}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}$$
$$-\frac{2\text{Subst}\left(\int\frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a+a\sec(c+dx)}\right)}{ad}$$
$$=\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{2\sqrt{a+a\sec(c+dx)}}{ad}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^2d}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int\frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$=\frac{2\left(-2-\sec(c+dx)+\sec^2(c+dx)+3\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{3d\sqrt{a(1+\sec(c+dx))}}$$

```
[In] Integrate[Tan[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*(-2 - Sec[c + d*x] + Sec[c + d*x]^2 + 3*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*
Sqrt[1 + Sec[c + d*x]]))/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\left(3\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+2-\sec(dx+c)\right)}{3da}$	74

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d/a*(a*(1+sec(d*x+c)))^(1/2)*(3*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2-sec(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.09

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{3\sqrt{a}\cos(dx+c)\log\left(-8\,a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\right.}{6\,ad\cos(dx+c)}, -\frac{3\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)\cos(dx+c)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(2\cos(dx+c)-1)}{3\,ad\cos(dx+c)}\right]$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d
*x + c) - a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) -
1))/(a*d*cos(d*x + c)), -1/3*(3*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c) +
 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) - 1))/(a*d*cos(d
*x + c))]
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan^3(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.17

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^2} + \frac{6\sqrt{a+\frac{a}{\cos(dx+c)}}}{a}}{3\,d}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
+ sqrt(a)))/sqrt(a) - 2*(a + a/cos(d*x + c))^(3/2)/a^2 + 6*sqrt(a + a/cos(d
*x + c))/a)/d
```

Giac [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.62

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\left(\frac{3\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)a+2\,a^2\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}\right)}{3\,ad\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*(3*sqrt(2)*a*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a)/sqrt(-a))/sqrt(-a) + 2*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a + 2*a^2)/(
(a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(a*d*s
gn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{\tan(c+d\,x)^3}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(1/2), x)
```

3.173 $\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=-\frac{2\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3965, 65, 213}

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=-\frac{2\mathrm{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[In] Int[Tan[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d}\\ &= \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{ad}\\ &= -\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}}{d\sqrt{a(1+\sec(c+dx))}}$$

```
[In] Integrate[Tan[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
[Out] (-2*ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]])/(d*Sqrt[a*(1 +
Sec[c + d*x])])
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(25) = 50$.

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.42

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\,\left(2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\,a\cos\left(dx+c\right)-a\right)}{2\,\sqrt{a}d},\ \frac{\sqrt{-\ldots}}{\ldots}\right.$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a)/(sqrt(a)*d)
, sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)/(2*a*cos(d*x + c) + a))/(a*d)]
```

Sympy [F]

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqrt(a
)))/(sqrt(a)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.78 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] 2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a
)*d*sgn(cos(d*x + c)))
```

Mupad [B] (verification not implemented)

Time = 15.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^(1/2),x)
[Out] -(2*atanh((a + a/cos(c + d*x))^(1/2)/a^(1/2)))/(a^(1/2)*d)
```

3.174 $\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{1}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-1/2*arctanh(1/2*(a+a*se
c(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)-1/d/(a+a*sec(d*x+c))^(1/
2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3965, 87, 162, 65, 213}

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{1}{d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Cot[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d) - 1/(d*Sqrt[a + a*
Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a^2\text{Subst}\left(\int \frac{1}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= -\frac{1}{d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{2a^2-a^2x}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{2ad}
\end{aligned}
$$

$$= -\frac{1}{d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{2d}$$

$$= -\frac{1}{d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{d} - \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{ad}$$

$$= \frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{1}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.62

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1+\sec(c+dx))\right) - 2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\sec(c+dx)\right)}{d\sqrt{a(1+\sec(c+dx))}}$$

```
[In] Integrate[Cot[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
[Out] (Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2F
1[-1/2, 1, 1/2, 1 + Sec[c + d*x]])/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(75) = 150.

Time = 1.80 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.49

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(-\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{3}{2}}+(1-\cos(dx+c))^2\sqrt{(1-c}\right.}{}$

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/d/a*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^2*c
sc(d*x+c)^2-1)^(1/2)*(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)+(1-cos(d*x+c
))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*x+c)^2+6*2^(1/2)*arctan(
1/2*2^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))+3*arctan(1/((1-cos(d*x
+c))^2*csc(d*x+c)^2-1)^(1/2))-4*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(75) = 150$.

Time = 0.35 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.17

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{2\sqrt{a}(\cos(dx+c)+1)\log\left(-8\,a\cos(dx+c)^2-4\left(2\,\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}-8\right.}{4\,(ad\cos(\right.$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a
*cos(d*x + c) - a) + sqrt(2)*(a*cos(d*x + c) + a)*log(-(2*sqrt(2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/sqrt(a) - 3*cos(d*x + c) - 1)/(
cos(d*x + c) - 1))/sqrt(a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c))/(a*d*cos(d*x + c) + a*d), 1/2*(sqrt(2)*(a*cos(d*x + c) + a)*sqrt(
-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos
(d*x + c)/(cos(d*x + c) + 1)) - 2*sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c)
 + a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(a*d*cos(d
*x + c) + a*d)]
```

Sympy [F]

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(dx+c)}{\sqrt{a\sec(dx+c)+a}}\,dx$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.75 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a}\right)}{2\,d\operatorname{sgn}(\cos(dx+c))}$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a)/sqrt(-a))/sqrt(-a) - arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a
))/sqrt(-a) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/a)/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)/(a + a/cos(c + d*x))^(1/2), x)
```

3.175 $\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+\frac{9\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d} - \frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{7}{8d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -1/12*a/d/(a+a*sec(d*x+c))^(3/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(3
/2)-2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)+9/16*arctanh(1/2*(a
+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)+7/8/d/(a+a*sec(d*x+
c))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

$= \{3965, 105, 157, 162, 65, 213\}$

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{9\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d} - \frac{a}{12d(a\sec(c+dx)+a)^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a\sec(c+dx)+a)^{3/2}} + \frac{7}{8d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Cot[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (9*ArcTanh[Sqr
t[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(8*Sqrt[2]*Sqrt[a]*d) - a/(12*d*(
a + a*Sec[c + d*x])^(3/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])
^(3/2)) + 7/(8*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^4\text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d} \\
&= \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} - \frac{a\text{Subst}\left(\int \frac{2a^2+\frac{5a^2x}{2}}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{2d} \\
&= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-6a^4-\frac{3a^4x}{4}}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{6a^2d} \\
&= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} \\
&\quad + \frac{7}{8d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{6a^6-\frac{21a^6x}{8}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{6a^5d} \\
&= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} \\
&\quad + \frac{7}{8d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} \\
&\quad - \frac{(9a)\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{16d}
\end{aligned}$$

$$= -\frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{7}{8d\sqrt{a+a\sec(c+dx)}} - \frac{9\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{8d} + \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{ad}$$

$$= -\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{9\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d} - \frac{a}{12d(a+a\sec(c+dx))^{3/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{3/2}} + \frac{7}{8d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.59

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{a\left(-6-9\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))+8\,\text{Hypergeometric2F}\right.}{12d(-1+\sec(c+dx))(a(1+\sec(c+dx)))^{3/2}}$$

```
[In] Integrate[Cot[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (a*(-6 - 9*Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sec[c + d*x])/2]*(-1 + Sec
[c + d*x]) + 8*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]]*(-1 + Sec
[c + d*x])))/(12*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(123) = 246.

Time = 1.75 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.79

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(27\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+27\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+}}}\right.\right.}{}$

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(27*arctan(1/2*2^(1/2)/(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)+27*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+96*arctan((-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+96*arctan((-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-62*cos(d*x+c)*
cot(d*x+c)^2-4*cot(d*x+c)^2+42*cot(d*x+c)*csc(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(121) = 242$.

Time = 0.35 (sec) , antiderivative size = 546, normalized size of antiderivative = 3.59

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{27\sqrt{2}\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)+3a\cos(dx+c)+a}{\cos(dx+c)-1}\right)\right.$$

$$-\frac{27\sqrt{2}\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-48}{48}$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(27*sqrt(2)*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt
(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)) + 48*(cos(d*x + c)^3 + cos
(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(
d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
- 8*a*cos(d*x + c) - a) + 4*(31*cos(d*x + c)^3 + 2*cos(d*x + c)^2 - 21*cos(
d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*
d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d), -1/48*(27*sqrt(2)*(cos(d*x + c)
^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) -
48*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(-a)*arctan(2*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x +
 c) + a)) - 2*(31*cos(d*x + c)^3 + 2*cos(d*x + c)^2 - 21*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)
^2 - a*d*cos(d*x + c) - a*d)]
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot^3\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot\left(dx+c\right)^3}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.84 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{48\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{27\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2} + \frac{2\left(\left(-a\tan\left(\frac{1}{2}d\right.\right.\right.}{}\right.}{48\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] 1/48*sqrt(2)*(48*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a)/sqrt(-a))/sqrt(-a) - 27*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqr
t(-a))/sqrt(-a) - 3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a*tan(1/2*d*x + 1/
2*c)^2) + 2*((-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^4 + 12*sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a)*a^5)/a^6)/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^3}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(1/2), x)
```

3.176 $\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 214

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{151\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d} + \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{17a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{23a}{192d(a+a\sec(c+dx))^{3/2}} - \frac{105}{128d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 87/160*a^2/d/(a+a*sec(d*x+c))^(5/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x
+c))^(5/2)-17/16*a^2/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)+23/192*a/d/(a+
a*sec(d*x+c))^(3/2)+2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-151
/256*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)-
105/128/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3965, 105, 156, 157, 162, 65, 213}

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{87a^2}{160d(a\sec(c+dx)+a)^{5/2}} - \frac{17a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{5/2}} + \frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{151\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d} + \frac{23a}{192d(a\sec(c+dx)+a)^{3/2}} - \frac{105}{128d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Cot[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (151*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(128*Sqrt[2]*Sqrt[a]*d) + (87*a^
2)/(160*d*(a + a*Sec[c + d*x])^(5/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
a*Sec[c + d*x])^(5/2)) - (17*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d
*x])^(5/2)) + (23*a)/(192*d*(a + a*Sec[c + d*x])^(3/2)) - 105/(128*d*Sqrt[a
 + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{a^6 \text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{a^3\text{Subst}\left(\int \frac{4a^2+\frac{9a^2x}{2}}{x(-a+ax)^2(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{17a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{\text{Subst}\left(\int \frac{8a^4+\frac{119a^4x}{4}}{x(-a+ax)(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{8d}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{17a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} - \frac{\text{Subst}\left(\int \frac{-40a^6-\frac{435a^6x}{8}}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{40a^3d}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{17a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{23a}{192d(a+a\sec(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{120a^8+\frac{345a^8x}{16}}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{120a^6d}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{17a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{23a}{192d(a+a\sec(c+dx))^{3/2}} - \frac{105}{128d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{-120a^{10}+\frac{1575a^{10}x}{32}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{120a^9d}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{17a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{23a}{192d(a+a\sec(c+dx))^{3/2}} - \frac{105}{128d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{d} + \frac{(151a)\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{256d}$$

$$= \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{17a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{23a}{192d(a+a\sec(c+dx))^{3/2}} - \frac{105}{128d\sqrt{a+a\sec(c+dx)}} + \frac{151\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{128d} - \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{ad}$$

$$= \frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{151\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d} + \frac{87a^2}{160d(a+a\sec(c+dx))^{5/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{5/2}} - \frac{17a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{23a}{192d(a+a\sec(c+dx))^{3/2}} - \frac{105}{128d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.48

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\cot^4(c+dx)\left(-2\left(105+32\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1+\sec(c+dx)\right)(-1+\sec(c+dx))^2-85\sec\right.\right.}{160d\sqrt{a(1+\sec(c+}}$$

```
[In] Integrate[Cot[c + d*x]^5/Sqrt[a + a*Sec[c + d*x]],x]
```

[Out] (Cot[c + d*x]^4*(-2*(105 + 32*Hypergeometric2F1[-5/2, 1, -3/2, 1 + Sec[c + d*x]]*(-1 + Sec[c + d*x])^2 - 85*Sec[c + d*x]) + 151*Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sec[c + d*x])/2]*(-1 + Sec[c + d*x])^2))/(160*d*Sqrt[a*(1 + Sec[c + d*x])])

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.44

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(2265\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+7680\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\cos(dx+c)\sqrt{\cdot}\right.}{}$

[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3840/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(2265*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+7680*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2265*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+7680*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+5642*cos(d*x+c)*cot(d*x+c)^4+556*cot(d*x+c)^4-7928*cot(d*x+c)^3*csc(d*x+c)-460*cot(d*x+c)^2*csc(d*x+c)^2+3150*csc(d*x+c)^3*cot(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.29

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{2265\sqrt{2}\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\cos(dx+c)+1\right)\sqrt{a}\log\left(-\cdots\right.}{}\right.$$

[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/7680*(2265*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 3840*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4

```
*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c)) - 8*a*cos(d*x + c) - a) - 4*(2821*cos(d*x + c)^5 + 278*cos(d*x + c)
^4 - 3964*cos(d*x + c)^3 - 230*cos(d*x + c)^2 + 1575*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 -
 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d), 1/3
840*(2265*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*c
os(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 3840*(
cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos
(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(2821*cos(d*x + c)^5 + 278*
cos(d*x + c)^4 - 3964*cos(d*x + c)^3 - 230*cos(d*x + c)^2 + 1575*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a*d*cos(d*x + c)^5 + a*d*cos(
d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)
 + a*d)]
```

Sympy [F]

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot^5\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)**5/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot\left(dx+c\right)^5}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^5/sqrt(a*sec(d*x + c) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.89 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.13

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\left(\frac{3840\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2265\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{15\left(25\left(-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{3}{2}}-23\sqrt{\cdots}\right.}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\cdots\right.}\right.}{3840\cdots}$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3840*sqrt(2)*(3840*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)/sqrt(-a))/sqrt(-a) - 2265*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a)/sqrt(-a))/sqrt(-a) + 15*(25*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 23*
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a)/(a^2*tan(1/2*d*x + 1/2*c)^4) + 8*(3*
(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^12 +
 25*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^13 + 240*sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a)*a^14)/a^15)/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{{\cot(c+d\,x)}^5}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(1/2), x)
```

3.177 $\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 189

$$
\begin{aligned}
\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = {} & -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} \\
& -\frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} \\
& +\frac{6a^3\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}+\frac{2a^4\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}}
\end{aligned}
$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+2*tan(d*x+c)
/d/(a+a*sec(d*x+c))^(1/2)-2/3*a*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2/5*a
^2*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+6/7*a^3*tan(d*x+c)^7/d/(a+a*sec(d*
x+c))^(7/2)+2/9*a^4*tan(d*x+c)^9/d/(a+a*sec(d*x+c))^(9/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 472, 209}

$$
\begin{aligned}
\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = {} & \frac{2a^4\tan^9(c+dx)}{9d(a\sec(c+dx)+a)^{9/2}}+\frac{6a^3\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}} \\
& +\frac{2a^2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}-\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d} \\
& -\frac{2a\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}+\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}
\end{aligned}
$$

```
[In] Int[Tan[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (2*a*Tan[c + d*x]^3)/(3*d*(
a + a*Sec[c + d*x])^(3/2)) + (2*a^2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x
])^(5/2)) + (6*a^3*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2)) + (2*a^
4*Tan[c + d*x]^9)/(9*d*(a + a*Sec[c + d*x])^(9/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(2a^3)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{(2a^3)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}+3x^6+ax^8-\frac{1}{a^3(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} \\
&\quad + \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{6a^3\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} \\
&\quad + \frac{2a^4\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}} + \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}
\end{aligned}
$$

$$= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{6a^3\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} + \frac{2a^4\tan^9(c+dx)}{9d(a+a\sec(c+dx))^{9/2}}$$

Mathematica [A] (warning: unable to verify)

Time = 14.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.88

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{-5040\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{\frac{\sec(c+dx)}{(1+\sec(c+dx))^2}}\sqrt{1+\sec(c+dx)} + (901+16\cdots}{1260d\sqrt{a(1+\cdots}}$$

```
[In] Integrate[Tan[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-5040*ArcTan[Tan[(c + d*x)/2]/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]]*Cos[(
c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*Sqrt[Sec[c + d*x]/(1 + Sec[c + d*x])^2]*Sq
rt[1 + Sec[c + d*x]] + (901 + 164*Cos[c + d*x] + 1004*Cos[2*(c + d*x)] + 68
*Cos[3*(c + d*x)] + 383*Cos[4*(c + d*x)])*Sec[c + d*x]^4*Tan[c + d*x])/(126
0*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.16

method	result
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\left(315\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+315\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{arctanh}\left(\cdots\right.\right.}{315\cdots}$

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(315*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))*cos(d*x+c)+315*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*
x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-383*sin(d*x+c)-34*t
an(d*x+c)+132*sec(d*x+c)*tan(d*x+c)+5*tan(d*x+c)*sec(d*x+c)^2-35*tan(d*x+c)
*sec(d*x+c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.88

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[-\frac{315\left(\cos\left(dx+c\right)^5+\cos\left(dx+c\right)^4\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{315\left(ad\cos\left(dx\right.\right.} \right.$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/315*(315*(cos(d*x + c)^5 + cos(d*x + c)^4)*sqrt(-a)*log((2*a*cos(d*x +
c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(383*cos(d*x + c)^4
+ 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 - 5*cos(d*x + c) + 35)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^5 + a*d*cos(d*
x + c)^4), 2/315*(315*(cos(d*x + c)^5 + cos(d*x + c)^4)*sqrt(a)*arctan(sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) +
(383*cos(d*x + c)^4 + 34*cos(d*x + c)^3 - 132*cos(d*x + c)^2 - 5*cos(d*x +
c) + 35)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x
 + c)^5 + a*d*cos(d*x + c)^4)]
```

Sympy [F]

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan^6\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**6/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan\left(dx+c\right)^6}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/630*(12*(105*sin(8*d*x + 8*c) + 280*sin(6*d*x + 6*c) + 546*sin(4*d*x + 4
*c) + 312*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) - 4*(315*cos(8*d*x + 8*c) + 840*cos(6*d*x + 6*c) + 1638*cos(4*d*x
 + 4*c) + 936*cos(2*d*x + 2*c) + 383)*sin(9/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) - 315*((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + 4*cos
(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x
 + 2*c)^2 + 6*cos(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4
 + 4*cos(2*d*x + 2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*s
in(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*arctan2(
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) + 2*(a*d*cos(2*d*x + 2*c)^4 + a*d*
sin(2*d*x + 2*c)^4 + 4*a*d*cos(2*d*x + 2*c)^3 + 6*a*d*cos(2*d*x + 2*c)^2 +
4*a*d*cos(2*d*x + 2*c) + 2*(a*d*cos(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c)
 + a*d)*sin(2*d*x + 2*c)^2 + a*d)*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(14*d*x + 14*c)*cos(2*d*x
+ 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(
2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*c
os(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2
+ sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*
c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x
 + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(
12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*
c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x +
2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x
+ 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(
8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*co
s(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((c
os(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c)
+ 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x +
8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x
+ 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d
*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*si
n(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*
sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (
cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c)
 + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x +
```

```
 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x
 + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(
12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) +
20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c)
 + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(11/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)))/(a*cos(14*d*x + 14*c)^2 + 36*a*cos(12*d*x + 12*c)^2
 + 225*a*cos(10*d*x + 10*c)^2 + 400*a*cos(8*d*x + 8*c)^2 + 225*a*cos(6*d*x
+ 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 12*a*cos(4*d*x + 4*c)*cos(2*d*x + 2*c)
 + a*cos(2*d*x + 2*c)^2 + a*sin(14*d*x + 14*c)^2 + 36*a*sin(12*d*x + 12*c)^
2 + 225*a*sin(10*d*x + 10*c)^2 + 400*a*sin(8*d*x + 8*c)^2 + 225*a*sin(6*d*x
 + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 12*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + a*sin(2*d*x + 2*c)^2 + 2*(6*a*cos(12*d*x + 12*c) + 15*a*cos(10*d*x + 10
*c) + 20*a*cos(8*d*x + 8*c) + 15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c)
+ a*cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + 12*(15*a*cos(10*d*x + 10*c) + 20
*a*cos(8*d*x + 8*c) + 15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(
2*d*x + 2*c))*cos(12*d*x + 12*c) + 30*(20*a*cos(8*d*x + 8*c) + 15*a*cos(6*d
*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) +
 40*(15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos
(8*d*x + 8*c) + 30*(6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(6*d*x +
6*c) + 2*(6*a*sin(12*d*x + 12*c) + 15*a*sin(10*d*x + 10*c) + 20*a*sin(8*d*x
 + 8*c) + 15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c)
)*sin(14*d*x + 14*c) + 12*(15*a*sin(10*d*x + 10*c) + 20*a*sin(8*d*x + 8*c)
+ 15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(12
*d*x + 12*c) + 30*(20*a*sin(8*d*x + 8*c) + 15*a*sin(6*d*x + 6*c) + 6*a*sin(
4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 40*(15*a*sin(6*d*x
+ 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 30*(
6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - 10*(a*d*
cos(2*d*x + 2*c)^4 + a*d*sin(2*d*x + 2*c)^4 + 4*a*d*cos(2*d*x + 2*c)^3 + 6*
a*d*cos(2*d*x + 2*c)^2 + 4*a*d*cos(2*d*x + 2*c) + 2*(a*d*cos(2*d*x + 2*c)^2
 + 2*a*d*cos(2*d*x + 2*c) + a*d)*sin(2*d*x + 2*c)^2 + a*d)*integrate(-(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(
14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 1
5*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c
) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2
*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d
*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*s
in(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(9/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x + 14*c) +
 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 1
0*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x
 + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*
x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*si
n(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)
```

```
*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(9/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2
*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*
d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(
2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos
(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) -
 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c
) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - (cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x +
12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*
d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(
4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin
(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*
c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x +
 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x +
2*c)^2)*sin(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a*cos(14*d*x + 14*c)^2 + 36*a
*cos(12*d*x + 12*c)^2 + 225*a*cos(10*d*x + 10*c)^2 + 400*a*cos(8*d*x + 8*c)
^2 + 225*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 12*a*cos(4*d*x +
4*c)*cos(2*d*x + 2*c) + a*cos(2*d*x + 2*c)^2 + a*sin(14*d*x + 14*c)^2 + 36*
a*sin(12*d*x + 12*c)^2 + 225*a*sin(10*d*x + 10*c)^2 + 400*a*sin(8*d*x + 8*c
)^2 + 225*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 12*a*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + a*sin(2*d*x + 2*c)^2 + 2*(6*a*cos(12*d*x + 12*c) +
 15*a*cos(10*d*x + 10*c) + 20*a*cos(8*d*x + 8*c) + 15*a*cos(6*d*x + 6*c) +
6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + 12*(15*a*co
s(10*d*x + 10*c) + 20*a*cos(8*d*x + 8*c) + 15*a*cos(6*d*x + 6*c) + 6*a*cos(
4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 30*(20*a*cos(8*d*x
+ 8*c) + 15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))
*cos(10*d*x + 10*c) + 40*(15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*
cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 30*(6*a*cos(4*d*x + 4*c) + a*cos(2*d*x
 + 2*c))*cos(6*d*x + 6*c) + 2*(6*a*sin(12*d*x + 12*c) + 15*a*sin(10*d*x + 1
0*c) + 20*a*sin(8*d*x + 8*c) + 15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c)
 + a*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + 12*(15*a*sin(10*d*x + 10*c) + 2
0*a*sin(8*d*x + 8*c) + 15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin
(2*d*x + 2*c))*sin(12*d*x + 12*c) + 30*(20*a*sin(8*d*x + 8*c) + 15*a*sin(6*
d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c)
+ 40*(15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*si
n(8*d*x + 8*c) + 30*(6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c)), x) + 20*(a*d*cos(2*d*x + 2*c)^4 + a*d*sin(2*d*x + 2*c)^4 + 4*a*d*co
s(2*d*x + 2*c)^3 + 6*a*d*cos(2*d*x + 2*c)^2 + 4*a*d*cos(2*d*x + 2*c) + 2*(a
*d*cos(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*sin(2*d*x + 2*c)^2 +
a*d)*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(((cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)
*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x +
```

```
 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x
 + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*
x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*si
n(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)
*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^
2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)
*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x
+ 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2
*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14
*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*
cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c)
- 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c
))*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(14*d*x + 14
*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*
x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(
6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin
(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*
c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x +
 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(7/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(14*d*x + 14*c)*cos(2*d*x + 2
*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d
*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(
2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + s
in(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c)
+ 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x +
2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + sin(2*d*x + 2*c)^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a*cos(14
*d*x + 14*c)^2 + 36*a*cos(12*d*x + 12*c)^2 + 225*a*cos(10*d*x + 10*c)^2 + 4
00*a*cos(8*d*x + 8*c)^2 + 225*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^
2 + 12*a*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + a*cos(2*d*x + 2*c)^2 + a*sin(1
4*d*x + 14*c)^2 + 36*a*sin(12*d*x + 12*c)^2 + 225*a*sin(10*d*x + 10*c)^2 +
400*a*sin(8*d*x + 8*c)^2 + 225*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)
^2 + 12*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a*sin(2*d*x + 2*c)^2 + 2*(6*a
*cos(12*d*x + 12*c) + 15*a*cos(10*d*x + 10*c) + 20*a*cos(8*d*x + 8*c) + 15*
a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(14*d*x
+ 14*c) + 12*(15*a*cos(10*d*x + 10*c) + 20*a*cos(8*d*x + 8*c) + 15*a*cos(6*
d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(12*d*x + 12*c)
+ 30*(20*a*cos(8*d*x + 8*c) + 15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c)
+ a*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 40*(15*a*cos(6*d*x + 6*c) + 6*a*
cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 30*(6*a*cos(4*d*x
 + 4*c) + a*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(6*a*sin(12*d*x + 12*c)
+ 15*a*sin(10*d*x + 10*c) + 20*a*sin(8*d*x + 8*c) + 15*a*sin(6*d*x + 6*c) +
 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + 12*(15*a*s
```

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in(10*d*x + 10*c) + 20*a*sin(8*d*x + 8*c) + 15*a*sin(6*d*x + 6*c) + 6*a*sin
(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 30*(20*a*sin(8*d*x
 + 8*c) + 15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c)
)*sin(10*d*x + 10*c) + 40*(15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a
*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 30*(6*a*sin(4*d*x + 4*c) + a*sin(2*d*
x + 2*c))*sin(6*d*x + 6*c)), x) - 20*(a*d*cos(2*d*x + 2*c)^4 + a*d*sin(2*d*
x + 2*c)^4 + 4*a*d*cos(2*d*x + 2*c)^3 + 6*a*d*cos(2*d*x + 2*c)^2 + 4*a*d*co
s(2*d*x + 2*c) + 2*(a*d*cos(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*
sin(2*d*x + 2*c)^2 + a*d)*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(14*d*x + 14*c)*cos(2*d*x + 2*c) +
 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x +
2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x
 + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14
*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*
sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c)
+ 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + sin(2*d*x + 2*c)^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + (cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x +
12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*
d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(
4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*s
in(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8
*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x +
 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x +
 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2
*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*
cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - c
os(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c)
- 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x +
2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x
+ 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(14*d*x
+ 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(1
0*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*
cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + c
os(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12
*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*
x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))/(a*cos(14*d*x + 14*c)^2 + 36*a*cos(12*d*x + 12*c)^2 + 225*a*cos
(10*d*x + 10*c)^2 + 400*a*cos(8*d*x + 8*c)^2 + 225*a*cos(6*d*x + 6*c)^2 + 3
6*a*cos(4*d*x + 4*c)^2 + 12*a*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + a*cos(2*d
*x + 2*c)^2 + a*sin(14*d*x + 14*c)^2 + 36*a*sin(12*d*x + 12*c)^2 + 225*a*si
n(10*d*x + 10*c)^2 + 400*a*sin(8*d*x + 8*c)^2 + 225*a*sin(6*d*x + 6*c)^2 +
```

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36*a*sin(4*d*x + 4*c)^2 + 12*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a*sin(2*
d*x + 2*c)^2 + 2*(6*a*cos(12*d*x + 12*c) + 15*a*cos(10*d*x + 10*c) + 20*a*c
os(8*d*x + 8*c) + 15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*
x + 2*c))*cos(14*d*x + 14*c) + 12*(15*a*cos(10*d*x + 10*c) + 20*a*cos(8*d*x
 + 8*c) + 15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c)
)*cos(12*d*x + 12*c) + 30*(20*a*cos(8*d*x + 8*c) + 15*a*cos(6*d*x + 6*c) +
6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 40*(15*a*co
s(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(8*d*x + 8*c
) + 30*(6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(6*
a*sin(12*d*x + 12*c) + 15*a*sin(10*d*x + 10*c) + 20*a*sin(8*d*x + 8*c) + 15
*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(14*d*x
 + 14*c) + 12*(15*a*sin(10*d*x + 10*c) + 20*a*sin(8*d*x + 8*c) + 15*a*sin(6
*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(12*d*x + 12*c)
 + 30*(20*a*sin(8*d*x + 8*c) + 15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c)
 + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 40*(15*a*sin(6*d*x + 6*c) + 6*a
*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 30*(6*a*sin(4*d*
x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) + 10*(a*d*cos(2*d*x +
2*c)^4 + a*d*sin(2*d*x + 2*c)^4 + 4*a*d*cos(2*d*x + 2*c)^3 + 6*a*d*cos(2*d*
x + 2*c)^2 + 4*a*d*cos(2*d*x + 2*c) + 2*(a*d*cos(2*d*x + 2*c)^2 + 2*a*d*cos
(2*d*x + 2*c) + a*d)*sin(2*d*x + 2*c)^2 + a*d)*integrate(-(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(14*d*x + 14*
c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x
 + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6
*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*
d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*s
in(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8
*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x
 + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*co
s(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*
cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6
*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*
c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x +
 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)) + ((cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d
*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*s
in(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)
*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12
*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*
x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*
d*x + 4*c)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) - (cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*
d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*c
```

```
os(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)
*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c
) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x
 + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*
d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)))/(a*cos(14*d*x + 14*c)^2 + 36*a*cos(12*d*x
+ 12*c)^2 + 225*a*cos(10*d*x + 10*c)^2 + 400*a*cos(8*d*x + 8*c)^2 + 225*a*c
os(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 12*a*cos(4*d*x + 4*c)*cos(2*d
*x + 2*c) + a*cos(2*d*x + 2*c)^2 + a*sin(14*d*x + 14*c)^2 + 36*a*sin(12*d*x
 + 12*c)^2 + 225*a*sin(10*d*x + 10*c)^2 + 400*a*sin(8*d*x + 8*c)^2 + 225*a*
sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 12*a*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + a*sin(2*d*x + 2*c)^2 + 2*(6*a*cos(12*d*x + 12*c) + 15*a*cos(10
*d*x + 10*c) + 20*a*cos(8*d*x + 8*c) + 15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*
x + 4*c) + a*cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + 12*(15*a*cos(10*d*x + 1
0*c) + 20*a*cos(8*d*x + 8*c) + 15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c)
 + a*cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 30*(20*a*cos(8*d*x + 8*c) + 15*
a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(10*d*x
+ 10*c) + 40*(15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*x +
2*c))*cos(8*d*x + 8*c) + 30*(6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos
(6*d*x + 6*c) + 2*(6*a*sin(12*d*x + 12*c) + 15*a*sin(10*d*x + 10*c) + 20*a*
sin(8*d*x + 8*c) + 15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin(2*d
*x + 2*c))*sin(14*d*x + 14*c) + 12*(15*a*sin(10*d*x + 10*c) + 20*a*sin(8*d*
x + 8*c) + 15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c
))*sin(12*d*x + 12*c) + 30*(20*a*sin(8*d*x + 8*c) + 15*a*sin(6*d*x + 6*c) +
 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 40*(15*a*s
in(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*
c) + 30*(6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) -
 2*(a*d*cos(2*d*x + 2*c)^4 + a*d*sin(2*d*x + 2*c)^4 + 4*a*d*cos(2*d*x + 2*c
)^3 + 6*a*d*cos(2*d*x + 2*c)^2 + 4*a*d*cos(2*d*x + 2*c) + 2*(a*d*cos(2*d*x
+ 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*sin(2*d*x + 2*c)^2 + a*d)*integrat
e(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*(((cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x +
2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d
*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2
*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*
sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c
) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x +
2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x +
 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10
*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*s
in(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*
sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x +
10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*
```

```
x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2
*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 2
0*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c)
+ 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c)
 - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x
+ 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d
*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - (cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(1
2*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 2
0*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c)
+ 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 1
4*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d
*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin
(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(
2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a*cos(14*d*x + 14*c)^
2 + 36*a*cos(12*d*x + 12*c)^2 + 225*a*cos(10*d*x + 10*c)^2 + 400*a*cos(8*d*
x + 8*c)^2 + 225*a*cos(6*d*x + 6*c)^2 + 36*a*cos(4*d*x + 4*c)^2 + 12*a*cos(
4*d*x + 4*c)*cos(2*d*x + 2*c) + a*cos(2*d*x + 2*c)^2 + a*sin(14*d*x + 14*c)
^2 + 36*a*sin(12*d*x + 12*c)^2 + 225*a*sin(10*d*x + 10*c)^2 + 400*a*sin(8*d
*x + 8*c)^2 + 225*a*sin(6*d*x + 6*c)^2 + 36*a*sin(4*d*x + 4*c)^2 + 12*a*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + a*sin(2*d*x + 2*c)^2 + 2*(6*a*cos(12*d*x +
 12*c) + 15*a*cos(10*d*x + 10*c) + 20*a*cos(8*d*x + 8*c) + 15*a*cos(6*d*x +
 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + 12*
(15*a*cos(10*d*x + 10*c) + 20*a*cos(8*d*x + 8*c) + 15*a*cos(6*d*x + 6*c) +
6*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 30*(20*a*co
s(8*d*x + 8*c) + 15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4*c) + a*cos(2*d*x
 + 2*c))*cos(10*d*x + 10*c) + 40*(15*a*cos(6*d*x + 6*c) + 6*a*cos(4*d*x + 4
*c) + a*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 30*(6*a*cos(4*d*x + 4*c) + a*c
os(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(6*a*sin(12*d*x + 12*c) + 15*a*sin(10
*d*x + 10*c) + 20*a*sin(8*d*x + 8*c) + 15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*
x + 4*c) + a*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + 12*(15*a*sin(10*d*x + 1
0*c) + 20*a*sin(8*d*x + 8*c) + 15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c)
 + a*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 30*(20*a*sin(8*d*x + 8*c) + 15*
a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x
+ 10*c) + 40*(15*a*sin(6*d*x + 6*c) + 6*a*sin(4*d*x + 4*c) + a*sin(2*d*x +
2*c))*sin(8*d*x + 8*c) + 30*(6*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c)), x))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4))/((d*cos(2*d*x + 2*c)^4 + d*sin(2*d*x + 2*c)^4 + 4*d*cos(
2*d*x + 2*c)^3 + 6*d*cos(2*d*x + 2*c)^2 + 2*(d*cos(2*d*x + 2*c)^2 + 2*d*cos
(2*d*x + 2*c) + d)*sin(2*d*x + 2*c)^2 + 4*d*cos(2*d*x + 2*c) + d)*(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a))
```

Giac [A] (verification not implemented)

none

Time = 2.84 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.41

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{315\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}|a|-6\,a\right|}\right)}{|a|}+\frac{4\left(315\,a^4-\left(1470\,a^4-\left(2772\,a^4+\left(257\,a^4\tan\left(\frac{1}{2}\,dx\right.\right.\right.\right.\right.}{\left(a\tan\right.}\right.}{630\,d\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/630*sqrt(2)*(315*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4
*sqrt(2)*abs(a) - 6*a))/abs(a) + 4*(315*a^4 - (1470*a^4 - (2772*a^4 + (257*
a^4*tan(1/2*d*x + 1/2*c)^2 - 1314*a^4)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x +
1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{{\tan\left(c+d\,x\right)}^6}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(1/2), x)
```

3.178 $\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}+\frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-2*tan(d*x+c)/
d/(a+a*sec(d*x+c))^(1/2)+2/3*a*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2/5*a^
2*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3972, 470, 308, 209}

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{2a^2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}}+\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2a\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}}-\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Tan[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Tan[c + d*x]^3)/(3*d*(a
```

```
 + a*Sec[c + d*x])^(3/2)) + (2*a^2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x]
)^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(2a^2)\,\text{Subst}\left(\int \frac{x^4(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+\frac{1}{a^2(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} \\
&\quad + \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} - \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}
\end{aligned}
$$

$$= \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a^2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.47 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.90

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{16\sqrt{2}\left(\frac{1}{1+\sec(c+dx)}\right)^{9/2}\left(-\frac{\cos(c+dx)(9+5\cos(c+dx))\csc^6\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(30\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)\cos^2(c+dx)+(-29+\right.}{480\sqrt{1-\sec(c+dx)}}\right.}{5d\sqrt{}}$$

```
[In] Integrate[Tan[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (16*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(9/2)*(-1/480*(Cos[c + d*x]*(9 + 5*Co
s[c + d*x])*Csc[(c + d*x)/2]^6*Sec[(c + d*x)/2]^2*(30*ArcTanh[Sqrt[1 - Sec[
c + d*x]]]*Cos[c + d*x]^2 + (-29 + 22*Cos[c + d*x] - 23*Cos[2*(c + d*x)])*S
qrt[1 - Sec[c + d*x]]))/Sqrt[1 - Sec[c + d*x]] - (4*Hypergeometric2F1[2, 9/
2, 11/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[c + d*x]*Tan[(c + d*x)/2]^
2)/9)*Tan[c + d*x]^5)/(5*d*Sqrt[a*(1 + Sec[c + d*x])]*(1 - Tan[(c + d*x)/2]
^2)^(7/2))
```

Maple [A] (verified)

Time = 3.69 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.50

method	result
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(15\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+15\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{arctanh}\left(\frac{}{(\cos(dx+\right.\right.}{15da(\cos(dx+c)+1)}$

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(15*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*cos(d*x+c)+15*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)
/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-17*sin(d*x+c)-tan(d*x+c
)+3*sec(d*x+c)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.49

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[-\frac{15\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right.}{15\left(ad\cos\left(dx+c\right)^3+ad\cos\left(dx+c\right.\right.}\right.$$
$$-\frac{2\left(15\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\left(17\cos\left(dx+c\right)^2+\cos\left(dx\right.\right.\right.}{15\left(ad\cos\left(dx+c\right)^3+ad\cos\left(dx+c\right)^2\right)}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/15*(15*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c)
^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(17*cos(d*x + c)^2 + c
os(d*x + c) - 3)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d
*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), -2/15*(15*(cos(d*x + c)^3 + cos(d*x
+ c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c))) + (17*cos(d*x + c)^2 + cos(d*x + c) - 3)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(
d*x + c)^2)]
```

Sympy [F]

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan^4\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan\left(dx+c\right)^4}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/30*(20*(3*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*(15*cos(4*d*x + 4*c) + 20*cos(2*d*x + 2*c) + 17)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 15*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) + 2*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a*cos(10*d*x + 10*c)^2 + 16*a*cos(8*d*x + 8*c)^2 + 36*a*cos

(6*d*x + 6*c)^2 + 16*a*cos(4*d*x + 4*c)^2 + 8*a*cos(4*d*x + 4*c)*cos(2*d*x
+ 2*c) + a*cos(2*d*x + 2*c)^2 + a*sin(10*d*x + 10*c)^2 + 16*a*sin(8*d*x + 8
*c)^2 + 36*a*sin(6*d*x + 6*c)^2 + 16*a*sin(4*d*x + 4*c)^2 + 8*a*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(8*d*x + 8*c) + 6
*a*cos(6*d*x + 6*c) + 4*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(10*d*x
 + 10*c) + 8*(6*a*cos(6*d*x + 6*c) + 4*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2
*c))*cos(8*d*x + 8*c) + 12*(4*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(
6*d*x + 6*c) + 2*(4*a*sin(8*d*x + 8*c) + 6*a*sin(6*d*x + 6*c) + 4*a*sin(4*d
*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 8*(6*a*sin(6*d*x + 6*c
) + 4*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 12*(4*a*s
in(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - 6*(a*d*cos(2*
d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*integ
rate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*(((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x +
 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x
+ 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8
*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*co
s(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*c
os(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*
cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4
*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (
(cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c)
+ 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c)
 - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c
) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*
c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(10*d*x + 10
*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x +
6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2
*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x
+ 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a*cos(1
0*d*x + 10*c)^2 + 16*a*cos(8*d*x + 8*c)^2 + 36*a*cos(6*d*x + 6*c)^2 + 16*a*
cos(4*d*x + 4*c)^2 + 8*a*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + a*cos(2*d*x +
2*c)^2 + a*sin(10*d*x + 10*c)^2 + 16*a*sin(8*d*x + 8*c)^2 + 36*a*sin(6*d*x
+ 6*c)^2 + 16*a*sin(4*d*x + 4*c)^2 + 8*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(8*d*x + 8*c) + 6*a*cos(6*d*x + 6*c) + 4
*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 8*(6*a*cos(6
*d*x + 6*c) + 4*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) +
 12*(4*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(4*a*s
in(8*d*x + 8*c) + 6*a*sin(6*d*x + 6*c) + 4*a*sin(4*d*x + 4*c) + a*sin(2*d*x
 + 2*c))*sin(10*d*x + 10*c) + 8*(6*a*sin(6*d*x + 6*c) + 4*a*sin(4*d*x + 4*c

```
) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 12*(4*a*sin(4*d*x + 4*c) + a*sin
(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) + 6*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(
2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*integrate(-(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(10*d*x + 10*
c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6
*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*
c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x +
 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + (cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*
x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d
*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*
d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2
*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(1
0*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*si
n(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*s
in(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*
sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) +
4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) +
 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10
*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x +
6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2
*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a*cos(10*d*x + 10*c)^2 + 16*a*
cos(8*d*x + 8*c)^2 + 36*a*cos(6*d*x + 6*c)^2 + 16*a*cos(4*d*x + 4*c)^2 + 8*
a*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + a*cos(2*d*x + 2*c)^2 + a*sin(10*d*x +
 10*c)^2 + 16*a*sin(8*d*x + 8*c)^2 + 36*a*sin(6*d*x + 6*c)^2 + 16*a*sin(4*d
*x + 4*c)^2 + 8*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a*sin(2*d*x + 2*c)^2
+ 2*(4*a*cos(8*d*x + 8*c) + 6*a*cos(6*d*x + 6*c) + 4*a*cos(4*d*x + 4*c) + a
*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 8*(6*a*cos(6*d*x + 6*c) + 4*a*cos(4
*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 12*(4*a*cos(4*d*x + 4*
c) + a*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(4*a*sin(8*d*x + 8*c) + 6*a*s
in(6*d*x + 6*c) + 4*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 1
0*c) + 8*(6*a*sin(6*d*x + 6*c) + 4*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))
*sin(8*d*x + 8*c) + 12*(4*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*
x + 6*c)), x) - 2*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 + 2*a*d*
cos(2*d*x + 2*c) + a*d)*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4
*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) +
4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*
c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6
*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*
c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2
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*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x
+ 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x
+ 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*
x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2
*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(
2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos
(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*co
s(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(
2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos
(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) +
4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) +
 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)))/(a*cos(10*d*x + 10*c)^2 + 16*a*cos(8*d*x + 8*c)^2 + 36
*a*cos(6*d*x + 6*c)^2 + 16*a*cos(4*d*x + 4*c)^2 + 8*a*cos(4*d*x + 4*c)*cos(
2*d*x + 2*c) + a*cos(2*d*x + 2*c)^2 + a*sin(10*d*x + 10*c)^2 + 16*a*sin(8*d
*x + 8*c)^2 + 36*a*sin(6*d*x + 6*c)^2 + 16*a*sin(4*d*x + 4*c)^2 + 8*a*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + a*sin(2*d*x + 2*c)^2 + 2*(4*a*cos(8*d*x + 8*
c) + 6*a*cos(6*d*x + 6*c) + 4*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(
10*d*x + 10*c) + 8*(6*a*cos(6*d*x + 6*c) + 4*a*cos(4*d*x + 4*c) + a*cos(2*d
*x + 2*c))*cos(8*d*x + 8*c) + 12*(4*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c)
)*cos(6*d*x + 6*c) + 2*(4*a*sin(8*d*x + 8*c) + 6*a*sin(6*d*x + 6*c) + 4*a*s
in(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 8*(6*a*sin(6*d*x
 + 6*c) + 4*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 12*
(4*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x))*(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4))/((d*cos(2
*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*c) + d)*(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(109) = 218$.

Time = 1.60 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.82

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{15\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{4\left(\left(13a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-40a^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\cdots}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}\cdots\right.}}\cdots\right.}{30\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(2)*(15*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*
sqrt(2)*abs(a) - 6*a))/abs(a) + 4*((13*a^2*tan(1/2*d*x + 1/2*c)^2 - 40*a^2)
*tan(1/2*d*x + 1/2*c)^2 + 15*a^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/
2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^4}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(1/2), x)
```

3.179 $\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}} dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+2*tan(d*x+c)
/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 327, 209}

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[In] Int[Tan[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(2a)\text{Subst}\left(\int \frac{x^2}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.89

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{16\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{5/2}\left(-\arcsin\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\cos(c+dx)}}}\right)\cos(c+dx)+\sqrt{\cos(c+dx)}\cdots\right.}{d\sqrt{a(1+\sec(c+dx))}}$$

```
[In] Integrate[Tan[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (16*Cos[(c + d*x)/2]^6*Sec[c + d*x]^4*((1 + Sec[c + d*x])^(-1))^(5/2)*(-(Ar
cSin[Tan[(c + d*x)/2]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Cos[c + d*x]) + Sqrt[C
os[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sin[c + d*x]))/(d*Sqrt[a*(1 + Se
c[c + d*x])])
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\cot(dx+c)-\csc(dx+c)\right)}{da}$	95

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a*(a*(1+sec(d*x+c)))^(1/2)*((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh
(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+cot(d*x+c)-c
sc(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.73

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left[-\frac{\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\sqrt{\frac{a\cos(dx+}{\cos(dx+}}}{ad\cos(dx+c)+ad}\right.$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c)
 - a)/(cos(d*x + c) + 1)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c))/(a*d*cos(d*x + c) + a*d), 2*(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) +
 sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) +
a*d)]
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan^2{(c+dx)}}{\sqrt{a\left(\sec{(c+dx)}+1\right)}}\,dx$$

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan{(dx+c)}^2}{\sqrt{a\sec{(dx+c)}+a}}\,dx$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 1/2*((2*a*d*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4
*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*
c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*
c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c
) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*
c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c
))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x + 6*c)
*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^
2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a*cos(6*d*x +
6*c)^2 + 4*a*cos(4*d*x + 4*c)^2 + 4*a*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + a
*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 4*a*sin(4*d*x + 4*c)^2 + 4*a*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + a*sin(2*d*x + 2*c)^2 + 2*(2*a*cos(4*d*x
+ 4*c) + a*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(2*a*sin(4*d*x + 4*c) + a
*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - 2*a*d*integrate(-(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*
c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c
)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*
c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c
```

```
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c
) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c)
 - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)
*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c)
+ 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)))/(a*cos(6*d*x + 6*c)^2 + 4*a*cos(4*d*x + 4*c)^2 + 4*a*
cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*
c)^2 + 4*a*sin(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a*s
in(2*d*x + 2*c)^2 + 2*(2*a*cos(4*d*x + 4*c) + a*cos(2*d*x + 2*c))*cos(6*d*x
 + 6*c) + 2*(2*a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)),
x) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sqrt(a)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(55) = 110$.

Time = 1.14 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.97

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\,\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\,\sqrt{2}|a|-6\,a\right|}\right)}{|a|}-\frac{4\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}\right)}{2\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(
```

```
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))/abs(a) - 4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*
x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^2}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(1/2), x)
```

3.180 $\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{7\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}d} - \frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4ad} - \frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{4ad}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+7/8*arctan(1
/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)-1/4
*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a/d-1/4*cos(d*x+c)*cot(d*x+c)*sec(1/2*d*
x+1/2*c)^2*(a+a*sec(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3972, 483, 597, 536, 209}

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{7\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{2}\sqrt{a}d} - \frac{\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4ad} - \frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a\sec(c+dx)+a}}{4ad}$$

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(7*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(4*Sq
rt[2]*Sqrt[a]*d) - (Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4*a*d) - (Cos[c
 + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d*x]])/(4*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} \\
&= -\frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{4ad} \\
&\quad - \frac{\text{Subst}\left(\int \frac{a-3a^2x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2a^2d} \\
&= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4ad} \\
&\quad - \frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{4ad} \\
&\quad + \frac{\text{Subst}\left(\int \frac{9a^2+a^3x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^2d} \\
&= -\frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4ad} \\
&\quad - \frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{4ad} \\
&\quad - \frac{7\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{7\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}d} \\
&\quad - \frac{\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4ad} \\
&\quad - \frac{\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{4ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.05 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$
\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx
$$

$$
= \frac{-2(3\cot(c+dx)+\csc(c+dx)) - 32\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\sec(c+dx)}}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{\frac{\sec(c+dx)}{(1+\sec(c+dx))^2}}}{8d\sqrt{a(1+\sec(c+dx))}}
$$

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*(3*Cot[c + d*x] + Csc[c + d*x]) - 32*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 +
Sec[c + d*x])^(-1)]]*Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*Sqrt[Sec[c + d*x
]/(1 + Sec[c + d*x])^2]*Sqrt[1 + Sec[c + d*x]] + (14*ArcSin[Tan[(c + d*x)/2
]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]])
/Sqrt[Sec[(c + d*x)/2]^2])/(8*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(139) = 278$.

Time = 2.01 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.04

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(7\sqrt{2}\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\cos(dx\right.}{}$

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(7*2^(1/2)*ln(csc(d*x+c)-co
t(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+7*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*
x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)-16*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x
+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-16*(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))-6*cot(d*x+c)*cos(d*x+c)-2*cot(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.05

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[-\frac{7\sqrt{2}\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\sin(dx}{}\right.$$

$$-\frac{7\sqrt{2}\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)+8\sqrt{a}(\cos(dx+c)+1)\arctan}{8(ad\cos(dx+c)+a}$$

[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/16*(7*sqrt(2)*sqrt(-a)*(cos(d*x + c) + 1)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 8*sqrt(-a)*(cos(d*x + c) + 1)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4*(3*cos(d*x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c) + a*d)*sin(d*x + c)), -1/8*(7*sqrt(2)*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 8*sqrt(a)*(cos(d*x + c) + 1)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(3*cos(d*x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c) + a*d)*sin(d*x + c))]

Sympy [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)

[Out] Integral(cot(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)

Maxima [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(dx+c)^2}{\sqrt{a\sec(dx+c)+a}}\,dx$$

[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cot(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)

Giac [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.63

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{2}\left(\frac{8\sqrt{2}\sqrt{-a}\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a}\right|\right)}{|a|} - \frac{7\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\cdots}\right.\right.}{\sqrt{-a}}\cdots\right)}{16\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(8*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sq
rt(2)*abs(a) - 6*a))/abs(a) - 7*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) + 2*sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)*tan(1/2*d*x + 1/2*c)/a + 8*sqrt(-a)/((sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{{\mathrm{cot}\left(c+d\,x\right)}^2}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(1/2), x)
```

3.181 $\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 251

$$\begin{aligned}
\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{107\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}d} \\
&\quad + \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{64ad} + \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2d} \\
&\quad - \frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{32a^2d} \\
&\quad - \frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{16a^2d}
\end{aligned}$$

```
[Out] 43/96*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^2/d-15/32*cos(d*x+c)*cot(d*x+c)
^3*sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a^2/d-1/16*cos(d*x+c)^2*cot(
d*x+c)^3*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(3/2)/a^2/d+2*arctan(a^(1/2)
*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-107/128*arctan(1/2*a^(1/2)*ta
n(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)+21/64*cot(d*x+c)
*(a+a*sec(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3972, 483, 593, 597, 536, 209}

$$\begin{aligned}
&\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx \\
&= \frac{43\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{96a^2d} \\
&\quad - \frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{16a^2d} \\
&\quad - \frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{32a^2d} \\
&\quad + \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d} - \frac{107\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{64\sqrt{2}\sqrt{a}d} \\
&\quad + \frac{21\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{64ad}
\end{aligned}$$

```
[In] Int[Cot[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
107*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(64*
Sqrt[2]*Sqrt[a]*d) + (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(64*a*d) +
(43*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(96*a^2*d) - (15*Cos[c + d*x
]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(32*a^2*d)
- (Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(3
/2))/(16*a^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} \\
&= -\frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{16a^2d} \\
&\quad -\frac{\text{Subst}\left(\int \frac{a-7a^2x^2}{x^4(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^3d}
\end{aligned}$$

$$= -\frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{32a^2d} - \frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{16a^2d} - \frac{\text{Subst}\left(\int \frac{-43a^2-75a^3x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{16a^4d}$$

$$= \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2d} - \frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{32a^2d} - \frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{16a^2d} + \frac{\text{Subst}\left(\int \frac{63a^3-129a^4x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{96a^4d}$$

$$= \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{64ad} + \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2d} - \frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{32a^2d} - \frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{16a^2d} - \frac{\text{Subst}\left(\int \frac{447a^4+63a^5x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{192a^4d}$$

$$= \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{64ad} + \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2d} - \frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{32a^2d} - \frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{16a^2d} + \frac{107\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} - \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{107\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}d} + \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{64ad} + \frac{43\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{96a^2d} - \frac{15\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{32a^2d} - \frac{\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{16a^2d}$$

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.91

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{-321\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sec(c+dx)}\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{1+\sec(c+dx)} + \frac{-\left(\frac{1}{1+\cos(c+dx)}\right)^{3/2}(-110+19\cos(c+}{}}{192d\sqrt{\sec^2\left(\frac{1}{2}(c+dx\right.}}$$

```
[In] Integrate[Cot[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-321*ArcSin[Tan[(c + d*x)/2]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 + Sec[c + d*x])^(
-1)]*Sqrt[1 + Sec[c + d*x]] + (-(((1 + Cos[c + d*x])^(-1))^(3/2)*(-110 + 19
*Cos[c + d*x] + 142*Cos[2*(c + d*x)] + 205*Cos[3*(c + d*x)])*Csc[(c + d*x)/
2]^3*Sec[(c + d*x)/2]) + 6144*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x
])^(-1)]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c +
 d*x]])/(8*Sqrt[2]))/(192*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a*(1 + Sec[c + d*
x])])
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.48

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(321\sqrt{2}\,\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.}{}$

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(321*2^(1/2)*ln(csc(d*x+
c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*
```

```
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+321*ln(csc(d*x+c)-cot(d*x+c)+
(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)-768*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh
(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-7
68*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))+410*cos(d*x+c)*cot(d*x+c)^3+142*cot(d*x+c)
^3-298*cot(d*x+c)^2*csc(d*x+c)-126*cot(d*x+c)*csc(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.65

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left[-\frac{321\sqrt{2}\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c}{\cos(dx+c)^2+2\cos(}\right.}{}\right.$$

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(321*sqrt(2)*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*s
qrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 384*(cos(d*x + c)^3 + cos(d
*x + c)^2 - cos(d*x + c) - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 + 4*(2*cos(
d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) - 4*
(205*cos(d*x + c)^4 + 71*cos(d*x + c)^3 - 149*cos(d*x + c)^2 - 63*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)^3 + a*d*co
s(d*x + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c)), 1/384*(321*sqrt(2)*(c
os(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sqrt(a)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
*sin(d*x + c) + 384*(cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sq
rt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(
205*cos(d*x + c)^4 + 71*cos(d*x + c)^3 - 149*cos(d*x + c)^2 - 63*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)^3 + a*d*cos
(d*x + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot^4\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot\left(dx+c\right)^4}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^4/sqrt(a*sec(d*x + c) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 1.21 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.53

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}\left(6\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a}-\frac{21}{a}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{384\,\sqrt{2}\sqrt{-a}\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots\right.}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots\right.}\right.\right.\cdots}{\cdots}\cdots\right.}{\cdots}$$

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] 1/768*sqrt(2)*(6*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*tan(1/2*d*x + 1/2*c
)^2/a - 21/a)*tan(1/2*d*x + 1/2*c) - 384*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2
)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/abs(a) + 321*log((sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) - 64*
(9*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*
sqrt(-a) - 15*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*sqrt(-a)*a + 8*sqrt(-a)*a^2)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3)/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^4}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(1/2), x)
```

3.182 $\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 335

$$
\begin{aligned}
&\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx \\
&= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{835\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{512\sqrt{2}\sqrt{a}d} \\
&\quad - \frac{189\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{512ad} - \frac{323\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2d} \\
&\quad + \frac{579\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{640a^3d} \\
&\quad - \frac{101\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^3d} \\
&\quad - \frac{23\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{192a^3d} \\
&\quad - \frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{48a^3d}
\end{aligned}
$$

```
[Out] -323/768*cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^2/d+579/640*cot(d*x+c)^5*(a+
a*sec(d*x+c))^(5/2)/a^3/d-101/128*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c
)^2*(a+a*sec(d*x+c))^(5/2)/a^3/d-23/192*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2*d
*x+1/2*c)^4*(a+a*sec(d*x+c))^(5/2)/a^3/d-1/48*cos(d*x+c)^3*cot(d*x+c)^5*sec
(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(5/2)/a^3/d-2*arctan(a^(1/2)*tan(d*x+c)/
(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+835/1024*arctan(1/2*a^(1/2)*tan(d*x+c)*2^
(1/2)/(a+a*sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)-189/512*cot(d*x+c)*(a+a*sec
(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3972, 483, 593, 597, 536, 209}

$$
\begin{aligned}
&\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx \\
&= \frac{579\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{640a^3d} \\
&\quad - \frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{48a^3d} \\
&\quad - \frac{23\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{192a^3d} \\
&\quad - \frac{101\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{128a^3d} \\
&\quad - \frac{323\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{768a^2d} - \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d} \\
&\quad + \frac{835\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{512\sqrt{2}\sqrt{a}d} - \frac{189\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{512ad}
\end{aligned}
$$

```
[In] Int[Cot[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) +
(835*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(51
2*Sqrt[2]*Sqrt[a]*d) - (189*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(512*a*d
) - (323*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(768*a^2*d) + (579*Cot[
c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(640*a^3*d) - (101*Cos[c + d*x]*Cot[
c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(128*a^3*d) - (23
*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec[c + d*x])^(5/2
))/(192*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]^5*Sec[(c + d*x)/2]^6*(a + a*S
ec[c + d*x])^(5/2))/(48*a^3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
```

```
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^3d}$$

$$= -\frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{48a^3d} - \frac{\text{Subst}\left(\int \frac{a-11a^2x^2}{x^6(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{6a^4d}$$

$$= -\frac{23\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{192a^3d} - \frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{48a^3d} - \frac{\text{Subst}\left(\int \frac{-111a^2-207a^3x^2}{x^6(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{48a^5d}$$

$$= -\frac{101\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^3d} - \frac{23\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{192a^3d} - \frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{48a^3d} - \frac{\text{Subst}\left(\int \frac{-1737a^3-2121a^4x^2}{x^6(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{192a^6d}$$

$$= \frac{579\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{640a^3d} - \frac{101\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^3d} - \frac{23\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{192a^3d} - \frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{48a^3d} + \frac{\text{Subst}\left(\int \frac{-4845a^4-8685a^5x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{1920a^6d}$$

$$= -\frac{323\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2d} + \frac{579\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{640a^3d} - \frac{101\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^3d} - \frac{23\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{192a^3d} - \frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{48a^3d} - \frac{\text{Subst}\left(\int \frac{8505a^5-14535a^6x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{11520a^6d}$$

$$
\begin{aligned}
&=-\frac{189 \cot(c+dx)\sqrt{a+a\sec(c+dx)}}{512ad}-\frac{323 \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2d} \\
&+\frac{579 \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{640a^3d} \\
&-\frac{101 \cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^3d} \\
&-\frac{23 \cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{192a^3d} \\
&-\frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{48a^3d} \\
&+\frac{\text{Subst}\left(\int \frac{54585a^6+8505a^7x^2}{(1+ax^2)(2+ax^2)}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{23040a^6d} \\
&=-\frac{189 \cot(c+dx)\sqrt{a+a\sec(c+dx)}}{512ad}-\frac{323 \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2d} \\
&+\frac{579 \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{640a^3d} \\
&-\frac{101 \cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^3d} \\
&-\frac{23 \cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{192a^3d} \\
&-\frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{48a^3d} \\
&-\frac{835\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{512d} \\
&+\frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&=-\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{835\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{512\sqrt{2}\sqrt{a}d} \\
&-\frac{189 \cot(c+dx)\sqrt{a+a\sec(c+dx)}}{512ad}-\frac{323 \cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{768a^2d} \\
&+\frac{579 \cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{640a^3d} \\
&-\frac{101 \cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^3d} \\
&-\frac{23 \cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{192a^3d} \\
&-\frac{\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{48a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.79

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\left(-\frac{\left(\frac{1}{1+\cos(c+dx)}\right)^{3/2}(21682+11948\cos(c+dx)+12791\cos(2(c+dx))-14754\cos(3(c+dx))+846\cos(4(c+dx))+6902\cos(5(c+d}{256\sqrt{2}}\right.}{}$$

[In] Integrate[Cot[c + d*x]^6/Sqrt[a + a*Sec[c + d*x]],x]

[Out] (Sqrt[Sec[c + d*x]]*(-1/256*(((1 + Cos[c + d*x])^(-1))^(3/2)*(21682 + 11948*Cos[c + d*x] + 12791*Cos[2*(c + d*x)] - 14754*Cos[3*(c + d*x)] + 846*Cos[4*(c + d*x)] + 6902*Cos[5*(c + d*x)] + 9737*Cos[6*(c + d*x)])*Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^3*Sqrt[Sec[c + d*x]])/Sqrt[2] + 12525*ArcSin[Tan[(c + d*x)/2]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] - 15360*Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]]))/(7680*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a*(1 + Sec[c + d*x])])

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(-12525\sqrt{2}\,\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+}}\right.}{}$

[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/15360/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(-12525*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+30720*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-12525*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+30720*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+19474*cos(d*x+c)*cot(d*x+c)^5+6902*cot(d*x+c)^5-28788*cot(d*x+c)^4*csc(d*x+c)-12316*cot(d*x+c)^3*csc(d*x+c)^2+12130*cot(d*x+c)^2*csc(d*x+c)^3+5670*cot(d*x+c)*csc(d*x+c)^4)

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 823, normalized size of antiderivative = 2.46

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/30720*(12525*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 15360*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4*(9737*cos(d*x + c)^6 + 3451*cos(d*x + c)^5 - 14394*cos(d*x + c)^4 - 6158*cos(d*x + c)^3 + 6065*cos(d*x + c)^2 + 2835*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)*sin(d*x + c)), -1/15360*(12525*sqrt(2)*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 15360*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(9737*cos(d*x + c)^6 + 3451*cos(d*x + c)^5 - 14394*cos(d*x + c)^4 - 6158*cos(d*x + c)^3 + 6065*cos(d*x + c)^2 + 2835*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)*sin(d*x + c))]

Sympy [F]

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cot^6{(c+dx)}}{\sqrt{a\,(\sec{(c+dx)}+1)}}\,dx$$

[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(1/2),x)

[Out] Integral(cot(c + d*x)**6/sqrt(a*(sec(c + d*x) + 1)), x)

Maxima [F]

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{\cot(dx+c)^6}{\sqrt{a\sec(dx+c)+a}}\,dx$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^6/sqrt(a*sec(d*x + c) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 1.32 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.49

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$=\frac{\sqrt{2}\left(10\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(2\left(\frac{4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a}-\frac{43}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{567}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\cdots\right.}{}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] 1/30720*sqrt(2)*(10*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*tan(1/2*d*x +
 1/2*c)^2/a - 43/a)*tan(1/2*d*x + 1/2*c)^2 + 567/a)*tan(1/2*d*x + 1/2*c) +
15360*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(
a) - 6*a))/abs(a) - 12525*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) + 192*(145*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a) - 500*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a + 710*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt
(-a)*a^2 - 460*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*sqrt(-a)*a^3 + 121*sqrt(-a)*a^4)/((sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5)/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{\cot(c+d\,x)^6}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(1/2), x)
```

3.183 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d} - \frac{2(a+a\sec(c+dx))^{3/2}}{a^3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^4d}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-2*(a+a*sec(d*x+c))^(3/
2)/a^3/d+2/5*(a+a*sec(d*x+c))^(5/2)/a^4/d+2*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3965, 90, 52, 65, 213}

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2(a\sec(c+dx)+a)^{5/2}}{5a^4d} - \frac{2(a\sec(c+dx)+a)^{3/2}}{a^3d} + \frac{2\sqrt{a\sec(c+dx)+a}}{a^2d}$$

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*Sqrt[a + a*
Sec[c + d*x]])/(a^2*d) - (2*(a + a*Sec[c + d*x])^(3/2))/(a^3*d) + (2*(a + a
*Sec[c + d*x])^(5/2))/(5*a^4*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-a+ax)^2\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{a^4 d} \\
&= \frac{\text{Subst}\left(\int \left(-3a^2\sqrt{a+ax} + \frac{a^2\sqrt{a+ax}}{x} + a(a+ax)^{3/2}\right)\,dx, x, \sec(c+dx)\right)}{a^4 d} \\
&= -\frac{2(a+a\sec(c+dx))^{3/2}}{a^3 d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^4 d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+ax}}{x}\,dx, x, \sec(c+dx)\right)}{a^2 d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d} - \frac{2(a+a\sec(c+dx))^{3/2}}{a^3d} \\
&\quad + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^4d} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{ad} \\
&= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d} - \frac{2(a+a\sec(c+dx))^{3/2}}{a^3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^4d} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^2d} \\
&= -\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d} \\
&\quad - \frac{2(a+a\sec(c+dx))^{3/2}}{a^3d} + \frac{2(a+a\sec(c+dx))^{5/2}}{5a^4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$
\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\left(1-2\sec(c+dx)-2\sec^2(c+dx)+\sec^3(c+dx)-5\text{arctanh}\left(\sqrt{1+\sec(c+\right.\right.}{5ad\sqrt{a(1+\sec(c+dx))}}
$$

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(1 - 2*Sec[c + d*x] - 2*Sec[c + d*x]^2 + Sec[c + d*x]^3 - 5*ArcTanh[Sqrt
[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(5*a*d*Sqrt[a*(1 + Sec[c + d*x
])])
```

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(5\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+1-3\sec(dx+c)+\sec(dx+c)^2\right)}{5d\,a^2}$	82

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/d/a^2*(a*(1+sec(d*x+c)))^(1/2)*(5*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1-3*sec(d*x+c)+sec(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.61

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{5\sqrt{a}\cos(dx+c)^2\log\left(-8\,a\cos(dx+c)^2+4\left(2\,\cos(dx+c)^2+\cos(dx+c\right.\right.}{}\right.$$

[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/10*(5*sqrt(a)*cos(d*x + c)^2*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(cos(d*x + c)^2 - 3*cos(d*x + c) + 1)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^2), 1/5*(5*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)^2 + 2*(cos(d*x + c)^2 - 3*cos(d*x + c) + 1)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^2)]

Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)

[Out] Integral(tan(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{5\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{5}{2}}}{a^4}-\frac{10\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^3}+\frac{10\sqrt{a+\frac{a}{\cos(dx+c)}}}{a^2}}{5\,d}$$

[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/5*(5*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqrt(a)))/a^(3/2) + 2*(a + a/cos(d*x + c))^(5/2)/a^4 - 10*(a + a/cos(d*x + c))^(3/2)/a^3 + 10*sqrt(a + a/cos(d*x + c))/a^2)/d

Giac [A] (verification not implemented)

none

Time = 2.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.54

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\left(\frac{5\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))}+\frac{\sqrt{2}\left(5\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2+10\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)a+4a^2\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,a\mathrm{sgn}(\cos(dx+c))}\right)}{5\,d}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*(5*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sq
rt(-a)*a*sgn(cos(d*x + c))) + sqrt(2)*(5*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2 +
 10*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a + 4*a^2)/((a*tan(1/2*d*x + 1/2*c)^2 -
a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^5}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(3/2), x)
```

3.184 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\sqrt{a+a\sec(c+dx)}}{a^2d}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*(a+a*sec(d*x+c))^(1/2
)/a^2/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3965, 81, 65, 213}

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\sqrt{a\sec(c+dx)+a}}{a^2d}$$

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*Sqrt[a + a*S
ec[c + d*x]])/(a^2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-a+ax}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d} \\
&= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{ad} \\
&= \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d} - \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^2d} \\
&= \frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2\sqrt{a+a\sec(c+dx)}}{a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\left(1+\sec(c+dx)+\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{ad\sqrt{a(1+\sec(c+dx))}}$$

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(1 + Sec[c + d*x] + ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x
]]))/(a*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] (verified)

Time = 3.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\left(\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-1\right)}{d\,a^2}$	65

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/a^2*(a*(1+sec(d*x+c)))^(1/2)*(arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.54

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{\sqrt{a}\log\left(-8\,a\cos\left(dx+c\right)^2-4\left(2\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx\cdots}{\cos(dx\cdots}}\cdots\right.}{2\,a^2d}\right.$$

$$\left.-\frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{a^2d}\right]$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) +
```

```
4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d), -(sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c
) + a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d)]
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(tan(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2\sqrt{a+\frac{a}{\cos(dx+c)}}}{a^2}}{d}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -(log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqrt
(a)))/a^(3/2) - 2*sqrt(a + a/cos(d*x + c))/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 1.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.57

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(\cos(dx+c))} - \frac{\sqrt{2}}{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\operatorname{sgn}(\cos(dx+c))}\right)}{ad}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] -2*(arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(
-a)*sgn(cos(d*x + c))) - sqrt(2)/(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*sgn(c
os(d*x + c))))/(a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(3/2), x)
```

3.185 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2}{ad\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2/a/d/(a+a*sec(d*x+c))
^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3965, 53, 65, 213}

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2}{ad\sqrt{a\sec(c+dx)+a}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d}$$

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + 2/(a*d*Sqrt[a
+ a*Sec[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{d} \\
&= \frac{2}{ad\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{ad} \\
&= \frac{2}{ad\sqrt{a+a\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^2d} \\
&= -\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2}{ad\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\sec(c+dx)\right)}{ad\sqrt{a(1+\sec(c+dx))}}$$

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + Sec[c + d*x]])/(a*d*Sqrt[a*(1 + Sec[
c + d*x])])
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+a\sec(dx+c)}}}{d}$	45
default	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+a\sec(dx+c)}}}{d}$	45

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^(3/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))+2/a/(a+a*sec(d*x+c)
)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(46) = 92$.

Time = 0.31 (sec) , antiderivative size = 244, normalized size of antiderivative = 4.52

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\left[\frac{\sqrt{a}(\cos{(dx+c)}+1)\log\left(-8\,a\cos{(dx+c)}^2+4\left(2\,\cos{(dx+c)}^2+\cos{(dx+}\right.\right.}{2\,(a^2d\cos{(dx}}\right.$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
```

```
os(d*x + c) - a) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/
(a^2*d*cos(d*x + c) + a^2*d), (sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
a)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(a^2*d*cos(d*
x + c) + a^2*d)]
```

Sympy [F]

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan{(c+dx)}}{(a(\sec{(c+dx)}+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(tan(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{a+\frac{a}{\cos(dx+c)}}\,a}}{d}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] (log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) + sqrt(
a)))/a^(3/2) + 2/(sqrt(a + a/cos(d*x + c))*a))/d
```

Giac [A] (verification not implemented)

none

Time = 0.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{2\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))} + \frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{a^2\mathrm{sgn}(\cos(dx+c))}}{d}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] (2*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-
a)*a*sgn(cos(d*x + c))) + sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^2*
sgn(cos(d*x + c))))/d
```

Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2}{a\,d\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}} - \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{a^{3/2}\,d}$$

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^(3/2),x)
[Out] 2/(a*d*(a + a/cos(c + d*x))^(1/2)) - (2*atanh((a + a/cos(c + d*x))^(1/2)/a^
(1/2)))/(a^(3/2)*d)
```

3.186 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-1/3/d/(a+a*sec(d*x+c))^
(3/2)-1/4*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(
1/2)-3/2/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3965, 87, 157, 162, 65, 213}

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{3}{2ad\sqrt{a\sec(c+dx)+a}} - \frac{1}{3d(a\sec(c+dx)+a)^{3/2}}$$

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*a^(3/2)*d) - 1/(3*d*(a + a*
Sec[c + d*x])^(3/2)) - 3/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a^2\text{Subst}\left(\int \frac{1}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{2a^2-a^2x}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{2ad}\\
&= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{-2a^4+\frac{3a^4x}{2}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{2a^4d}\\
&= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{4d} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{ad}\\
&= -\frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}}\\
&\quad - \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^2d}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{2ad}\\
&= \frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d}\\
&\quad - \frac{1}{3d(a+a\sec(c+dx))^{3/2}} - \frac{3}{2ad\sqrt{a+a\sec(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$
\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, \frac{1}{2}(1+\sec(c+dx))\right) - 2\,\text{Hypergeometric2F1}}{3d(a(1+\sec(c+dx)))^{3/2}}
$$

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
[Out] (Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2
F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]])/(3*d*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(95) = 190.

Time = 1.58 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.53

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(3\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{5}{2}}-3(1-\cos(dx+c))^4\sqrt{(1-}\right.}{}$

[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/60/d/a^2*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(3*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(5/2)-3*(1-cos(d*x+c))^4*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*x+c)^4-5*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)+16*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*x+c)^2+60*2^(1/2)*arctan(1/2*2^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))+15*arctan(1/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))-58*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(95) = 190.

Time = 0.36 (sec) , antiderivative size = 485, normalized size of antiderivative = 4.04

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c}{\cos(dx+c)-1}\right.}{}\right.$$

[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/24*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) - 4*(11*cos(d*x + c)^2 + 9*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/12*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(11*cos(d*x + c)^2 + 9*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]

Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot{\left(c+dx \right)}}{\left(a\left(\sec{\left(c+dx \right)}+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot\left(dx+c\right)}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.92 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.37

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))} - \frac{24\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))} - \frac{\sqrt{2}\left(\left(-a\tan\left(\frac{1}{2}\right.\right.\right.}{}}{12\,d}$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] 1/12*(3*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(
-a)*a*sgn(cos(d*x + c))) - 24*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))) - sqrt(2)*((-a*tan(1/2*d
*x + 1/2*c)^2 + a)^(3/2)*a^6 + 9*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^7)/(
a^9*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)/(a + a/cos(c + d*x))^(3/2), x)
```

3.187 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 176

$$\begin{aligned}\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = &-\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}\\ &+\frac{11\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d}-\frac{3a}{20d(a+a\sec(c+dx))^{5/2}}\\ &+\frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}}\\ &+\frac{5}{24d(a+a\sec(c+dx))^{3/2}}+\frac{21}{16ad\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-3/20*a/d/(a+a*sec(d*x+
c))^(5/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)+5/24/d/(a+a*sec(d*x
+c))^(3/2)+11/32*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2
)/d*2^(1/2)+21/16/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

$= \{3965, 105, 157, 162, 65, 213\}$

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{11\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d} - \frac{3a}{20d(a\sec(c+dx)+a)^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a\sec(c+dx)+a)^{5/2}} + \frac{5}{24d(a\sec(c+dx)+a)^{3/2}} + \frac{21}{16ad\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (11*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*a^(3/2)*d) - (3*a)/(
20*d*(a + a*Sec[c + d*x])^(5/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(5/2)) + 5/(24*d*(a + a*Sec[c + d*x])^(3/2)) + 21/(16*a*d*Sqrt[a + a
*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^4 \text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{d} \\
&= \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} - \frac{a\text{Subst}\left(\int \frac{2a^2+\frac{7a^2x}{2}}{x(-a+ax)(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{2d} \\
&= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-10a^4-\frac{15a^4x}{4}}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{10a^2d} \\
&= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} \\
&\quad + \frac{5}{24d(a+a\sec(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{30a^6-\frac{75a^6x}{8}}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{30a^5d}
\end{aligned}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{5}{24d(a+a\sec(c+dx))^{3/2}} + \frac{21}{16ad\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{-30a^8+\frac{315a^8x}{16}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{30a^8d}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{5}{24d(a+a\sec(c+dx))^{3/2}} + \frac{21}{16ad\sqrt{a+a\sec(c+dx)}} - \frac{11\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{32d} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= -\frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{5}{24d(a+a\sec(c+dx))^{3/2}} + \frac{21}{16ad\sqrt{a+a\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^2d} - \frac{11\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{16ad}$$

$$= -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{11\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d} - \frac{3a}{20d(a+a\sec(c+dx))^{5/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{5/2}} + \frac{5}{24d(a+a\sec(c+dx))^{3/2}} + \frac{21}{16ad\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.51

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{a\left(-10-11\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c}{20d(-1+\sec(c+dx}$$

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*(-10 - 11*Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sec[c + d*x])/2]*(-1 + S
ec[c + d*x]) + 8*Hypergeometric2F1[-5/2, 1, -3/2, 1 + Sec[c + d*x]]*(-1 + S
ec[c + d*x])))/(20*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(143) = 286.

Time = 1.92 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.27

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(165\cos(dx+c)^2\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+330\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)\sqrt{}\right.}{}$

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/480/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2*(165*cos(d*x+c)^2*arc
tan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)+330*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+960*cos(d*x+c)^2*
arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)+165*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)/(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))+1920*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+960*arctan((-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-898*cos(d*x+c)^2*co
t(d*x+c)^2-702*cos(d*x+c)*cot(d*x+c)^2+730*cot(d*x+c)^2+630*cot(d*x+c)*csc(
d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(141) = 282$.

Time = 0.38 (sec) , antiderivative size = 592, normalized size of antiderivative = 3.36

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{165\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{2\sqrt{2}\sqrt{a}\sqrt{}}{}\right.}{}\right.$$

$$-\frac{165\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a\cos(dx+c)+a}\right)-4}{}$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/960*(165*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1
)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c) + 3*a*cos(d*x + c) + a)/(cos(d*x + c) - 1)) + 480*(cos(d*x + c)^
4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2
+ 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(449*cos(d*x + c)^4 + 351*cos(d*x +
c)^3 - 365*cos(d*x + c)^2 - 315*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c)))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*
x + c) - a^2*d), -1/480*(165*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2
*cos(d*x + c) - 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 480*(cos(d*x + c)^4 +
 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(4
49*cos(d*x + c)^4 + 351*cos(d*x + c)^3 - 365*cos(d*x + c)^2 - 315*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^4 + 2*a^2
*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c) - a^2*d)]
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.44

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{165\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))} - \frac{960\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))} + \frac{15\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^2\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2} - \frac{2\sqrt{2}\left(3\left(a\tan\cdots\right.\right.}{\cdots}}{480\,d}$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] -1/480*(165*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(s
qrt(-a)*a*sgn(cos(d*x + c))) - 960*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))) + 15*sqrt(2)*sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^2*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2
) - 2*sqrt(2)*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a)*a^16 + 20*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^17 + 165*sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^18)/(a^20*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(3/2), x)
```

3.188 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 238

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{203\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d} + \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{15a}{64d(a+a\sec(c+dx))^{5/2}} - \frac{53}{384d(a+a\sec(c+dx))^{3/2}} - \frac{309}{256ad\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+139/224*a^2/d/(a+a*sec(
d*x+c))^(7/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+c))^(7/2)-19/16*a^2/d
/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(7/2)+15/64*a/d/(a+a*sec(d*x+c))^(5/2)-53/
384/d/(a+a*sec(d*x+c))^(3/2)-203/512*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(
1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)-309/256/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3965, 105, 156, 157, 162, 65, 213}

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{203\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d} + \frac{139a^2}{224d(a\sec(c+dx)+a)^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{7/2}} + \frac{15a}{64d(a\sec(c+dx)+a)^{5/2}} - \frac{53}{384d(a\sec(c+dx)+a)^{3/2}} - \frac{309}{256ad\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - (203*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(256*Sqrt[2]*a^(3/2)*d) + (139*a
^2)/(224*d*(a + a*Sec[c + d*x])^(7/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
 a*Sec[c + d*x])^(7/2)) - (19*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
d*x])^(7/2)) + (15*a)/(64*d*(a + a*Sec[c + d*x])^(5/2)) - 53/(384*d*(a + a*
Sec[c + d*x])^(3/2)) - 309/(256*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{a^6 \text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{9/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{a^3\text{Subst}\left(\int \frac{4a^2+\frac{11a^2x}{2}}{x(-a+ax)^2(a+ax)^{9/2}}\,dx, x, \sec(c+dx)\right)}{4d}$$

$$= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{\text{Subst}\left(\int \frac{8a^4+\frac{171a^4x}{4}}{x(-a+ax)(a+ax)^{9/2}}\,dx, x, \sec(c+dx)\right)}{8d}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} - \frac{\text{Subst}\left(\int \frac{-56a^6-\frac{973a^6x}{8}}{x(-a+ax)(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{56a^3d}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{15a}{64d(a+a\sec(c+dx))^{5/2}} + \frac{\text{Subst}\left(\int \frac{280a^8+\frac{2625a^8x}{16}}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{280a^6d}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{15a}{64d(a+a\sec(c+dx))^{5/2}} - \frac{53}{384d(a+a\sec(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{-840a^{10}+\frac{5565a^{10}x}{32}}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{840a^9d}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{15a}{64d(a+a\sec(c+dx))^{5/2}} - \frac{53}{384d(a+a\sec(c+dx))^{3/2}} - \frac{309}{256ad\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{840a^{12}-\frac{32445a^{12}x}{64}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{840a^{12}d}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{15a}{64d(a+a\sec(c+dx))^{5/2}} - \frac{53}{384d(a+a\sec(c+dx))^{3/2}} - \frac{309}{256ad\sqrt{a+a\sec(c+dx)}} + \frac{203\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{512d} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{ad}$$

$$= \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{15a}{64d(a+a\sec(c+dx))^{5/2}} - \frac{53}{384d(a+a\sec(c+dx))^{3/2}} - \frac{309}{256ad\sqrt{a+a\sec(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^2d} + \frac{203\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{256ad}$$

$$= \frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{203\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d} + \frac{139a^2}{224d(a+a\sec(c+dx))^{7/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{7/2}} - \frac{19a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{15a}{64d(a+a\sec(c+dx))^{5/2}} - \frac{53}{384d(a+a\sec(c+dx))^{3/2}} - \frac{309}{256ad\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.42

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\cot^4(c+dx)\left(-322+203\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},\frac{1}{2}(1+\sec(c+dx)\right)\right.}{}$$

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cot[c + d*x]^4*(-322 + 203*Hypergeometric2F1[-7/2, 1, -5/2, (1 + Sec[c + d
*x])/2]*(-1 + Sec[c + d*x])^2 - 64*Hypergeometric2F1[-7/2, 1, -5/2, 1 + Sec
[c + d*x]]*(-1 + Sec[c + d*x])^2 + 266*Sec[c + d*x]))/(224*d*(a*(1 + Sec[c
+ d*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(197) = 394.

Time = 1.82 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.83

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(4263\cos(dx+c)^2\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+8526\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+\right.}{}$

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10752/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2*(4263*cos(d*x+c)^2
*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)+8526*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+21504*cos(d*
x+c)^2*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)+4263*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+43008*arctan((-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+21504*arctan((-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+20726*co
s(d*x+c)^2*cot(d*x+c)^4+16074*cos(d*x+c)*cot(d*x+c)^4-33076*cot(d*x+c)^4-28
476*cot(d*x+c)^3*csc(d*x+c)+14462*cot(d*x+c)^2*csc(d*x+c)^2+12978*csc(d*x+c
)^3*cot(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(193) = 386$.

Time = 0.40 (sec) , antiderivative size = 837, normalized size of antiderivative = 3.52

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{4263\sqrt{2}\left(\cos(dx+c)^6+2\cos(dx+c)^5-\cos(dx+c)^4-4\cos(dx+c)^3-\right.}{}\right.$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/21504*(4263*sqrt(2)*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4
- 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(2*s
qrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) - 3*a*c
os(d*x + c) - a)/(cos(d*x + c) - 1)) + 10752*(cos(d*x + c)^6 + 2*cos(d*x +
c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*
sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) - 4
*(10363*cos(d*x + c)^6 + 8037*cos(d*x + c)^5 - 16538*cos(d*x + c)^4 - 14238
*cos(d*x + c)^3 + 7231*cos(d*x + c)^2 + 6489*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c)))/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a
^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2
*d*cos(d*x + c) + a^2*d), 1/10752*(4263*sqrt(2)*(cos(d*x + c)^6 + 2*cos(d*x
 + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)/(a*cos(d*x + c) + a)) - 10752*(cos(d*x + c)^6 + 2*cos(d*
x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(10363*cos(d*x + c)^6 + 8037*cos(
d*x + c)^5 - 16538*cos(d*x + c)^4 - 14238*cos(d*x + c)^3 + 7231*cos(d*x + c
)^2 + 6489*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^2*d*co
s(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(
d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{\cot\left(dx+c\right)^5}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(cot(d*x + c)^5/(a*sec(d*x + c) + a)^(3/2), x)

Giac [A] (verification not implemented)

none

Time = 1.01 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.35

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{\frac{4263\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))}-\frac{21504\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))}-\frac{21\left(29\sqrt{\right.}}{}}{}$$

[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] 1/10752*(4263*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))) - 21504*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))) - 21*(29*sqrt(2)*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 27*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a)/(a^3*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^4) + 8*sqrt(2)*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^30 - 21*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^31 - 112*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^32 - 882*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^33)/(a^35*sgn(cos(d*x + c))))/d

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{{\mathrm{cot}\left(c+d\,x\right)}^5}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(3/2),x)

[Out] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(3/2), x)

3.189 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{2a^2\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+2*tan(d*x+c)
/a/d/(a+a*sec(d*x+c))^(1/2)-2/3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)+2/5*a
*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)+2/7*a^2*tan(d*x+c)^7/d/(a+a*sec(d*x+
c))^(7/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3972, 470, 308, 209}

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d} + \frac{2a^2\tan^7(c+dx)}{7d(a\sec(c+dx)+a)^{7/2}} + \frac{2a\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}} - \frac{2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}} + \frac{2\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(2*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Tan[c + d*x]^3)/(3*d*(
```

```
a + a*Sec[c + d*x])^(3/2)) + (2*a*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x])
^(5/2)) + (2*a^2*Tan[c + d*x]^7)/(7*d*(a + a*Sec[c + d*x])^(7/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(2a^2)\,\text{Subst}\left(\int \frac{x^6(2+ax^2)}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{2a^2\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{2a^2\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} - \frac{(2a^2)\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}-\frac{1}{a^3(1+ax^2)}\right)dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} \\
&\quad + \frac{2a^2\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}} + \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}
\end{aligned}
$$

$$= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}} + \frac{2a\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}} + \frac{2a^2\tan^7(c+dx)}{7d(a+a\sec(c+dx))^{7/2}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.93 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.58

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{32\sqrt{2}\left(\frac{1}{1+\sec(c+dx)}\right)^{11/2}\left(\frac{\cos(c+dx)(11+7\cos(c+dx))\csc^8\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(105\text{arctanh}\right.}{}\right.}{}$$

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (32*Sqrt[2]*((1 + Sec[c + d*x])^(-1))^(11/2)*((Cos[c + d*x]*(11 + 7*Cos[c +
 d*x])*Csc[(c + d*x)/2]^8*Sec[(c + d*x)/2]^2*(105*ArcTanh[Sqrt[1 - Sec[c +
d*x]]]*Cos[c + d*x]^3 + (76 - 198*Cos[c + d*x] + 61*Cos[2*(c + d*x)] - 44*C
os[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]))/(3360*Sqrt[1 - Sec[c + d*x]]) - (
4*Hypergeometric2F1[2, 11/2, 13/2, -2*Sec[c + d*x]*Sin[(c + d*x)/2]^2]*Sec[
c + d*x]*Tan[(c + d*x)/2]^2)/11)*Tan[c + d*x]^7)/(7*d*(a*(1 + Sec[c + d*x])
)^(3/2)*(1 - Tan[(c + d*x)/2]^2)^(9/2))
```

Maple [A] (warning: unable to verify)

Time = 3.74 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.48

method	result
default	$-\frac{\left(105\sqrt{2}\ \text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{7}{2}}-278(1-\cos(dx+c))^7\csc(dx+c)^7+1078(1-c\right.}{105d\,a^2(-\cot(dx+c)+\csc(dx+c)-1)}$

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105/d/a^2*(105*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)
^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(7/2)-27
8*(1-cos(d*x+c))^7*csc(d*x+c)^7+1078*(1-cos(d*x+c))^5*csc(d*x+c)^5-770*(1-c
os(d*x+c))^3*csc(d*x+c)^3+210*csc(d*x+c)-210*cot(d*x+c))*(-2*a/((1-cos(d*x+
c))^2*csc(d*x+c)^2-1))^(1/2)/(-cot(d*x+c)+csc(d*x+c)-1)^3/(csc(d*x+c)-cot(d
*x+c)+1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.18

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{105\left(\cos\left(dx+c\right)^4+\cos\left(dx+c\right)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+}{\cos(dx+c)}}}{\cos(dx}\right.}{}\right.$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/105*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(-a)*log((2*a*cos(d*x +
c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(146*cos(d*x + c)^3
- 32*cos(d*x + c)^2 - 24*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3), 2/105
*(105*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*arctan(sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (146*cos(d*x + c
)^3 - 32*cos(d*x + c)^2 - 24*cos(d*x + c) + 15)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c)^3)]
```

Sympy [F]

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^6\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**6/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan\left(dx+c\right)^6}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/210*(105*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*
```

d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - 1) + 2*(a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(2*d*x + 2*c)
^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*integrate(-(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(((cos(14*d*x + 14*c)*cos(2
*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)
*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6
*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*
c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x
 + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(
2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*
sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x
+ 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*
x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*
d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*
cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) -
6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(9/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
((cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*
c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x
 + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d
*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(
2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)
*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*
c)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 (cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*
c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x
 + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d
*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*si
n(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c)
+ 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*
c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(9/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)))/(a^2*cos(14*d*x + 14*c)^2 + 36*a^2*cos(12*d*x + 12
*c)^2 + 225*a^2*cos(10*d*x + 10*c)^2 + 400*a^2*cos(8*d*x + 8*c)^2 + 225*a^2
*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 12*a^2*cos(4*d*x + 4*c)*c
os(2*d*x + 2*c) + a^2*cos(2*d*x + 2*c)^2 + a^2*sin(14*d*x + 14*c)^2 + 36*a^
2*sin(12*d*x + 12*c)^2 + 225*a^2*sin(10*d*x + 10*c)^2 + 400*a^2*sin(8*d*x +
 8*c)^2 + 225*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 12*a^2*s

in(4*d*x + 4*c)*sin(2*d*x + 2*c) + a^2*sin(2*d*x + 2*c)^2 + 2*(6*a^2*cos(12
*d*x + 12*c) + 15*a^2*cos(10*d*x + 10*c) + 20*a^2*cos(8*d*x + 8*c) + 15*a^2
*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(14*d
*x + 14*c) + 12*(15*a^2*cos(10*d*x + 10*c) + 20*a^2*cos(8*d*x + 8*c) + 15*a
^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(12
*d*x + 12*c) + 30*(20*a^2*cos(8*d*x + 8*c) + 15*a^2*cos(6*d*x + 6*c) + 6*a^
2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 40*(15*a^2*
cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(8*d*x
 + 8*c) + 30*(6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(6*d*x + 6*
c) + 2*(6*a^2*sin(12*d*x + 12*c) + 15*a^2*sin(10*d*x + 10*c) + 20*a^2*sin(8
*d*x + 8*c) + 15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) + a^2*sin(2*
d*x + 2*c))*sin(14*d*x + 14*c) + 12*(15*a^2*sin(10*d*x + 10*c) + 20*a^2*sin
(8*d*x + 8*c) + 15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) + a^2*sin(
2*d*x + 2*c))*sin(12*d*x + 12*c) + 30*(20*a^2*sin(8*d*x + 8*c) + 15*a^2*sin
(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(10*d*x +
 10*c) + 40*(15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d
*x + 2*c))*sin(8*d*x + 8*c) + 30*(6*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x +
2*c))*sin(6*d*x + 6*c)), x) - 12*(a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(2*d*
x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*integrate(-(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(((cos(14*d*x + 14*
c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x
 + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6
*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*
d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*s
in(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8
*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x
 + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*co
s(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*
cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6
*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*
c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x +
 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)) + ((cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*sin(12*d
*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x + 2*c)*s
in(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*x + 2*c)
*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*d*x + 12
*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*cos(8*d*
x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 6*cos(4*
d*x + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) - (cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*
d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*c
os(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)

```
*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c
) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x
 + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*
d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(
5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)))/(a^2*cos(14*d*x + 14*c)^2 + 36*a^2*cos(12*
d*x + 12*c)^2 + 225*a^2*cos(10*d*x + 10*c)^2 + 400*a^2*cos(8*d*x + 8*c)^2 +
 225*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 12*a^2*cos(4*d*x
+ 4*c)*cos(2*d*x + 2*c) + a^2*cos(2*d*x + 2*c)^2 + a^2*sin(14*d*x + 14*c)^2
 + 36*a^2*sin(12*d*x + 12*c)^2 + 225*a^2*sin(10*d*x + 10*c)^2 + 400*a^2*sin
(8*d*x + 8*c)^2 + 225*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 +
12*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a^2*sin(2*d*x + 2*c)^2 + 2*(6*a^
2*cos(12*d*x + 12*c) + 15*a^2*cos(10*d*x + 10*c) + 20*a^2*cos(8*d*x + 8*c)
+ 15*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*
cos(14*d*x + 14*c) + 12*(15*a^2*cos(10*d*x + 10*c) + 20*a^2*cos(8*d*x + 8*c
) + 15*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c)
)*cos(12*d*x + 12*c) + 30*(20*a^2*cos(8*d*x + 8*c) + 15*a^2*cos(6*d*x + 6*c
) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 40*
(15*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*c
os(8*d*x + 8*c) + 30*(6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(6*
d*x + 6*c) + 2*(6*a^2*sin(12*d*x + 12*c) + 15*a^2*sin(10*d*x + 10*c) + 20*a
^2*sin(8*d*x + 8*c) + 15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) + a^
2*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + 12*(15*a^2*sin(10*d*x + 10*c) + 20
*a^2*sin(8*d*x + 8*c) + 15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) +
a^2*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 30*(20*a^2*sin(8*d*x + 8*c) + 15
*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(
10*d*x + 10*c) + 40*(15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) + a^2
*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 30*(6*a^2*sin(4*d*x + 4*c) + a^2*sin(
2*d*x + 2*c))*sin(6*d*x + 6*c)), x) + 16*(a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*
sin(2*d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*integrate(-(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(((cos(14*d
*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 15*co
s(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) +
15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c)
+ cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(12*d*x +
 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 20*sin(8
*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 6*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*c
os(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c)
 + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6
*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x +
2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*
d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin
(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*
```

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d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x + 2*c)*
sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos(2*d*x
+ 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*cos(2*d*
x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*cos(12*
d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 20*
cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) -
6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - (cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c
)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x
+ 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*
x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d
*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*s
in(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c
)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)
^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a^2*cos(14*d*x + 14*c)^2 + 36*a^2
*cos(12*d*x + 12*c)^2 + 225*a^2*cos(10*d*x + 10*c)^2 + 400*a^2*cos(8*d*x +
8*c)^2 + 225*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 12*a^2*co
s(4*d*x + 4*c)*cos(2*d*x + 2*c) + a^2*cos(2*d*x + 2*c)^2 + a^2*sin(14*d*x +
 14*c)^2 + 36*a^2*sin(12*d*x + 12*c)^2 + 225*a^2*sin(10*d*x + 10*c)^2 + 400
*a^2*sin(8*d*x + 8*c)^2 + 225*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4
*c)^2 + 12*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a^2*sin(2*d*x + 2*c)^2 +
 2*(6*a^2*cos(12*d*x + 12*c) + 15*a^2*cos(10*d*x + 10*c) + 20*a^2*cos(8*d*x
 + 8*c) + 15*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x
+ 2*c))*cos(14*d*x + 14*c) + 12*(15*a^2*cos(10*d*x + 10*c) + 20*a^2*cos(8*d
*x + 8*c) + 15*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*
x + 2*c))*cos(12*d*x + 12*c) + 30*(20*a^2*cos(8*d*x + 8*c) + 15*a^2*cos(6*d
*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(10*d*x + 10*
c) + 40*(15*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x +
 2*c))*cos(8*d*x + 8*c) + 30*(6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c)
)*cos(6*d*x + 6*c) + 2*(6*a^2*sin(12*d*x + 12*c) + 15*a^2*sin(10*d*x + 10*c
) + 20*a^2*sin(8*d*x + 8*c) + 15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4
*c) + a^2*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + 12*(15*a^2*sin(10*d*x + 10
*c) + 20*a^2*sin(8*d*x + 8*c) + 15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x +
 4*c) + a^2*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 30*(20*a^2*sin(8*d*x + 8
*c) + 15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*
c))*sin(10*d*x + 10*c) + 40*(15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*
c) + a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 30*(6*a^2*sin(4*d*x + 4*c) +
a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - 6*(a^2*d*cos(2*d*x + 2*c)^2 +
 a^2*d*sin(2*d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*integrate(-(c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(((c
os(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x + 12*c)*cos(2*d*x + 2*c)
+ 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos(8*d*x + 8*c)*cos(2*d*x +
2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*cos(4*d*x + 4*c)*cos(2*d*x
```

+ 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*sin(2*d*x + 2*c) + 6*sin(1
2*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 2
0*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*x + 6*c)*sin(2*d*x + 2*c)
+ 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(14*d*x + 14*c
) + 6*cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x
+ 10*c) + 20*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*
d*x + 6*c) + 6*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2
*d*x + 2*c) - 6*cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)
*sin(2*d*x + 2*c) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6
*c)*sin(2*d*x + 2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(14*d*x + 14*c) + 6*cos(2*d*x
+ 2*c)*sin(12*d*x + 12*c) + 15*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 20*cos
(2*d*x + 2*c)*sin(8*d*x + 8*c) + 15*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 6*c
os(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(14*d*x + 14*c)*sin(2*d*x + 2*c) - 6*
cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 15*cos(10*d*x + 10*c)*sin(2*d*x + 2*c
) - 20*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 15*cos(6*d*x + 6*c)*sin(2*d*x +
2*c) - 6*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - (cos(14*d*x + 14*c)*cos(2*d*x + 2*c) + 6*cos(12*d*x
+ 12*c)*cos(2*d*x + 2*c) + 15*cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 20*cos
(8*d*x + 8*c)*cos(2*d*x + 2*c) + 15*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 6*c
os(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(14*d*x + 14*c)*
sin(2*d*x + 2*c) + 6*sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 15*sin(10*d*x +
10*c)*sin(2*d*x + 2*c) + 20*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 15*sin(6*d*
x + 6*c)*sin(2*d*x + 2*c) + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x
+ 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a^2*cos(14*d*x + 14*c)^2 +
36*a^2*cos(12*d*x + 12*c)^2 + 225*a^2*cos(10*d*x + 10*c)^2 + 400*a^2*cos(8
*d*x + 8*c)^2 + 225*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 12
*a^2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + a^2*cos(2*d*x + 2*c)^2 + a^2*sin(1
4*d*x + 14*c)^2 + 36*a^2*sin(12*d*x + 12*c)^2 + 225*a^2*sin(10*d*x + 10*c)^
2 + 400*a^2*sin(8*d*x + 8*c)^2 + 225*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*
d*x + 4*c)^2 + 12*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a^2*sin(2*d*x + 2
c)^2 + 2(6*a^2*cos(12*d*x + 12*c) + 15*a^2*cos(10*d*x + 10*c) + 20*a^2*co
s(8*d*x + 8*c) + 15*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos
(2*d*x + 2*c))*cos(14*d*x + 14*c) + 12*(15*a^2*cos(10*d*x + 10*c) + 20*a^2*
cos(8*d*x + 8*c) + 15*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*c
os(2*d*x + 2*c))*cos(12*d*x + 12*c) + 30*(20*a^2*cos(8*d*x + 8*c) + 15*a^2*
cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(10*d*
x + 10*c) + 40*(15*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + a^2*cos(
2*d*x + 2*c))*cos(8*d*x + 8*c) + 30*(6*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x
+ 2*c))*cos(6*d*x + 6*c) + 2*(6*a^2*sin(12*d*x + 12*c) + 15*a^2*sin(10*d*x
+ 10*c) + 20*a^2*sin(8*d*x + 8*c) + 15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*
d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + 12*(15*a^2*sin(10*d

```
*x + 10*c) + 20*a^2*sin(8*d*x + 8*c) + 15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(
4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 30*(20*a^2*sin(8*
d*x + 8*c) + 15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d
*x + 2*c))*sin(10*d*x + 10*c) + 40*(15*a^2*sin(6*d*x + 6*c) + 6*a^2*sin(4*d
*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 30*(6*a^2*sin(4*d*x +
4*c) + a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x))*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*sqrt(a) - 8*(7*(15*sin(6*
d*x + 6*c) + 25*sin(4*d*x + 4*c) + 29*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (105*cos(6*d*x + 6*c) + 175*cos(4*d
*x + 4*c) + 203*cos(2*d*x + 2*c) + 73)*sin(7/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)))*sqrt(a))/((a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(2*d*x
 + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(137) = 274$.

Time = 3.75 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.89

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{105\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^2\mathrm{sgn}(\cos(dx+c))}-\frac{\log\left(\left|\left(\sqrt{-\right.\right.\right.}{}\right.}{}$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/105*(105*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^2*sgn(cos(d*x + c))) - l
og(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 + a*(2*sqrt(2) - 3)))/(a^2*sgn(cos(d*x + c)))) + 2*(((139*sqrt(2)*a^2*t
an(1/2*d*x + 1/2*c)^2/sgn(cos(d*x + c)) - 539*sqrt(2)*a^2/sgn(cos(d*x + c))
)*tan(1/2*d*x + 1/2*c)^2 + 385*sqrt(2)*a^2/sgn(cos(d*x + c)))*tan(1/2*d*x +
 1/2*c)^2 - 105*sqrt(2)*a^2/sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a*tan
(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(3/2), x)
```

3.190 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} + \frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-2*tan(d*x+c)/
a/d/(a+a*sec(d*x+c))^(1/2)+2/3*tan(d*x+c)^3/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3972, 308, 209}

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d} + \frac{2\tan^3(c+dx)}{3d(a\sec(c+dx)+a)^{3/2}} - \frac{2\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
2*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Tan[c + d*x]^3)/(3*d*(a
 + a*Sec[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(2a)\text{Subst}\left(\int \frac{x^4}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{(2a)\text{Subst}\left(\int \left(-\frac{1}{a^2}+\frac{x^2}{a}+\frac{1}{a^2(1+ax^2)}\right)\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= -\frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}+\frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}-\frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} \\
&= \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}+\frac{2\tan^3(c+dx)}{3d(a+a\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 3.32 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.71

$$
\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{64\cos^6\left(\frac{1}{2}(c+dx)\right)\cot^4\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{7/2}\left(3\arcsin\left(\right.\right.}{3d\left(-1+\cot^2\left(\frac{1}{2}(c\right.\right.}
$$

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (64*Cos[(c + d*x)/2]^6*Cot[(c + d*x)/2]^4*Sec[c + d*x]^5*((1 + Sec[c + d*x]
)^(-1))^(7/2)*(3*ArcSin[Tan[(c + d*x)/2]/Sqrt[(1 + Cos[c + d*x])^(-1)]]*Cos
```

```
[c + d*x]^2 + Sqrt[Cos[c + d*x]]*Sqrt[(1 + Cos[c + d*x])^(-1)]*(Sin[c + d*x
] - 2*Sin[2*(c + d*x)])))/(3*d*(-1 + Cot[(c + d*x)/2]^2)^2*(a*(1 + Sec[c +
d*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(83) = 166$.

Time = 3.76 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.83

method	result
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\left(-3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)-3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{}{(\cos(d}\right.\right.}{3d\,a^2(\cos(dx+c)+1)}$

```
[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(-3*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)-3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c
)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4*sin(d*x+c)-tan(d*x+c
))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.11

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{3\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(}{\cos(dx+c)+}\right.}{3\left(a^2d\cos(dx+c)^2\right.}\right.$$

$$-\frac{2\left(3\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(4\cos(dx+c)-1)\,\mathrm{s}\right.}{3\left(a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x + c)^2 +
 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*(4*cos(d*x + c) - 1)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + a^
2*d*cos(d*x + c)), -2/3*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(s
```

```
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 + sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(4*cos(d*x + c) - 1)*sin(d*x + c
))/(a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^4\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(tan(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan\left(dx+c\right)^4}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -1/6*(3*(2*a^2*d*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(3/4)*(((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*
d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4
*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(
2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin
(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*c
os(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin
(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*
sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)
*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c
)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x +
2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x +
 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x
+ 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x
 + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x +
 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x
+ 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8
*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d
```

```
*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)))/(a^2*cos(10*d*x + 10*c)^2 + 16*a^2*cos(8*d*x + 8*c)^2 + 36*a^
2*cos(6*d*x + 6*c)^2 + 16*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*cos(4*d*x + 4*c)*c
os(2*d*x + 2*c) + a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*d*x + 10*c)^2 + 16*a^
2*sin(8*d*x + 8*c)^2 + 36*a^2*sin(6*d*x + 6*c)^2 + 16*a^2*sin(4*d*x + 4*c)^
2 + 8*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a^2*sin(2*d*x + 2*c)^2 + 2*(4
*a^2*cos(8*d*x + 8*c) + 6*a^2*cos(6*d*x + 6*c) + 4*a^2*cos(4*d*x + 4*c) + a
^2*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 8*(6*a^2*cos(6*d*x + 6*c) + 4*a^2
*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 12*(4*a^2*cos(
4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(4*a^2*sin(8*d*x
+ 8*c) + 6*a^2*sin(6*d*x + 6*c) + 4*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x +
2*c))*sin(10*d*x + 10*c) + 8*(6*a^2*sin(6*d*x + 6*c) + 4*a^2*sin(4*d*x + 4*
c) + a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 12*(4*a^2*sin(4*d*x + 4*c) +
a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) + 4*a^2*d*integrate(-(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(((cos(10*d*
x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*
d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d
*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(
2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*s
in(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*
sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)
*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*c
os(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c
)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x +
2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x +
10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x
+ 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(3/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x +
2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x +
 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d
*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6
*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*
d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a^2*cos(10*d*x + 10*c)^
2 + 16*a^2*cos(8*d*x + 8*c)^2 + 36*a^2*cos(6*d*x + 6*c)^2 + 16*a^2*cos(4*d*
x + 4*c)^2 + 8*a^2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + a^2*cos(2*d*x + 2*c)
^2 + a^2*sin(10*d*x + 10*c)^2 + 16*a^2*sin(8*d*x + 8*c)^2 + 36*a^2*sin(6*d*
x + 6*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + a^2*sin(2*d*x + 2*c)^2 + 2*(4*a^2*cos(8*d*x + 8*c) + 6*a^2*cos(6*d*
x + 6*c) + 4*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(10*d*x + 10*c
) + 8*(6*a^2*cos(6*d*x + 6*c) + 4*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*
c))*cos(8*d*x + 8*c) + 12*(4*a^2*cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*c
```

```
os(6*d*x + 6*c) + 2*(4*a^2*sin(8*d*x + 8*c) + 6*a^2*sin(6*d*x + 6*c) + 4*a^
2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 8*(6*a^2*si
n(6*d*x + 6*c) + 4*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(8*d*x +
 8*c) + 12*(4*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
), x) - 6*a^2*d*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(3/4)*(((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d
*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*
d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2
*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(
2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(
10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*s
in(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*
sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)
*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2
*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x +
2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x +
 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x +
2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x +
 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*
d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)))/(a^2*cos(10*d*x + 10*c)^2 + 16*a^2*cos(8*d*x + 8*c)^2 + 36*a^2
*cos(6*d*x + 6*c)^2 + 16*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*cos(4*d*x + 4*c)*co
s(2*d*x + 2*c) + a^2*cos(2*d*x + 2*c)^2 + a^2*sin(10*d*x + 10*c)^2 + 16*a^2
*sin(8*d*x + 8*c)^2 + 36*a^2*sin(6*d*x + 6*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2
 + 8*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a^2*sin(2*d*x + 2*c)^2 + 2*(4*
a^2*cos(8*d*x + 8*c) + 6*a^2*cos(6*d*x + 6*c) + 4*a^2*cos(4*d*x + 4*c) + a^
2*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 8*(6*a^2*cos(6*d*x + 6*c) + 4*a^2*
cos(4*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 12*(4*a^2*cos(4
*d*x + 4*c) + a^2*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(4*a^2*sin(8*d*x +
 8*c) + 6*a^2*sin(6*d*x + 6*c) + 4*a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2
*c))*sin(10*d*x + 10*c) + 8*(6*a^2*sin(6*d*x + 6*c) + 4*a^2*sin(4*d*x + 4*c
) + a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 12*(4*a^2*sin(4*d*x + 4*c) + a
^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) + arctan2((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
```

```
c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*sqrt(a
) - 8*(3*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(2*d*x
 + 2*c) - (3*cos(2*d*x + 2*c) + 2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(3/4)*a^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(83) = 166$.

Time = 2.44 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.42

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{3\sqrt{-a}\Bigg(\frac{\log\Big(\Big|\Big(\sqrt{-a}\tan\big(\frac{1}{2}dx+\frac{1}{2}c\big)-\sqrt{-a\tan\big(\frac{1}{2}dx+\frac{1}{2}c\big)^2+a}\Big)^2-a\big(2\sqrt{2}+3\big)\Big|\Big)}{a^2\mathrm{sgn}(\cos(dx+c))} - \frac{\log\Big(\Big|\Big(\sqrt{-a}\tan\big(\frac{1}{2}dx+\frac{1}{2}c\big)-\sqrt{-a\tan\big(\frac{1}{2}dx+\frac{1}{2}c\big)^2+a}\Big)}{a^2\mathrm{sgn}(\cos(dx+c))}}{3d}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^2*sgn(cos(d*x + c))) - log(
abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
 + a*(2*sqrt(2) - 3)))/(a^2*sgn(cos(d*x + c)))) + 2*(5*sqrt(2)*tan(1/2*d*x
+ 1/2*c)^2/sgn(cos(d*x + c)) - 3*sqrt(2)/sgn(cos(d*x + c)))*tan(1/2*d*x + 1
/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))
/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(3/2), x)
```

3.191 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+2*arctan(1/2
*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 492, 209}

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d} - \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}$$

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(2*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(a^(3/2)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} - \frac{4\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} \\
&= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.55

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{4\left(2\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}\right)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sec^{\frac{3}{2}}(c}{d\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}(a(1+\sec(c+dx)))^{3/2}}$$

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*(2*ArcSin[Tan[(c + d*x)/2]] - Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]])*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[c +
 d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]])/(d*(Sec[(c + d*x)/2]^2)^(3/2)*(a*(1 + S
ec[c + d*x]))^(3/2))
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.60

method	result
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(-\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^{2}-2\,\mathrm{co}}\right.\right.}{d\,a^{2}}$

```
[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(a*(1+sec(d*x+c)))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-arcta
nh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+ln(csc(d*x
+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))
*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.47

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{a^2}\right.$$

$$\left.-\frac{2\left(\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\right)}{a^2d}\right]$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c
) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a^2*d), -2*(sqrt(2)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c))) - sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a^2*d)]
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(tan(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 1.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a+\frac{a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2\arctan\left(\frac{\sqrt{-a+\frac{a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)}{d}$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a + a/tan(1/2*d*x + 1/2*c)^2)/sqr
t(a))/a^(3/2) - 2*arctan(sqrt(-a + a/tan(1/2*d*x + 1/2*c)^2)/sqrt(a))/a^(3/
2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(3/2), x)
```

3.192 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 215

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = & -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} \\
& + \frac{71\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{32\sqrt{2}a^{3/2}d} + \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32a^2d} \\
& - \frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{32a^2d} \\
& - \frac{\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{16a^2d}
\end{aligned}
$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+71/64*arctan
(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d+7
/32*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^2/d-13/32*cos(d*x+c)*cot(d*x+c)*sec
(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(1/2)/a^2/d-1/16*cos(d*x+c)^2*cot(d*x+c)
*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

$= \{3972, 483, 593, 597, 536, 209\}$

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d} + \frac{71\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{32\sqrt{2}a^{3/2}d} + \frac{7\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{32a^2d} - \frac{\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a\sec(c+dx)+a}}{16a^2d} - \frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a\sec(c+dx)+a}}{32a^2d}$$

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(71*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(32*
Sqrt[2]*a^(3/2)*d) + (7*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(32*a^2*d) -
 (13*Cos[c + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d*x]])
/(32*a^2*d) - (Cos[c + d*x]^2*Cot[c + d*x]*Sec[(c + d*x)/2]^4*Sqrt[a + a*Se
c[c + d*x]])/(16*a^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} \\
&= -\frac{\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{16a^2d} \\
&\quad -\frac{\text{Subst}\left(\int \frac{3a-5a^2x^2}{x^2(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^3d} \\
&= -\frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{32a^2d} \\
&\quad -\frac{\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{16a^2d} \\
&\quad -\frac{\text{Subst}\left(\int \frac{-7a^2-39a^3x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{16a^4d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32a^2d} \\
&\quad - \frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{32a^2d} \\
&\quad - \frac{\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{16a^2d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{57a^3-7a^4x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{32a^4d} \\
&= \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32a^2d} \\
&\quad - \frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{32a^2d} \\
&\quad - \frac{\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{16a^2d} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} \\
&\quad - \frac{71\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{32ad} \\
&= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{71\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{32\sqrt{2}a^{3/2}d} \\
&\quad + \frac{7\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32a^2d} \\
&\quad - \frac{13\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{32a^2d} \\
&\quad - \frac{\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{16a^2d}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 6.51 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.98

$$
\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{27\cot(2(c+dx)) + 12\csc(c+dx) + 13\csc(2(c+dx)) + 256\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\sec(c+dx)}}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(
$$

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -1/32*(27*Cot[2*(c + d*x)] + 12*Csc[c + d*x] + 13*Csc[2*(c + d*x)] + 256*Ar
cTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Cos[(c + d*x)/2]^4*Sec
```

```
[c + d*x]^(3/2)*Sqrt[Sec[c + d*x]/(1 + Sec[c + d*x])^2]*Sqrt[1 + Sec[c + d*
x]] - 142*ArcSin[Tan[(c + d*x)/2]]*Cos[(c + d*x)/2]^4*Sqrt[Sec[(c + d*x)/2]
^2]*Sec[c + d*x]^(3/2)*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]]
)/(d*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(184) = 368$.

Time = 2.00 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.31

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(24\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{9}{2}}\sin(dx+c)-24(1-\cos(dx+c\right.}{}$

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192/d/a^2*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^
2*csc(d*x+c)^2-1)^(1/2)/(1-cos(d*x+c))*(24*((1-cos(d*x+c))^2*csc(d*x+c)^2-1
)^(9/2)*sin(d*x+c)-24*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(7
/2)*csc(d*x+c)+28*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(5/2)*
csc(d*x+c)-4*(1-cos(d*x+c))^6*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d
*x+c)^5-35*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)*csc(d*x
+c)+25*(1-cos(d*x+c))^4*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*x+c)^
3-192*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot
(d*x+c)+csc(d*x+c)))*(1-cos(d*x+c))-42*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*c
sc(d*x+c)^2-1)^(1/2)*csc(d*x+c)+213*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c)
)^2*csc(d*x+c)^2-1)^(1/2))*(1-cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.80

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{71\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(\cdots}{\cos\cdots}\right.}{}\right.$$

$$-\frac{71\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)+64\left(\cos(dx+\cdots\right.}{64\left(a^2\cdots\right.}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(71*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*s
qrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos
(d*x + c) + 1))*sin(d*x + c) + 64*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqr
t(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)
+ a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4*(27*cos(d*x + c)^3 + 12*cos(d*x +
 c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^2*d*co
s(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c)), -1/64*(71*sqrt(
2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x +
 c) + 64*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x
 + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(27*cos(d*x + c)^3 + 12*cos
(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.67

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^2\mathrm{sgn}(\cos(dx+c))}-\frac{17\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{\cdots}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\,\mathrm{ta}\cdots}\right.\right.}}{64\,d}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/64*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^
2/(a^2*sgn(cos(d*x + c))) - 17*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x
 + 1/2*c) + 16*sqrt(2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 - a)*sqrt(-a)*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(3/2), x)
```

3.193 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 303

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{533\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d} - \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{256a^2d} + \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3d} - \frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^3d} - \frac{7\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{64a^3d} - \frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{48a^3d}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+277/384*cot(d
*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^3/d-81/128*cos(d*x+c)*cot(d*x+c)^3*sec(1/2
*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a^3/d-7/64*cos(d*x+c)^2*cot(d*x+c)^3*s
ec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(3/2)/a^3/d-1/48*cos(d*x+c)^3*cot(d*x+
c)^3*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(3/2)/a^3/d-533/512*arctan(1/2*a
^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-21/256*
cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3972, 483, 593, 597, 536, 209}

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d} - \frac{533\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{256\sqrt{2}a^{3/2}d} + \frac{277\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{384a^3d} - \frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{48a^3d} - \frac{7\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{64a^3d} - \frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{128a^3d} - \frac{21\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{256a^2d}$$

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
533*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(256
*Sqrt[2]*a^(3/2)*d) - (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(256*a^2*d
) + (277*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(384*a^3*d) - (81*Cos[c
 + d*x]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/2))/(128*
a^3*d) - (7*Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a + a*Sec[c +
 d*x])^(3/2))/(64*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]^3*Sec[(c + d*x)/2]^
6*(a + a*Sec[c + d*x])^(3/2))/(48*a^3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^3d} \\
&= -\frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{48a^3d} \\
&\quad -\frac{\text{Subst}\left(\int \frac{3a-9a^2x^2}{x^4(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{6a^4d}
\end{aligned}
$$

$$= -\frac{7\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{64a^3d}$$
$$-\frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{48a^3d}$$
$$-\frac{\text{Subst}\left(\int \frac{-51a^2-147a^3x^2}{x^4(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{48a^5d}$$
$$= -\frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^3d}$$
$$-\frac{7\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{64a^3d}$$
$$-\frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{48a^3d}$$
$$-\frac{\text{Subst}\left(\int \frac{-831a^3-1215a^4x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{192a^6d}$$
$$= \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3d}$$
$$-\frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^3d}$$
$$-\frac{7\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{64a^3d}$$
$$-\frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{48a^3d}$$
$$+\frac{\text{Subst}\left(\int \frac{-189a^4-2493a^5x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{1152a^6d}$$
$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{256a^2d} + \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3d}$$
$$-\frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^3d}$$
$$-\frac{7\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{64a^3d}$$
$$-\frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{48a^3d}$$
$$-\frac{\text{Subst}\left(\int \frac{4419a^5-189a^6x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2304a^6d}$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{256a^2d} + \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3d} - \frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^3d} - \frac{7\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{64a^3d} - \frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{48a^3d} - \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} + \frac{533\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{256ad}$$

$$= \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{533\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d} - \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{256a^2d} + \frac{277\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{384a^3d} - \frac{81\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^3d} - \frac{7\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{64a^3d} - \frac{\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{48a^3d}$$

Mathematica [A] (warning: unable to verify)

Time = 2.52 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.79

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{-\frac{1}{16}(201-472\cos(c+dx)+516\cos(2(c+dx))+984\cos(3(c+dx))+819\,\mathrm{c}}{}$$

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-1/16*((201 - 472*Cos[c + d*x] + 516*Cos[2*(c + d*x)] + 984*Cos[3*(c + d*x
)] + 819*Cos[4*(c + d*x)])*Csc[c + d*x]^3*Sec[c + d*x]) + 3072*ArcTan[Tan[(
c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^
(3/2)*Sqrt[Sec[c + d*x]/(1 + Sec[c + d*x])^2]*Sqrt[1 + Sec[c + d*x]] - 1599
*ArcSin[Tan[(c + d*x)/2]]*Cos[(c + d*x)/2]^4*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c
 + d*x]^(3/2)*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]])/(384*d*
(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(263) = 526$.

Time = 1.87 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.79

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(1599\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\right.}{}$

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1536/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2*(1599*2^(1/2)*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(
d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*cos(d*x+c)^2-3072*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2))*cos(d*x+c)^2+3198*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c
)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*cos(d*x+c)-6144*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d
*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+1599*ln
(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1
)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1638*cot(d*x+c)^3*cos(d
*x+c)^2-3072*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x
+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+984*cos(d*x+c)*cot(d*x+c)^3-1380
*cot(d*x+c)^3-856*cot(d*x+c)^2*csc(d*x+c)+126*cot(d*x+c)*csc(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.35

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{1599\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)-1\right)\sqrt{-a}\log\left(-\right.}{}\right.$$

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3072*(1599*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)
- 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 1536*(cos(d*x + c)^4
+ 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3
+ 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
```

```
s(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*
x + c) - 4*(819*cos(d*x + c)^5 + 492*cos(d*x + c)^4 - 690*cos(d*x + c)^3 -
428*cos(d*x + c)^2 + 63*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
)))/((a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)
- a^2*d)*sin(d*x + c)), 1/1536*(1599*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x +
c)^3 - 2*cos(d*x + c) - 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 1536*(co
s(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c) - 1)*sqrt(a)*arctan(2*sqrt
(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*
cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(819*cos(d*x + c)^5
+ 492*cos(d*x + c)^4 - 690*cos(d*x + c)^3 - 428*cos(d*x + c)^2 + 63*cos(d*x
 + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^2*d*cos(d*x + c)^4 + 2*
a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^4/(a*sec(d*x + c) + a)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.86

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(2\left(\frac{4\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2\mathrm{sgn}(\cos(dx+c))}-\frac{37\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{417\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan}{1}$$

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/1536*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*sqrt(2)*tan(1/2*d*x + 1/
2*c)^2/(a^2*sgn(cos(d*x + c))) - 37*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/
2*d*x + 1/2*c)^2 + 417*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c
) - 32*sqrt(2)*(21*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^4 - 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2*a + 19*a^2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2 - a)^3*sqrt(-a)*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{\cot(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(3/2), x)
```

3.194 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 387

$$
\begin{aligned}
\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} \\
&+ \frac{16363\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{8192\sqrt{2}a^{3/2}d} - \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8192a^2d} \\
&- \frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12288a^3d} \\
&+ \frac{12267\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{10240a^4d} \\
&- \frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{2048a^4d} \\
&- \frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^4d} \\
&- \frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{768a^4d} \\
&- \frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d}
\end{aligned}
$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-8171/12288*c
ot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^3/d+12267/10240*cot(d*x+c)^5*(a+a*sec(
d*x+c))^(5/2)/a^4/d-2045/2048*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2*
(a+a*sec(d*x+c))^(5/2)/a^4/d-511/3072*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2*d*x
+1/2*c)^4*(a+a*sec(d*x+c))^(5/2)/a^4/d-29/768*cos(d*x+c)^3*cot(d*x+c)^5*sec
(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(5/2)/a^4/d-1/128*cos(d*x+c)^4*cot(d*x+c
)^5*sec(1/2*d*x+1/2*c)^8*(a+a*sec(d*x+c))^(5/2)/a^4/d+16363/16384*arctan(1/
2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-21/8
192*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3972, 483, 593, 597, 536, 209}

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d} + \frac{16363\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{8192\sqrt{2}a^{3/2}d} + \frac{12267\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{10240a^4d} - \frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{128a^4d} - \frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{768a^4d} - \frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{3072a^4d} - \frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{2048a^4d} - \frac{8171\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{12288a^3d} - \frac{21\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{8192a^2d}$$

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(16363*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
8192*Sqrt[2]*a^(3/2)*d) - (21*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(8192*
a^2*d) - (8171*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(12288*a^3*d) + (
12267*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(10240*a^4*d) - (2045*Cos[
c + d*x]*Cot[c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))/(204
8*a^4*d) - (511*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a + a*Sec
[c + d*x])^(5/2))/(3072*a^4*d) - (29*Cos[c + d*x]^3*Cot[c + d*x]^5*Sec[(c +
 d*x)/2]^6*(a + a*Sec[c + d*x])^(5/2))/(768*a^4*d) - (Cos[c + d*x]^4*Cot[c
+ d*x]^5*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(5/2))/(128*a^4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
```

```
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^5}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^4 d}$$

$$= -\frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d} - \frac{\text{Subst}\left(\int \frac{3a-13a^2x^2}{x^6(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8a^5d}$$

$$= -\frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{768a^4d} - \frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d} - \frac{\text{Subst}\left(\int \frac{-127a^2-319a^3x^2}{x^6(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{96a^6d}$$

$$= -\frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^4d} - \frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{768a^4d} - \frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d} - \frac{\text{Subst}\left(\int \frac{-3063a^3-4599a^4x^2}{x^6(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{768a^7d}$$

$$= -\frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{2048a^4d} - \frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^4d} - \frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{768a^4d} - \frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d} - \frac{\text{Subst}\left(\int \frac{-36801a^4-42945a^5x^2}{x^6(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{3072a^8d}$$

$$
\begin{aligned}
&= \frac{12267\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{10240a^4d} \\
&\quad - \frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{2048a^4d} \\
&\quad - \frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^4d} \\
&\quad - \frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{768a^4d} \\
&\quad - \frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d} \\
&\quad + \frac{\mathrm{Subst}\left(\int \frac{-122565a^5-184005a^6x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{30720a^8d} \\
&= -\frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12288a^3d} \\
&\quad + \frac{12267\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{10240a^4d} \\
&\quad - \frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{2048a^4d} \\
&\quad - \frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^4d} \\
&\quad - \frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{768a^4d} \\
&\quad - \frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d} \\
&\quad - \frac{\mathrm{Subst}\left(\int \frac{945a^6-367695a^7x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{184320a^8d} \\
&= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8192a^2d} - \frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12288a^3d} \\
&\quad + \frac{12267\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{10240a^4d} \\
&\quad - \frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{2048a^4d} \\
&\quad - \frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^4d} \\
&\quad - \frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{768a^4d} \\
&\quad - \frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d} \\
&\quad + \frac{\mathrm{Subst}\left(\int \frac{738225a^7+945a^8x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{368640a^8d}
\end{aligned}
$$

$$= -\frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8192a^2d} - \frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12288a^3d} + \frac{12267\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{10240a^4d} - \frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{2048a^4d} - \frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^4d} - \frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{768a^4d} - \frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d} - \frac{16363\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8192ad} + \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{16363\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{8192\sqrt{2}a^{3/2}d} - \frac{21\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{8192a^2d} - \frac{8171\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{12288a^3d} + \frac{12267\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{10240a^4d} - \frac{2045\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{2048a^4d} - \frac{511\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^4d} - \frac{29\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{768a^4d} - \frac{\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{128a^4d}$$

Mathematica [A] (verified)

Time = 5.13 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.68

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\left(245445\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{1+\sec(c+dx)}\right.}{}$$

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(245445*ArcSin[Tan[(c + d*x)/2]]*Sqrt[(1 + Sec[c + d*x]
)^(-1)]*Sqrt[1 + Sec[c + d*x]] - 245760*Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqr
t[(1 + Sec[c + d*x])^(-1)]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c +
d*x]] - (Sqrt[(2 + 2*Cos[c + d*x])^(-1)]*(355216 + 831963*Cos[c + d*x] - 59
592*Cos[2*(c + d*x)] + 153657*Cos[3*(c + d*x)] - 207760*Cos[4*(c + d*x)] +
141291*Cos[5*(c + d*x)] + 207048*Cos[6*(c + d*x)] + 151041*Cos[7*(c + d*x)]
)*Csc[c + d*x]^6*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2])/32))/(61440*d*(Sec[(c
 + d*x)/2]^2)^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.49

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(-245445\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)\right.\right.}{}$

```
[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/245760/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2*(-245445*2^(1/2)*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2
*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*cos(d*x+c)^2+491520*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))*cos(d*x+c)^2-490890*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(
cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*cos(d*x+c)+983040*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ar
ctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x
+c)-245445*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+c
sc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+491520*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))+302082*cos(d*x+c)^2*cot(d*x+c)^5+207048*cos(d*x+
c)*cot(d*x+c)^5-457998*cot(d*x+c)^5-362512*cot(d*x+c)^4*csc(d*x+c)+195222*c
ot(d*x+c)^3*csc(d*x+c)^2+164680*cot(d*x+c)^2*csc(d*x+c)^3+630*cot(d*x+c)*cs
c(d*x+c)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 955, normalized size of antiderivative = 2.47

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/491520*(245445*sqrt(2)*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c
)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(
(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*si
n(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1))*sin(d*x + c) + 245760*(cos(d*x + c)^6 + 2*cos(d*x + c)^
5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x
+ c) + a)/(cos(d*x + c) + 1))*sin(d*x + c) + 4*(151041*cos(d*x + c)^7 + 103
524*cos(d*x + c)^6 - 228999*cos(d*x + c)^5 - 181256*cos(d*x + c)^4 + 97611*
cos(d*x + c)^3 + 82340*cos(d*x + c)^2 + 315*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c)))/((a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a
^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2
*d*cos(d*x + c) + a^2*d)*sin(d*x + c)), -1/245760*(245445*sqrt(2)*(cos(d*x
+ c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(d*x + c
)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 245760*(c
os(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3 - cos(
d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*co
s(d*x + c) - a))*sin(d*x + c) + 2*(151041*cos(d*x + c)^7 + 103524*cos(d*x +
 c)^6 - 228999*cos(d*x + c)^5 - 181256*cos(d*x + c)^4 + 97611*cos(d*x + c)^
3 + 82340*cos(d*x + c)^2 + 315*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c)))/((a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x
+ c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x +
c) + a^2*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^6(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)**6/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 1.33 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.98

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx =$$

$$-\frac{5\left(2\left(4\left(\frac{6\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2\mathrm{sgn}(\cos(dx+c))}-\frac{65\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{1451\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{13}{a^2\mathrm{sgn}(}\right.}{}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] -1/245760*(5*(2*(4*(6*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^2*sgn(cos(d*x + c))
) - 65*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 1451*sqrt(
2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 13503*sqrt(2)/(a^2*sgn
(cos(d*x + c))))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) +
 256*sqrt(2)*(555*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^8 - 1950*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^6*a + 2780*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^4*a^2 - 1810*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^3 + 473*a^4)/(((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*sqrt(-a)*sgn(cos(d*x
+ c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(3/2), x)
```

3.195 $\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{6\sqrt{a+a\sec(c+dx)}}{a^3d} + \frac{2(a+a\sec(c+dx))^{3/2}}{3a^4d}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+2/3*(a+a*sec(d*x+c))^(
3/2)/a^4/d-6*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3965, 90, 65, 213}

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{2(a\sec(c+dx)+a)^{3/2}}{3a^4d} - \frac{6\sqrt{a\sec(c+dx)+a}}{a^3d}$$

```
[In] Int[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - (6*Sqrt[a + a*
Sec[c + d*x]])/(a^3*d) + (2*(a + a*Sec[c + d*x])^(3/2))/(3*a^4*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-a+ax)^2}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^4d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{3a^2}{\sqrt{a+ax}}+\frac{a^2}{x\sqrt{a+ax}}+a\sqrt{a+ax}\right)\,dx, x, \sec(c+dx)\right)}{a^4d}\\
&= -\frac{6\sqrt{a+a\sec(c+dx)}}{a^3d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^4d}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d}\\
&= -\frac{6\sqrt{a+a\sec(c+dx)}}{a^3d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^4d}\\
&\quad+\frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^3d}\\
&= -\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{6\sqrt{a+a\sec(c+dx)}}{a^3d}+\frac{2(a+a\sec(c+dx))^{3/2}}{3a^4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\left(-8-7\sec(c+dx)+\sec^2(c+dx)-3\text{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{3a^2d\sqrt{a(1+\sec(c+dx))}}$$

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
[Out] (2*(-8 - 7*Sec[c + d*x] + Sec[c + d*x]^2 - 3*ArcTanh[Sqrt[1 + Sec[c + d*x]]
]*Sqrt[1 + Sec[c + d*x]]))/(3*a^2*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(3\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-8+\sec(dx+c)\right)}{3d\,a^3}$	72

```
[In] int(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/3/d/a^3*(a*(1+sec(d*x+c)))^(1/2)*(3*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-8+sec(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.09

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{3\sqrt{a}\cos(dx+c)\log\left(-8\,a\cos(dx+c)^2+4\left(2\cos(dx+c)^2+\cos(dx+c)\right)\right.}{6\,a^3d\,c}\right.$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] [1/6*(3*sqrt(a)*cos(d*x + c)*log(-8*a*cos(d*x + c)^2 + 4*(2*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*cos(d
*x + c) - a) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(8*cos(d*x + c) -
1))/(a^3*d*cos(d*x + c)), 1/3*(3*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + a))*cos(d*x + c)
- 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(8*cos(d*x + c) - 1))/(a^3*d*co
s(d*x + c))]
```

Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(66) = 132$.

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.09

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}}{a^4} - \frac{18\sqrt{a+\frac{a}{\cos(dx+c)}}}{a^3} + \frac{2\left(4a+\frac{3a}{\cos(dx+c)}\right)}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a^2} - \frac{}{\sqrt{a}}}{3d}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) +
 sqrt(a)))/a^(5/2) + 2*(a + a/cos(d*x + c))^(3/2)/a^4 - 18*sqrt(a + a/cos(d
*x + c))/a^3 + 2*(4*a + 3*a/cos(d*x + c))/((a + a/cos(d*x + c))^(3/2)*a^2)
- 6/(sqrt(a + a/cos(d*x + c))*a^2) - 2/((a + a/cos(d*x + c))^(3/2)*a))/d
```

Giac [A] (verification not implemented)

none

Time = 2.79 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.62

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\left(\frac{3\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))} - \frac{\sqrt{2}\left(9a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7a\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}a^2\mathrm{sgn}(\cos}\right.}{3d}$$

```
[In] integrate(tan(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sq
rt(-a)*a^2*sgn(cos(d*x + c))) - sqrt(2)*(9*a*tan(1/2*d*x + 1/2*c)^2 - 7*a)/
((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^2*sgn
(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+d\,x)^5}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^5/(a + a/cos(c + d*x))^(5/2), x)
```

3.196 $\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{4}{a^2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-4/a^2/d/(a+a*sec(d*x+c)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3965, 79, 65, 213}

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d}-\frac{4}{a^2d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - 4/(a^2*d*Sqrt[a
 + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-a+ax}{x(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{a^2d}\\
&= -\frac{4}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d}\\
&= -\frac{4}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^3d}\\
&= \frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{4}{a^2d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\left(-2+\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)\sqrt{1+\sec(c+dx)}\right)}{a^2d\sqrt{a(1+\sec(c+dx))}}$$

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(-2 + ArcTanh[Sqrt[1 + Sec[c + d*x]]]*Sqrt[1 + Sec[c + d*x]]))/(a^2*d*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

method	result	size
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\,\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\right)}{d\,a^3}$	87

```
[In] int(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(a*(1+sec(d*x+c)))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(2*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)-arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(46) = 92$.

Time = 0.31 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{\sqrt{a}(\cos(dx+c)+1)\log\left(-8\,a\cos(dx+c)^2-4\left(2\cos(dx+c)^2+\cos(dx+\cdots\right.\right.}{2\left(a^3d\cos(dx\cdots\right.}\right.$$

$$\left.-\frac{\sqrt{-a}(\cos(dx+c)+1)\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+a}\right)+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{a^3d\cos(dx+c)+a^3d}\right]$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*(cos(d*x + c) + 1)*log(-8*a*cos(d*x + c)^2 - 4*(2*cos(d*x + c
)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)) - 8*a*c
```

```
os(d*x + c) - a) - 8*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/
(a^3*d*cos(d*x + c) + a^3*d), -(sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
 a)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c))/(a^3*d*cos(d
*x + c) + a^3*d)]
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
[Out] Integral(tan(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(46) = 92$.

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.31

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4a+\frac{3a}{\cos(dx+c)}\right)}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a^2}+\frac{6}{\sqrt{a+\frac{a}{\cos(dx+c)}}a^2}-\frac{2}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a}}{3d}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] -1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c))
+ sqrt(a)))/a^(5/2) + 2*(4*a + 3*a/cos(d*x + c))/((a + a/cos(d*x + c))^(3/2
)*a^2) + 6/(sqrt(a + a/cos(d*x + c))*a^2) - 2/((a + a/cos(d*x + c))^(3/2)*a
))/d
```

Giac [A] (verification not implemented)

none

Time = 1.71 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}(\cos(dx+c))}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)}{ad}$$

```
[In] integrate(tan(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2*(arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(
-a)*a*sgn(cos(d*x + c))) + sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^2
*sgn(cos(d*x + c))))/(a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{\tan(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^3/(a + a/cos(c + d*x))^(5/2), x)
```

3.197 $\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2}{a^2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+2/3/a/d/(a+a*sec(d*x+c
))^(3/2)+2/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3965, 53, 65, 213}

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{2}{a^2d\sqrt{a\sec(c+dx)+a}} + \frac{2}{3ad(a\sec(c+dx)+a)^{3/2}}$$

```
[In] Int[Tan[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + 2/(3*a*d*(a +
a*Sec[c + d*x])^(3/2)) + 2/(a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{d} \\
&= \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{ad} \\
&= \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2}{a^2d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d} \\
&= \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2}{a^2d\sqrt{a+a\sec(c+dx)}} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^3d} \\
&= -\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{2}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2}{a^2d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.51

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1+\sec(c+dx)\right)}{3ad(a(1+\sec(c+dx)))^{3/2}}$$

```
[In] Integrate[Tan[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + Sec[c + d*x]])/(3*a*d*(a*(1 + Sec[c
 + d*x]))^(3/2))
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^{2}\sqrt{a+a\sec(dx+c)}}+\frac{2}{3a(a+a\sec(dx+c))^{\frac{3}{2}}}}{d}$	62
default	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^{2}\sqrt{a+a\sec(dx+c)}}+\frac{2}{3a(a+a\sec(dx+c))^{\frac{3}{2}}}}{d}$	62

```
[In] int(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/d*(-2/a^(5/2)*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))+2/a^2/(a+a*sec(d*x+
c))^(1/2)+2/3/a/(a+a*sec(d*x+c))^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(66) = 132$.

Time = 0.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 4.12

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{3\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-8\,a\cos\left(dx+c\right)^{2}+4\,(2\,\cos\right.}{6}\right.$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] [1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)
^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
```

```
cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(4*cos(d*x + c)^2 + 3*cos(d*x + c
))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d
*cos(d*x + c) + a^3*d), 1/3*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
2*a*cos(d*x + c) + a)) + 2*(4*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) +
 a^3*d)]
```

Sympy [F]

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{3\log\left(\frac{\sqrt{a+\frac{a}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{a}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4\,a+\frac{3\,a}{\cos(dx+c)}\right)}{\left(a+\frac{a}{\cos(dx+c)}\right)^{\frac{3}{2}}a^{2}}}{3\,d}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*log((sqrt(a + a/cos(d*x + c)) - sqrt(a))/(sqrt(a + a/cos(d*x + c)) +
 sqrt(a)))/a^(5/2) + 2*(4*a + 3*a/cos(d*x + c))/((a + a/cos(d*x + c))^(3/2)
*a^2))/d
```

Giac [A] (verification not implemented)

none

Time = 1.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{12\,\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}a^{2}\mathrm{sgn}(\cos(dx+c))}+\frac{\sqrt{2}\left(\left(-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{3}{2}}a^{8}+6\,\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)}{a^{12}\mathrm{sgn}(\cos(dx+c))}}{6\,d}$$

```
[In] integrate(tan(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(12*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(s
qrt(-a)*a^2*sgn(cos(d*x + c))) + sqrt(2)*((-a*tan(1/2*d*x + 1/2*c)^2 + a)^(
3/2)*a^8 + 6*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^9)/(a^12*sgn(cos(d*x + c
))))/d
```

Mupad [B] (verification not implemented)

Time = 14.76 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{\tan(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{2\left(a+\frac{a}{\cos(c+d\,x)}\right)}{a^2}+\frac{2}{3\,a}}{d\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{a^{5/2}\,d}$$

```
[In] int(tan(c + d*x)/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] ((2*(a + a/cos(c + d*x)))/a^2 + 2/(3*a))/(d*(a + a/cos(c + d*x))^(3/2)) - (
2*atanh((a + a/cos(c + d*x))^(1/2)/a^(1/2)))/(a^(5/2)*d)
```

3.198 $\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} - \frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-1/5/d/(a+a*sec(d*x+c))^
(5/2)-1/2/a/d/(a+a*sec(d*x+c))^(3/2)-1/8*arctanh(1/2*(a+a*sec(d*x+c))^(1/2)
*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)-7/4/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3965, 87, 157, 162, 65, 213}

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} - \frac{7}{4a^2d\sqrt{a\sec(c+dx)+a}} - \frac{1}{2ad(a\sec(c+dx)+a)^{3/2}} - \frac{1}{5d(a\sec(c+dx)+a)^{5/2}}$$

```
[In] Int[Cot[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - ArcTanh[Sqrt[a
+ a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(4*Sqrt[2]*a^(5/2)*d) - 1/(5*d*(a + a*
Sec[c + d*x])^(5/2)) - 1/(2*a*d*(a + a*Sec[c + d*x])^(3/2)) - 7/(4*a^2*d*Sq
rt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{a^2\text{Subst}\left(\int \frac{1}{x(-a+ax)(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} + \frac{\text{Subst}\left(\int \frac{2a^2-a^2x}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{2ad}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{-6a^4+\frac{9a^4x}{2}}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{6a^4d}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{6a^6-\frac{21a^6x}{4}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{6a^7d}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{8ad}$$

$$= -\frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^3d} + \frac{\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{4a^2d}$$

$$= \frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} - \frac{1}{5d(a+a\sec(c+dx))^{5/2}} - \frac{1}{2ad(a+a\sec(c+dx))^{3/2}} - \frac{7}{4a^2d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1+\sec(c+dx))\right)-2\,\text{Hypergeometric2F1}}{5d(a(1+\sec(c+dx)))^{5/2}}$$

```
[In] Integrate[Cot[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2
F1[-5/2, 1, -3/2, 1 + Sec[c + d*x]])/(5*d*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(115) = 230$.

Time = 1.73 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.62

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(-15\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{7}{2}}+15(1-\cos(dx+c))^6\right.}{}$

```
[In] int(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/840/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))
^2*csc(d*x+c)^2-1)^(1/2)*(-15*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(7/2)+15*(1
-cos(d*x+c))^6*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*x+c)^6+21*((1-
cos(d*x+c))^2*csc(d*x+c)^2-1)^(5/2)-87*(1-cos(d*x+c))^4*((1-cos(d*x+c))^2*c
sc(d*x+c)^2-1)^(1/2)*csc(d*x+c)^4-35*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2
)+269*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*x+c)^2
+840*2^(1/2)*arctan(1/2*2^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))+10
5*arctan(1/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))-932*((1-cos(d*x+c))^2*c
sc(d*x+c)^2-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(115) = 230$.

Time = 0.36 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.98

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{2\sqrt{2}\sqrt{a}\sqrt{}}{}\right.}{}\right.$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/80*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 40*(cos(d*x + c)^3 +
 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)^2 - 4
*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c)) - 8*a*cos(d*x + c) - a) - 4*(49*cos(d*x + c)^3 + 80*cos(d*x + c)^2
+ 35*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/40*(5*sq
rt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arc
tan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
a*cos(d*x + c) + a)) - 40*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x +
c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)/(2*a*cos(d*x + c) + a)) - 2*(49*cos(d*x + c)^3 + 80*cos(d*x +
c)^2 + 35*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos
(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cot(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot\left(dx+c\right)}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.93 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.43

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{5\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\operatorname{sgn}(\cos(dx+c))} - \frac{80\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\operatorname{sgn}(\cos(dx+c))} - \frac{\sqrt{2}\left(\left(a\tan\left(\frac{1}{2}dx \cdots\right.\right.}{\cdots}}{\cdots}$$

```
[In] integrate(cot(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/40*(5*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(
-a)*a^2*sgn(cos(d*x + c))) - 80*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(cos(d*x + c))) - sqrt(2)*((a*tan(1/
2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^20 + 5*(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^21 + 35*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a)*a^22)/(a^25*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

```
[In] int(cot(c + d*x)/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)/(a + a/cos(c + d*x))^(5/2), x)
```

3.199 $\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 200

$$\begin{aligned}\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = & -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} \\ & + \frac{13\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d} - \frac{5a}{28d(a+a\sec(c+dx))^{7/2}} \\ & + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} + \frac{3}{40d(a+a\sec(c+dx))^{5/2}} \\ & + \frac{19}{48ad(a+a\sec(c+dx))^{3/2}} + \frac{51}{32a^2d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

```
[Out] -2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-5/28*a/d/(a+a*sec(d*x+
c))^(7/2)+1/2*a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(7/2)+3/40/d/(a+a*sec(d*x
+c))^(5/2)+19/48/a/d/(a+a*sec(d*x+c))^(3/2)+13/64*arctanh(1/2*(a+a*sec(d*x+
c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)+51/32/a^2/d/(a+a*sec(d*x+c))^(
1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

$= \{3965, 105, 157, 162, 65, 213\}$

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{13\text{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d} + \frac{51}{32a^2d\sqrt{a\sec(c+dx)+a}} - \frac{5a}{28d(a\sec(c+dx)+a)^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a\sec(c+dx)+a)^{7/2}} + \frac{3}{40d(a\sec(c+dx)+a)^{5/2}} + \frac{19}{48ad(a\sec(c+dx)+a)^{3/2}}$$

```
[In] Int[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) + (13*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*a^(5/2)*d) - (5*a)/(
28*d*(a + a*Sec[c + d*x])^(7/2)) + a/(2*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(7/2)) + 3/(40*d*(a + a*Sec[c + d*x])^(5/2)) + 19/(48*a*d*(a + a*Sec
[c + d*x])^(3/2)) + 51/(32*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a^4\text{Subst}\left(\int \frac{1}{x(-a+ax)^2(a+ax)^{9/2}}\,dx, x, \sec(c+dx)\right)}{d}\\
&= \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} - \frac{a\text{Subst}\left(\int \frac{2a^2+\frac{9a^2x}{2}}{x(-a+ax)(a+ax)^{9/2}}\,dx, x, \sec(c+dx)\right)}{2d}\\
&= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{-14a^4-\frac{35a^4x}{4}}{x(-a+ax)(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{14a^2d}\\
&= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}}\\
&\quad + \frac{3}{40d(a+a\sec(c+dx))^{5/2}} - \frac{\text{Subst}\left(\int \frac{70a^6-\frac{105a^6x}{8}}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{70a^5d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} \\
&\quad + \frac{3}{40d(a+a\sec(c+dx))^{5/2}} + \frac{19}{48ad(a+a\sec(c+dx))^{3/2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-210a^8+\frac{1995a^8x}{16}}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{210a^8d} \\
&= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} \\
&\quad + \frac{3}{40d(a+a\sec(c+dx))^{5/2}} + \frac{19}{48ad(a+a\sec(c+dx))^{3/2}} \\
&\quad + \frac{51}{32a^2d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{210a^{10}-\frac{5355a^{10}x}{32}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{210a^{11}d} \\
&= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} \\
&\quad + \frac{3}{40d(a+a\sec(c+dx))^{5/2}} + \frac{19}{48ad(a+a\sec(c+dx))^{3/2}} \\
&\quad + \frac{51}{32a^2d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d} \\
&\quad - \frac{13\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{64ad} \\
&= -\frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} \\
&\quad + \frac{3}{40d(a+a\sec(c+dx))^{5/2}} + \frac{19}{48ad(a+a\sec(c+dx))^{3/2}} \\
&\quad + \frac{51}{32a^2d\sqrt{a+a\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^3d} \\
&\quad - \frac{13\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{32a^2d} \\
&= -\frac{2\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{13\text{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d} \\
&\quad - \frac{5a}{28d(a+a\sec(c+dx))^{7/2}} + \frac{a}{2d(1-\sec(c+dx))(a+a\sec(c+dx))^{7/2}} \\
&\quad + \frac{3}{40d(a+a\sec(c+dx))^{5/2}} + \frac{19}{48ad(a+a\sec(c+dx))^{3/2}} \\
&\quad + \frac{51}{32a^2d\sqrt{a+a\sec(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{a\left(-14-13\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c-\right.}{28d(-1+\sec(c+dx)}$$

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a*(-14 - 13*Hypergeometric2F1[-7/2, 1, -5/2, (1 + Sec[c + d*x])/2]*(-1 + S
ec[c + d*x]) + 8*Hypergeometric2F1[-7/2, 1, -5/2, 1 + Sec[c + d*x]]*(-1 + S
ec[c + d*x])))/(28*d*(-1 + Sec[c + d*x])*(a*(1 + Sec[c + d*x]))^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(163) = 326.

Time = 1.76 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.64

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(1365\cos(dx+c)^3\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+13440\cos(dx+c)^3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\sqrt{\right.\right.}{}$

```
[In] int(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6720/d/a^3*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^3*(1365*cos(d*x+c)^3*2
^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))+13440*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+4095*cos(d*x+c)^2*arctan(1/2*2^
(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)+40320*cos(d*x+c)^2*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)+4095*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4032
0*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)+1365*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*
2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-16034*cos(d*x+c)^3*cot(d*x+c)^2
+13440*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)-25280*cos(d*x+c)^2*cot(d*x+c)^2+3164*cos(d*x+c)*cot(d*x+c)^2+2408
0*cot(d*x+c)^2+10710*cot(d*x+c)*csc(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(161) = 322$.

Time = 0.38 (sec) , antiderivative size = 748, normalized size of antiderivative = 3.74

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{1365\,\sqrt{2}\left(\cos(dx+c)^5 + 3\,\cos(dx+c)^4 + 2\,\cos(dx+c)^3 - 2\,\cos(dx+c)^2\right.}{}\right.$$

$$-\frac{1365\,\sqrt{2}\left(\cos(dx+c)^5 + 3\,\cos(dx+c)^4 + 2\,\cos(dx+c)^3 - 2\,\cos(dx+c)^2 - 3\,\cos(dx+c) - 1\right)\sqrt{-a}\,\mathrm{ar}}{}$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/13440*(1365*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^
3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(a)*log((2*sqrt(2)*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c) + 3*a*cos(d*x + c) + a)
/(cos(d*x + c) - 1)) + 6720*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x
+ c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(a)*log(-8*a*cos(d*x +
c)^2 + 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c)) - 8*a*cos(d*x + c) - a) + 4*(8017*cos(d*x + c)^5 + 12640*co
s(d*x + c)^4 - 1582*cos(d*x + c)^3 - 12040*cos(d*x + c)^2 - 5355*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^5 + 3*a^3*
d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*
d*cos(d*x + c) - a^3*d), -1/6720*(1365*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x
+ c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)
*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(a*cos(d*x + c) + a)) - 6720*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(
d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)*arctan(2*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c)
+ a)) - 2*(8017*cos(d*x + c)^5 + 12640*cos(d*x + c)^4 - 1582*cos(d*x + c)^3
 - 12040*cos(d*x + c)^2 - 5355*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c)))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x
 + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d*cos(d*x + c) - a^3*d)]
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
[Out] Integral(cot(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^3/(a*sec(d*x + c) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 1.08 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.48

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\frac{1365\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))} - \frac{13440\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))} + \frac{105\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{a^3\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2} + \frac{2\sqrt{2}\left(15\left(a \right.\right.}{}}{}$$

```
[In] integrate(cot(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] -1/6720*(1365*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/
(sqrt(-a)*a^2*sgn(cos(d*x + c))) - 13440*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(cos(d*x + c))) + 105*sqrt(
2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/(a^3*sgn(cos(d*x + c))*tan(1/2*d*x +
 1/2*c)^2) + 2*sqrt(2)*(15*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a)*a^36 - 84*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)*a^37 - 385*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a
^38 - 2730*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^39)/(a^42*sgn(cos(d*x + c)
)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)^3}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^3/(a + a/cos(c + d*x))^(5/2), x)
```

3.200 $\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 262

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} \\ &- \frac{263\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}a^{5/2}d} + \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} \\ &- \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} \\ &- \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{9/2}} \\ &+ \frac{135a}{448d(a+a\sec(c+dx))^{7/2}} + \frac{7}{640d(a+a\sec(c+dx))^{5/2}} \\ &- \frac{83}{256ad(a+a\sec(c+dx))^{3/2}} - \frac{761}{512a^2d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

```
[Out] 2*arctanh((a+a*sec(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+199/288*a^2/d/(a+a*sec(
d*x+c))^(9/2)-1/4*a^2/d/(1-sec(d*x+c))^2/(a+a*sec(d*x+c))^(9/2)-21/16*a^2/d
/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(9/2)+135/448*a/d/(a+a*sec(d*x+c))^(7/2)+7
/640/d/(a+a*sec(d*x+c))^(5/2)-83/256/a/d/(a+a*sec(d*x+c))^(3/2)-263/1024*ar
ctanh(1/2*(a+a*sec(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)-761/512
/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3965, 105, 156, 157, 162, 65, 213}

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{263\operatorname{arctanh}\left(\frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}a^{5/2}d} + \frac{199a^2}{288d(a\sec(c+dx)+a)^{9/2}} - \frac{21a^2}{16d(1-\sec(c+dx))(a\sec(c+dx)+a)^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{9/2}} - \frac{761}{512a^2d\sqrt{a\sec(c+dx)+a}} + \frac{135a}{448d(a\sec(c+dx)+a)^{7/2}} + \frac{7}{640d(a\sec(c+dx)+a)^{5/2}} - \frac{83}{256ad(a\sec(c+dx)+a)^{3/2}}$$

```
[In] Int[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + a*Sec[c + d*x]]/Sqrt[a]])/(a^(5/2)*d) - (263*ArcTanh[Sq
rt[a + a*Sec[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(512*Sqrt[2]*a^(5/2)*d) + (199*a
^2)/(288*d*(a + a*Sec[c + d*x])^(9/2)) - a^2/(4*d*(1 - Sec[c + d*x])^2*(a +
 a*Sec[c + d*x])^(9/2)) - (21*a^2)/(16*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
d*x])^(9/2)) + (135*a)/(448*d*(a + a*Sec[c + d*x])^(7/2)) + 7/(640*d*(a + a
*Sec[c + d*x])^(5/2)) - 83/(256*a*d*(a + a*Sec[c + d*x])^(3/2)) - 761/(512*
a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a^6 \text{Subst}\left(\int \frac{1}{x(-a+ax)^3(a+ax)^{11/2}}\,dx, x, \sec(c+dx)\right)}{d} \\
&= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} \\
&\quad - \frac{a^3 \text{Subst}\left(\int \frac{4a^2+\frac{13a^2x}{2}}{x(-a+ax)^2(a+ax)^{11/2}}\,dx, x, \sec(c+dx)\right)}{4d} \\
&= -\frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} \\
&\quad - \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{9/2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{8a^4+\frac{231a^4x}{4}}{x(-a+ax)(a+ax)^{11/2}}\,dx, x, \sec(c+dx)\right)}{8d} \\
&= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} \\
&\quad - \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{9/2}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{-72a^6-\frac{1791a^6x}{8}}{x(-a+ax)(a+ax)^{9/2}}\,dx, x, \sec(c+dx)\right)}{72a^3d} \\
&= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} \\
&\quad - \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{9/2}} + \frac{135a}{448d(a+a\sec(c+dx))^{7/2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{504a^8+\frac{8505a^8x}{16}}{x(-a+ax)(a+ax)^{7/2}}\,dx, x, \sec(c+dx)\right)}{504a^6d} \\
&= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} \\
&\quad - \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{9/2}} + \frac{135a}{448d(a+a\sec(c+dx))^{7/2}} \\
&\quad + \frac{7}{640d(a+a\sec(c+dx))^{5/2}} - \frac{\text{Subst}\left(\int \frac{-2520a^{10}-\frac{2205a^{10}x}{32}}{x(-a+ax)(a+ax)^{5/2}}\,dx, x, \sec(c+dx)\right)}{2520a^9d}
\end{aligned}
$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{9/2}} + \frac{135a}{448d(a+a\sec(c+dx))^{7/2}} + \frac{7}{640d(a+a\sec(c+dx))^{5/2}} - \frac{83}{256ad(a+a\sec(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{7560a^{12}-\frac{235305a^{12}x}{64}}{x(-a+ax)(a+ax)^{3/2}}\,dx, x, \sec(c+dx)\right)}{7560a^{12}d}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{9/2}} + \frac{135a}{448d(a+a\sec(c+dx))^{7/2}} + \frac{7}{640d(a+a\sec(c+dx))^{5/2}} - \frac{83}{256ad(a+a\sec(c+dx))^{3/2}} - \frac{761}{512a^2d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{-7560a^{14}+\frac{719145a^{14}x}{128}}{x(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{7560a^{15}d}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{9/2}} + \frac{135a}{448d(a+a\sec(c+dx))^{7/2}} + \frac{7}{640d(a+a\sec(c+dx))^{5/2}} - \frac{83}{256ad(a+a\sec(c+dx))^{3/2}} - \frac{761}{512a^2d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{a^2d} + \frac{263\text{Subst}\left(\int \frac{1}{(-a+ax)\sqrt{a+ax}}\,dx, x, \sec(c+dx)\right)}{1024ad}$$

$$= \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{9/2}} + \frac{135a}{448d(a+a\sec(c+dx))^{7/2}} + \frac{7}{640d(a+a\sec(c+dx))^{5/2}} - \frac{83}{256ad(a+a\sec(c+dx))^{3/2}} - \frac{761}{512a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{a^3d} + \frac{263\text{Subst}\left(\int \frac{1}{-2a+x^2}\,dx, x, \sqrt{a+a\sec(c+dx)}\right)}{512a^2d}$$

$$= \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{263\operatorname{arctanh}\left(\frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}a^{5/2}d} + \frac{199a^2}{288d(a+a\sec(c+dx))^{9/2}} - \frac{a^2}{4d(1-\sec(c+dx))^2(a+a\sec(c+dx))^{9/2}} - \frac{21a^2}{16d(1-\sec(c+dx))(a+a\sec(c+dx))^{9/2}} + \frac{135a}{448d(a+a\sec(c+dx))^{7/2}} + \frac{7}{640d(a+a\sec(c+dx))^{5/2}} - \frac{83}{256ad(a+a\sec(c+dx))^{3/2}} - \frac{761}{512a^2d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.38

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\cot^4(c+dx)\left(-450+263\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},\frac{1}{2}(1+\sec(c+dx)\right)\right.}{}$$

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cot[c + d*x]^4*(-450 + 263*Hypergeometric2F1[-9/2, 1, -7/2, (1 + Sec[c + d
*x])/2]*(-1 + Sec[c + d*x])^2 - 64*Hypergeometric2F1[-9/2, 1, -7/2, 1 + Sec
[c + d*x]]*(-1 + Sec[c + d*x])^2 + 378*Sec[c + d*x]))/(288*d*(a*(1 + Sec[c
+ d*x]))^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(217) = 434.

Time = 1.68 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.15

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(82845\cos(dx+c)^3\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+645120\cos(dx+c)^3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{arc}\right.}{}$

```
[In] int(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/322560/d/a^3*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^3*(82845*cos(d*x+c)
^3*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)/(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2))+645120*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+248535*cos(d*x+c)^2*arctan
(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(
```

```
d*x+c)+1))^(1/2)+1935360*cos(d*x+c)^2*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+248535*arctan(1/2*2^(1/2)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)+1935360*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)+82845*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+764402*cos(d*x+c)
^3*cot(d*x+c)^4+645120*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)+1183040*cos(d*x+c)^2*cot(d*x+c)^4-807214*cos(d*x+
c)*cot(d*x+c)^4-2224080*cot(d*x+c)^4-378378*cot(d*x+c)^3*csc(d*x+c)+1063440
*cot(d*x+c)^2*csc(d*x+c)^2+479430*csc(d*x+c)^3*cot(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(213) = 426$.

Time = 0.39 (sec) , antiderivative size = 905, normalized size of antiderivative = 3.45

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/645120*(82845*sqrt(2)*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^
5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) +
 1)*sqrt(a)*log(-(2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c) - 3*a*cos(d*x + c) - a)/(cos(d*x + c) - 1)) + 322560*(cos(d*x
 + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x
+ c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log(-8*a*cos(d*x + c)
^2 - 4*(2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c)) - 8*a*cos(d*x + c) - a) - 4*(382201*cos(d*x + c)^7 + 591520*c
os(d*x + c)^6 - 403607*cos(d*x + c)^5 - 1112040*cos(d*x + c)^4 - 189189*cos
(d*x + c)^3 + 531720*cos(d*x + c)^2 + 239715*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c)))/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a
^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3
*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/322560*(82845*sqrt(2)*
(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*
cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(a*cos(d*x
 + c) + a)) - 322560*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 -
5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)/(2*a*cos(d*x + c) + a)) - 2*(382201*cos(d*x + c)^7 + 591520*cos(d*x +
c)^6 - 403607*cos(d*x + c)^5 - 1112040*cos(d*x + c)^4 - 189189*cos(d*x + c)
^3 + 531720*cos(d*x + c)^2 + 239715*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c)))/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(
d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*
x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
[Out] Integral(cot(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 1.27 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.39

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{82845\sqrt{2}\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))} - \frac{645120\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))} - \frac{315\left(3\right.}{}}{}$$

```
[In] integrate(cot(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] 1/322560*(82845*sqrt(2)*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a)
)/(sqrt(-a)*a^2*sgn(cos(d*x + c))) - 645120*arctan(1/2*sqrt(2)*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(cos(d*x + c))) - 315*(3
3*sqrt(2)*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 31*sqrt(2)*sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)*a)/(a^4*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^4) - 8
*sqrt(2)*(35*(a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)*a^56 - 225*(a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)*a^57 + 1008*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a)*a^58 + 4410*(-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)*a^59 +
31185*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^60)/(a^63*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)^5}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^5/(a + a/cos(c + d*x))^(5/2), x)
```

3.201 $\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\tan^3(c+dx)}{3ad(a+a\sec(c+dx))^{3/2}} + \frac{2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+2*tan(d*x+c)
/a^2/d/(a+a*sec(d*x+c))^(1/2)-2/3*tan(d*x+c)^3/a/d/(a+a*sec(d*x+c))^(3/2)+2
/5*tan(d*x+c)^5/d/(a+a*sec(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3972, 308, 209}

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d} + \frac{2\tan(c+dx)}{a^2d\sqrt{a\sec(c+dx)+a}} + \frac{2\tan^5(c+dx)}{5d(a\sec(c+dx)+a)^{5/2}} - \frac{2\tan^3(c+dx)}{3ad(a\sec(c+dx)+a)^{3/2}}$$

```
[In] Int[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(2*Tan[c + d*x])/(a^2*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Tan[c + d*x]^3)/(3*a
```

```
*d*(a + a*Sec[c + d*x])^(3/2)) + (2*Tan[c + d*x]^5)/(5*d*(a + a*Sec[c + d*x
])^(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(2a)\text{Subst}\left(\int \frac{x^6}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= -\frac{(2a)\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{x^2}{a^2}+\frac{x^4}{a}-\frac{1}{a^3(1+ax^2)}\right)\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}}-\frac{2\tan^3(c+dx)}{3ad(a+a\sec(c+dx))^{3/2}}\\
&\quad+\frac{2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}+\frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d}\\
&= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}}\\
&\quad-\frac{2\tan^3(c+dx)}{3ad(a+a\sec(c+dx))^{3/2}}+\frac{2\tan^5(c+dx)}{5d(a+a\sec(c+dx))^{5/2}}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 447 vs. $2(127) = 254$.

Time = 6.09 (sec) , antiderivative size = 447, normalized size of antiderivative = 3.52

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\cot^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{9/2}\sqrt{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(-1+\frac{2\tan^2\left(\frac{1}{2}(c+\right.}{1+\tan^2\left(\frac{1}{2}(c-\right.}\right.}{}$$

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[2]*Cot[(c + d*x)/2]^8*((1 + Sec[c + d*x])^(-1))^(9/2)*Sqrt[1 + Tan[(c
 + d*x)/2]^2]*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))^3*((Sq
rt[2]*ArcSin[(Sqrt[2]*Tan[(c + d*x)/2])/Sqrt[1 + Tan[(c + d*x)/2]^2]]*Tan[(
c + d*x)/2])/(Sqrt[1 + Tan[(c + d*x)/2]^2]*Sqrt[1 - (2*Tan[(c + d*x)/2]^2)/
(1 + Tan[(c + d*x)/2]^2)]) + (8*Tan[(c + d*x)/2]^6)/(5*(1 + Tan[(c + d*x)/2
]^2)^3*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))^3) + (4*Tan[(
c + d*x)/2]^4)/(3*(1 + Tan[(c + d*x)/2]^2)^2*(-1 + (2*Tan[(c + d*x)/2]^2)/(
1 + Tan[(c + d*x)/2]^2))^2) + (2*Tan[(c + d*x)/2]^2)/((1 + Tan[(c + d*x)/2]
^2)*(-1 + (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))))*Tan[c + d*x]^7
)/(d*(a*(1 + Sec[c + d*x]))^(5/2)*(1 - (2*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
 d*x)/2]^2))^(5/2))
```

Maple [A] (warning: unable to verify)

Time = 3.70 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\left(15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{5}{2}}-74(1-\cos(dx+c))^5\csc(dx+c)^5+80(1-\cos\right.}{15d\,a^3(\csc(dx+c)-\cot(dx+c)+1)^2(-\cot(dx+c)+\csc(}$

```
[In] int(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/d/a^3*(15*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(
1/2)*(-cot(d*x+c)+csc(d*x+c)))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(5/2)-74*(
1-cos(d*x+c))^5*csc(d*x+c)^5+80*(1-cos(d*x+c))^3*csc(d*x+c)^3-30*csc(d*x+c)
+30*cot(d*x+c))*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)/(csc(d*x+c)-
cot(d*x+c)+1)^2/(-cot(d*x+c)+csc(d*x+c)-1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.54

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{15\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\mathrm{co}}{\cos(dx+c}\right.}{15\left(a^3d\,\mathrm{c}\right.}\right.$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/15*(15*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*log((2*a*cos(d*x + c)
^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(23*cos(d*x + c)^2 - 1
1*cos(d*x + c) + 3)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(
a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2), 2/15*(15*(cos(d*x + c)^3 + co
s(d*x + c)^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)/(sqrt(a)*sin(d*x + c))) + (23*cos(d*x + c)^2 - 11*cos(d*x + c) + 3)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + a^3*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan^6\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)**6/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**6/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan\left(dx+c\right)^6}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/30*(20*(9*sin(4*d*x + 4*c) + 16*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*(45*cos(4*d*x + 4*c) + 80*cos(2*d*x
+ 2*c) + 23)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 15*
```

((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2
((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - 1) + 2*(a^3*d*cos(2*d*x + 2*c)^2 + a^3*d*sin(2*d*x + 2*c)^2 + 2*a^3*
d*cos(2*d*x + 2*c) + a^3*d)*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*
c)^4 + 2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)*sin(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + 6*(c
os(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
)^2 + cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(cos(2*d*x + 2*c)^3 + cos(2
*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*c
os(10*d*x + 10*c) + 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(cos
(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^
2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x
+ 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*
d*x + 4*c) + cos(2*d*x + 2*c)^2 + (sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + 6*(sin(2*
d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*
c))*sin(12*d*x + 12*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 20*(sin(2*d*x +
 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*s
in(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(sin(2*d*x + 2*c)^3 +
(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x +
 4*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - ((sin(2*d*x +
 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*c
os(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(sin(2*d*x + 2*c)^
3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(10*
d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)^3 + (co
s(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(6*d*x + 6*
c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
sin(2*d*x + 2*c))*cos(4*d*x + 4*c) - (cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)
*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(14*d*x +
 14*c) - 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(12*d*x + 12*c) - 15*(cos(2*d*x + 2

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*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*
d*x + 2*c))*sin(10*d*x + 10*c) - 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(8*d*x + 8
*c) - 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(6*d*x + 6*c) - 6*(cos(2*d*x + 2*c)^3
 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x +
 2*c))*sin(4*d*x + 4*c))*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (((sin(2*d*x
 + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))
*cos(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(sin(2*d*x + 2*c
)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(1
0*d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)^3 + (
cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(6*d*x +
6*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*sin(2*d*x + 2*c))*cos(4*d*x + 4*c) - (cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*
c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(14*d*x
 + 14*c) - 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(12*d*x + 12*c) - 15*(cos(2*d*x +
 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(
2*d*x + 2*c))*sin(10*d*x + 10*c) - 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c
)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(8*d*x +
 8*c) - 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(6*d*x + 6*c) - 6*(cos(2*d*x + 2*c)
^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x
 + 2*c))*sin(4*d*x + 4*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + 2*cos(2*d*x + 2*c)^3 + (2
*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + (cos(2*d
*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 +
cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x +
2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(12*d
*x + 12*c) + 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 20*(cos(2*d*
x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + c
os(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*
c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x
+ 6*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2
+ (sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2
*d*x + 2*c))*sin(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 15*(sin
(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x +
 2*c))*sin(10*d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 15*(sin(2*d*x
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+ 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(7/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))/(a^3*cos(14*d*x + 14*c)^2 + 36*a^3*cos(12*d*x + 12*c)^2 + 225*a
^3*cos(10*d*x + 10*c)^2 + 400*a^3*cos(8*d*x + 8*c)^2 + 225*a^3*cos(6*d*x +
6*c)^2 + 36*a^3*cos(4*d*x + 4*c)^2 + 12*a^3*cos(4*d*x + 4*c)*cos(2*d*x + 2*
c) + a^3*cos(2*d*x + 2*c)^2 + a^3*sin(14*d*x + 14*c)^2 + 36*a^3*sin(12*d*x
+ 12*c)^2 + 225*a^3*sin(10*d*x + 10*c)^2 + 400*a^3*sin(8*d*x + 8*c)^2 + 225
*a^3*sin(6*d*x + 6*c)^2 + 36*a^3*sin(4*d*x + 4*c)^2 + 12*a^3*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + a^3*sin(2*d*x + 2*c)^2 + 2*(6*a^3*cos(12*d*x + 12*c)
+ 15*a^3*cos(10*d*x + 10*c) + 20*a^3*cos(8*d*x + 8*c) + 15*a^3*cos(6*d*x +
6*c) + 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(14*d*x + 14*c) +
12*(15*a^3*cos(10*d*x + 10*c) + 20*a^3*cos(8*d*x + 8*c) + 15*a^3*cos(6*d*x
+ 6*c) + 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(12*d*x + 12*c)
+ 30*(20*a^3*cos(8*d*x + 8*c) + 15*a^3*cos(6*d*x + 6*c) + 6*a^3*cos(4*d*x +
 4*c) + a^3*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 40*(15*a^3*cos(6*d*x + 6
*c) + 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 30*
(6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(6*a^3
*sin(12*d*x + 12*c) + 15*a^3*sin(10*d*x + 10*c) + 20*a^3*sin(8*d*x + 8*c) +
 15*a^3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*x + 2*c))*s
in(14*d*x + 14*c) + 12*(15*a^3*sin(10*d*x + 10*c) + 20*a^3*sin(8*d*x + 8*c)
 + 15*a^3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*x + 2*c))
*sin(12*d*x + 12*c) + 30*(20*a^3*sin(8*d*x + 8*c) + 15*a^3*sin(6*d*x + 6*c)
 + 6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 40*(
15*a^3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*x + 2*c))*si
n(8*d*x + 8*c) + 30*(6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c)), x) + 18*(a^3*d*cos(2*d*x + 2*c)^2 + a^3*d*sin(2*d*x + 2*c)^2 +
2*a^3*d*cos(2*d*x + 2*c) + a^3*d)*integrate(-(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(2*d*x + 2*c)^4 + sin(2*d*
x + 2*c)^4 + 2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)*sin(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(14*d*x + 14*c)
+ 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c)^2 + cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(cos(2*d*x + 2*c)^3 +
 cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2
*c))*cos(10*d*x + 10*c) + 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 1
5*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(cos(2*d*x + 2*c)^3 + cos(
2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*
cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + (sin(2*d*x + 2*c)^3 + (cos(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + 6*(
sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*
x + 2*c))*sin(12*d*x + 12*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2
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 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 20*(sin(2
*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2
*c))*sin(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(sin(2*d*x + 2*c
)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4
*d*x + 4*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - ((sin(2
*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2
*c))*cos(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(sin(2*d*x +
 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*c
os(10*d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)^3
 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(6*d*
x + 6*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)*sin(2*d*x + 2*c))*cos(4*d*x + 4*c) - (cos(2*d*x + 2*c)^3 + cos(2*d*x
+ 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(14
*d*x + 14*c) - 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(12*d*x + 12*c) - 15*(cos(2*d
*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 +
cos(2*d*x + 2*c))*sin(10*d*x + 10*c) - 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x +
 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(8*d
*x + 8*c) - 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(6*d*x + 6*c) - 6*(cos(2*d*x +
2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2
*d*x + 2*c))*sin(4*d*x + 4*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (((sin
(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x +
 2*c))*cos(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(sin(2*d*x
 + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))
*cos(10*d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)
^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(6*
d*x + 6*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)*sin(2*d*x + 2*c))*cos(4*d*x + 4*c) - (cos(2*d*x + 2*c)^3 + cos(2*d*
x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(
14*d*x + 14*c) - 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(12*d*x + 12*c) - 15*(cos(2
*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2
+ cos(2*d*x + 2*c))*sin(10*d*x + 10*c) - 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x
 + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(8
*d*x + 8*c) - 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(6*d*x + 6*c) - 6*(cos(2*d*x
+ 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos
(2*d*x + 2*c))*sin(4*d*x + 4*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
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x + 2*c))) + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + 2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + (sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a^3*cos(14*d*x + 14*c)^2 + 36*a^3*cos(12*d*x + 12*c)^2 + 225*a^3*cos(10*d*x + 10*c)^2 + 400*a^3*cos(8*d*x + 8*c)^2 + 225*a^3*cos(6*d*x + 6*c)^2 + 36*a^3*cos(4*d*x + 4*c)^2 + 12*a^3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + a^3*cos(2*d*x + 2*c)^2 + a^3*sin(14*d*x + 14*c)^2 + 36*a^3*sin(12*d*x + 12*c)^2 + 225*a^3*sin(10*d*x + 10*c)^2 + 400*a^3*sin(8*d*x + 8*c)^2 + 225*a^3*sin(6*d*x + 6*c)^2 + 36*a^3*sin(4*d*x + 4*c)^2 + 12*a^3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a^3*sin(2*d*x + 2*c)^2 + 2*(6*a^3*cos(12*d*x + 12*c) + 15*a^3*cos(10*d*x + 10*c) + 20*a^3*cos(8*d*x + 8*c) + 15*a^3*cos(6*d*x + 6*c) + 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + 12*(15*a^3*cos(10*d*x + 10*c) + 20*a^3*cos(8*d*x + 8*c) + 15*a^3*cos(6*d*x + 6*c) + 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 30*(20*a^3*cos(8*d*x + 8*c) + 15*a^3*cos(6*d*x + 6*c) + 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 40*(15*a^3*cos(6*d*x + 6*c) + 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 30*(6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(6*a^3*sin(12*d*x + 12*c) + 15*a^3*sin(10*d*x + 10*c) + 20*a^3*sin(8*d*x + 8*c) + 15*a^3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + 12*(15*a^3*sin(10*d*x + 10*c) + 20*a^3*sin(8*d*x + 8*c) + 15*a^3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 30*(20*a^3*sin(8*d*x + 8*c) + 15*a^3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 40*(15*a^3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 30*(6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*x + 2*c))*s

```
in(6*d*x + 6*c)), x) - 10*(a^3*d*cos(2*d*x + 2*c)^2 + a^3*d*sin(2*d*x + 2*c
)^2 + 2*a^3*d*cos(2*d*x + 2*c) + a^3*d)*integrate(-(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(2*d*x + 2*c)^4 + si
n(2*d*x + 2*c)^4 + 2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(14*d*x +
14*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(cos(2*d*x + 2*
c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d
*x + 2*c))*cos(10*d*x + 10*c) + 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*
c) + 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(cos(2*d*x + 2*c)^3
+ cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x +
2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + (sin(2*d*x + 2*c)^3 + (cos(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(14*d*x + 14*c)
 + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*si
n(2*d*x + 2*c))*sin(12*d*x + 12*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 20*
(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d
*x + 2*c))*sin(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(sin(2*d*x
 + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))
*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (
(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d
*x + 2*c))*cos(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(sin(2
*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2
*c))*cos(10*d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(sin(2*d*x +
2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*co
s(6*d*x + 6*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)*sin(2*d*x + 2*c))*cos(4*d*x + 4*c) - (cos(2*d*x + 2*c)^3 + cos(
2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*
sin(14*d*x + 14*c) - 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(12*d*x + 12*c) - 15*(c
os(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
)^2 + cos(2*d*x + 2*c))*sin(10*d*x + 10*c) - 20*(cos(2*d*x + 2*c)^3 + cos(2
*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*s
in(8*d*x + 8*c) - 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(6*d*x + 6*c) - 6*(cos(2*
d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 +
 cos(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
(((sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2
```

*d*x + 2*c))*cos(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(4*d*x + 4*c) - (cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(14*d*x + 14*c) - 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(12*d*x + 12*c) - 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(10*d*x + 10*c) - 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(8*d*x + 8*c) - 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(6*d*x + 6*c) - 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + 2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + (sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(a^3*cos(14*d*x + 14*c)^2 + 36*a^3*cos(12*d*x + 12*c)^2 + 225*a^3*cos(10*d*x + 10*c)^2 + 400*a^3*cos(8*d*x + 8*c)^2 + 225*a^3*cos(6*d*x + 6*c)^2 + 36*a^3*cos(4*d*x + 4*c)^2 + 12*a^3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + a^3*cos(2*d*x + 2*c)^2 + a^3*sin(14*d*x + 14*c)^2 + 36*a^3*sin(12*d*x + 12*c)^2 + 225*a^3*sin(10*d*x + 10*c)^2 + 400*a^3*sin(8*d*x +

8*c)^2 + 225*a^3*sin(6*d*x + 6*c)^2 + 36*a^3*sin(4*d*x + 4*c)^2 + 12*a^3*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + a^3*sin(2*d*x + 2*c)^2 + 2*(6*a^3*cos(12*
d*x + 12*c) + 15*a^3*cos(10*d*x + 10*c) + 20*a^3*cos(8*d*x + 8*c) + 15*a^3*
cos(6*d*x + 6*c) + 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(14*d*
x + 14*c) + 12*(15*a^3*cos(10*d*x + 10*c) + 20*a^3*cos(8*d*x + 8*c) + 15*a^
3*cos(6*d*x + 6*c) + 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(12*
d*x + 12*c) + 30*(20*a^3*cos(8*d*x + 8*c) + 15*a^3*cos(6*d*x + 6*c) + 6*a^3
*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 40*(15*a^3*c
os(6*d*x + 6*c) + 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(8*d*x
+ 8*c) + 30*(6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(6*d*x + 6*c
) + 2*(6*a^3*sin(12*d*x + 12*c) + 15*a^3*sin(10*d*x + 10*c) + 20*a^3*sin(8*
d*x + 8*c) + 15*a^3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d
*x + 2*c))*sin(14*d*x + 14*c) + 12*(15*a^3*sin(10*d*x + 10*c) + 20*a^3*sin(
8*d*x + 8*c) + 15*a^3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*sin(2
*d*x + 2*c))*sin(12*d*x + 12*c) + 30*(20*a^3*sin(8*d*x + 8*c) + 15*a^3*sin(
6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*x + 2*c))*sin(10*d*x +
10*c) + 40*(15*a^3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*
x + 2*c))*sin(8*d*x + 8*c) + 30*(6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*x + 2
*c))*sin(6*d*x + 6*c)), x) - 10*(a^3*d*cos(2*d*x + 2*c)^2 + a^3*d*sin(2*d*x
 + 2*c)^2 + 2*a^3*d*cos(2*d*x + 2*c) + a^3*d)*integrate(-(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(2*d*x + 2*c)^
4 + sin(2*d*x + 2*c)^4 + 2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c)^3 + cos(2*d*x +
 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(14*
d*x + 14*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*(cos(2*d*
x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + c
os(2*d*x + 2*c))*cos(10*d*x + 10*c) + 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x +
2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*
x + 8*c) + 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(cos(2*d*x + 2
*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*
d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + (sin(2*d*x + 2*c)^3 + (
cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(14*d*x +
 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(10*d*x + 10*c)
 + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*s
in(2*d*x + 2*c))*sin(8*d*x + 8*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(sin
(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x +
 2*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - ((sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*s
in(2*d*x + 2*c))*cos(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 15*

```
(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d
*x + 2*c))*cos(10*d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(sin(2*
d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*
c))*cos(6*d*x + 6*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(4*d*x + 4*c) - (cos(2*d*x + 2*c)^3
+ cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x +
2*c))*sin(14*d*x + 14*c) - 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(12*d*x + 12*c) -
 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c)^2 + cos(2*d*x + 2*c))*sin(10*d*x + 10*c) - 20*(cos(2*d*x + 2*c)^3 +
 cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2
*c))*sin(8*d*x + 8*c) - 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(6*d*x + 6*c) - 6*(
cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c)^2 + cos(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - (((sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 1
5*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2
*d*x + 2*c))*cos(10*d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(sin(
2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x +
2*c))*cos(6*d*x + 6*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*cos(4*d*x + 4*c) - (cos(2*d*x + 2*c)^
3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x
+ 2*c))*sin(14*d*x + 14*c) - 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(12*d*x + 12*c)
 - 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(10*d*x + 10*c) - 20*(cos(2*d*x + 2*c)^3
 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x +
 2*c))*sin(8*d*x + 8*c) - 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*sin(6*d*x + 6*c) - 6
*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c)^2 + cos(2*d*x + 2*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + 2*cos(2
*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x +
 2*c)^2 + (cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(14*d*x + 14*c) + 6*(cos(2*d*x + 2*c
)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*
x + 2*c))*cos(12*d*x + 12*c) + 15*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(10*d*x + 10
*c) + 20*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 15*(cos(2*d*x + 2*c)^
```

```
3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x
+ 2*c))*cos(6*d*x + 6*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + c
os(2*d*x + 2*c)^2 + (sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) + 6*(sin(2*d*x + 2*c)^3 +
 (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(12*d*x
 + 12*c) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 20*(sin(2*d*x + 2*c)^3 + (cos
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c
) + 15*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)))/(a^3*cos(14*d*x + 14*c)^2 + 36*a^3*cos(12*d*x
 + 12*c)^2 + 225*a^3*cos(10*d*x + 10*c)^2 + 400*a^3*cos(8*d*x + 8*c)^2 + 22
5*a^3*cos(6*d*x + 6*c)^2 + 36*a^3*cos(4*d*x + 4*c)^2 + 12*a^3*cos(4*d*x + 4
*c)*cos(2*d*x + 2*c) + a^3*cos(2*d*x + 2*c)^2 + a^3*sin(14*d*x + 14*c)^2 +
36*a^3*sin(12*d*x + 12*c)^2 + 225*a^3*sin(10*d*x + 10*c)^2 + 400*a^3*sin(8*
d*x + 8*c)^2 + 225*a^3*sin(6*d*x + 6*c)^2 + 36*a^3*sin(4*d*x + 4*c)^2 + 12*
a^3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + a^3*sin(2*d*x + 2*c)^2 + 2*(6*a^3*c
os(12*d*x + 12*c) + 15*a^3*cos(10*d*x + 10*c) + 20*a^3*cos(8*d*x + 8*c) + 1
5*a^3*cos(6*d*x + 6*c) + 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos
(14*d*x + 14*c) + 12*(15*a^3*cos(10*d*x + 10*c) + 20*a^3*cos(8*d*x + 8*c) +
 15*a^3*cos(6*d*x + 6*c) + 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*c
os(12*d*x + 12*c) + 30*(20*a^3*cos(8*d*x + 8*c) + 15*a^3*cos(6*d*x + 6*c) +
 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 40*(15
*a^3*cos(6*d*x + 6*c) + 6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(
8*d*x + 8*c) + 30*(6*a^3*cos(4*d*x + 4*c) + a^3*cos(2*d*x + 2*c))*cos(6*d*x
 + 6*c) + 2*(6*a^3*sin(12*d*x + 12*c) + 15*a^3*sin(10*d*x + 10*c) + 20*a^3*
sin(8*d*x + 8*c) + 15*a^3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*s
in(2*d*x + 2*c))*sin(14*d*x + 14*c) + 12*(15*a^3*sin(10*d*x + 10*c) + 20*a^
3*sin(8*d*x + 8*c) + 15*a^3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3
*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 30*(20*a^3*sin(8*d*x + 8*c) + 15*a^
3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d*x + 2*c))*sin(10*
d*x + 10*c) + 40*(15*a^3*sin(6*d*x + 6*c) + 6*a^3*sin(4*d*x + 4*c) + a^3*si
n(2*d*x + 2*c))*sin(8*d*x + 8*c) + 30*(6*a^3*sin(4*d*x + 4*c) + a^3*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c)), x))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4))/((a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(2*d
*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*d)*(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(111) = 222$.

Time = 4.29 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.02

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{15\sqrt{-a}\left(\frac{\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^3\mathrm{sgn}(\cos(dx+c))} - \frac{\log\left(\left|\left(\sqrt{-a}\cdots\right.\right.\right.}{\cdots}\cdots\right.}{\cdots}$$

```
[In] integrate(tan(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/15*(15*sqrt(-a)*(log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(a^3*sgn(cos(d*x + c))) - log
(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2 + a*(2*sqrt(2) - 3)))/(a^3*sgn(cos(d*x + c)))) + 2*((37*sqrt(2)*tan(1/2*d
*x + 1/2*c)^2/sgn(cos(d*x + c)) - 40*sqrt(2)/sgn(cos(d*x + c)))*tan(1/2*d*x
 + 1/2*c)^2 + 15*sqrt(2)/sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/
2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(tan(c + d*x)^6/(a + a/cos(c + d*x))^(5/2), x)
```

3.202 $\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{4\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-4*arctan(1/2*
a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+2*tan(
d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3972, 490, 536, 209}

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d} - \frac{4\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d} + \frac{2\tan(c+dx)}{a^2d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
4*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(a^(5/2)*d) + (2*Tan[c + d*x])/(a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} \\
&= \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{2+3ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} \\
&= \frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} \\
&\quad + \frac{8\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d}
\end{aligned}
$$

$$
=\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{4\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}+\frac{2\tan(c+dx)}{a^2d\sqrt{a+a\sec(c+dx)}}
$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.66

$$
\int\frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\frac{8\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-4\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{1\cdots}\right.}{d\cdots}
$$

[In] Integrate[Tan[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]

[Out] (8*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*(-4*ArcSin[Tan[(c + d*x)/2]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] + Sqrt[2]*(ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] + Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])))/(d*Sqrt[Sec[(c + d*x)/2]^2]*(a*(1 + Sec[c + d*x]))^(5/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(96) = 192$.

Time = 3.73 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.81

method	result
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\left(-2\sqrt{2}\,\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos\cdots\right.}{\cdots}$

[In] int(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/d/a^3*(a*(1+sec(d*x+c)))^(1/2)*(-2*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-2*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+sin(d*x+c))/(cos(d*x+c)+1)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.66

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{2\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos\ldots}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)\ldots}{\ldots}\right.$$

$$-\frac{2\left(\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\frac{2\sqrt{2}(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}-\ldots\right)}{a^3d\cos(dx+c)+a^3d}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [(2*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x +
c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - sqrt(-a
)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos
(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(
a^3*d*cos(d*x + c) + a^3*d), -2*(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*sq
rt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a) - sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
[Out] Integral(tan(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^4/(a*sec(d*x + c) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 2.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.58

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a^2d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

```
[In] integrate(tan(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] -2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/((a*tan
(1/2*d*x + 1/2*c)^2 - a)*a^2*d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^4/(a + a/cos(c + d*x))^(5/2), x)
```

3.203 $\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{3\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2a^2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+3/2*arctan(1
/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+1/2
*sec(1/2*d*x+1/2*c)^2*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3972, 482, 536, 209}

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d} + \frac{3\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{2}a^{5/2}d} + \frac{\sin(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}{2a^2d\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt
```

```
[2]*a^(5/2)*d) + (Sec[(c + d*x)/2]^2*Sin[c + d*x])/(2*a^2*d*Sqrt[a + a*Sec[
c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} \\
&= \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2a^2d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1-ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} \\
&= \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2a^2d\sqrt{a+a\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} \\
&\quad - \frac{3\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d}
\end{aligned}
$$

$$= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{3\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2a^2d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.52

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(3\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{1+\cdots}\right.}{d\sqrt{\sec^2\cdots}}$$

[In] Integrate[Tan[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]

[Out] (4*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*(3*ArcSin[Tan[(c + d*x)/2]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] + Sqrt[2]*(-2*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] + (Sqrt[(1 + Cos[c + d*x])^(-1)]*Tan[(c + d*x)/2])/Sqrt[Sec[c + d*x]])))/(d*Sqrt[Sec[(c + d*x)/2]^2]*(a*(1 + Sec[c + d*x]))^(5/2))

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.57

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\,(-\cot(dx+c)+\csc(dx+c))\cdots\right.}{2d\,a^3}$

[In] int(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/2/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))+2*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))-3*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.87

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \Bigg[-\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(d\ldots}{\cos(\ldots}\right.}{\ldots}$$

$$-\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-4\left(\cos(dx+c)^2+2\cos(d\ldots\right.}{2\left(a^3d\cos(dx+c)^2+2a^3d\cos(\ldots\right.}$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*co
s(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^3*d*cos(d*x + c)^
2 + 2*a^3*d*cos(d*x + c) + a^3*d), -1/2*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 4*(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c))/(a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\tan^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{\tan\left(dx+c\right)^2}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 1.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.37

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\,a^3d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

```
[In] integrate(tan(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] 1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^3*d
*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{\tan(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(tan(c + d*x)^2/(a + a/cos(c + d*x))^(5/2), x)
```

3.204 $\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 265

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} \\
&+ \frac{319\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d} + \frac{63\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128a^3d} \\
&- \frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{384a^3d} \\
&- \frac{19\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{192a^3d} \\
&- \frac{\cos^3(c+dx)\cot(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{48a^3d}
\end{aligned}
$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+319/256*arct
an(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d
+63/128*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^3/d-191/384*cos(d*x+c)*cot(d*x+
c)*sec(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(1/2)/a^3/d-19/192*cos(d*x+c)^2*co
t(d*x+c)*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(1/2)/a^3/d-1/48*cos(d*x+c)^
3*cot(d*x+c)*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3972, 483, 593, 597, 536, 209}

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d} + \frac{319\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{128\sqrt{2}a^{5/2}d} + \frac{63\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{128a^3d} - \frac{\cos^3(c+dx)\cot(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{a\sec(c+dx)+a}}{48a^3d} - \frac{19\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a\sec(c+dx)+a}}{192a^3d} - \frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a\sec(c+dx)+a}}{384a^3d}$$

```
[In] Int[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(319*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(12
8*Sqrt[2]*a^(5/2)*d) + (63*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(128*a^3*
d) - (191*Cos[c + d*x]*Cot[c + d*x]*Sec[(c + d*x)/2]^2*Sqrt[a + a*Sec[c + d
*x]])/(384*a^3*d) - (19*Cos[c + d*x]^2*Cot[c + d*x]*Sec[(c + d*x)/2]^4*Sqrt
[a + a*Sec[c + d*x]])/(192*a^3*d) - (Cos[c + d*x]^3*Cot[c + d*x]*Sec[(c + d
*x)/2]^6*Sqrt[a + a*Sec[c + d*x]])/(48*a^3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^3d} \\
&= -\frac{\cos^3(c+dx)\cot(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{48a^3d} \\
&\quad -\frac{\text{Subst}\left(\int \frac{5a-7a^2x^2}{x^2(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{6a^4d}
\end{aligned}$$

$$= -\frac{19\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{192a^3d} - \frac{\cos^3(c+dx)\cot(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{48a^3d} - \frac{\text{Subst}\left(\int\frac{a^2-95a^3x^2}{x^2(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{48a^5d}$$

$$= -\frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{384a^3d} - \frac{19\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{192a^3d} - \frac{\cos^3(c+dx)\cot(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{48a^3d} - \frac{\text{Subst}\left(\int\frac{-189a^3-573a^4x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{192a^6d}$$

$$= \frac{63\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128a^3d} - \frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{384a^3d} - \frac{19\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{192a^3d} - \frac{\cos^3(c+dx)\cot(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{48a^3d} + \frac{\text{Subst}\left(\int\frac{579a^4-189a^5x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{384a^6d}$$

$$= \frac{63\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128a^3d} - \frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{384a^3d} - \frac{19\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{192a^3d} - \frac{\cos^3(c+dx)\cot(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{48a^3d} + \frac{2\text{Subst}\left(\int\frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} - \frac{319\text{Subst}\left(\int\frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128a^2d}$$

$$= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{319\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d}$$
$$+ \frac{63\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{128a^3d}$$
$$- \frac{191\cos(c+dx)\cot(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{384a^3d}$$
$$- \frac{19\cos^2(c+dx)\cot(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{192a^3d}$$
$$- \frac{\cos^3(c+dx)\cot(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{a+a\sec(c+dx)}}{48a^3d}$$

Mathematica [A] (warning: unable to verify)

Time = 5.56 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.88

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-((-58+487\cos(c+dx)+698\cos(2(c+dx)\right.}{}$$

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*Sec[c + d*x]^2*(-((-58 + 487*Cos[c + d*x] + 698*Cos[2*(
c + d*x)] + 409*Cos[3*(c + d*x)])*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]^7) - 49
152*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Sqrt[Sec[c + d*x
]]*Sqrt[Sec[c + d*x]/(1 + Sec[c + d*x])^2]*Sqrt[1 + Sec[c + d*x]] + 30624*A
rcSin[Tan[(c + d*x)/2]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*Sqrt[(1
 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]]))/(3072*d*(a*(1 + Sec[c + d*x
]))^(5/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(229) = 458.

Time = 1.88 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.22

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(-192\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{11}{2}}\sin(dx+c)+192(1-\cos\right.}{}$

```
[In] int(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3072/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))
^2*csc(d*x+c)^2-1)^(1/2)/(1-cos(d*x+c))*(-192*((1-cos(d*x+c))^2*csc(d*x+c)^
```

```
2-1)^(11/2)*sin(d*x+c)+192*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-
1)^(9/2)*csc(d*x+c)-216*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^
(7/2)*csc(d*x+c)+24*(1-cos(d*x+c))^8*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2
)*csc(d*x+c)^7+252*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(5/2)
*csc(d*x+c)-164*(1-cos(d*x+c))^6*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*cs
c(d*x+c)^5-315*(1-cos(d*x+c))^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)*csc
(d*x+c)+563*(1-cos(d*x+c))^4*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*
x+c)^3-3072*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)
*(-cot(d*x+c)+csc(d*x+c)))*(1-cos(d*x+c))-1179*(1-cos(d*x+c))^2*((1-cos(d*x
+c))^2*csc(d*x+c)^2-1)^(1/2)*csc(d*x+c)+3828*ln(csc(d*x+c)-cot(d*x+c)+((1-c
os(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))*(1-cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.61

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{957\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{957\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx\cdots}{\cdots}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/1536*(957*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 768*(cos(d*x + c)^3 + 3
*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3 - 4
*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d*x +
 c) + 4*(409*cos(d*x + c)^4 + 349*cos(d*x + c)^3 - 185*cos(d*x + c)^2 - 189
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c)),
 -1/768*(957*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 768*(cos(d*x + c)^3 + 3*cos(d*x
+ c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*
```

```
x + c) - a))*sin(d*x + c) + 2*(409*cos(d*x + c)^4 + 349*cos(d*x + c)^3 - 18
5*cos(d*x + c)^2 - 189*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
))/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
 a^3*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
[Out] Integral(cot(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 1.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.67

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(2\left(\frac{4\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{31\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\right.\right.}{}$$

```
[In] integrate(cot(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] 1/768*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*(4*sqrt(2)*tan(1/2*d*x + 1/2*
c)^2/(a^3*sgn(cos(d*x + c))) - 31*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*
d*x + 1/2*c)^2 + 291*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)
- 96*sqrt(2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 - a)*sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^2/(a + a/cos(c + d*x))^(5/2), x)
```

3.205 $\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 355

$$\begin{aligned}\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} \\ &- \frac{9683\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d} - \frac{1491\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4096a^3d} \\ &+ \frac{5587\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{6144a^4d} \\ &- \frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{2048a^4d} \\ &- \frac{145\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{1024a^4d} \\ &- \frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{256a^4d} \\ &- \frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^4d}\end{aligned}$$

```
[Out] 2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+5587/6144*cot
(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^4/d-1527/2048*cos(d*x+c)*cot(d*x+c)^3*se
c(1/2*d*x+1/2*c)^2*(a+a*sec(d*x+c))^(3/2)/a^4/d-145/1024*cos(d*x+c)^2*cot(d
*x+c)^3*sec(1/2*d*x+1/2*c)^4*(a+a*sec(d*x+c))^(3/2)/a^4/d-9/256*cos(d*x+c)^
3*cot(d*x+c)^3*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(3/2)/a^4/d-1/128*cos(
d*x+c)^4*cot(d*x+c)^3*sec(1/2*d*x+1/2*c)^8*(a+a*sec(d*x+c))^(3/2)/a^4/d-968
3/8192*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2
)/a^(5/2)/d-1491/4096*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3972, 483, 593, 597, 536, 209}

$$
\begin{aligned}
\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d} \\
&- \frac{9683\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{4096\sqrt{2}a^{5/2}d} + \frac{5587\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{6144a^4d} \\
&- \frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{128a^4d} \\
&- \frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{256a^4d} \\
&- \frac{145\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{1024a^4d} \\
&- \frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{3/2}}{2048a^4d} \\
&- \frac{1491\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{4096a^3d}
\end{aligned}
$$

```
[In] Int[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
9683*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(40
96*Sqrt[2]*a^(5/2)*d) - (1491*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(4096*
a^3*d) + (5587*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(6144*a^4*d) - (1
527*Cos[c + d*x]*Cot[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(3/
2))/(2048*a^4*d) - (145*Cos[c + d*x]^2*Cot[c + d*x]^3*Sec[(c + d*x)/2]^4*(a
 + a*Sec[c + d*x])^(3/2))/(1024*a^4*d) - (9*Cos[c + d*x]^3*Cot[c + d*x]^3*S
ec[(c + d*x)/2]^6*(a + a*Sec[c + d*x])^(3/2))/(256*a^4*d) - (Cos[c + d*x]^4
*Cot[c + d*x]^3*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(3/2))/(128*a^4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
```

```
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^5}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^4d}$$

$$= -\frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^4d} - \frac{\text{Subst}\left(\int \frac{5a-11a^2x^2}{x^4(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8a^5d}$$

$$= -\frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{256a^4d} - \frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^4d} - \frac{\text{Subst}\left(\int \frac{-51a^2-243a^3x^2}{x^4(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{96a^6d}$$

$$= -\frac{145\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{1024a^4d} - \frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{256a^4d} - \frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^4d} - \frac{\text{Subst}\left(\int \frac{-1509a^3-3045a^4x^2}{x^4(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{768a^7d}$$

$$= -\frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{2048a^4d} - \frac{145\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{1024a^4d} - \frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{256a^4d} - \frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^4d} - \frac{\text{Subst}\left(\int \frac{-16761a^4-22905a^5x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{3072a^8d}$$

$$= \frac{5587\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{6144a^4d} - \frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{2048a^4d} - \frac{145\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{1024a^4d} - \frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{256a^4d} - \frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^4d} + \frac{\text{Subst}\left(\int \frac{-13419a^5-50283a^6x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{18432a^8d}$$

$$= -\frac{1491\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4096a^3d} + \frac{5587\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{6144a^4d} - \frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{2048a^4d} - \frac{145\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{1024a^4d} - \frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{256a^4d} - \frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^4d} - \frac{\text{Subst}\left(\int \frac{60309a^6-13419a^7x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{36864a^8d}$$

$$= -\frac{1491\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4096a^3d} + \frac{5587\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{6144a^4d} - \frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{2048a^4d} - \frac{145\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{1024a^4d} - \frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{256a^4d} - \frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^4d} - \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} + \frac{9683\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4096a^2d}$$

$$= \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{9683\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d}$$
$$- \frac{1491\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{4096a^3d} + \frac{5587\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{6144a^4d}$$
$$- \frac{1527\cos(c+dx)\cot^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{2048a^4d}$$
$$- \frac{145\cos^2(c+dx)\cot^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{1024a^4d}$$
$$- \frac{9\cos^3(c+dx)\cot^3(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{256a^4d}$$
$$- \frac{\cos^4(c+dx)\cot^3(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{3/2}}{128a^4d}$$

Mathematica [A] (warning: unable to verify)

Time = 3.44 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.81

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\left(-29258+3200\csc^2\left(\frac{1}{2}(c+dx)\right)-128\csc^4\left(\right.\right.\right.}{}$$

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*Sec[c + d*x]^(5/2)*((-29258 + 3200*Csc[(c + d*x)/2]^2 -
 128*Csc[(c + d*x)/2]^4 + 18225*Sec[(c + d*x)/2]^2 - 4470*Sec[(c + d*x)/2]^
4 + 696*Sec[(c + d*x)/2]^6 - 48*Sec[(c + d*x)/2]^8)*Sqrt[Sec[c + d*x]] + 49
152*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Cot[(c + d*x)/2]
*Sqrt[Sec[c + d*x]/(1 + Sec[c + d*x])^2]*Sqrt[1 + Sec[c + d*x]] - 29049*Arc
Sin[Tan[(c + d*x)/2]]*Cot[(c + d*x)/2]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[(1 + S
ec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]])*Sin[(c + d*x)/2])/(3072*d*(a*(1
+ Sec[c + d*x]))^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(310) = 620$.

Time = 1.93 (sec) , antiderivative size = 710, normalized size of antiderivative = 2.00

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(29049\cos(dx+c)^3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\mathrm{cs}\right.\right.}{}$

```
[In] int(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24576/d/a^3*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^3*(29049*cos(d*x+c)^
3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(
d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))-49152*cos(d*x+c)^3*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))+87147*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+
c)^2-1)^(1/2))*cos(d*x+c)^2-147456*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcta
nh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)
^2+87147*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*
x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-1
47456*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+29049*ln(csc(d*x+c)-cot(d*x+
c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)+29258*cos(d*x+c)^3*cot(d*x+c)^3-49152*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))+28466*cot(d*x+c)^3*cos(d*x+c)^2-28116*cos(d*x+c)*c
ot(d*x+c)^3-34852*cot(d*x+c)^3+4490*cot(d*x+c)^2*csc(d*x+c)+8946*cot(d*x+c)
*csc(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 868, normalized size of antiderivative = 2.45

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/49152*(29049*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c
)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a
*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1))*sin(d*x + c) + 24576*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c
)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(-a)*log(-(8*a*cos(d*x + c
)^3 + 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*si
n(d*x + c) - 4*(14629*cos(d*x + c)^6 + 14233*cos(d*x + c)^5 - 14058*cos(d*x
 + c)^4 - 17426*cos(d*x + c)^3 + 2245*cos(d*x + c)^2 + 4473*cos(d*x + c))*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^5 + 3*a^3*d*co
s(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d*co
s(d*x + c) - a^3*d)*sin(d*x + c)), 1/24576*(29049*sqrt(2)*(cos(d*x + c)^5 +
 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) -
```

```
1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 24576*(cos(d*x + c)^5 + 3*cos(d*
x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*sqrt(a
)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*sin(d*x + c) + 2*(1462
9*cos(d*x + c)^6 + 14233*cos(d*x + c)^5 - 14058*cos(d*x + c)^4 - 17426*cos(
d*x + c)^3 + 2245*cos(d*x + c)^2 + 4473*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 2*a^3
*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d*cos(d*x + c) - a^3*d)*
sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{\cot^4\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
[Out] Integral(cot(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{\cot\left(dx+c\right)^4}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^4/(a*sec(d*x + c) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 1.39 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.83

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\frac{3\left(2\left(4\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{19\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{369\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c}}\right.\right.}{}$$

```
[In] integrate(cot(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*(3*(2*(4*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(cos(d*x + c)))
- 19*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 369*sqrt(2)/
(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 2989*sqrt(2)/(a^3*sgn(cos
(d*x + c))))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) + 512
*sqrt(2)*(12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^4 - 21*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*a + 11*a^2)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 - a)^3*sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{\cot(c+d\,x)^4}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^4/(a + a/cos(c + d*x))^(5/2), x)
```

3.206 $\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 439

$$
\begin{aligned}
\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = &-\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} \\
&+\frac{74461\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{32768\sqrt{2}a^{5/2}d}+\frac{8925\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32768a^3d} \\
&-\frac{41693\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{49152a^4d} \\
&+\frac{58077\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{40960a^5d} \\
&-\frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{8192a^5d} \\
&-\frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{12288a^5d} \\
&-\frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^5d} \\
&-\frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5d} \\
&-\frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5d}
\end{aligned}
$$

```
[Out] -2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-41693/49152*
cot(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)/a^4/d+58077/40960*cot(d*x+c)^5*(a+a*sec
(d*x+c))^(5/2)/a^5/d-9467/8192*cos(d*x+c)*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^2
*(a+a*sec(d*x+c))^(5/2)/a^5/d-2473/12288*cos(d*x+c)^2*cot(d*x+c)^5*sec(1/2*
d*x+1/2*c)^4*(a+a*sec(d*x+c))^(5/2)/a^5/d-155/3072*cos(d*x+c)^3*cot(d*x+c)^
5*sec(1/2*d*x+1/2*c)^6*(a+a*sec(d*x+c))^(5/2)/a^5/d-7/512*cos(d*x+c)^4*cot(
```

```
d*x+c)^5*sec(1/2*d*x+1/2*c)^8*(a+a*sec(d*x+c))^(5/2)/a^5/d-1/320*cos(d*x+c)
^5*cot(d*x+c)^5*sec(1/2*d*x+1/2*c)^10*(a+a*sec(d*x+c))^(5/2)/a^5/d+74461/65
536*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/a
^(5/2)/d+8925/32768*cot(d*x+c)*(a+a*sec(d*x+c))^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3972, 483, 593, 597, 536, 209}

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}\\ &+\frac{74461\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{32768\sqrt{2}a^{5/2}d}+\frac{58077\cot^5(c+dx)(a\sec(c+dx)+a)^{5/2}}{40960a^5d}\\ &-\frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{320a^5d}\\ &-\frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{512a^5d}\\ &-\frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{3072a^5d}\\ &-\frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{12288a^5d}\\ &-\frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a\sec(c+dx)+a)^{5/2}}{8192a^5d}\\ &-\frac{41693\cot^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{49152a^4d}\\ &+\frac{8925\cot(c+dx)\sqrt{a\sec(c+dx)+a}}{32768a^3d}\end{aligned}$$

```
[In] Int[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(74461*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
32768*Sqrt[2]*a^(5/2)*d) + (8925*Cot[c + d*x]*Sqrt[a + a*Sec[c + d*x]])/(32
768*a^3*d) - (41693*Cot[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2))/(49152*a^4*d
) + (58077*Cot[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2))/(40960*a^5*d) - (9467
*Cos[c + d*x]*Cot[c + d*x]^5*Sec[(c + d*x)/2]^2*(a + a*Sec[c + d*x])^(5/2))
/(8192*a^5*d) - (2473*Cos[c + d*x]^2*Cot[c + d*x]^5*Sec[(c + d*x)/2]^4*(a +
 a*Sec[c + d*x])^(5/2))/(12288*a^5*d) - (155*Cos[c + d*x]^3*Cot[c + d*x]^5*
Sec[(c + d*x)/2]^6*(a + a*Sec[c + d*x])^(5/2))/(3072*a^5*d) - (7*Cos[c + d*
x]^4*Cot[c + d*x]^5*Sec[(c + d*x)/2]^8*(a + a*Sec[c + d*x])^(5/2))/(512*a^5
```

```
*d) - (Cos[c + d*x]^5*Cot[c + d*x]^5*Sec[(c + d*x)/2]^10*(a + a*Sec[c + d*x
])^(5/2))/(320*a^5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)^6}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^5d}\\
&= -\frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5d}\\
&\quad-\frac{\text{Subst}\left(\int \frac{5a-15a^2x^2}{x^6(1+ax^2)(2+ax^2)^5}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{10a^6d}\\
&= -\frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^{8}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5d}\\
&\quad-\frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5d}\\
&\quad-\frac{\text{Subst}\left(\int \frac{-135a^2-455a^3x^2}{x^6(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{160a^7d}\\
&= -\frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^{6}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^5d}\\
&\quad-\frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^{8}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5d}\\
&\quad-\frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5d}\\
&\quad-\frac{\text{Subst}\left(\int \frac{-4685a^3-8525a^4x^2}{x^6(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{1920a^8d}\\
&= -\frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^{4}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{12288a^5d}\\
&\quad-\frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^{6}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^5d}\\
&\quad-\frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^{8}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5d}\\
&\quad-\frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5d}\\
&\quad-\frac{\text{Subst}\left(\int \frac{-80565a^4-111285a^5x^2}{x^6(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{15360a^9d}
\end{aligned}
$$

$$= -\frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{8192a^5d}$$
$$- \frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{12288a^5d}$$
$$- \frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^5d}$$
$$- \frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5d}$$
$$- \frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5d}$$
$$- \frac{\text{Subst}\left(\int \frac{-871155a^5-994035a^6x^2}{x^6(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{61440a^{10}d}$$

$$= \frac{58077\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{40960a^5d}$$
$$- \frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{8192a^5d}$$
$$- \frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{12288a^5d}$$
$$- \frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^5d}$$
$$- \frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5d}$$
$$- \frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5d}$$
$$+ \frac{\text{Subst}\left(\int \frac{-3126975a^6-4355775a^7x^2}{x^4(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{614400a^{10}d}$$

$$= -\frac{41693\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{49152a^4d} + \frac{58077\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{40960a^5d} - \frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{8192a^5d} - \frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{12288a^5d} - \frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^5d} - \frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5d} - \frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5d} - \frac{\text{Subst}\left(\int \frac{-2008125a^7-9380925a^8x^2}{x^2(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{3686400a^{10}d}$$

$$= \frac{8925\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32768a^3d} - \frac{41693\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{49152a^4d} + \frac{58077\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{40960a^5d} - \frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{8192a^5d} - \frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{12288a^5d} - \frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^5d} - \frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5d} - \frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5d} + \frac{\text{Subst}\left(\int \frac{12737475a^8-2008125a^9x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{7372800a^{10}d}$$

$$= \frac{8925\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32768a^3d} - \frac{41693\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{49152a^4d} + \frac{58077\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{40960a^5d} - \frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{8192a^5d} - \frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{12288a^5d} - \frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^5d} - \frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5d} - \frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5d} + \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2d} - \frac{74461\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{32768a^2d}$$

$$= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{74461\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{32768\sqrt{2}a^{5/2}d} + \frac{8925\cot(c+dx)\sqrt{a+a\sec(c+dx)}}{32768a^3d} - \frac{41693\cot^3(c+dx)(a+a\sec(c+dx))^{3/2}}{49152a^4d} + \frac{58077\cot^5(c+dx)(a+a\sec(c+dx))^{5/2}}{40960a^5d} - \frac{9467\cos(c+dx)\cot^5(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{8192a^5d} - \frac{2473\cos^2(c+dx)\cot^5(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{12288a^5d} - \frac{155\cos^3(c+dx)\cot^5(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{3072a^5d} - \frac{7\cos^4(c+dx)\cot^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{512a^5d} - \frac{\cos^5(c+dx)\cot^5(c+dx)\sec^{10}\left(\frac{1}{2}(c+dx)\right)(a+a\sec(c+dx))^{5/2}}{320a^5d}$$

Mathematica [A] (verified)

Time = 5.53 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.67

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-\frac{\sqrt{\frac{1}{2+2\cos(c+dx)}}(3364685+2115266\cos(c+dx)+3550428\cos(2\cdots}{\cdots}\right.}{\cdots}$$

[In] Integrate[Cot[c + d*x]^6/(a + a*Sec[c + d*x])^(5/2),x]

[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*(-1/8192*(Sqrt[(2 + 2*Cos[c + d*x])^(-1)]*(3364685 + 2115266*Cos[c + d*x] + 3550428*Cos[2*(c + d*x)] + 1005782*Cos[3*(c + d*x)] + 714844*Cos[4*(c + d*x)] - 1338430*Cos[5*(c + d*x)] + 1168164*Cos[6*(c + d*x)] + 1363110*Cos[7*(c + d*x)] + 639063*Cos[8*(c + d*x)])*Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^9*Sqrt[Sec[c + d*x]]) + 1116915*ArcSin[Tan[(c + d*x)/2]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] - 983040*Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1 + Sec[c + d*x])^(-1)]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]]))/(122880*d*Sqrt[Sec[(c + d*x)/2]^2]*(a*(1 + Sec[c + d*x]))^(5/2))

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.70

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(1116915\cos(dx+c)^3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(\cdots}\right.\right.}{\cdots}$

[In] int(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/983040/d/a^3*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^3*(1116915*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))+3350745*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*cos(d*x+c)^2-1966080*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+3350745*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-5898240*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+1116915*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-1278126*cos(d*x+c)^3*cot(d*x+c)^5-5898240*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arc

```
tanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+
c)-1363110*cos(d*x+c)^2*cot(d*x+c)^5-1966080*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+
1972170*cos(d*x+c)*cot(d*x+c)^5+2720050*cot(d*x+c)^5-810890*cot(d*x+c)^4*cs
c(d*x+c)-1673842*cot(d*x+c)^3*csc(d*x+c)^2-30610*cot(d*x+c)^2*csc(d*x+c)^3+
267750*cot(d*x+c)*csc(d*x+c)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 1023, normalized size of antiderivative = 2.33

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/1966080*(1116915*sqrt(2)*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x +
 c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x +
c) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))*sin(d*x + c) + 983040*(cos(d*x + c
)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)
^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log(-(8*a*cos(d*x + c)^3
 - 4*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c) + a)/(cos(d*x + c) + 1))*sin(d
*x + c) + 4*(639063*cos(d*x + c)^8 + 681555*cos(d*x + c)^7 - 986085*cos(d*x
 + c)^6 - 1360025*cos(d*x + c)^5 + 405445*cos(d*x + c)^4 + 836921*cos(d*x +
 c)^3 + 15305*cos(d*x + c)^2 - 133875*cos(d*x + c))*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*c
os(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos
(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c)), -1/983040*(11169
15*sqrt(2)*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x
+ c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*ar
ctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c)))*sin(d*x + c) + 983040*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 + c
os(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*co
s(d*x + c) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)*sin(d*x + c)/(2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a))*
sin(d*x + c) + 2*(639063*cos(d*x + c)^8 + 681555*cos(d*x + c)^7 - 986085*co
s(d*x + c)^6 - 1360025*cos(d*x + c)^5 + 405445*cos(d*x + c)^4 + 836921*cos(
d*x + c)^3 + 15305*cos(d*x + c)^2 - 133875*cos(d*x + c))*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^
3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*
d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot^6(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)**6/(a+a*sec(d*x+c))**(5/2),x)
[Out] Integral(cot(c + d*x)**6/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 1.62 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.94

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\left(2\left(4\left(6\left(\frac{8\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{91\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{3043\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c}\right.\right.\right.}{}$$

```
[In] integrate(cot(d*x+c)^6/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] 1/983040*((2*(4*(6*(8*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a^3*sgn(cos(d*x + c))
) - 91*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 3043*sqrt(
2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 47185*sqrt(2)/(a^3*sgn
(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 349965*sqrt(2)/(a^3*sgn(cos(d*x +
 c))))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c) - 1024*sqrt
(2)*(345*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^8 - 1230*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^6*a + 1760*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^4*a^2 - 1150*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^2*a^3 + 299*a^4)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a)^5*sqrt(-a)*a*sgn(cos(d*x + c))))
/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)^6}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cot(c + d*x)^6/(a + a/cos(c + d*x))^(5/2), x)
```

3.207 $\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 177

$$\begin{aligned}\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx &= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{9/2}f}\\ &+\frac{91\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{32\sqrt{2}a^{9/2}f}+\frac{\tan(e+fx)}{3af(a+a\sec(e+fx))^{7/2}}\\ &+\frac{11\tan(e+fx)}{24a^2f(a+a\sec(e+fx))^{5/2}}+\frac{27\tan(e+fx)}{32a^3f(a+a\sec(e+fx))^{3/2}}\end{aligned}$$

```
[Out] -2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(9/2)/f+91/64*arctan
(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(9/2)/f*2^(1/2)+1
/3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(7/2)+11/24*tan(f*x+e)/a^2/f/(a+a*sec(f*
x+e))^(5/2)+27/32*tan(f*x+e)/a^3/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3972, 482, 541, 536, 209}

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{9/2}f} + \frac{91\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{32\sqrt{2}a^{9/2}f} + \frac{27\sin(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}{64a^4f\sqrt{a\sec(e+fx)+a}} + \frac{\sin(e+fx)\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)}{24a^4f\sqrt{a\sec(e+fx)+a}} + \frac{11\sin(e+fx)\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}{96a^4f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Tan[e + f*x]^2/(a + a*Sec[e + f*x])^(9/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^(9/2)*f) +
(91*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(32*
Sqrt[2]*a^(9/2)*f) + (27*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(64*a^4*f*Sqrt[a
+ a*Sec[e + f*x]]) + (11*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Sin[e + f*x])/(96*
a^4*f*Sqrt[a + a*Sec[e + f*x]]) + (Cos[e + f*x]^2*Sec[(e + f*x)/2]^6*Sin[e
+ f*x])/(24*a^4*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^4}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^3 f} \\
&= \frac{\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{24a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1-5ax^2}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{3a^4 f} \\
&= \frac{11\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{96a^4 f\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{24a^4 f\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{15a-33a^2x^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{24a^5 f} \\
&= \frac{27\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{64a^4 f\sqrt{a+a\sec(e+fx)}} + \frac{11\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{96a^4 f\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{24a^4 f\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{111a^2-81a^3x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{96a^6 f}
\end{aligned}$$

$$= \frac{27\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{64a^4f\sqrt{a+a\sec(e+fx)}} + \frac{11\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{96a^4f\sqrt{a+a\sec(e+fx)}} + \frac{\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{24a^4f\sqrt{a+a\sec(e+fx)}} + \frac{2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^4f} - \frac{91\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{32a^4f}$$

$$= -\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{9/2}f} + \frac{91\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{32\sqrt{2}a^{9/2}f} + \frac{27\sec^2\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{64a^4f\sqrt{a+a\sec(e+fx)}} + \frac{11\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{96a^4f\sqrt{a+a\sec(e+fx)}} + \frac{\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)}{24a^4f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (warning: unable to verify)

Time = 6.36 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.36

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = \frac{\cos^3\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{9}{2}}(e+fx)\sqrt{a(1+\sec(e+fx))}\left(-384\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx\right.}{\sqrt{\frac{1}{1+\sec(e+f}}}\right.\right.}{}$$

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sec[e + f*x])^(9/2),x]
```

```
[Out] (Cos[(e + f*x)/2]^3*Sec[e + f*x]^(9/2)*Sqrt[a*(1 + Sec[e + f*x])]*(-384*Arc
Tan[Tan[(e + f*x)/2]/Sqrt[(1 + Sec[e + f*x])^(-1)]]*Cos[(e + f*x)/2]^7*Sqrt
[Cos[e + f*x]/(1 + Cos[e + f*x])^2]*Sqrt[1 + Sec[e + f*x]] + 273*ArcSin[Tan
[(e + f*x)/2]]*Cos[(e + f*x)/2]^7*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[(1 + Sec[e
+ f*x])^(-1)]*Sqrt[1 + Sec[e + f*x]] + ((319 + 412*Cos[e + f*x] + 157*Cos[2
*(e + f*x)])*Sin[(e + f*x)/2])/(8*Sqrt[Sec[e + f*x]])))/(6*a^5*f*(1 + Sec[e
 + f*x])^5)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(148) = 296$.

Time = 3.07 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.87

method	result
default	$\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(-8\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{5}{2}}(-\cot(fx+e)+\csc(fx+\right.$

```
[In] int(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192/f/a^5*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2)*(-8*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(5/2)*(-cot(f
*x+e)+csc(f*x+e))+10*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(-cot(f*x+e)+c
sc(f*x+e))+12*(1-cos(f*x+e))^3*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*csc(
f*x+e)^3-192*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2
)*(-cot(f*x+e)+csc(f*x+e)))-93*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-co
t(f*x+e)+csc(f*x+e))+273*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x
+e)^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 674, normalized size of antiderivative = 3.81

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = \left[-\frac{273\sqrt{2}\left(\cos(fx+e)^4+4\cos(fx+e)^3+6\cos(fx+e)^2+4\cos(fx+e)\right.}{}\right.$$

$$-\frac{273\sqrt{2}\left(\cos(fx+e)^4+4\cos(fx+e)^3+6\cos(fx+e)^2+4\cos(fx+e)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{\sqrt{a}\sin(fx+}\right.}{}$$

```
[In] integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(273*sqrt(2)*(cos(f*x + e)^4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2
+ 4*cos(f*x + e) + 1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*co
s(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 384*(cos(f*x + e)^
4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2 + 4*cos(f*x + e) + 1)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(157*
cos(f*x + e)^3 + 206*cos(f*x + e)^2 + 81*cos(f*x + e))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 4*a^5*f*cos(f*x +
 e)^3 + 6*a^5*f*cos(f*x + e)^2 + 4*a^5*f*cos(f*x + e) + a^5*f), -1/192*(273
*sqrt(2)*(cos(f*x + e)^4 + 4*cos(f*x + e)^3 + 6*cos(f*x + e)^2 + 4*cos(f*x
+ e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)/(sqrt(a)*sin(f*x + e))) - 384*(cos(f*x + e)^4 + 4*cos(f*x + e)^3
 + 6*cos(f*x + e)^2 + 4*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(157*cos(f*x
 + e)^3 + 206*cos(f*x + e)^2 + 81*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 4*a^5*f*cos(f*x + e)^3 +
 6*a^5*f*cos(f*x + e)^2 + 4*a^5*f*cos(f*x + e) + a^5*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(f*x+e)**2/(a+a*sec(f*x+e))**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = \int \frac{\tan\left(fx+e\right)^2}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^2/(a*sec(f*x + e) + a)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 1.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.62

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = \frac{\sqrt{-a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}\left(2\left(\frac{4\,\sqrt{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}{a^5\mathrm{sgn}(\cos(fx+e))}-\frac{19\,\sqrt{2}}{a^5\mathrm{sgn}(\cos(fx+e))}\right)\tan\left(\frac{1}{2}\,fx\right.\right.}{192\,f}$$

```
[In] integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="giac")
[Out] 1/192*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)*(2*(4*sqrt(2)*tan(1/2*f*x + 1/2*e
)^2/(a^5*sgn(cos(f*x + e))) - 19*sqrt(2)/(a^5*sgn(cos(f*x + e))))*tan(1/2*f
*x + 1/2*e)^2 + 111*sqrt(2)/(a^5*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+a\sec(e+fx))^{9/2}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

```
[In] int(tan(e + f*x)^2/(a + a/cos(e + f*x))^(9/2),x)
[Out] int(tan(e + f*x)^2/(a + a/cos(e + f*x))^(9/2), x)
```

3.208 $\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 125

$$\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \frac{2^{1+m+n}\operatorname{AppellF1}\left(\frac{1+m}{2},m+n,1,\frac{3+m}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{1+m+n}(a+a\sec(c+dx))^n}{de(1+m)}$$

```
[Out] 2^(1+m+n)*AppellF1(1/2+1/2*m,m+n,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(d*x
+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(1+m+n)*(a+a*sec
(d*x+c))^n*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3974}

$$\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \frac{2^{m+n+1}(a\sec(c+dx)+a)^n(e\tan(c+dx))^{m+1}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+n+1}\operatorname{AppellF1}\left(\frac{m+1}{2},m+n,1,\frac{m+3}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)\ldots}\right.}{de(m+1)}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
```

```
[Out] (2^(1 + m + n)*AppellF1[(1 + m)/2, m + n, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(1 + m + n)*(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^(1
 + m))/(d*e*(1 + m))
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{2^{1+m+n} \text{AppellF1}\left(\frac{1+m}{2}, m+n, 1, \frac{3+m}{2}, -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right) \left(\frac{1}{1+\sec(c+dx)}\right)^{1+m+n} (a+a\sec(c+dx))}{de(1+m)}$$

Mathematica [F]

$$\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
[Out] Integrate[(a + a*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

Maple [F]

$$\int (a+a\sec(dx+c))^n (e\tan(dx+c))^m\,dx$$

```
[In] int((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \int (a\sec(dx+c)+a)^n(e\tan(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx=\int (a(\sec{(c+dx)}+1))^n\,(e\tan{(c+dx)})^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**n*(e*tan(d*x+c))**m,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*(e*tan(c + d*x))**m, x)
```

Maxima [F]

$$\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx=\int (a\sec{(dx+c)}+a)^n(e\tan{(dx+c)})^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Giac [F]

$$\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx=\int (a\sec{(dx+c)}+a)^n(e\tan{(dx+c)})^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^n(e\tan(c+dx))^m\,dx=\int (e\tan(c+d\,x))^m\,\left(a+\frac{a}{\cos{(c+d\,x)}}\right)^n\,dx$$

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^n,x)
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^n, x)
```

3.209 $\int (a+a\sec(c+dx))^3(e\tan(c+dx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 243

$$\begin{aligned}\int (a+a\sec(c+dx))^3(e\tan(c+dx))^m\,dx &= \frac{3a^3(e\tan(c+dx))^{1+m}}{de(1+m)}\\ &+ \frac{a^3\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)(e\tan(c+dx))^{1+m}}{de(1+m)}\\ &+ \frac{3a^3\cos^2(c+dx)^{\frac{2+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{2+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx))^{1+m}}{de(1+m)}\\ &+ \frac{a^3\cos^2(c+dx)^{\frac{4+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{4+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec^3(c+dx)(e\tan(c+dx))^{1+m}}{de(1+m)}\end{aligned}$$

```
[Out] 3*a^3*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+a^3*hypergeom([1, 1/2+1/2*m],[3/2+1/2*
m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+3*a^3*(cos(d*x+c)^2)^(1+1/
2*m)*hypergeom([1+1/2*m, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e
*tan(d*x+c))^(1+m)/d/e/(1+m)+a^3*(cos(d*x+c)^2)^(2+1/2*m)*hypergeom([2+1/2*
m, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)^3*(e*tan(d*x+c))^(1+m)/d
/e/(1+m)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

$$= \{3971, 3557, 371, 2697, 2687, 32\}$$

$$\int (a+a\sec(c+dx))^3(e\tan(c+dx))^m\,dx$$
$$= \frac{a^3(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{de(m+1)}$$
$$+ \frac{a^3\sec^3(c+dx)\cos^2(c+dx)^{\frac{m+4}{2}}(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{m+4}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$
$$+ \frac{3a^3\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{m+2}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$
$$+ \frac{3a^3(e\tan(c+dx))^{m+1}}{de(m+1)}$$

```
[In] Int[(a + a*Sec[c + d*x])^3*(e*Tan[c + d*x])^m,x]
```

```
[Out] (3*a^3*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)) + (a^3*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 +
m)) + (3*a^3*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometric2F1[(1 + m)/2, (2 +
 m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^(1 + m))/(d
*e*(1 + m)) + (a^3*(Cos[c + d*x]^2)^((4 + m)/2)*Hypergeometric2F1[(1 + m)/2
, (4 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]^3*(e*Tan[c + d*x])^(1
+ m))/(d*e*(1 + m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
```

```
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3(e\tan(c+dx))^m + 3a^3\sec(c+dx)(e\tan(c+dx))^m \right. \\
&\qquad \left. + 3a^3\sec^2(c+dx)(e\tan(c+dx))^m + a^3\sec^3(c+dx)(e\tan(c+dx))^m\right)\,dx \\
&= a^3\int (e\tan(c+dx))^m\,dx + a^3\int \sec^3(c+dx)(e\tan(c+dx))^m\,dx \\
&\quad + \left(3a^3\right)\int \sec(c+dx)(e\tan(c+dx))^m\,dx + \left(3a^3\right)\int \sec^2(c+dx)(e\tan(c+dx))^m\,dx \\
&= \frac{3a^3\cos^2(c+dx)^{\frac{2+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{2+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx)}{de(1+m)} \\
&\quad + \frac{a^3\cos^2(c+dx)^{\frac{4+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{4+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec^3(c+dx)(e\tan(c+d}{de(1+m)} \\
&\quad + \frac{\left(3a^3\right)\text{Subst}\left(\int (ex)^m\,dx, x, \tan(c+dx)\right)}{d} \\
&\quad + \frac{\left(a^3e\right)\text{Subst}\left(\int \frac{x^m}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{d} \\
&= \frac{3a^3(e\tan(c+dx))^{1+m}}{de(1+m)} \\
&\quad + \frac{a^3\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)(e\tan(c+dx))^{1+m}}{de(1+m)} \\
&\quad + \frac{3a^3\cos^2(c+dx)^{\frac{2+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{2+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+d}{de(1+m)} \\
&\quad + \frac{a^3\cos^2(c+dx)^{\frac{4+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{4+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec^3(c+dx)(e\tan(c+d}{de(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.93

$$\int (a+a\sec(c+dx))^3(e\tan(c+dx))^m\,dx$$
$$= \frac{a^3e(e\tan(c+dx))^{-1+m}\left(-\tan^2(c+dx)\right)^{-m/2}\left(9\sqrt{-\tan^2(c+dx)}+9(1+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2}\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^3*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a^3*e*(e*Tan[c + d*x])^(-1 + m)*(9*Sqrt[-Tan[c + d*x]^2] + 9*(1 + m)*Hyper
geometric2F1[1/2, (1 - m)/2, 3/2, Sec[c + d*x]^2]*Sec[c + d*x]*Sqrt[-Tan[c
+ d*x]^2] + (1 + m)*Hypergeometric2F1[3/2, (1 - m)/2, 5/2, Sec[c + d*x]^2]*
Sec[c + d*x]^3*Sqrt[-Tan[c + d*x]^2] - 9*(-Tan[c + d*x]^2)^((2 + m)/2) - 3*
Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(-Tan[c + d*x]^
2)^((2 + m)/2)))/(3*d*(1 + m)*(-Tan[c + d*x]^2)^(m/2))
```

Maple [F]

$$\int \left(a+a\sec\left(dx+c\right)\right)^{3}\left(e\tan\left(dx+c\right)\right)^{m}dx$$

```
[In] int((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))^3(e\tan(c+dx))^m\,dx = \int \left(a\sec\left(dx+c\right)+a\right)^{3}\left(e\tan\left(dx+c\right)\right)^{m}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)*(e*tan(d*x + c))^m, x)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^3(e\tan(c+dx))^m\,dx = a^3\left(\int (e\tan(c+dx))^m\;dx + \int 3(e\tan(c+dx))^m\sec(c+dx)\,dx + \int 3(e\tan(c+dx))^m\sec^2(c+dx)\,dx + \int (e\tan(c+dx))^m\sec^3(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**3*(e*tan(d*x+c))**m,x)
```

```
[Out] a**3*(Integral((e*tan(c + d*x))**m, x) + Integral(3*(e*tan(c + d*x))**m*sec
(c + d*x), x) + Integral(3*(e*tan(c + d*x))**m*sec(c + d*x)**2, x) + Integr
al((e*tan(c + d*x))**m*sec(c + d*x)**3, x))
```

Maxima [F]

$$\int (a+a\sec(c+dx))^3(e\tan(c+dx))^m\,dx = \int (a\sec(dx+c)+a)^3(e\tan(dx+c))^m\;dx$$

```
[In] integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*tan(d*x + c))^m, x)
```

Giac [F]

$$\int (a+a\sec(c+dx))^3(e\tan(c+dx))^m\,dx = \int (a\sec(dx+c)+a)^3(e\tan(dx+c))^m\;dx$$

```
[In] integrate((a+a*sec(d*x+c))^3*(e*tan(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*tan(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^3 (e\tan(c + dx))^m \, dx = \int (e\tan(c + d\,x))^m \left(a + \frac{a}{\cos(c + d\,x)}\right)^3 dx$$

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^3,x)
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^3, x)
```

3.210 $\int (a+a\sec(c+dx))^2(e\tan(c+dx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 161

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^m\,dx = \frac{a^2(e\tan(c+dx))^{1+m}}{de(1+m)} + \frac{a^2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)(e\tan(c+dx))^{1+m}}{de(1+m)} + \frac{2a^2\cos^2(c+dx)^{\frac{2+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{2+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx))^{1+m}}{de(1+m)}$$

```
[Out] a^2*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+a^2*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m]
,-tan(d*x+c)^2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)+2*a^2*(cos(d*x+c)^2)^(1+1/2*
m)*hypergeom([1+1/2*m, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*t
an(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3971, 3557, 371, 2697, 2687, 32}

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^m\,dx$$
$$= \frac{a^2(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{de(m+1)} + \frac{2a^2\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{m+2}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)} + \frac{a^2(e\tan(c+dx))^{m+1}}{de(m+1)}$$

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a^2*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)) + (a^2*Hypergeometric2F1[1, (1
 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)
) + (2*a^2*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometric2F1[(1 + m)/2, (2 + m
)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^(1 + m))/(d*e
*(1 + m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2(e\tan(c+dx))^m + 2a^2\sec(c+dx)(e\tan(c+dx))^m + a^2\sec^2(c+dx)(e\tan(c+dx))^m\right) dx$$

$$= a^2\int (e\tan(c+dx))^m\,dx + a^2\int \sec^2(c+dx)(e\tan(c+dx))^m\,dx + \left(2a^2\right)\int \sec(c+dx)(e\tan(c+dx))^m\,dx$$

$$= \frac{2a^2\cos^2(c+dx)^{\frac{2+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{2+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx)}{de(1+m)} + \frac{a^2\text{Subst}\left(\int(ex)^m\,dx,x,\tan(c+dx)\right)}{d} + \frac{(a^2e)\,\text{Subst}\left(\int\frac{x^m}{e^2+x^2}\,dx,x,e\tan(c+dx)\right)}{d}$$

$$= \frac{a^2(e\tan(c+dx))^{1+m}}{de(1+m)} + \frac{a^2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)(e\tan(c+dx))^{1+m}}{de(1+m)} + \frac{2a^2\cos^2(c+dx)^{\frac{2+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{2+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+d}{de(1+m)}$$

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06

$$\int (a+a\sec(c+dx))^2(e\tan(c+dx))^m\,dx = \frac{a^2e(e\tan(c+dx))^{-1+m}\left(-\tan^2(c+dx)\right)^{-m/2}\left(\sqrt{-\tan^2(c+dx)}+2(1+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2}\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a^2*e*(e*Tan[c + d*x])^(-1 + m)*(Sqrt[-Tan[c + d*x]^2] + 2*(1 + m)*Hyperge
ometric2F1[1/2, (1 - m)/2, 3/2, Sec[c + d*x]^2]*Sec[c + d*x]*Sqrt[-Tan[c +
d*x]^2] - (-Tan[c + d*x]^2)^((2 + m)/2) - Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -Tan[c + d*x]^2]*(-Tan[c + d*x]^2)^((2 + m)/2)))/(d*(1 + m)*(-Tan
[c + d*x]^2)^(m/2))
```

Maple [F]

$$\int (a + a\sec(dx + c))^2 (e\tan(dx + c))^m dx$$

```
[In] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x)
```

Fricas [F]

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^m dx = \int (a\sec(dx + c) + a)^2 (e\tan(dx + c))^m dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*(e*tan(d*x + c))^m
, x)
```

Sympy [F]

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^m dx = a^2 \left(\int (e\tan(c + dx))^m dx + \int 2(e\tan(c + dx))^m \sec(c + dx) dx + \int (e\tan(c + dx))^m \sec^2(c + dx) dx \right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*(e*tan(d*x+c))**m,x)
```

```
[Out] a**2*(Integral((e*tan(c + d*x))**m, x) + Integral(2*(e*tan(c + d*x))**m*sec
(c + d*x), x) + Integral((e*tan(c + d*x))**m*sec(c + d*x)**2, x))
```

Maxima [F]

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^m dx = \int (a\sec(dx + c) + a)^2 (e\tan(dx + c))^m dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^m, x)
```

Giac [F]

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^m \, dx = \int (a\sec(dx + c) + a)^2 (e\tan(dx + c))^m \, dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2*(e*tan(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^2 (e\tan(c + dx))^m \, dx = \int (e\tan(c + d\,x))^m \left(a + \frac{a}{\cos(c + d\,x)}\right)^2 \, dx$$

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^2,x)
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^2, x)
```

3.211 $\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 129

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx = \frac{a\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)(e\tan(c+dx))^{1+m}}{de(1+m)} + \frac{a\cos^2(c+dx)^{\frac{2+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{2+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx))^{1+m}}{de(1+m)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(1+m)/
d/e/(1+m)+a*(cos(d*x+c)^2)^(1+1/2*m)*hypergeom([1+1/2*m, 1/2+1/2*m],[3/2+1/
2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3969, 3557, 371, 2697}

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx = \frac{a(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{de(m+1)} + \frac{a\sec(c+dx)\cos^2(c+dx)^{\frac{m+2}{2}}(e\tan(c+dx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{m+2}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}$$

```
[In] Int[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^m,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d
*x])^(1 + m))/(d*e*(1 + m)) + (a*(Cos[c + d*x]^2)^((2 + m)/2)*Hypergeometri
c2F1[(1 + m)/2, (2 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c
 + d*x])^(1 + m))/(d*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\text{integral} = a\int (e\tan(c+dx))^m\,dx + a\int \sec(c+dx)(e\tan(c+dx))^m\,dx$$

$$= \frac{a\cos^2(c+dx)^{\frac{2+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{2+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx))^{1-}}{de(1+m)}$$

$$+ \frac{(ae)\text{Subst}\left(\int \frac{x^m}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{d}$$

$$= \frac{a\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)(e\tan(c+dx))^{1+m}}{de(1+m)}$$

$$+ \frac{a\cos^2(c+dx)^{\frac{2+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{2+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx)}{de(1+m)}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx = \frac{a(e\tan(c+dx))^m\left(\frac{\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan(c+dx)}{1+m}+\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2}\right.\right.}{d}$$

[In] Integrate[(a + a*Sec[c + d*x])*(e*Tan[c + d*x])^m,x]

[Out] (a*(e*Tan[c + d*x])^m*((Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(1 + m) + Csc[c + d*x]*Hypergeometric2F1[1/2, (1 - m)/2, 3/2, Sec[c + d*x]^2]*(-Tan[c + d*x]^2)^((1 - m)/2)))/d

Maple [F]

$$\int (a+a\sec(dx+c))\,(e\tan(dx+c))^m\,dx$$

[In] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x)

[Out] int((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x)

Fricas [F]

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx = \int (a\sec(dx+c)+a)(e\tan(dx+c))^m\,dx$$

[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="fricas")

[Out] integral((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)

Sympy [F]

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx = a\left(\int (e\tan(c+dx))^m\,dx + \int (e\tan(c+dx))^m\sec(c+dx)\,dx\right)$$

[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))**m,x)

[Out] a*(Integral((e*tan(c + d*x))**m, x) + Integral((e*tan(c + d*x))**m*sec(c + d*x), x))

Maxima [F]

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx = \int (a\sec(dx+c)+a)(e\tan(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Giac [F]

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx = \int (a\sec(dx+c)+a)(e\tan(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))(e\tan(c+dx))^m\,dx = \int (e\tan(c+d\,x))^m\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x)),x)
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x)), x)
```

3.212 $\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \frac{e\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+m),\frac{1+m}{2},-\tan^2(c+dx)\right)(e\tan(c+dx))^{-1+m}}{ad(1-m)} - \frac{e\cos^2(c+dx)^{m/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+m),\frac{m}{2},\frac{1+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx))^{-1+m}}{ad(1-m)}$$

```
[Out] e*hypergeom([1, -1/2+1/2*m],[1/2+1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(-1+m
)/a/d/(1-m)-e*(cos(d*x+c)^2)^(1/2*m)*hypergeom([1/2*m, -1/2+1/2*m],[1/2+1/2
*m],sin(d*x+c)^2)*sec(d*x+c)*(e*tan(d*x+c))^(-1+m)/a/d/(1-m)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3973, 3969, 3557, 371, 2697}

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \frac{e(e\tan(c+dx))^{m-1}\,\text{Hypergeometric2F1}\left(1,\frac{m-1}{2},\frac{m+1}{2},-\tan^2(c+dx)\right)}{ad(1-m)} - \frac{e\sec(c+dx)\cos^2(c+dx)^{m/2}(e\tan(c+dx))^{m-1}\,\text{Hypergeometric2F1}\left(\frac{m-1}{2},\frac{m}{2},\frac{m+1}{2},\sin^2(c+dx)\right)}{ad(1-m)}$$

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c +
d*x])^(-1 + m))/(a*d*(1 - m)) - (e*(Cos[c + d*x]^2)^(m/2)*Hypergeometric2F1
[(-1 + m)/2, m/2, (1 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])^
(-1 + m))/(a*d*(1 - m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{e^2 \int (-a + a\sec(c+dx))(e\tan(c+dx))^{-2+m}\,dx}{a^2} \\ &= -\frac{e^2 \int (e\tan(c+dx))^{-2+m}\,dx}{a} + \frac{e^2 \int \sec(c+dx)(e\tan(c+dx))^{-2+m}\,dx}{a}\end{aligned}$$

$$= -\frac{e\cos^2(c+dx)^{m/2}\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+m),\frac{m}{2},\frac{1+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx)}{ad(1-m)}$$

$$-\frac{e^3\text{Subst}\left(\int\frac{x^{-2+m}}{e^2+x^2}\,dx,x,e\tan(c+dx)\right)}{ad}$$

$$= \frac{e\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+m),\frac{1+m}{2},-\tan^2(c+dx)\right)(e\tan(c+dx))^{-1+m}}{ad(1-m)}$$

$$-\frac{e\cos^2(c+dx)^{m/2}\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+m),\frac{m}{2},\frac{1+m}{2},\sin^2(c+dx)\right)\sec(c+dx)(e\tan(c+dx)}{ad(1-m)}$$

Mathematica [F]

$$\int\frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int\frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx$$

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]),x]
[Out] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x]), x]
```

Maple [F]

$$\int\frac{(e\tan(dx+c))^m}{a+a\sec(dx+c)}dx$$

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x)
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

Fricas [F]

$$\int\frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int\frac{(e\tan(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\tan(c+dx))^m}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**m/(sec(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{(e\tan(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^m}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,(e\tan(c+d\,x))^m}{a\,(\cos(c+d\,x)+1)}\,dx$$

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^m)/(a*(cos(c + d*x) + 1)), x)
```

3.213 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 169

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = -\frac{e^3(e\tan(c+dx))^{-3+m}}{a^2d(3-m)} - \frac{e^3\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),-\tan^2(c+dx)\right)(e\tan(c+dx))^{-3+m}}{a^2d(3-m)} + \frac{2e^3\cos^2(c+dx)^{\frac{1}{2}(-2+m)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+m),\frac{1}{2}(-2+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)\sec(c+}{a^2d(3-m)}$$

```
[Out] -e^3*(e*tan(d*x+c))^(-3+m)/a^2/d/(3-m)-e^3*hypergeom([1, -3/2+1/2*m],[-1/2+
1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(-3+m)/a^2/d/(3-m)+2*e^3*(cos(d*x+c)^2
)^(-1+1/2*m)*hypergeom([-1+1/2*m, -3/2+1/2*m],[-1/2+1/2*m],sin(d*x+c)^2)*se
c(d*x+c)*(e*tan(d*x+c))^(-3+m)/a^2/d/(3-m)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3973, 3971, 3557, 371, 2697, 2687, 32}

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$$
$$= -\frac{e^3(e\tan(c+dx))^{m-3}\,\text{Hypergeometric2F1}\left(1,\frac{m-3}{2},\frac{m-1}{2},-\tan^2(c+dx)\right)}{a^2d(3-m)} + \frac{2e^3\sec(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+dx))^{m-3}\,\text{Hypergeometric2F1}\left(\frac{m-3}{2},\frac{m-2}{2},\frac{m-1}{2},\sin^2(c+dx)\right)}{a^2d(3-m)} - \frac{e^3(e\tan(c+dx))^{m-3}}{a^2d(3-m)}$$

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((e^3*(e*Tan[c + d*x])^(-3 + m))/(a^2*d*(3 - m))) - (e^3*Hypergeometric2F1
[1, (-3 + m)/2, (-1 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(-3 + m))/(a^
2*d*(3 - m)) + (2*e^3*(Cos[c + d*x]^2)^((-2 + m)/2)*Hypergeometric2F1[(-3 +
 m)/2, (-2 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x]
)^(-3 + m))/(a^2*d*(3 - m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{e^4 \int (-a + a\sec(c+dx))^2 (e\tan(c+dx))^{-4+m}\,dx}{a^4}$$

$$= \frac{e^4 \int \left(a^2(e\tan(c+dx))^{-4+m} - 2a^2\sec(c+dx)(e\tan(c+dx))^{-4+m} + a^2\sec^2(c+dx)(e\tan(c+dx)\right.}{a^4}$$

$$= \frac{e^4 \int (e\tan(c+dx))^{-4+m}\,dx}{a^2} + \frac{e^4 \int \sec^2(c+dx)(e\tan(c+dx))^{-4+m}\,dx}{a^2} - \frac{(2e^4)\int \sec(c+dx)(e\tan(c+dx))^{-4+m}\,dx}{a^2}$$

$$= \frac{2e^3\cos^2(c+dx)^{\frac{1}{2}(-2+m)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+m),\frac{1}{2}(-2+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)\text{sec}}{a^2d(3-m)} + \frac{e^4\text{Subst}\left(\int (ex)^{-4+m}\,dx, x, \tan(c+dx)\right)}{a^2d} + \frac{e^5\text{Subst}\left(\int \frac{x^{-4+m}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{a^2d}$$

$$= -\frac{e^3(e\tan(c+dx))^{-3+m}}{a^2d(3-m)} - \frac{e^3\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),-\tan^2(c+dx)\right)(e\tan(c+dx))^{-3+m}}{a^2d(3-m)} + \frac{2e^3\cos^2(c+dx)^{\frac{1}{2}(-2+m)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+m),\frac{1}{2}(-2+m),\frac{1}{2}(-1+m),\sin^2(c+dx)\right)}{a^2d(3-m)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.84 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.95

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^m\tan\left(\frac{1}{2}(c+dx)\right)\left(-3(3+m)\,\text{Hypergeometric2F1}\left(m,\frac{1+m}{2},\frac{3+m}{2},\tan^2\left(\frac{1}{2}(c\right.\right.\right.}{2a^2d(1+}$$
$$+ \frac{i2^{1-m}\left(-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}\right)^m\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2^m\,\text{Hypergeometric2F1}\left(1,m,1+m,-\frac{-1+e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)-\left(1+e^{2i(\right.\right.}{dm(a+a\,\text{s}}$$

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*Tan[(c + d*x)/2]*(-3*(3 + m)*Hypergeom
etric2F1[m, (1 + m)/2, (3 + m)/2, Tan[(c + d*x)/2]^2] + (1 + m)*Hypergeomet
ric2F1[m, (3 + m)/2, (5 + m)/2, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)*(e*
Tan[c + d*x])^m)/(2*a^2*d*(1 + m)*(3 + m)) + (I*2^(1 - m)*(((-I)*(-1 + E^((
2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x))))^m*Cos[c/2 + (d*x)/2]^4*(2^m*Hy
pergeometric2F1[1, m, 1 + m, -((-1 + E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c
+ d*x))))] - (1 + E^((2*I)*(c + d*x)))^m*Hypergeometric2F1[m, m, 1 + m, (1
- E^((2*I)*(c + d*x)))/2])*Sec[c + d*x]^2*(e*Tan[c + d*x])^m)/(d*m*(a + a*S
ec[c + d*x])^2*Tan[c + d*x]^m)
```

Maple [F]

$$\int \frac{(e\tan(dx+c))^m}{(a+a\sec(dx+c))^2}dx$$

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

```
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

Fricas [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((e*tan(d*x + c))^m/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

Sympy [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{(e\tan(c+dx))^m}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*tan(c + d*x))**m/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

Maxima [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan{(dx+c)})^m}{(a\sec{(dx+c)}+a)^2}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{(e\tan{(dx+c)})^m}{(a\sec{(dx+c)}+a)^2}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos{(c+d\,x)}^2\,(e\tan(c+d\,x))^m}{a^2\,(\cos{(c+d\,x)}+1)^2}\,dx$$

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*tan(c + d*x))^m)/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.214 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 252

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \frac{3e^5(e\tan(c+dx))^{-5+m}}{a^3d(5-m)} + \frac{e^5\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-5+m),\frac{1}{2}(-3+m),-\tan^2(c+dx)\right)(e\tan(c+dx))^{-5+m}}{a^3d(5-m)} - \frac{3e^5\cos^2(c+dx)^{\frac{1}{2}(-4+m)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-5+m),\frac{1}{2}(-4+m),\frac{1}{2}(-3+m),\sin^2(c+dx)\right)\sec(c-}{a^3d(5-m)} - \frac{e^5\cos^2(c+dx)^{\frac{1}{2}(-2+m)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-5+m),\frac{1}{2}(-2+m),\frac{1}{2}(-3+m),\sin^2(c+dx)\right)\sec^3(c+}{a^3d(5-m)}$$

```
[Out] 3*e^5*(e*tan(d*x+c))^(-5+m)/a^3/d/(5-m)+e^5*hypergeom([1, -5/2+1/2*m],[-3/2
+1/2*m],-tan(d*x+c)^2)*(e*tan(d*x+c))^(-5+m)/a^3/d/(5-m)-3*e^5*(cos(d*x+c)^
2)^(-2+1/2*m)*hypergeom([-2+1/2*m, -5/2+1/2*m],[-3/2+1/2*m],sin(d*x+c)^2)*s
ec(d*x+c)*(e*tan(d*x+c))^(-5+m)/a^3/d/(5-m)-e^5*(cos(d*x+c)^2)^(-1+1/2*m)*h
ypergeom([-1+1/2*m, -5/2+1/2*m],[-3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)^3*(e*
tan(d*x+c))^(-5+m)/a^3/d/(5-m)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

$= \{3973, 3971, 3557, 371, 2697, 2687, 32\}$

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{e^5(e\tan(c+dx))^{m-5}\,\text{Hypergeometric2F1}\left(1,\frac{m-5}{2},\frac{m-3}{2},-\tan^2(c+dx)\right)}{a^3d(5-m)}$$
$$-\frac{e^5\sec^3(c+dx)\cos^2(c+dx)^{\frac{m-2}{2}}(e\tan(c+dx))^{m-5}\,\text{Hypergeometric2F1}\left(\frac{m-5}{2},\frac{m-2}{2},\frac{m-3}{2},\sin^2(c+dx)\right)}{a^3d(5-m)}$$
$$-\frac{3e^5\sec(c+dx)\cos^2(c+dx)^{\frac{m-4}{2}}(e\tan(c+dx))^{m-5}\,\text{Hypergeometric2F1}\left(\frac{m-5}{2},\frac{m-4}{2},\frac{m-3}{2},\sin^2(c+dx)\right)}{a^3d(5-m)}$$
$$+\frac{3e^5(e\tan(c+dx))^{m-5}}{a^3d(5-m)}$$

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*e^5*(e*Tan[c + d*x])^(-5 + m))/(a^3*d*(5 - m)) + (e^5*Hypergeometric2F1[
1, (-5 + m)/2, (-3 + m)/2, -Tan[c + d*x]^2]*(e*Tan[c + d*x])^(-5 + m))/(a^3
*d*(5 - m)) - (3*e^5*(Cos[c + d*x]^2)^((-4 + m)/2)*Hypergeometric2F1[(-5 +
m)/2, (-4 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Tan[c + d*x])
^(-5 + m))/(a^3*d*(5 - m)) - (e^5*(Cos[c + d*x]^2)^((-2 + m)/2)*Hypergeomet
ric2F1[(-5 + m)/2, (-2 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]^3*(
e*Tan[c + d*x])^(-5 + m))/(a^3*d*(5 - m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
```

```
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{e^6 \int (-a + a\sec(c+dx))^3 (e\tan(c+dx))^{-6+m}\,dx}{a^6}$$

$$= \frac{e^6 \int \left(-a^3 (e\tan(c+dx))^{-6+m} + 3a^3\sec(c+dx)(e\tan(c+dx))^{-6+m} - 3a^3\sec^2(c+dx)(e\tan(c+\right.}{a^6}$$

$$= -\frac{e^6 \int (e\tan(c+dx))^{-6+m}\,dx}{a^3} + \frac{e^6 \int \sec^3(c+dx)(e\tan(c+dx))^{-6+m}\,dx}{a^3} + \frac{(3e^6) \int \sec(c+dx)(e\tan(c+dx))^{-6+m}\,dx}{a^3} - \frac{(3e^6) \int \sec^2(c+dx)(e\tan(c+dx))^{-6+m}\,dx}{a^3}$$

$$= -\frac{3e^5\cos^2(c+dx)^{\frac{1}{2}(-4+m)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-5+m),\frac{1}{2}(-4+m),\frac{1}{2}(-3+m),\sin^2(c+dx)\right.}{a^3d(5-m)} - \frac{e^5\cos^2(c+dx)^{\frac{1}{2}(-2+m)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-5+m),\frac{1}{2}(-2+m),\frac{1}{2}(-3+m),\sin^2(c+dx)\right)}{a^3d(5-m)} - \frac{(3e^6)\,\text{Subst}\left(\int (ex)^{-6+m}\,dx, x, \tan(c+dx)\right)}{a^3d} - \frac{e^7\text{Subst}\left(\int \frac{x^{-6+m}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{a^3d}$$

$$= \frac{3e^5(e\tan(c+dx))^{-5+m}}{a^3d(5-m)} + \frac{e^5\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-5+m),\frac{1}{2}(-3+m),-\tan^2(c+dx)\right)(e\tan(c+dx))^{-5+m}}{a^3d(5-m)} - \frac{3e^5\cos^2(c+dx)^{\frac{1}{2}(-4+m)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-5+m),\frac{1}{2}(-4+m),\frac{1}{2}(-3+m),\sin^2(c+dx)\right)}{a^3d(5-m)} - \frac{e^5\cos^2(c+dx)^{\frac{1}{2}(-2+m)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-5+m),\frac{1}{2}(-2+m),\frac{1}{2}(-3+m),\sin^2(c+dx)\right)\,\text{s}}{a^3d(5-m)}$$

Mathematica [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$$

[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]

[Out] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^3, x]

Maple [F]

$$\int \frac{(e\tan(dx+c))^m}{(a+a\sec(dx+c))^3}dx$$

[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x)

[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x)

Fricas [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral((e*tan(d*x + c))^m/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) + a^3), x)

Sympy [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{(e\tan(c+dx))^m}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**3,x)

[Out] Integral((e*tan(c + d*x))**m/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)/a**3

Maxima [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos(c+d\,x)^3\,(e\tan(c+d\,x))^m}{a^3\,(\cos(c+d\,x)+1)^3}\,dx$$

[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^3,x)

[Out] int((cos(c + d*x)^3*(e*tan(c + d*x))^m)/(a^3*(cos(c + d*x) + 1)^3), x)

3.215 $\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \frac{2^{\frac{5}{2}+m}\operatorname{AppellF1}\left(\frac{1+m}{2},\frac{3}{2}+m,1,\frac{3+m}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{5}{2}}}{de(1+m)}$$

```
[Out] 2^(5/2+m)*AppellF1(1/2+1/2*m,3/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(d
*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(5/2+m)*(a+a*s
ec(d*x+c))^(3/2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3974}

$$\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \frac{2^{m+\frac{5}{2}}(a\sec(c+dx)+a)^{3/2}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{5}{2}}(e\tan(c+dx))^{m+1}\operatorname{AppellF1}\left(\frac{m+}{2}\right.}{de(m+1)}$$

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
```

```
[Out] (2^(5/2 + m)*AppellF1[(1 + m)/2, 3/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(5/2 + m)*(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^
(1 + m))/(d*e*(1 + m))
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{2^{\frac{5}{2}+m}\operatorname{AppellF1}\left(\frac{1+m}{2},\frac{3}{2}+m,1,\frac{3+m}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{5}{2}+m}(a+a\sec(c+dx))^{3/2}(\cdots}{de(1+m)}$$

Mathematica [F]

$$\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx=\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]

[Out] Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]

Maple [F]

$$\int (a+a\sec(dx+c))^{\frac{3}{2}}(e\tan(dx+c))^m\,dx$$

[In] int((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)

[Out] int((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)

Fricas [F]

$$\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx=\int (a\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^m\,dx$$

[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")

[Out] integral((a*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)

Sympy [F]

$$\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\left(e\tan\left(c+dx\right)\right)^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(e*tan(d*x+c))**m,x)
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(e*tan(c + d*x))**m, x)
```

Maxima [F]

$$\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int \left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\left(e\tan\left(dx+c\right)\right)^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Giac [F]

$$\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int \left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\left(e\tan\left(dx+c\right)\right)^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int (e\tan(c+d\,x))^m\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(3/2),x)
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(3/2), x)
```

3.216 $\int \sqrt{a+a\sec(c+dx)}(e\tan(c+dx))^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \sqrt{a+a\sec(c+dx)}(e\tan(c+dx))^m\,dx = \frac{2^{\frac{3}{2}+m}\operatorname{AppellF1}\left(\frac{1+m}{2},\frac{1}{2}+m,1,\frac{3+m}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{3}{2}+m}\sqrt{a+a\sec(c+dx)}(e\,t}{de(1+m)}$$

```
[Out] 2^(3/2+m)*AppellF1(1/2+1/2*m,1/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(d
*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(3/2+m)*(a+a*s
ec(d*x+c))^(1/2)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {3974}

$$\int \sqrt{a+a\sec(c+dx)}(e\tan(c+dx))^m\,dx = \frac{2^{m+\frac{3}{2}}\sqrt{a\sec(c+dx)+a}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{3}{2}}(e\tan(c+dx))^{m+1}\operatorname{AppellF1}\left(\frac{m+1}{2},m+\frac{1}{2},1,\frac{m+3}{2},-\frac{a-a\sec(c+dx}{\sec(c+dx)a+}\right.}{de(m+1)}$$

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```

```
[Out] (2^(3/2 + m)*AppellF1[(1 + m)/2, 1/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d*
x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 +
 Sec[c + d*x])^(-1))^(3/2 + m)*Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^(1
 + m))/(d*e*(1 + m))
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{2^{\frac{3}{2}+m}\operatorname{AppellF1}\left(\frac{1+m}{2},\frac{1}{2}+m,1,\frac{3+m}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{3}{2}+m}\sqrt{a+a\sec(c+dx)}(e\,ta}{de(1+m)}$$

Mathematica [F]

$$\int \sqrt{a+a\sec(c+dx)}(e\tan(c+dx))^m\,dx=\int \sqrt{a+a\sec(c+dx)}(e\tan(c+dx))^m\,dx$$

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
[Out] Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

Maple [F]

$$\int \sqrt{a+a\sec(dx+c)}\,(e\tan(dx+c))^m\,dx$$

```
[In] int((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

Fricas [F]

$$\int \sqrt{a+a\sec(c+dx)}(e\tan(c+dx))^m\,dx=\int \sqrt{a\sec(dx+c)+a}(e\tan(dx+c))^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Sympy [F]

$$\int \sqrt{a+a\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(e\tan\left(c+dx\right)\right)^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*(e*tan(d*x+c))**m,x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(e*tan(c + d*x))**m, x)
```

Maxima [F]

$$\int \sqrt{a+a\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\left(e\tan\left(dx+c\right)\right)^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Giac [F]

$$\int \sqrt{a+a\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\left(e\tan\left(dx+c\right)\right)^m\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int (e\tan(c+d\,x))^m\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(1/2),x)
[Out] int((e*tan(c + d*x))^m*(a + a/cos(c + d*x))^(1/2), x)
```

3.217 $\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1+m}{2},-\frac{1}{2}+m,1,\frac{3+m}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+m}(e\tan(c+dx))^{1+m}}{de(1+m)\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2^(1/2+m)*AppellF1(1/2+1/2*m,-1/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(1/2+m)*(e*tan(d*x+c))^(1+m)/d/e/(1+m)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3974}

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2^{m+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m+\frac{1}{2}}(e\tan(c+dx))^{m+1}\operatorname{AppellF1}\left(\frac{m+1}{2},m-\frac{1}{2},1,\frac{m+3}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{de(m+1)\sqrt{a\sec(c+dx)+a}}$$

```
[In] Int[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2^(1/2 + m)*AppellF1[(1 + m)/2, -1/2 + m, 1, (3 + m)/2, -((a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1
```

```
+ Sec[c + d*x])^(-1))^(1/2 + m)*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt
[a + a*Sec[c + d*x]])
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{2^{\frac{1}{2}+m} \operatorname{AppellF1}\left(\frac{1+m}{2}, -\frac{1}{2}+m, 1, \frac{3+m}{2}, -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right) \left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+m} (e\tan(c+dx))^{1+m}}{de(1+m)\sqrt{a+a\sec(c+dx)}}$$

Mathematica [F]

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$$

```
[In] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
[Out] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]], x]
```

Maple [F]

$$\int \frac{(e\tan(dx+c))^m}{\sqrt{a+a\sec(dx+c)}}dx$$

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\tan(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\tan(c+dx))^m}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral((e*tan(c + d*x))**m/sqrt(a*(sec(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\tan(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\tan(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\tan(c+d\,x))^m}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(1/2),x)
[Out] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(1/2), x)
```

3.218 $\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2^{-\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1+m}{2},-\frac{3}{2}+m,1,\frac{3+m}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right.\cdots}{de(1+m)(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 2^(-1/2+m)*AppellF1(1/2+1/2*m,-3/2+m,1,3/2+1/2*m,(-a+a*sec(d*x+c))/(a+a*sec
(d*x+c)),(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(-1/2+m)*(e*
tan(d*x+c))^(1+m)/d/e/(1+m)/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {3974}

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2^{m-\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{m-\frac{1}{2}}(e\tan(c+dx))^{m+1}\operatorname{AppellF1}\left(\frac{m+1}{2},m-\frac{3}{2},1,\frac{m+3}{2},-\cdots\right.}{de(m+1)(a\sec(c+dx)+a)^{3/2}}$$

```
[In] Int[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2^(-1/2 + m)*AppellF1[(1 + m)/2, -3/2 + m, 1, (3 + m)/2, -((a - a*Sec[c +
d*x])/(a + a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1
 + Sec[c + d*x])^(-1))^(-1/2 + m)*(e*Tan[c + d*x])^(1 + m))/(d*e*(1 + m)*(a
 + a*Sec[c + d*x])^(3/2))
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
```

```
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{2^{-\frac{1}{2}+m}\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{3}{2}+m,1,\frac{3+m}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{-\frac{1}{2}+m}(e\tan(c+dx))^{1+m}}{de(1+m)(a+a\sec(c+dx))^{3/2}}$$

Mathematica [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$$

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
[Out] Integrate[(e*Tan[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2), x]
```

Maple [F]

$$\int \frac{(e\tan(dx+c))^m}{(a+a\sec(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
[Out] int((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*tan(d*x + c))^m/(a^2*sec(d*x + c)^2 +
2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(c+dx))^m}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*tan(d*x+c))**m/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral((e*tan(c + d*x))**m/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(c+d\,x))^m}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(3/2),x)
[Out] int((e*tan(c + d*x))^m/(a + a/cos(c + d*x))^(3/2), x)
```

3.219 $\int (a+a\sec(c+dx))^n \tan^7(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int (a+a\sec(c+dx))^n \tan^7(c+dx)\,dx = \frac{7(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)} + \frac{\text{Hypergeometric2F1}(1,4+n,5+n,1+\sec(c+dx))(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)} - \frac{5(a+a\sec(c+dx))^{5+n}}{a^5d(5+n)} + \frac{(a+a\sec(c+dx))^{6+n}}{a^6d(6+n)}$$

```
[Out] 7*(a+a*sec(d*x+c))^(4+n)/a^4/d/(4+n)+hypergeom([1, 4+n],[5+n],1+sec(d*x+c))
*(a+a*sec(d*x+c))^(4+n)/a^4/d/(4+n)-5*(a+a*sec(d*x+c))^(5+n)/a^5/d/(5+n)+(a
+a*sec(d*x+c))^(6+n)/a^6/d/(6+n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3965, 90, 67}

$$\int (a+a\sec(c+dx))^n \tan^7(c+dx)\,dx = \frac{(a\sec(c+dx)+a)^{n+6}}{a^6d(n+6)} - \frac{5(a\sec(c+dx)+a)^{n+5}}{a^5d(n+5)} + \frac{(a\sec(c+dx)+a)^{n+4}\,\text{Hypergeometric2F1}(1,n+4,n+5,\sec(c+dx)+1)}{a^4d(n+4)} + \frac{7(a\sec(c+dx)+a)^{n+4}}{a^4d(n+4)}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^7,x]
```

```
[Out] (7*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(4 + n)) + (Hypergeometric2F1[1, 4
+ n, 5 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(4 + n))
 - (5*(a + a*Sec[c + d*x])^(5 + n))/(a^5*d*(5 + n)) + (a + a*Sec[c + d*x])^
(6 + n)/(a^6*d*(6 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-a+ax)^3(a+ax)^{3+n}}{x}\,dx, x, \sec(c+dx)\right)}{a^6 d} \\
&= \frac{\text{Subst}\left(\int \left(7a^3(a+ax)^{3+n} - \frac{a^3(a+ax)^{3+n}}{x} - 5a^2(a+ax)^{4+n} + a(a+ax)^{5+n}\right)\,dx, x, \sec(c+dx)\right)}{a^6 d} \\
&= \frac{7(a+a\sec(c+dx))^{4+n}}{a^4 d(4+n)} - \frac{5(a+a\sec(c+dx))^{5+n}}{a^5 d(5+n)} \\
&\quad + \frac{(a+a\sec(c+dx))^{6+n}}{a^6 d(6+n)} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{3+n}}{x}\,dx, x, \sec(c+dx)\right)}{a^3 d} \\
&= \frac{7(a+a\sec(c+dx))^{4+n}}{a^4 d(4+n)} \\
&\quad + \frac{\text{Hypergeometric2F1}(1, 4+n, 5+n, 1+\sec(c+dx))(a+a\sec(c+dx))^{4+n}}{a^4 d(4+n)} \\
&\quad - \frac{5(a+a\sec(c+dx))^{5+n}}{a^5 d(5+n)} + \frac{(a+a\sec(c+dx))^{6+n}}{a^6 d(6+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int (a+a\sec(c+dx))^n\tan^7(c+dx)\,dx$$
$$=\frac{(1+\sec(c+dx))^4(a(1+\sec(c+dx)))^n\left(\frac{7}{4+n}+\frac{\text{Hypergeometric2F1}(1,4+n,5+n,1+\sec(c+dx))}{4+n}-\frac{5(1+\sec(c+dx))}{5+n}+\frac{(1+\text{se}}{}\right.}{d}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^7,x]
```

```
[Out] ((1 + Sec[c + d*x])^4*(a*(1 + Sec[c + d*x]))^n*(7/(4 + n) + Hypergeometric2
F1[1, 4 + n, 5 + n, 1 + Sec[c + d*x]]/(4 + n) - (5*(1 + Sec[c + d*x]))/(5 +
 n) + (1 + Sec[c + d*x])^2/(6 + n)))/d
```

Maple [F]

$$\int (a+a\sec(dx+c))^n\tan(dx+c)^7\,dx$$

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))^n\tan^7(c+dx)\,dx=\int (a\sec(dx+c)+a)^n\tan(dx+c)^7\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^n\tan^7(c+dx)\,dx=\int (a(\sec(c+dx)+1))^n\tan^7(c+dx)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**7,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**7, x)
```

Maxima [F]

$$\int (a + a\sec(c + dx))^n \tan^7(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^7\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

Giac [F]

$$\int (a + a\sec(c + dx))^n \tan^7(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^7\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^7,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \tan^7(c + dx)\, dx = \int \tan(c + d\,x)^7 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

```
[In] int(tan(c + d*x)^7*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^7*(a + a/cos(c + d*x))^n, x)
```

3.220 $\int (a+a\sec(c+dx))^n \tan^5(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int (a+a\sec(c+dx))^n \tan^5(c+dx)\,dx = -\frac{3(a+a\sec(c+dx))^{3+n}}{a^3d(3+n)} - \frac{\text{Hypergeometric2F1}(1,3+n,4+n,1+\sec(c+dx))(a+a\sec(c+dx))^{3+n}}{a^3d(3+n)} + \frac{(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)}$$

```
[Out] -3*(a+a*sec(d*x+c))^(3+n)/a^3/d/(3+n)-hypergeom([1, 3+n],[4+n],1+sec(d*x+c)
)*(a+a*sec(d*x+c))^(3+n)/a^3/d/(3+n)+(a+a*sec(d*x+c))^(4+n)/a^4/d/(4+n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3965, 90, 67}

$$\int (a+a\sec(c+dx))^n \tan^5(c+dx)\,dx = \frac{(a\sec(c+dx)+a)^{n+4}}{a^4d(n+4)} - \frac{(a\sec(c+dx)+a)^{n+3}\,\text{Hypergeometric2F1}(1,n+3,n+4,\sec(c+dx)+1)}{a^3d(n+3)} - \frac{3(a\sec(c+dx)+a)^{n+3}}{a^3d(n+3)}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] (-3*(a + a*Sec[c + d*x])^(3 + n))/(a^3*d*(3 + n)) - (Hypergeometric2F1[1, 3
 + n, 4 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3 + n))/(a^3*d*(3 + n)
) + (a + a*Sec[c + d*x])^(4 + n)/(a^4*d*(4 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-a+ax)^2(a+ax)^{2+n}}{x}\,dx, x, \sec(c+dx)\right)}{a^4d} \\
&= \frac{\text{Subst}\left(\int \left(-3a^2(a+ax)^{2+n}+\frac{a^2(a+ax)^{2+n}}{x}+a(a+ax)^{3+n}\right)\,dx, x, \sec(c+dx)\right)}{a^4d} \\
&= -\frac{3(a+a\sec(c+dx))^{3+n}}{a^3d(3+n)}+\frac{(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)}+\frac{\text{Subst}\left(\int \frac{(a+ax)^{2+n}}{x}\,dx, x, \sec(c+dx)\right)}{a^2d} \\
&= -\frac{3(a+a\sec(c+dx))^{3+n}}{a^3d(3+n)} \\
&\quad -\frac{\text{Hypergeometric2F1}(1,3+n,4+n,1+\sec(c+dx))(a+a\sec(c+dx))^{3+n}}{a^3d(3+n)} \\
&\quad +\frac{(a+a\sec(c+dx))^{4+n}}{a^4d(4+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.74

$$\int (a+a\sec(c+dx))^n\tan^5(c+dx)\,dx$$
$$=\frac{(1+\sec(c+dx))^3(a(1+\sec(c+dx)))^n(-9-2n-(4+n)\,\text{Hypergeometric2F1}(1,3+n,4+n,1+\sec(c$$
$$d(3+n)(4+n)$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
[Out] ((1 + Sec[c + d*x])^3*(a*(1 + Sec[c + d*x]))^n*(-9 - 2*n - (4 + n)*Hypergeo
metric2F1[1, 3 + n, 4 + n, 1 + Sec[c + d*x]] + (3 + n)*Sec[c + d*x]))/(d*(3
 + n)*(4 + n))
```

Maple [F]

$$\int (a+a\sec(dx+c))^n\tan(dx+c)^5\,dx$$

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x)
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))^n\tan^5(c+dx)\,dx=\int (a\sec(dx+c)+a)^n\tan(dx+c)^5\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^n\tan^5(c+dx)\,dx=\int (a(\sec(c+dx)+1))^n\tan^5(c+dx)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**5,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**5, x)
```

Maxima [F]

$$\int (a + a \sec(c + dx))^n \tan^5(c + dx) \, dx = \int (a \sec(dx + c) + a)^n \tan(dx + c)^5 \, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Giac [F]

$$\int (a + a \sec(c + dx))^n \tan^5(c + dx) \, dx = \int (a \sec(dx + c) + a)^n \tan(dx + c)^5 \, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a \sec(c + dx))^n \tan^5(c + dx) \, dx = \int \tan(c + d\,x)^5 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n \, dx$$

```
[In] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^5*(a + a/cos(c + d*x))^n, x)
```

3.221 $\int (a+a\sec(c+dx))^n \tan^3(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int (a+a\sec(c+dx))^n \tan^3(c+dx)\,dx = \frac{(a+a\sec(c+dx))^{2+n}}{a^2d(2+n)} + \frac{\text{Hypergeometric2F1}(1,2+n,3+n,1+\sec(c+dx))(a+a\sec(c+dx))^{2+n}}{a^2d(2+n)}$$

```
[Out] (a+a*sec(d*x+c))^(2+n)/a^2/d/(2+n)+hypergeom([1, 2+n],[3+n],1+sec(d*x+c))*(
a+a*sec(d*x+c))^(2+n)/a^2/d/(2+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3965, 81, 67}

$$\int (a+a\sec(c+dx))^n \tan^3(c+dx)\,dx = \frac{(a\sec(c+dx)+a)^{n+2}\,\text{Hypergeometric2F1}(1,n+2,n+3,\sec(c+dx)+1)}{a^2d(n+2)} + \frac{(a\sec(c+dx)+a)^{n+2}}{a^2d(n+2)}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] (a + a*Sec[c + d*x])^(2 + n)/(a^2*d*(2 + n)) + (Hypergeometric2F1[1, 2 + n,
 3 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2 + n))/(a^2*d*(2 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-a+ax)(a+ax)^{1+n}}{x}\,dx, x, \sec(c+dx)\right)}{a^2 d}\\
&= \frac{(a+a\sec(c+dx))^{2+n}}{a^2 d(2+n)} - \frac{\text{Subst}\left(\int \frac{(a+ax)^{1+n}}{x}\,dx, x, \sec(c+dx)\right)}{ad}\\
&= \frac{(a+a\sec(c+dx))^{2+n}}{a^2 d(2+n)}\\
&\quad + \frac{\text{Hypergeometric2F1}(1,2+n,3+n,1+\sec(c+dx))(a+a\sec(c+dx))^{2+n}}{a^2 d(2+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.71

$$\int (a+a\sec(c+dx))^n \tan^3(c+dx)\,dx$$
$$= \frac{(1+\text{Hypergeometric2F1}(1,2+n,3+n,1+\sec(c+dx)))(1+\sec(c+dx))^2(a(1+\sec(c+dx)))^n}{d(2+n)}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
[Out] ((1 + Hypergeometric2F1[1, 2 + n, 3 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*
x])^2*(a*(1 + Sec[c + d*x]))^n)/(d*(2 + n))
```

Maple [F]

$$\int (a + a\sec(dx + c))^n \tan(dx + c)^3\, dx$$

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x)
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

Fricas [F]

$$\int (a + a\sec(c + dx))^n \tan^3(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^3\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Sympy [F]

$$\int (a + a\sec(c + dx))^n \tan^3(c + dx)\, dx = \int (a(\sec(c + dx) + 1))^n \tan^3(c + dx)\, dx$$

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**3,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**3, x)
```

Maxima [F]

$$\int (a + a\sec(c + dx))^n \tan^3(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^3\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Giac [F]

$$\int (a + a\sec(c + dx))^n \tan^3(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^3\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \tan^3(c + dx)\, dx = \int \tan(c + d\,x)^3 \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

```
[In] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^3*(a + a/cos(c + d*x))^n, x)
```

3.222 $\int (a+a\sec(c+dx))^n \tan(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int (a+a\sec(c+dx))^n \tan(c+dx)\,dx = -\frac{\text{Hypergeometric2F1}(1,1+n,2+n,1+\sec(c+dx))(a+a\sec(c+dx))^{1+n}}{ad(1+n)}$$

```
[Out] -hypergeom([1, 1+n],[2+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(1+n)/a/d/(1+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3965, 67}

$$\int (a+a\sec(c+dx))^n \tan(c+dx)\,dx = -\frac{(a\sec(c+dx)+a)^{n+1}\,\text{Hypergeometric2F1}(1,n+1,n+2,\sec(c+dx)+1)}{ad(n+1)}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x]
)^(1 + n))/(a*d*(1 + n)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
```

```
|| GtQ[-d/(b*c), 0])
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+ax)^n}{x}\,dx, x, \sec(c+dx)\right)}{d}\\
&= -\frac{\text{Hypergeometric2F1}(1, 1+n, 2+n, 1+\sec(c+dx))(a+a\sec(c+dx))^{1+n}}{ad(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int (a+a\sec(c+dx))^n\tan(c+dx)\,dx\\
&= -\frac{\text{Hypergeometric2F1}(1, 1+n, 2+n, 1+\sec(c+dx))(a(1+\sec(c+dx)))^{1+n}}{ad(1+n)}
\end{aligned}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x],x]
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a*(1 + Sec[c + d*x
]))^(1 + n))/(a*d*(1 + n)))
```

Maple [F]

$$\int (a+a\sec(dx+c))^n\tan(dx+c)\,dx$$

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c),x)
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c),x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))^n\tan(c+dx)\,dx=\int (a\sec{(dx+c)}+a)^n\tan{(dx+c)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^n\tan(c+dx)\,dx=\int (a(\sec{(c+dx)}+1))^n\tan{(c+dx)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x), x)
```

Maxima [F]

$$\int (a+a\sec(c+dx))^n\tan(c+dx)\,dx=\int (a\sec{(dx+c)}+a)^n\tan{(dx+c)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Giac [F]

$$\int (a+a\sec(c+dx))^n\tan(c+dx)\,dx=\int (a\sec{(dx+c)}+a)^n\tan{(dx+c)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^n\tan(c+dx)\,dx=\int \tan(c+d\,x)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

```
[In] int(tan(c + d*x)*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)*(a + a/cos(c + d*x))^n, x)
```

3.223 $\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn} + \frac{\text{Hypergeometric2F1}(1,n,1+n,1+\sec(c+dx))(a+a\sec(c+dx))^n}{dn}$$

```
[Out] -1/2*hypergeom([1, n],[1+n],1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^n/d/n+hype
rgeom([1, n],[1+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^n/d/n
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3965, 88, 67, 70}

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \frac{(a\sec(c+dx)+a)^n\,\text{Hypergeometric2F1}(1,n,n+1,\sec(c+dx)+1)}{dn} - \frac{(a\sec(c+dx)+a)^n\,\text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{2dn}$$

```
[In] Int[Cot[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d
*x])^n)/(d*n) + (Hypergeometric2F1[1, n, 1 + n, 1 + Sec[c + d*x]]*(a + a*Se
c[c + d*x])^n)/(d*n)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a^2\text{Subst}\left(\int \frac{(a+ax)^{-1+n}}{x(-a+ax)}\,dx, x, \sec(c+dx)\right)}{d}\\
&= -\frac{a\text{Subst}\left(\int \frac{(a+ax)^{-1+n}}{x}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\text{Subst}\left(\int \frac{(a+ax)^{-1+n}}{-a+ax}\,dx, x, \sec(c+dx)\right)}{d}\\
&= -\frac{\text{Hypergeometric2F1}\left(1, n, 1+n, \frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn}\\
&\quad + \frac{\text{Hypergeometric2F1}(1, n, 1+n, 1+\sec(c+dx))(a+a\sec(c+dx))^n}{dn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{\left(\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+\sec(c+dx))\right)-2\,\text{Hypergeometric2F1}(1,n,1+n,1+\sec(c+$$
$$2dn$$

[In] Integrate[Cot[c + d*x]*(a + a*Sec[c + d*x])^n,x]

[Out] -1/2*((Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2] - 2*Hypergeometric2F1[1, n, 1 + n, 1 + Sec[c + d*x]])*(a*(1 + Sec[c + d*x]))^n)/(d*n)

Maple [F]

$$\int \cot(dx+c)\,(a+a\sec(dx+c))^n\,dx$$

[In] int(cot(d*x+c)*(a+a*sec(d*x+c))^n,x)

[Out] int(cot(d*x+c)*(a+a*sec(d*x+c))^n,x)

Fricas [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="fricas")

[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c), x)

Sympy [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a(\sec(c+dx)+1))^n\cot(c+dx)\,dx$$

[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))**n,x)

[Out] Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x), x)

Maxima [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Giac [F]

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

```
[In] integrate(cot(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+a\sec(c+dx))^n\,dx = \int \cot(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int(cot(c + d*x)*(a + a/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)*(a + a/cos(c + d*x))^n, x)
```

3.224 $\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{a(4-n)\,\text{Hypergeometric2F1}\left(1,-1+n,n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)} + \frac{a\,\text{Hypergeometric2F1}(1,-1+n,n,1+\sec(c+dx))(a+a\sec(c+dx))^{-1+n}}{d(1-n)} + \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))}$$

```
[Out] -1/4*a*(4-n)*hypergeom([1, -1+n],[n],1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^(
-1+n)/d/(1-n)+a*hypergeom([1, -1+n],[n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(-1+
n)/d/(1-n)+1/2*a*(a+a*sec(d*x+c))^(-1+n)/d/(1-sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3965, 105, 162, 67, 70}

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{a(4-n)(a\sec(c+dx)+a)^{n-1}\,\text{Hypergeometric2F1}\left(1,n-1,n,\frac{1}{2}(\sec(c+dx)+1)\right)}{4d(1-n)} + \frac{a(a\sec(c+dx)+a)^{n-1}\,\text{Hypergeometric2F1}(1,n-1,n,\sec(c+dx)+1)}{d(1-n)} + \frac{a(a\sec(c+dx)+a)^{n-1}}{2d(1-\sec(c+dx))}$$

```
[In] Int[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/4*(a*(4 - n)*Hypergeometric2F1[1, -1 + n, n, (1 + Sec[c + d*x])/2]*(a +
a*Sec[c + d*x])^(-1 + n))/(d*(1 - n)) + (a*Hypergeometric2F1[1, -1 + n, n,
1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(-1 + n))/(d*(1 - n)) + (a*(a + a*Se
c[c + d*x])^(-1 + n))/(2*d*(1 - Sec[c + d*x]))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 3965

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(d*b^(m - 1))^(-1), Subst[Int[(-a + b*x)^((m - 1)/2)
*((a + b*x)^((m - 1)/2 + n)/x), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{a^4\text{Subst}\left(\int \frac{(a+ax)^{-2+n}}{x(-a+ax)^2}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} - \frac{a\text{Subst}\left(\int \frac{(a+ax)^{-2+n}(2a^2+a^2(2-n)x)}{x(-a+ax)}\,dx, x, \sec(c+dx)\right)}{2d}$$
$$= \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} + \frac{a^2\text{Subst}\left(\int \frac{(a+ax)^{-2+n}}{x}\,dx, x, \sec(c+dx)\right)}{d} - \frac{(a^3(4-n))\,\text{Subst}\left(\int \frac{(a+ax)^{-2+n}}{-a+ax}\,dx, x, \sec(c+dx)\right)}{2d}$$
$$= -\frac{a(4-n)\,\text{Hypergeometric2F1}\left(1,-1+n,n,\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)} + \frac{a\,\text{Hypergeometric2F1}(1,-1+n,n,1+\sec(c+dx))(a+a\sec(c+dx))^{-1+n}}{d(1-n)} + \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{a\left(-2+2n+(-4+n)\,\text{Hypergeometric2F1}\left(1,-1+n,n,\frac{1}{2}(1+\sec(c+dx))\right)(-1+\sec(c+dx))+4\,\text{I}\right.}{4d(-1+n)(-1+\text{s}}$$

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/4*(a*(-2 + 2*n + (-4 + n)*Hypergeometric2F1[1, -1 + n, n, (1 + Sec[c + d
*x])/2]*(-1 + Sec[c + d*x]) + 4*Hypergeometric2F1[1, -1 + n, n, 1 + Sec[c +
 d*x]]*(-1 + Sec[c + d*x]))*(a*(1 + Sec[c + d*x]))^(-1 + n))/(d*(-1 + n)*(-
1 + Sec[c + d*x]))
```

Maple [F]

$$\int \cot(dx+c)^3 (a+a\sec(dx+c))^n \, dx$$

```
[In] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
[Out] int(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a\sec(dx+c)+a)^n \cot(dx+c)^3 \, dx$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Sympy [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a(\sec(c+dx)+1))^n \cot^3(c+dx) \, dx$$

```
[In] integrate(cot(d*x+c)**3*(a+a*sec(d*x+c))**n,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x)**3, x)
```

Maxima [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a\sec(dx+c)+a)^n \cot(dx+c)^3 \, dx$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Giac [F]

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n\cot(dx+c)^3\,dx$$

```
[In] integrate(cot(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+a\sec(c+dx))^n\,dx = \int \cot(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^3*(a + a/cos(c + d*x))^n, x)
```

3.225 $\int (a+a\sec(c+dx))^n \tan^4(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int (a+a\sec(c+dx))^n \tan^4(c+dx)\,dx = \frac{2^{5+n}\operatorname{AppellF1}\left(\frac{5}{2},4+n,1,\frac{7}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{5+n}(a+a\sec(c+dx))^n\tan^5(c+dx)}{5d}$$

```
[Out] 1/5*2^(5+n)*AppellF1(5/2,4+n,1,7/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a*
sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(5+n)*(a+a*sec(d*x+c))^n*t
an(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3974}

$$\int (a+a\sec(c+dx))^n \tan^4(c+dx)\,dx = \frac{2^{n+5}\tan^5(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+5}(a\sec(c+dx)+a)^n\operatorname{AppellF1}\left(\frac{5}{2},n+4,1,\frac{7}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{5d}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
```

```
[Out] (2^(5 + n)*AppellF1[5/2, 4 + n, 1, 7/2, -((a - a*Sec[c + d*x])/(a + a*Sec[c
 + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x])^(
-1))^(5 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^5)/(5*d)
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{2^{5+n}\,\text{AppellF1}\left(\frac{5}{2},4+n,1,\frac{7}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{5+n}(a+a\sec(c+dx))^n\tan^5(c+d}{5d}$$

Mathematica [F]

$$\int (a+a\sec(c+dx))^n\tan^4(c+dx)\,dx=\int (a+a\sec(c+dx))^n\tan^4(c+dx)\,dx$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
[Out] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^4, x]
```

Maple [F]

$$\int (a+a\sec(dx+c))^n\tan(dx+c)^4\,dx$$

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x)
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

Fricas [F]

$$\int (a+a\sec(c+dx))^n\tan^4(c+dx)\,dx=\int (a\sec(dx+c)+a)^n\tan(dx+c)^4\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Sympy [F]

$$\int (a+a\sec(c+dx))^n\tan^4(c+dx)\,dx=\int (a(\sec{(c+dx)}+1))^n\tan^4{(c+dx)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**4,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**4, x)
```

Maxima [F]

$$\int (a+a\sec(c+dx))^n\tan^4(c+dx)\,dx=\int (a\sec{(dx+c)}+a)^n\tan{(dx+c)}^4\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Giac [F]

$$\int (a+a\sec(c+dx))^n\tan^4(c+dx)\,dx=\int (a\sec{(dx+c)}+a)^n\tan{(dx+c)}^4\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^n\tan^4(c+dx)\,dx=\int \tan(c+d\,x)^4\left(a+\frac{a}{\cos{(c+d\,x)}}\right)^n\,dx$$

```
[In] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^4*(a + a/cos(c + d*x))^n, x)
```

3.226 $\int (a+a\sec(c+dx))^n \tan^2(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int (a+a\sec(c+dx))^n \tan^2(c+dx)\,dx = \frac{2^{3+n}\operatorname{AppellF1}\left(\frac{3}{2},2+n,1,\frac{5}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{3+n}(a+a\sec(c+dx))^n\tan^3(c+d}{3d}$$

```
[Out] 1/3*2^(3+n)*AppellF1(3/2,2+n,1,5/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a*
sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(3+n)*(a+a*sec(d*x+c))^n*t
an(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3974}

$$\int (a+a\sec(c+dx))^n \tan^2(c+dx)\,dx = \frac{2^{n+3}\tan^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+3}(a\sec(c+dx)+a)^n\operatorname{AppellF1}\left(\frac{3}{2},n+2,1,\frac{5}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx}{\sec(c+dx)a+a}\right.}{3d}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

```
[Out] (2^(3 + n)*AppellF1[3/2, 2 + n, 1, 5/2, -((a - a*Sec[c + d*x])/(a + a*Sec[c
 + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x])^(
-1))^(3 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^3)/(3*d)
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{2^{3+n}\,\mathrm{AppellF1}\left(\frac{3}{2},2+n,1,\frac{5}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{3+n}(a+a\sec(c+dx))^n\tan^3(c+dx)}{3d}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 910 vs. $2(106) = 212$.

Time = 6.33 (sec) , antiderivative size = 910, normalized size of antiderivative = 8.58

$$\int (a+a\sec(c+dx))^n\tan^2(c+dx)\,dx$$

$$= \frac{(a(1+\sec(c+dx)))^n\left(-\frac{4\,\mathrm{Hypergeometric2F1}\left(-1-n,n,-n,\frac{1}{2}\left(1-\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^n(1+\sec(c+dx))^{-n}(1+\mathrm{ta}}{(1+n)\left(-1+\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^n*((-4*Hypergeometric2F1[-1 - n, n, -n, (1 - Tan[(c
 + d*x)/2])/2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(1 + Tan[(c + d*x)/2])^n
)/((1 + n)*(1 + Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])) - (Hypergeometric2
F1[1 - n, 2 + n, 2 - n, (1 - Tan[(c + d*x)/2])/2]*(Cos[(c + d*x)/2]^2*Sec[c
 + d*x])^n*(-1 + Tan[(c + d*x)/2])*(1 + Tan[(c + d*x)/2])^n)/((-1 + n)*(1 +
 Sec[c + d*x])^n) - (120*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*Cos[c + d*x]*Sin[c + d*x]*(3*AppellF1[1/
2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(AppellF1[3/2, n
, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(45
*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c
 + d*x)/2]^2*(1 + 2*n - 2*n*Cos[c + d*x] + Cos[2*(c + d*x)]) + 6*AppellF1[1
/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c + d*x)/2]^2*
(-5*AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 +
2*n - 2*(2 + n)*Cos[c + d*x] + Cos[2*(c + d*x)]) + 5*n*AppellF1[3/2, 1 + n,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + 2*n - 2*(2 + n)*Cos[
c + d*x] + Cos[2*(c + d*x)]) - 48*(2*AppellF1[5/2, n, 3, 7/2, Tan[(c + d*x)
```

```
/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF1[5/2, 1 + n, 2, 7/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] + n*(1 + n)*AppellF1[5/2, 2 + n, 1, 7/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cot[c + d*x]*Csc[c + d*x]*Sin[(c + d*x
)/2]^4) + 40*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2])^2*Cos[c + d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)))/(4*d)
```

Maple [F]

$$\int (a+a\sec (dx+c))^{n} \tan (dx+c)^{2} dx$$

[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x)

[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x)

Fricas [F]

$$\int (a+a\sec(c+dx))^n \tan^2(c+dx)\,dx = \int (a\sec (dx+c)+a)^{n} \tan (dx+c)^{2}\,dx$$

[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="fricas")

[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)

Sympy [F]

$$\int (a+a\sec(c+dx))^n \tan^2(c+dx)\,dx = \int (a(\sec (c+dx)+1))^{n} \tan^{2} (c+dx)\,dx$$

[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**2,x)

[Out] Integral((a*(sec(c + d*x) + 1))**n*tan(c + d*x)**2, x)

Maxima [F]

$$\int (a+a\sec(c+dx))^n \tan^2(c+dx)\,dx = \int (a\sec (dx+c)+a)^{n} \tan (dx+c)^{2}\,dx$$

[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)

Giac [F]

$$\int (a+a\sec(c+dx))^n\tan^2(c+dx)\,dx=\int (a\sec{(dx+c)}+a)^n\tan{(dx+c)}^2\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(c+dx))^n\tan^2(c+dx)\,dx=\int \tan(c+d\,x)^2\,\left(a+\frac{a}{\cos{(c+d\,x)}}\right)^n\,dx$$

```
[In] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^2*(a + a/cos(c + d*x))^n, x)
```

3.227 $\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{2^{-1+n}\operatorname{AppellF1}\left(-\frac{1}{2},-2+n,1,\frac{1}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{-1+n}(a+a\sec(}{d}$$

```
[Out] -2^(-1+n)*AppellF1(-1/2,-2+n,1,1/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a*
sec(d*x+c))/(a+a*sec(d*x+c)))*cot(d*x+c)*(1/(1+sec(d*x+c)))^(-1+n)*(a+a*sec
(d*x+c))^n/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3974}

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{2^{n-1}\cot(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-1}(a\sec(c+dx)+a)^n\operatorname{AppellF1}\left(-\frac{1}{2},n-2,1,\frac{1}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c}{\sec(c+dx)}\right.}{d}$$

```
[In] Int[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((2^(-1 + n)*AppellF1[-1/2, -2 + n, 1, 1/2, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*Cot[c + d*x]*((1
 + Sec[c + d*x])^(-1))^(-1 + n)*(a + a*Sec[c + d*x])^n)/d)
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

integral =

$$-\frac{2^{-1+n}\operatorname{AppellF1}\left(-\frac{1}{2},-2+n,1,\frac{1}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{-1+n}(a+a\sec(c}{d}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 893 vs. $2(102) = 204$.

Time = 3.28 (sec) , antiderivative size = 893, normalized size of antiderivative = 8.75

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx$$

$$=\frac{(a(1+\sec(c+dx)))^n\left(-2^n\cot\left(\frac{1}{2}(c+dx)\right)\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},n,\frac{1}{2},\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(\cos(c+dx)\right.}{}$$

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^n*(-((2^n*Cot[(c + d*x)/2]*Hypergeometric2F1[-1/2,
n, 1/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d
*x)/2]^2*Sec[c + d*x])^n)/(1 + Sec[c + d*x])^n) + (2^n*Hypergeometric2F1[1/
2, n, 3/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c
+ d*x)/2]^2*Sec[c + d*x])^n*Tan[(c + d*x)/2])/(1 + Sec[c + d*x])^n - (60*Ap
pellF1[1/2, n, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*
x)/2]^2*Cos[c + d*x]*Sin[c + d*x]*(3*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] - 2*(AppellF1[3/2, n, 2, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(45*AppellF1[1/2, n, 1, 3/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c + d*x)/2]^2*(1 + 2*n - 2*n
*Cos[c + d*x] + Cos[2*(c + d*x)]) + 6*AppellF1[1/2, n, 1, 3/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c + d*x)/2]^2*(-5*AppellF1[3/2, n, 2, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + 2*n - 2*(2 + n)*Cos[c + d*x
] + Cos[2*(c + d*x)]) + 5*n*AppellF1[3/2, 1 + n, 1, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*(1 + 2*n - 2*(2 + n)*Cos[c + d*x] + Cos[2*(c + d*x)]
) - 48*(2*AppellF1[5/2, n, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
```

```
 - 2*n*AppellF1[5/2, 1 + n, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
] + n*(1 + n)*AppellF1[5/2, 2 + n, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2])*Cot[c + d*x]*Csc[c + d*x]*Sin[(c + d*x)/2]^4) + 40*(AppellF1[3/2,
 n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - n*AppellF1[3/2, 1 +
n, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])^2*Cos[c + d*x]*Sin[(c
+ d*x)/2]^2*Tan[(c + d*x)/2]^2)))/(2*d)
```

Maple [F]

$$\int \cot (dx + c)^2 (a + a \sec (dx + c))^n \, dx$$

[In] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x)

[Out] int(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x)

Fricas [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a \sec (dx + c) + a)^n \cot (dx + c)^2 \, dx$$

[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="fricas")

[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)

Sympy [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a(\sec (c + dx) + 1))^n \cot^2 (c + dx) \, dx$$

[In] integrate(cot(d*x+c)**2*(a+a*sec(d*x+c))**n,x)

[Out] Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x)**2, x)

Maxima [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n \, dx = \int (a \sec (dx + c) + a)^n \cot (dx + c)^2 \, dx$$

[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)

Giac [F]

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n\cot(dx+c)^2\,dx$$

```
[In] integrate(cot(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+dx)(a+a\sec(c+dx))^n\,dx = \int \cot(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^2*(a + a/cos(c + d*x))^n, x)
```

3.228 $\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{2^{-3+n}\operatorname{AppellF1}\left(-\frac{3}{2},-4+n,1,-\frac{1}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot^3(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{-3+n}(a+a\,s}{3d}$$

```
[Out] -1/3*2^(-3+n)*AppellF1(-3/2,-4+n,1,-1/2,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),
(a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*cot(d*x+c)^3*(1/(1+sec(d*x+c)))^(-3+n)*(
a+a*sec(d*x+c))^n/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3974}

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{2^{n-3}\cot^3(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n-3}(a\sec(c+dx)+a)^n\operatorname{AppellF1}\left(-\frac{3}{2},n-4,1,-\frac{1}{2},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\,s}{\sec(c}\right.}{3d}$$

```
[In] Int[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/3*(2^(-3 + n)*AppellF1[-3/2, -4 + n, 1, -1/2, -((a - a*Sec[c + d*x])/(a
+ a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*Cot[c + d*x]
^3*((1 + Sec[c + d*x])^(-1))^(-3 + n)*(a + a*Sec[c + d*x])^n)/d
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

integral =

$$-\frac{2^{-3+n}\,\mathrm{AppellF1}\left(-\frac{3}{2},-4+n,1,-\frac{1}{2},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\cot^3(c+dx)\left(\frac{1}{1+\sec(c+dx)}\right)^{-3+n}(a+a\,\mathrm{se}}{3d}$$

Mathematica [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx=\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx$$

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
[Out] Integrate[Cot[c + d*x]^4*(a + a*Sec[c + d*x])^n, x]
```

Maple [F]

$$\int \cot\left(dx+c\right)^4\left(a+a\sec\left(dx+c\right)\right)^n dx$$

```
[In] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
[Out] int(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx=\int \left(a\sec\left(dx+c\right)+a\right)^n\cot\left(dx+c\right)^4\,dx$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Sympy [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a(\sec(c+dx)+1))^n \cot^4(c+dx)\,dx$$

```
[In] integrate(cot(d*x+c)**4*(a+a*sec(d*x+c))**n,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*cot(c + d*x)**4, x)
```

Maxima [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \cot(dx+c)^4\,dx$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Giac [F]

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int (a\sec(dx+c)+a)^n \cot(dx+c)^4\,dx$$

```
[In] integrate(cot(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+a\sec(c+dx))^n\,dx = \int \cot(c+d\,x)^4\left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

```
[In] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^4*(a + a/cos(c + d*x))^n, x)
```

3.229 $\int (a+a\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int (a+a\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx = \frac{2^{\frac{7}{2}+n}\operatorname{AppellF1}\left(\frac{5}{4},\frac{3}{2}+n,1,\frac{9}{4},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{5}{2}+n}(a+a\sec(c+dx))^n\tan^{\frac{5}{2}}(c+d}{5d}$$

```
[Out] 1/5*2^(7/2+n)*AppellF1(5/4,3/2+n,1,9/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(
a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(5/2+n)*(a+a*sec(d*x+c
))^n*tan(d*x+c)^(5/2)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3974}

$$\int (a+a\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx = \frac{2^{n+\frac{7}{2}}\tan^{\frac{5}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{5}{2}}(a\sec(c+dx)+a)^n\operatorname{AppellF1}\left(\frac{5}{4},n+\frac{3}{2},1,\frac{9}{4},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right.}{5d}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
```

```
[Out] (2^(7/2 + n)*AppellF1[5/4, 3/2 + n, 1, 9/4, -((a - a*Sec[c + d*x])/(a + a*S
ec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x
])^(-1))^(5/2 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(5/2))/(5*d)
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{2^{\frac{7}{2}+n} \operatorname{AppellF1}\left(\frac{5}{4}, \frac{3}{2}+n, 1, \frac{9}{4}, -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right) \left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{5}{2}+n} (a+a\sec(c+dx))^n \tan^{\frac{5}{2}}(c+}{5d}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2072 vs. $2(114) = 228$.

Time = 27.34 (sec) , antiderivative size = 2072, normalized size of antiderivative = 18.18

$$\int (a+a\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\, dx = \text{Result too large to show}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
[Out] (2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c + d*x]))^n*(-1
 + Tan[(c + d*x)/2])^(-1/2 - n)*(-2*AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2 + n
)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2,
 Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2
, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-
1 + Tan[(c + d*x)/2]^2)^(1/2 + n))*Tan[c + d*x]^2)/(d*((2^n*Sec[c + d*x]^2*
(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-1/2 - n)*(-2*
AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Co
s[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n)
+ (AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]
] + AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2
]])*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n)))/
Sqrt[Tan[c + d*x]] + 2^n*(-1/2 - n)*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*
Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-3/2 - n)*(-2*AppellF1[1/4, 1/2 +
n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c +
d*x)/2]^2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (AppellF1[1/2, 1/2
 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/
2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*(1 - Tan[(c + d*
x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n))*Sqrt[Tan[c + d*x]] +
```

```
2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(-1 + Tan[(c + d*x)/2])^(-1/2
 - n)*(-((1/2 + n)*AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/2 +
 n)*(-1 + Tan[(c + d*x)/2])^(-1/2 + n)) - 2*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n)*(-1/5*(AppellF1[5/4, 1/2 + n
, 2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]) + ((1/2 + n)*AppellF1[5/4, 3/2 + n, 1, 9/4, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) - 2*(1/2 + n
)*AppellF1[1/4, 1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(
Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 +
n)*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]) + (1/2 + n)*(AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c +
d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c +
 d*x)/2], -Tan[(c + d*x)/2]])*Sec[(c + d*x)/2]^2*(1 - Tan[(c + d*x)/2])^(1/
2 + n)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)^(-1/2 + n) - ((1/2 + n)*(
AppellF1[1/2, 1/2 + n, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] +
 AppellF1[1/2, 3/2 + n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])
*Sec[(c + d*x)/2]^2*(1 - Tan[(c + d*x)/2])^(-1/2 + n)*(-1 + Tan[(c + d*x)/2
]^2)^(1/2 + n))/2 + (-1/6*((3/2 + n)*AppellF1[3/2, 1/2 + n, 5/2 + n, 5/2, T
an[(c + d*x)/2], -Tan[(c + d*x)/2]]*Sec[(c + d*x)/2]^2) + ((3/2 + n)*Appell
F1[3/2, 5/2 + n, 1/2 + n, 5/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]]*Sec[(c
+ d*x)/2]^2)/6)*(1 - Tan[(c + d*x)/2])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^
(1/2 + n))*Sqrt[Tan[c + d*x]] + 2^(1 + n)*n*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(-1 + n)*(-1 + Tan[(c + d*x)/2])^(-1/2 - n)*(-2*AppellF1[1/4, 1/2 + n, 1
, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)
/2]^2)^(1/2 + n)*(-1 + Tan[(c + d*x)/2])^(1/2 + n) + (AppellF1[1/2, 1/2 + n
, 3/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]] + AppellF1[1/2, 3/2 +
n, 1/2 + n, 3/2, Tan[(c + d*x)/2], -Tan[(c + d*x)/2]])*(1 - Tan[(c + d*x)/2
])^(1/2 + n)*(-1 + Tan[(c + d*x)/2]^2)^(1/2 + n))*Sqrt[Tan[c + d*x]]*(-(Cos
[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d
*x]*Tan[c + d*x])))
```

Maple [F]

$$\int (a + a\sec(dx + c))^n \tan(dx + c)^{\frac{3}{2}}\, dx$$

[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)

[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)

Fricas [F]

$$\int (a + a\sec(c + dx))^n \tan^{\frac{3}{2}}(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^{\frac{3}{2}}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \tan^{\frac{3}{2}}(c + dx)\, dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a + a\sec(c + dx))^n \tan^{\frac{3}{2}}(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^{\frac{3}{2}}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Giac [F]

$$\int (a + a\sec(c + dx))^n \tan^{\frac{3}{2}}(c + dx)\, dx = \int (a\sec(dx + c) + a)^n \tan(dx + c)^{\frac{3}{2}}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \tan^{\frac{3}{2}}(c + dx)\, dx = \int \tan(c + d\,x)^{3/2} \left(a + \frac{a}{\cos(c + d\,x)}\right)^n \, dx$$

```
[In] int(tan(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n, x)
```

3.230 $\int (a+a\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int (a+a\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx = \frac{2^{\frac{5}{2}+n}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2}+n,1,\frac{7}{4},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{3}{2}+n}(a+a\sec(c+dx))^n\tan^{\frac{3}{2}}(c+}{3d}$$

```
[Out] 1/3*2^(5/2+n)*AppellF1(3/4,1/2+n,1,7/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(
a-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(3/2+n)*(a+a*sec(d*x+c
))^n*tan(d*x+c)^(3/2)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3974}

$$\int (a+a\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx = \frac{2^{n+\frac{5}{2}}\tan^{\frac{3}{2}}(c+dx)\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{3}{2}}(a\sec(c+dx)+a)^n\operatorname{AppellF1}\left(\frac{3}{4},n+\frac{1}{2},1,\frac{7}{4},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+d}{\sec(c+dx)a-}\right.}{3d}$$

```
[In] Int[(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
```

```
[Out] (2^(5/2 + n)*AppellF1[3/4, 1/2 + n, 1, 7/4, -((a - a*Sec[c + d*x])/(a + a*S
ec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*x
])^(-1))^(3/2 + n)*(a + a*Sec[c + d*x])^n*Tan[c + d*x]^(3/2))/(3*d)
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{2^{\frac{5}{2}+n}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2}+n,1,\frac{7}{4},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{3}{2}+n}(a+a\sec(c+dx))^n\tan^{\frac{3}{2}}(c+d}{3d}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 238 vs. $2(114) = 228$.

Time = 4.88 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.09

$$\int (a+a\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$$

$$= \frac{56\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2}+n,1,\frac{7}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-}{d\left(6\left(2\,\text{AppellF1}\left(\frac{7}{4},\frac{1}{2}+n,2,\frac{11}{4},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-(1+2n)\,\text{AppellF1}\left(\frac{7}{4},\frac{3}{2}+n,1,\frac{1}{4}\right.\right.\right.}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
[Out] (56*AppellF1[3/4, 1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d*x]))^n*Sin[(c + d*x)/2]*Sqrt[Tan[c +
d*x]])/(d*(6*(2*AppellF1[7/4, 1/2 + n, 2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] - (1 + 2*n)*AppellF1[7/4, 3/2 + n, 1, 11/4, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 21*AppellF1[3/4, 1/2 + n, 1,
 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F]

$$\int (a+a\sec(dx+c))^n\sqrt{\tan(dx+c)}dx$$

```
[In] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
[Out] int((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int (a + a\sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx = \int (a\sec(dx + c) + a)^n \sqrt{\tan(dx + c)}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Sympy [F]

$$\int (a + a\sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx = \int (a(\sec(c + dx) + 1))^n \sqrt{\tan(c + dx)}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))**n*tan(d*x+c)**(1/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*sqrt(tan(c + d*x)), x)
```

Maxima [F]

$$\int (a + a\sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx = \int (a\sec(dx + c) + a)^n \sqrt{\tan(dx + c)}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Giac [F]

$$\int (a + a\sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx = \int (a\sec(dx + c) + a)^n \sqrt{\tan(dx + c)}\, dx$$

```
[In] integrate((a+a*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^n \sqrt{\tan(c + dx)}\, dx = \int \sqrt{\tan(c + d\,x)} \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

```
[In] int(tan(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n, x)
```

3.231 $\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \frac{2^{\frac{3}{2}+n}\operatorname{AppellF1}\left(\frac{1}{4},-\frac{1}{2}+n,1,\frac{5}{4},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+n}(a+a\sec(c+dx))^n\sqrt{\tan(c}}{d}$$

```
[Out] 2^(3/2+n)*AppellF1(1/4,-1/2+n,1,5/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a-a
*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(1/2+n)*(a+a*sec(d*x+c))^
n*tan(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3974}

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \frac{2^{n+\frac{3}{2}}\sqrt{\tan(c+dx)}\left(\frac{1}{\sec(c+dx)+1}\right)^{n+\frac{1}{2}}(a\sec(c+dx)+a)^n\operatorname{AppellF1}\left(\frac{1}{4},n-\frac{1}{2},1,\frac{5}{4},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+}{\sec(c+dx)a-}\right.}{d}$$

```
[In] Int[(a + a*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (2^(3/2 + n)*AppellF1[1/4, -1/2 + n, 1, 5/4, -((a - a*Sec[c + d*x])/(a + a*
Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c + d*
x])^(-1))^(1/2 + n)*(a + a*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]])/d
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

integral

$$= \frac{2^{\frac{3}{2}+n} \operatorname{AppellF1}\left(\frac{1}{4}, -\frac{1}{2}+n, 1, \frac{5}{4}, -\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}, \frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{\frac{1}{2}+n}(a+a\sec(c+dx))^n\sqrt{\tan(c+}{d}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 229 vs. $2(111) = 222$.

Time = 9.02 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.06

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \frac{10\operatorname{AppellF1}\left(\frac{1}{4}, -\frac{1}{2}+n, 1, \frac{5}{4}, \tan^2\left(\frac{1}{2}(c+dx)\right)}{d\left(2\left(2\operatorname{AppellF1}\left(\frac{5}{4}, -\frac{1}{2}+n, 2, \frac{9}{4}, \tan^2\left(\frac{1}{2}(c+dx)\right), -\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+(1-2n)\operatorname{AppellF1}\left(\frac{5}{4}, \frac{1}{2}+n, 1,\right.\right.\right.}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] (10*AppellF1[1/4, -1/2 + n, 1, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Cos[c + d*x]*(1 + Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sqrt[Tan[c + d*x
]])/(d*(2*(2*AppellF1[5/4, -1/2 + n, 2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2] + (1 - 2*n)*AppellF1[5/4, 1/2 + n, 1, 9/4, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 5*AppellF1[1/4, -1/2 + n, 1, 5/4
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F]

$$\int \frac{(a+a\sec(dx+c))^n}{\sqrt{\tan(dx+c)}}dx$$

```
[In] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(a\sec\left(dx+c\right)+a)^n}{\sqrt{\tan\left(dx+c\right)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(a(\sec\left(c+dx\right)+1))^n}{\sqrt{\tan\left(c+dx\right)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**n/tan(d*x+c)**(1/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n/sqrt(tan(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(a\sec\left(dx+c\right)+a)^n}{\sqrt{\tan\left(dx+c\right)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(a\sec\left(dx+c\right)+a)^n}{\sqrt{\tan\left(dx+c\right)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sqrt{\tan\left(c+d\,x\right)}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^n/tan(c + d*x)^(1/2),x)
[Out] int((a + a/cos(c + d*x))^n/tan(c + d*x)^(1/2), x)
```

3.232 $\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2^{\frac{1}{2}+n}\operatorname{AppellF1}\left(-\frac{1}{4},-\frac{3}{2}+n,1,\frac{3}{4},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{-\frac{1}{2}+n}(a+a\sec(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

```
[Out] -2^(1/2+n)*AppellF1(-1/4,-3/2+n,1,3/4,(-a+a*sec(d*x+c))/(a+a*sec(d*x+c)),(a
-a*sec(d*x+c))/(a+a*sec(d*x+c)))*(1/(1+sec(d*x+c)))^(-1/2+n)*(a+a*sec(d*x+c
))^n/d/tan(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3974}

$$\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2^{n+\frac{1}{2}}\left(\frac{1}{\sec(c+dx)+1}\right)^{n-\frac{1}{2}}(a\sec(c+dx)+a)^n\operatorname{AppellF1}\left(-\frac{1}{4},n-\frac{3}{2},1,\frac{3}{4},-\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a},\frac{a-a\sec(c+dx)}{\sec(c+dx)a+a}\right)}{d\sqrt{\tan(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] -((2^(1/2 + n)*AppellF1[-1/4, -3/2 + n, 1, 3/4, -((a - a*Sec[c + d*x])/(a +
 a*Sec[c + d*x])), (a - a*Sec[c + d*x])/(a + a*Sec[c + d*x])]*((1 + Sec[c +
 d*x])^(-1))^(-1/2 + n)*(a + a*Sec[c + d*x])^n)/(d*Sqrt[Tan[c + d*x]]))
```

Rule 3974

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Simp[(-2^(m + n + 1))*(e*Cot[c + d*x])^(m + 1)*((a
+ b*Csc[c + d*x])^n/(d*e*(m + 1)))*(a/(a + b*Csc[c + d*x]))^(m + n + 1)*App
ellF1[(m + 1)/2, m + n, 1, (m + 3)/2, -(a - b*Csc[c + d*x])/(a + b*Csc[c +
d*x]), (a - b*Csc[c + d*x])/(a + b*Csc[c + d*x])], x] /; FreeQ[{a, b, c, d,
 e, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

integral =

$$-\frac{2^{\frac{1}{2}+n}\text{AppellF1}\left(-\frac{1}{4},-\frac{3}{2}+n,1,\frac{3}{4},-\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)},\frac{a-a\sec(c+dx)}{a+a\sec(c+dx)}\right)\left(\frac{1}{1+\sec(c+dx)}\right)^{-\frac{1}{2}+n}(a+a\sec(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2164 vs. $2(112) = 224$.

Time = 17.52 (sec) , antiderivative size = 2164, normalized size of antiderivative = 19.32

$$\int \frac{(a+a\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] -1/21*(2^(1/2 + n)*Cot[c + d*x]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c + d*x]))^n*(21*Hypergeometric2
F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n, 1,
7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hyperg
eometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*
AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*T
an[(c + d*x)/2]^4))/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*((2^(-1/2 + n)
*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*Sec[c + d*x]^2*(Cos[(c + d*x)/2]^2*Sec
[c + d*x])^n*(21*Hypergeometric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2]
 + 7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Tan[(c + d*x)/2]^2 + 7*Hypergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*
x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^4))/(21*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Tan[c + d*x]^(3/2)) + (2^(-1/2 + n)*(Cos[c + d*x]*Sec[(c
 + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*((Cos[c + d*x]*Sin[c +
d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*(21*Hypergeom
etric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 +
n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*
```

```
Hypergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^
2 - 3*AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Tan[(c + d*x)/2]^4))/(21*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[
Tan[c + d*x]]) - (2^(1/2 + n)*n*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c
 + d*x)/2]^2*Sec[c + d*x])^(1 + n)*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Co
s[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(21*Hypergeometric2F1[-1/4,
 -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hypergeometric
2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*AppellF1
[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c +
d*x)/2]^4))/(21*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Tan[c + d*x]]) -
 (2^(1/2 + n)*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c
 + d*x])^n*(7*AppellF1[3/4, -1/2 + n, 1, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 7*Hypergeometric2F1[3/4,
-1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 6*
AppellF1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^3 + 7*Tan[(c + d*x)/2]^2*((-3*AppellF1[7
/4, -1/2 + n, 2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])/7 + (3*(-1/2 + n)*AppellF1[7/4, 1/2 + n, 1, 11/4,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/7) - 3*Tan[(c + d*x)/2]^4*((-7*AppellF1[11/4, -1/2 + n, 2, 15/4, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/11
+ (7*(-1/2 + n)*AppellF1[11/4, 1/2 + n, 1, 15/4, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/11) + (21*Csc[(c + d*x)
/2]*Sec[(c + d*x)/2]*(Hypergeometric2F1[-1/4, -1/2 + n, 3/4, Tan[(c + d*x)/
2]^2] - (1 - Tan[(c + d*x)/2]^2)^(1/2 - n)))/4 + (21*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2]*(-Hypergeometric2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2] +
 (1 - Tan[(c + d*x)/2]^2)^(1/2 - n)))/4))/(21*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[Tan[c + d*x]]) - (2^(1/2 + n)*n*(Cos[c + d*x]*Sec[(c + d*x)/2
]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*(21*Hypergeometric2F1[-1/
4, -1/2 + n, 3/4, Tan[(c + d*x)/2]^2] + 7*AppellF1[3/4, -1/2 + n, 1, 7/4, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 + 7*Hypergeometr
ic2F1[3/4, -1/2 + n, 7/4, Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2 - 3*Appell
F1[7/4, -1/2 + n, 1, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c
+ d*x)/2]^4)*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c +
d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(21*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[Tan[c + d*x]])))
```

Maple [F]

$$\int \frac{(a + a\sec(dx + c))^n}{\tan(dx + c)^{\frac{3}{2}}}dx$$

```
[In] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
[Out] int((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\tan^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^n}{\tan(dx + c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Sympy [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\tan^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a(\sec(c + dx) + 1))^n}{\tan^{\frac{3}{2}}(c + dx)}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))**n/tan(d*x+c)**(3/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n/tan(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\tan^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^n}{\tan(dx + c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a + a\sec(c + dx))^n}{\tan^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^n}{\tan(dx + c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c + dx))^n}{\tan^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{\left(a + \frac{a}{\cos(c+d\,x)}\right)^n}{\tan(c + d\,x)^{3/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^n/tan(c + d*x)^(3/2),x)
[Out] int((a + a/cos(c + d*x))^n/tan(c + d*x)^(3/2), x)
```

3.233 $\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 320

$$\begin{aligned}
&\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \\
&-\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} \\
&-\frac{a(e\cot(c+dx))^{5/2}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)}{3d} \\
&+\frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}{\sqrt{2}d} \\
&-\frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}{\sqrt{2}d} \\
&+\frac{a(e\cot(c+dx))^{5/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d} \\
&-\frac{a(e\cot(c+dx))^{5/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d}
\end{aligned}$$

```
[Out] -2/3*(e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))*tan(d*x+c)/d+1/3*a*(e*cot(d*x+c)
)^(5/2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*P
i+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)*tan(d*x+c)^2/d-1/2*a*arctan
(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(5/2)*tan(d*x+c)^(5/2)/d*2^(1/
2)-1/2*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(5/2)*tan(d*x+c)
^(5/2)/d*2^(1/2)+1/4*a*(e*cot(d*x+c))^(5/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c))*tan(d*x+c)^(5/2)/d*2^(1/2)-1/4*a*(e*cot(d*x+c))^(5/2)*ln(1+2^(1/
2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(5/2)/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3985, 3967, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \frac{a\tan^{\frac{5}{2}}(c+dx)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}}{\sqrt{2}d} - \frac{a\tan^{\frac{5}{2}}(c+dx)\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)(e\cot(c+dx))^{5/2}}{\sqrt{2}d} + \frac{a\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} - \frac{a\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} - \frac{2\tan(c+dx)(a\sec(c+dx)+a)(e\cot(c+dx))^{5/2}}{3d} - \frac{a\sqrt{\sin(2c+2dx)}\tan^2(c+dx)\sec(c+dx)\operatorname{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)(e\cot(c+dx))^{5/2}}{3d}$$

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])*Tan[c + d*x])/(3*d) - (a*(e
*Cot[c + d*x])^(5/2)*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c
 + 2*d*x]]*Tan[c + d*x]^2)/(3*d) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) - (a*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[
2]*d) + (a*(e*Cot[c + d*x])^(5/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt[2]*d) - (a*(e*Cot[c + d*x])^(5/2)*Log
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt
[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left((e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int\frac{a+a\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} + \frac{1}{3}\left(2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int\frac{-\frac{3a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx$$

$$
\begin{aligned}
&= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} \\
&\quad -\frac{1}{3}\left(a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int\frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx \\
&\quad -\left(a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int\frac{1}{\sqrt{\tan(c+dx)}}\,dx \\
&= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} \\
&\quad -\frac{\left(a(e\cot(c+dx))^{5/2}\sin^{\frac{5}{2}}(c+dx)\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3\cos^{\frac{5}{2}}(c+dx)} \\
&\quad -\frac{\left(a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{x}(1+x^2)}\,dx,x,\tan(c+dx)\right)}{d} \\
&= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} \\
&\quad -\frac{1}{3}\left(a(e\cot(c+dx))^{5/2}\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)\right)\int\frac{1}{\sqrt{\sin(2c+2dx)}}\,dx \\
&\quad -\frac{\left(2a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d} \\
&= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} \\
&\quad -\frac{a(e\cot(c+dx))^{5/2}\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)}{3d} \\
&\quad -\frac{\left(a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d} \\
&\quad -\frac{\left(a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} \\
&\quad -\frac{a(e\cot(c+dx))^{5/2}\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)}{3d} \\
&\quad -\frac{\left(a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d} \\
&\quad -\frac{\left(a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d} \\
&\quad +\frac{\left(a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad +\frac{\left(a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} \\
&\quad -\frac{a(e\cot(c+dx))^{5/2}\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)}{3d} \\
&\quad +\frac{a(e\cot(c+dx))^{5/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d} \\
&\quad -\frac{a(e\cot(c+dx))^{5/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d} \\
&\quad -\frac{\left(a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad +\frac{\left(a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\tan(c+dx)}{3d} \\
&\quad - \frac{a(e\cot(c+dx))^{5/2}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)}{3d} \\
&\quad + \frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}{\sqrt{2}d} \\
&\quad - \frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}{\sqrt{2}d} \\
&\quad + \frac{a(e\cot(c+dx))^{5/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d} \\
&\quad - \frac{a(e\cot(c+dx))^{5/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.43 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.58

$$
\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx =
$$

$$
-\frac{a(e\cot(c+dx))^{5/2}\sec(c+dx)\left(\sqrt{\cot(c+dx)}\left(4(1+\cos(c+dx))\cot(c+dx)-3\arcsin(\cos(c+dx)-\sin\right.\right.}{}
$$

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/6*(a*(e*Cot[c + d*x])^(5/2)*Sec[c + d*x]*(Sqrt[Cot[c + d*x]]*(4*(1 + Cos
[c + d*x])*Cot[c + d*x] - 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sqrt[Sin[2*
(c + d*x)]] + 3*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]]*S
qrt[Sin[2*(c + d*x)]]) + 2*(-1)^(1/4)*Sqrt[Csc[c + d*x]^2]*EllipticF[I*ArcS
inh[(-1)^(1/4)*Sqrt[Cot[c + d*x]]], -1]*Sin[2*(c + d*x)]))/(d*Cot[c + d*x]^
(5/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.54

method	result
default	$-\frac{e^{2} a \sqrt{2}\, (\cos(dx+c)+1) \left(3 i \sin(dx+c) \sqrt{\csc(dx+c)-\cot(dx+c)+1}\, \sqrt{\cot(dx+c)-\csc(dx+c)+1}\, \sqrt{\cot(dx+c)-\csc(dx+c)}\, \mathrm{EllipticPi}\left(\right.\right.}{}$
parts	$-\frac{2ae \left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^{2}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+}}{\left(e^{2}\right)^{\frac{1}{4}}}\right.\right.}{8\left(e^{2}\right)^{\frac{1}{4}}}\right.}{d}$

[In] int((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -1/6*e^2*a/d*2^(1/2)*(cos(d*x+c)+1)*(3*I*sin(d*x+c)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*sin(d*x+c)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-4*sin(d*x+c)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*2^(1/2)*cos(d*x+c))*(e*cot(d*x+c))^(1/2)*sec(d*x+c)*csc(d*x+c)

Fricas [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\, dx = \text{Timed out}$$

[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \int (e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)\,dx$$

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \int (e\cot(c+d\,x))^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

```
[In] int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x)),x)
[Out] int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x)), x)
```

3.234 $\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 346

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} - \frac{2a(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{d\sqrt{\sin(2c+2dx)}} + \frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d} - \frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d} - \frac{a(e\cot(c+dx))^{3/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d} + \frac{a(e\cot(c+dx))^{3/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d} + \frac{2a(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{d}$$

```
[Out] -2*(e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))*tan(d*x+c)/d+2*a*(e*cot(d*x+c))^(3
/2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*
x),2^(1/2))*sin(d*x+c)*tan(d*x+c)/d/sin(2*d*x+2*c)^(1/2)-1/2*a*arctan(-1+2^
(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/d*2^(1/2)-1/2
*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)
/d*2^(1/2)-1/4*a*(e*cot(d*x+c))^(3/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x
+c))*tan(d*x+c)^(3/2)/d*2^(1/2)+1/4*a*(e*cot(d*x+c))^(3/2)*ln(1+2^(1/2)*tan
```

```
(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/d*2^(1/2)+2*a*(e*cot(d*x+c))^(3/
2)*sin(d*x+c)*tan(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3985, 3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\begin{aligned}
&\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \frac{a\tan^{\frac{3}{2}}(c+dx)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}}{\sqrt{2}d} \\
&-\frac{a\tan^{\frac{3}{2}}(c+dx)\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)(e\cot(c+dx))^{3/2}}{\sqrt{2}d} \\
&-\frac{a\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} \\
&+\frac{a\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} \\
&+\frac{2a\sin(c+dx)\tan^2(c+dx)(e\cot(c+dx))^{3/2}}{d} \\
&-\frac{2\tan(c+dx)(a\sec(c+dx)+a)(e\cot(c+dx))^{3/2}}{d} \\
&-\frac{2a\sin(c+dx)\tan(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)(e\cot(c+dx))^{3/2}}{d\sqrt{\sin(2c+2dx)}}
\end{aligned}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])*Tan[c + d*x])/d - (2*a*(e*C
ot[c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*x, 2]*Sin[c + d*x]*Tan[c + d*x])/
(d*Sqrt[Sin[2*c + 2*d*x]]) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*C
ot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d) - (a*ArcTan[1 + Sqrt[2]*
Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d)
- (a*(e*Cot[c + d*x])^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d) + (a*(e*Cot[c + d*x])^(3/2)*Log[1 + S
qrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d)
 + (2*a*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x]*Tan[c + d*x]^2)/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\frac{a+a\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx\\
&= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d}\\
&\quad+\left(2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\left(-\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)\sqrt{\tan(c+dx)}\,dx\\
&= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d}\\
&\quad-\left(a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\sqrt{\tan(c+dx)}\,dx\\
&\quad+\left(a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\sec(c+dx)\sqrt{\tan(c+dx)}\,dx\\
&= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d}\\
&\quad+\frac{2a(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{d}\\
&\quad-\left(2a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\cos(c+dx)\sqrt{\tan(c+dx)}\,dx-\frac{\left(a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c\right.}{}\\
&= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d}\\
&\quad+\frac{2a(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{d}\\
&\quad-\frac{\left(2a(e\cot(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)\right)\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{\cos^{\frac{3}{2}}(c+dx)}\\
&\quad-\frac{\left(2a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} \\
&\quad + \frac{2a(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{d} \\
&\quad - \frac{\left(2a(e\cot(c+dx))^{3/2}\sin(c+dx)\tan(c+dx)\right)\int\sqrt{\sin(2c+2dx)}\,dx}{\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{\left(a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d} \\
&\quad - \frac{\left(a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d} \\
&= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} \\
&\quad - \frac{2a(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{d\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{2a(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{d} \\
&\quad - \frac{\left(a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d} \\
&\quad - \frac{\left(a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d} \\
&\quad - \frac{\left(a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad - \frac{\left(a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} \\
&\quad - \frac{2a(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{d\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{a(e\cot(c+dx))^{3/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d} \\
&\quad + \frac{a(e\cot(c+dx))^{3/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d} \\
&\quad + \frac{2a(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{d} \\
&\quad - \frac{\left(a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad + \frac{\left(a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\
&= -\frac{2(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\tan(c+dx)}{d} \\
&\quad - \frac{2a(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{d\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d} \\
&\quad - \frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d} \\
&\quad - \frac{a(e\cot(c+dx))^{3/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d} \\
&\quad + \frac{a(e\cot(c+dx))^{3/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d} \\
&\quad + \frac{2a(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.55

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \frac{ae(1+\cos(c+dx))\sqrt{e\cot(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(8\cot^2(c+dx)\,\mathrm{Hyperge}\right.}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*e*(1 + Cos[c + d*x])*Sqrt[e*Cot[c + d*x]]*Sec[(c + d*x)/2]^2*Sec[c + d*x
]*(8*Cot[c + d*x]^2*Hypergeometric2F1[3/4, 3/2, 7/4, -Cot[c + d*x]^2] + 3*S
qrt[Csc[c + d*x]^2]*(-4*Cos[c + d*x] - 4*Cos[c + d*x]^2 + ArcSin[Cos[c + d*
x] - Sin[c + d*x]]*Sqrt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x]
 + Sqrt[Sin[2*(c + d*x)]]]*Sqrt[Sin[2*(c + d*x)]])))/(12*d*Sqrt[Csc[c + d*x
]^2])
```

Maple [A] (verified)

Time = 10.76 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.45

method	result
parts	$-\frac{2ae\left(\sqrt{e\cot(dx+c)}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(d}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8}\right.}{d}$
default	Expression too large to display

```
[In] int((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a/d*e*((e*cot(d*x+c))^(1/2)-1/8*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1)))-a/d*2^(1/2)*e*(e*cot(d*x+c))^(1/2)*(-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)
*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((c
sc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*
(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((cs
c(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)
*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((c
sc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+(csc(d*x+c)-cot(d*x+c
```

```
)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*sec(d*x+c)+2^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = a\left(\int (e\cot{(c+dx)})^{\frac{3}{2}}\,dx + \int (e\cot{(c+dx)})^{\frac{3}{2}}\sec{(c+dx)}\,dx\right)$$

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*sec(d*x+c)),x)
[Out] a*(Integral((e*cot(c + d*x))**(3/2), x) + Integral((e*cot(c + d*x))**(3/2)*
sec(c + d*x), x))
```

Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \int (e\cot(dx+c))^{\frac{3}{2}}\,(a\sec(dx+c)+a)\,dx$$

[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \int (e\cot(c+d\,x))^{3/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

[In] int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x)),x)

[Out] int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x)), x)

3.235 $\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 274

$$
\begin{aligned}
\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx
&= \frac{a\sqrt{e\cot(c+dx)}\,\mathrm{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} \\
&\quad - \frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}d} \\
&\quad + \frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}d} \\
&\quad - \frac{a\sqrt{e\cot(c+dx)}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d} \\
&\quad + \frac{a\sqrt{e\cot(c+dx)}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d}
\end{aligned}
$$

```
[Out] -a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x
),2^(1/2))*sec(d*x+c)*(e*cot(d*x+c))^(1/2)*sin(2*d*x+2*c)^(1/2)/d+1/2*a*arc
tan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^
(1/2)+1/2*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x
+c)^(1/2)/d*2^(1/2)-1/4*a*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(e*cot(
d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)+1/4*a*ln(1+2^(1/2)*tan(d*x+c)^(1/2
)+tan(d*x+c))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3985, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\begin{aligned}
&\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx \\
&= -\frac{a\sqrt{\tan(c+dx)}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}d} \\
&+ \frac{a\sqrt{\tan(c+dx)}\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}d} \\
&- \frac{a\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} \\
&+ \frac{a\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} \\
&+ \frac{a\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)\sqrt{e\cot(c+dx)}}{d}
\end{aligned}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sqrt[e*Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[
2*c + 2*d*x]])/d - (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*Cot[c +
 d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) + (a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
 d*x]]]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) - (a*Sqrt[e*Co
t[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c +
 d*x]])/(2*Sqrt[2]*d) + (a*Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int\frac{a+a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx\\
&= \left(a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int\frac{1}{\sqrt{\tan(c+dx)}}\,dx\\
&\quad+\left(a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int\frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx\\
&= \frac{\left(a\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}\\
&\quad+\frac{\left(a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{x}(1+x^2)}\,dx,x,\tan(c+dx)\right)}{d}\\
&= \left(a\sqrt{e\cot(c+dx)}\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int\frac{1}{\sqrt{\sin(2c+2dx)}}\,dx\\
&\quad+\frac{\left(2a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\operatorname{Subst}\left(\int\frac{1}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} \\
&\quad + \frac{\left(a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d} \\
&\quad + \frac{\left(a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d} \\
&= \frac{a\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} \\
&\quad + \frac{\left(a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d} \\
&\quad + \frac{\left(a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d} \\
&\quad - \frac{\left(a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad - \frac{\left(a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&= \frac{a\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} \\
&\quad - \frac{a\sqrt{e\cot(c+dx)}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d} \\
&\quad + \frac{a\sqrt{e\cot(c+dx)}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d} \\
&\quad + \frac{\left(a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad - \frac{\left(a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}
\end{aligned}
$$

$$= \frac{a\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} - \frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}d} + \frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}d} - \frac{a\sqrt{e\cot(c+dx)}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d} + \frac{a\sqrt{e\cot(c+dx)}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.62

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx = \frac{a(1+\cos(c+dx))\sqrt{e\cot(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(4\sqrt[4]{-1}\sqrt{\cot(c+dx)}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{}\right.\right.\right.}{}$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sqrt[e*Cot[c + d*x]]*Sec[(c + d*x)/2]^2*Sec[c + d*x]*
(4*(-1)^(1/4)*Sqrt[Cot[c + d*x]]*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Cot[c
+ d*x]]], -1] + Sqrt[Csc[c + d*x]^2]*(-ArcSin[Cos[c + d*x] - Sin[c + d*x]]
+ Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]])*Sqrt[Sin[2*(c
+ d*x)]]))/(4*d*Sqrt[Csc[c + d*x]^2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.51

method	result
default	$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)a\sqrt{2}\left(i\,\mathrm{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)+\mathrm{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\right)\sqrt{e\cot(dx+c}}{d}$
parts	$-\frac{ae\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)\right)}{4d\left(e^2\right)^{\frac{1}{4}}}+\frac{a}{}$

[In] int((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] (1/2-1/2*I)*a/d*2^(1/2)*(I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2)))*(e*cot(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(1+sec(d*x+c))

Fricas [F(-1)]

Timed out.

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx = \text{Timed out}$$

[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{e\cot(c+dx)}\,dx + \int \sqrt{e\cot(c+dx)}\sec(c+dx)\,dx\right)$$

[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))**(1/2),x)

[Out] a*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(sqrt(e*cot(c + d*x))*sec(c + d*x), x))

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx = \int \sqrt{e\cot(dx+c)}(a\sec(dx+c)+a)\,dx$$

```
[In] integrate((a+a*sec(d*x+c))*(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))\,dx = \int \sqrt{e\cot(c+d\,x)}\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

```
[In] int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x)), x)
```

3.236 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 299

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx &= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}\\&\quad-\frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\&\quad+\frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\&\quad+\frac{a\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\&\quad-\frac{a\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\end{aligned}$$

```
[Out] 2*a*sin(d*x+c)/d/(e*cot(d*x+c))^(1/2)+2*a*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^
(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/d/(e*cot(d*x+c
))^(1/2)/sin(2*d*x+2*c)^(1/2)+1/2*a*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d*2
^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/2*a*arctan(1+2^(1/2)*tan(d*x
+c)^(1/2))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/4*a*ln(1-2^(1/
2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(
1/2)-1/4*a*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/(e*cot(d*x+c
))^(1/2)/tan(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3985, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx=-\frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{a\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}+\frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}+\frac{a\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}-\frac{a\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}}-\frac{2a\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{d\sqrt{\sin(2c+2dx)}\sqrt{e\cot(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[e*Cot[c + d*x]]) - (2*a*Cos[c + d*x]*EllipticE[c
 - Pi/4 + d*x, 2])/(d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - (a*Arc
Tan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[T
an[c + d*x]]) + (a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[
e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (a*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - (
a*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
```

```
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (a+a\sec(c+dx))\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{a\int \sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{a\int \sec(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{(2a)\int \cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{a\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
\end{aligned}
$$

$$
= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{\left(2a\sqrt{\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}}
+ \frac{(2a)\mathrm{Subst}\left(\int\frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
$$

$$
= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{(2a\cos(c+dx))\int\sqrt{\sin(2c+2dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}
- \frac{a\mathrm{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{a\mathrm{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
$$

$$
= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}
+ \frac{a\mathrm{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
+ \frac{a\mathrm{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
+ \frac{a\mathrm{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
+ \frac{a\mathrm{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
$$

$$
= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}
+ \frac{a\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
- \frac{a\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
+ \frac{a\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
- \frac{a\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
$$

$$= \frac{2a\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{2a\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}$$
$$- \frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$
$$+ \frac{a\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$
$$- \frac{a\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.42 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.63

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$$
$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(8\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\cot^2(c+dx\right)\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sec[c + d*x]*(8*Cot[c + d*x]^3*Hyp
ergeometric2F1[3/4, 3/2, 7/4, -Cot[c + d*x]^2] - 3*Cot[c + d*x]*Sqrt[Csc[c
+ d*x]^2]*(-2 + 2*Cos[2*(c + d*x)] + ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Sq
rt[Sin[2*(c + d*x)]] + Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*
x)]]]*Sqrt[Sin[2*(c + d*x)]])))/(12*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Csc[c + d*x
]^2])
```

Maple [A] (verified)

Time = 10.38 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.70

method	result
parts	$-\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)\right)}{4de}-$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/d/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*
2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-
2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-a/d*2^(1/2)*(-2*(csc
(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc
(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d
*x+c)+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(
d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1
/2))*cos(d*x+c)-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)
^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1
/2),1/2*2^(1/2))+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^
(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/
2),1/2*2^(1/2))+2^(1/2)*cos(d*x+c)-2^(1/2))/(e*cot(d*x+c))^(1/2)*csc(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\cot(c + dx)}}\, dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{e\cot(c + dx)}}\, dx = a\left(\int \frac{1}{\sqrt{e\cot{(c + dx)}}}\, dx + \int \frac{\sec{(c + dx)}}{\sqrt{e\cot{(c + dx)}}}\, dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*cot(c
 + d*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \int \frac{a\sec\left(dx+c\right)+a}{\sqrt{e\cot\left(dx+c\right)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*cot(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \int \frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{e\cot\left(c+d\,x\right)}}\,dx$$

```
[In] int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(1/2), x)
```

3.237 $\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 320

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx &= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}}\\
&-\frac{a\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}}\\
&+\frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}-\frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}\\
&+\frac{a\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}\\
&-\frac{a\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}\end{aligned}$$

```
[Out] 2/3*cot(d*x+c)*(3*a+a*sec(d*x+c))/d/(e*cot(d*x+c))^(3/2)+1/3*a*cot(d*x+c)*c
sc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4
*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/d/(e*cot(d*x+c))^(3/2)-1/2*a*arctan(
-1+2^(1/2)*tan(d*x+c)^(1/2))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2
)-1/2*a*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/t
an(d*x+c)^(3/2)+1/4*a*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d/(e*cot(d*
x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/4*a*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+ta
n(d*x+c))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3985, 3966, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}} - \frac{a\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}} + \frac{2\cot(c+dx)(a\sec(c+dx)+3a)}{3d(e\cot(c+dx))^{3/2}} + \frac{a\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}} - \frac{a\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}} - \frac{a\sqrt{\sin(2c+2dx)}\cot(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3d(e\cot(c+dx))^{3/2}}$$

```
[In] Int[(a + a*Sec[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (2*Cot[c + d*x]*(3*a + a*Sec[c + d*x]))/(3*d*(e*Cot[c + d*x])^(3/2)) - (a*C
ot[c + d*x]*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]
])/(3*d*(e*Cot[c + d*x])^(3/2)) + (a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
)/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - (a*ArcTan[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3
/2)) + (a*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*
(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - (a*Log[1 + Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(
3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}\\
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}\\
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}\\
&\quad - \frac{a\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{\left(a\cos^{\frac{3}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{a\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{\left(a\cot(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{(2a)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{a\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{a\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{a\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{a\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{a\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{a\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{a\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{a\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}} + \frac{a\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{a\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{a\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{a\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\cot(c+dx)(3a+a\sec(c+dx))}{3d(e\cot(c+dx))^{3/2}} - \frac{a\cot(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}} + \frac{a\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{a\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{a\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{a\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.70

$$\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{a(1+\cos(c+dx))\cos(2(c+dx))\csc(c+dx)\sqrt{\csc^2(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-4\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Cos[2*(c + d*x)]*Csc[c + d*x]*Sqrt[Csc[c + d*x]^2]*Se
c[(c + d*x)/2]^2*(-4*(-1)^(1/4)*Cot[c + d*x]^(3/2)*EllipticF[I*ArcSinh[(-1)
^(1/4)*Sqrt[Cot[c + d*x]]], -1] + Sqrt[Csc[c + d*x]^2]*(4 + 12*Cos[c + d*x]
```

```
 + 3*ArcSin[Cos[c + d*x] - Sin[c + d*x]]*Cot[c + d*x]*Sqrt[Sin[2*(c + d*x)]
] - 3*Cot[c + d*x]*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]
]*Sqrt[Sin[2*(c + d*x)]]])))/(12*d*(e*Cot[c + d*x])^(3/2)*(-1 + Cot[c + d*x]
^2))
```

Maple [A] (verified)

Time = 10.12 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.19

method	result
parts	$-\frac{2ae\left(-\frac{1}{e^2\sqrt{e\cot(dx+c)}}-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx-}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e^2\left(e^2\right)^{\frac{1}{4}}}\right.}{d}$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a/d*e*(-1/e^2/(e*cot(d*x+c))^(1/2)-1/8/e^2/(e^2)^(1/4)*2^(1/2)*(ln((e*co
t(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c
)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1)))-1/3*a/d*2^(1/2)*(-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-
cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-(csc(d*x+c)-cot(d*x+c)+1)^(1/
2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF(
(csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)+2^(1/2)*sin(d*x+c))
/e/(e*cot(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*tan(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = a\left(\int \frac{1}{(e\cot{(c+dx)})^{\frac{3}{2}}}\,dx+\int \frac{\sec{(c+dx)}}{(e\cot{(c+dx)})^{\frac{3}{2}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))**(3/2),x)
[Out] a*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*cot(c +
 d*x))**(3/2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{a\sec{(dx+c)}+a}{(e\cot{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*cot(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{a+\frac{a}{\cos(c+d\,x)}}{(e\cot{(c+d\,x)})^{3/2}}\,dx$$

```
[In] int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))/(e*cot(c + d*x))^(3/2), x)
```

3.238 $\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 357

$$\begin{aligned}
\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx &= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} \\
&- \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(c+dx)}{3d} \\
&- \frac{2a^2(e\cot(c+dx))^{5/2}\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)}{3d} \\
&+ \frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}{\sqrt{2}d} \\
&- \frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}{\sqrt{2}d} \\
&+ \frac{a^2(e\cot(c+dx))^{5/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d} \\
&- \frac{a^2(e\cot(c+dx))^{5/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d}
\end{aligned}$$

```
[Out] -4/3*a^2*(e*cot(d*x+c))^(5/2)*tan(d*x+c)/d-4/3*a^2*(e*cot(d*x+c))^(5/2)*sec
(d*x+c)*tan(d*x+c)/d+2/3*a^2*(e*cot(d*x+c))^(5/2)*(sin(c+1/4*Pi+d*x)^2)^(1/
2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*
d*x+2*c)^(1/2)*tan(d*x+c)^2/d-1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(
e*cot(d*x+c))^(5/2)*tan(d*x+c)^(5/2)/d*2^(1/2)-1/2*a^2*arctan(1+2^(1/2)*tan
(d*x+c)^(1/2))*(e*cot(d*x+c))^(5/2)*tan(d*x+c)^(5/2)/d*2^(1/2)+1/4*a^2*(e*c
ot(d*x+c))^(5/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(5/2)
/d*2^(1/2)-1/4*a^2*(e*cot(d*x+c))^(5/2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d
*x+c))*tan(d*x+c)^(5/2)/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3985, 3971, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2689, 2694, 2653, 2720, 2687, 30}

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \frac{a^2\tan^{\frac{5}{2}}(c+dx)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}}{\sqrt{2}d} - \frac{a^2\tan^{\frac{5}{2}}(c+dx)\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)(e\cot(c+dx))^{5/2}}{\sqrt{2}d} - \frac{4a^2\tan(c+dx)(e\cot(c+dx))^{5/2}}{3d} + \frac{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} - \frac{a^2\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d} - \frac{4a^2\tan(c+dx)\sec(c+dx)(e\cot(c+dx))^{5/2}}{3d} - \frac{2a^2\sqrt{\sin(2c+2dx)}\tan^2(c+dx)\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)(e\cot(c+dx))^{5/2}}{3d}$$

```
[In] Int[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x])/(3*d) - (4*a^2*(e*Cot[c + d*x]
)^(5/2)*Sec[c + d*x]*Tan[c + d*x])/(3*d) - (2*a^2*(e*Cot[c + d*x])^(5/2)*El
lipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]]*Tan[c + d*x]
^2)/(3*d) + (a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(5
/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]]]*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))/(Sqrt[2]*d) + (a^2*(e*Cot
[c + d*x])^(5/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c +
 d*x]^(5/2))/(2*Sqrt[2]*d) - (a^2*(e*Cot[c + d*x])^(5/2)*Log[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(5/2))/(2*Sqrt[2]*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int\frac{(a+a\sec(c+dx))^2}{\tan^{\frac{5}{2}}(c+dx)}\,dx\\
&= \left((e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int\left(\frac{a^2}{\tan^{\frac{5}{2}}(c+dx)}+\frac{2a^2\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}+\frac{a^2\sec^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\right)dx\\
&= \left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int\frac{1}{\tan^{\frac{5}{2}}(c+dx)}\,dx\\
&\quad+\left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int\frac{\sec^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx\\
&\quad+\left(2a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int\frac{\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx\\
&= -\frac{2a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d}-\frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(c+dx)}{3d}\\
&\quad-\frac{1}{3}\left(2a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\int\frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx-\left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\\
&= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d}-\frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(c+dx)}{3d}\\
&\quad-\frac{\left(2a^2(e\cot(c+dx))^{5/2}\sin^{\frac{5}{2}}(c+dx)\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3\cos^{\frac{5}{2}}(c+dx)}\\
&\quad-\frac{\left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{x}(1+x^2)}\,dx,x,\tan(c+dx)\right)}{d}\\
&= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d}-\frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(c+dx)}{3d}\\
&\quad-\frac{1}{3}\left(2a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)\right)\int\frac{1}{\sqrt{\sin(2c+2dx)}}\,dx-\frac{\left(2a^2(e\right.}{}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(c+dx)}{3d} \\
&\quad - \frac{2a^2(e\cot(c+dx))^{5/2}\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)}{3d} \\
&\quad - \frac{\left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d} \\
&\quad - \frac{\left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(c+dx)}{3d} \\
&\quad - \frac{2a^2(e\cot(c+dx))^{5/2}\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)}{3d} \\
&\quad - \frac{\left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d} \\
&\quad - \frac{\left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d} \\
&\quad + \frac{\left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d} \\
&\quad + \frac{\left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(c+dx)}{3d} \\
&\quad - \frac{2a^2(e\cot(c+dx))^{5/2}\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)}{3d} \\
&\quad + \frac{a^2(e\cot(c+dx))^{5/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d} \\
&\quad - \frac{a^2(e\cot(c+dx))^{5/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d} \\
&\quad - \frac{\left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad + \frac{\left(a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}
\end{aligned}
$$

$$= -\frac{4a^2(e\cot(c+dx))^{5/2}\tan(c+dx)}{3d} - \frac{4a^2(e\cot(c+dx))^{5/2}\sec(c+dx)\tan(c+dx)}{3d}$$
$$- \frac{2a^2(e\cot(c+dx))^{5/2}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}\tan^2(c+dx)}{3d}$$
$$+ \frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}{\sqrt{2}d}$$
$$- \frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}{\sqrt{2}d}$$
$$+ \frac{a^2(e\cot(c+dx))^{5/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d}$$
$$- \frac{a^2(e\cot(c+dx))^{5/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)}{2\sqrt{2}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.83 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.26

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{2a^2e\cos^4\left(\frac{1}{2}(c+dx)\right)(e\cot(c+dx))^{3/2}\left(2+2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\tan^2(c+dx)\right)-\text{Hypergeo}\right.}{3d}$$

```
[In] Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*e*Cos[(c + d*x)/2]^4*(e*Cot[c + d*x])^(3/2)*(2 + 2*Hypergeometric2F
1[-3/4, 1/2, 1/4, -Tan[c + d*x]^2] - Hypergeometric2F1[3/4, 1, 7/4, -Cot[c
+ d*x]^2])*Sec[ArcCot[Cot[c + d*x]]/2]^4)/(3*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 17.61 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.39

method	result
default	$-\frac{a^2 e^2 \sqrt{2}\,(\cos(dx+c)+1)\left(-3i\sin(dx+c)\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{\cot(dx+c)-\csc(dx+c)+1}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\ \mathrm{EllipticPi}\right.}{}$
parts	$-\frac{2a^2 e\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^2\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8(e^2)^{\frac{1}{4}}}\right.}{d}$

```
[In] int((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^2*e^2/d*2^(1/2)*(cos(d*x+c)+1)*(-3*I*sin(d*x+c)*(csc(d*x+c)-cot(d*x+
c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*E
llipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*sin(d*
x+c)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d
*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*
I,1/2*2^(1/2))+3*sin(d*x+c)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d
*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d
*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*sin(d*x+c)*(cot(d*x+c)-csc(d*x+c))^
(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*Ellip
ticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-2*sin(d*x+c)*(
cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-cs
c(d*x+c)+1)^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+4*
2^(1/2)*cos(d*x+c))*(e*cot(d*x+c))^(1/2)*sec(d*x+c)*csc(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))**(5/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \int (e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2\,dx$$

```
[In] integrate((e*cot(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \int (e\cot(c+d\,x))^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2 dx$$

```
[In] int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2, x)
```

3.239 $\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 343

$$\begin{aligned}
\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx &= \\
&-\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d}-\frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d} \\
&-\frac{4a^2(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{d\sqrt{\sin(2c+2dx)}} \\
&+\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d} \\
&-\frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d} \\
&-\frac{a^2(e\cot(c+dx))^{3/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d} \\
&+\frac{a^2(e\cot(c+dx))^{3/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d}
\end{aligned}$$

```
[Out] -4*a^2*(e*cot(d*x+c))^(3/2)*sin(d*x+c)/d-4*a^2*(e*cot(d*x+c))^(3/2)*tan(d*x
+c)/d+4*a^2*(e*cot(d*x+c))^(3/2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d
*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*sin(d*x+c)*tan(d*x+c)/d/sin(2*d*x+
2*c)^(1/2)-1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)
*tan(d*x+c)^(3/2)/d*2^(1/2)-1/2*a^2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*c
ot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/d*2^(1/2)-1/4*a^2*(e*cot(d*x+c))^(3/2)*ln
(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/d*2^(1/2)+1/4*a^2*
(e*cot(d*x+c))^(3/2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(
3/2)/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3985, 3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2688, 2695, 2652, 2719, 2687, 30}

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx=\frac{a^2\tan^{\frac{3}{2}}(c+dx)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}}{\sqrt{2}d}-\frac{a^2\tan^{\frac{3}{2}}(c+dx)\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)(e\cot(c+dx))^{3/2}}{\sqrt{2}d}-\frac{4a^2\sin(c+dx)(e\cot(c+dx))^{3/2}}{d}-\frac{4a^2\tan(c+dx)(e\cot(c+dx))^{3/2}}{d}-\frac{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d}+\frac{a^2\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d}-\frac{4a^2\sin(c+dx)\tan(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)(e\cot(c+dx))^{3/2}}{d\sqrt{\sin(2c+2dx)}}$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x])/d - (4*a^2*(e*Cot[c + d*x])^(3
/2)*Tan[c + d*x])/d - (4*a^2*(e*Cot[c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*
x, 2]*Sin[c + d*x]*Tan[c + d*x])/(d*Sqrt[Sin[2*c + 2*d*x]]) + (a^2*ArcTan[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(
Sqrt[2]*d) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(
3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*d) - (a^2*(e*Cot[c + d*x])^(3/2)*Log[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d
) + (a^2*(e*Cot[c + d*x])^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
```

```
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\frac{(a+a\sec(c+dx))^2}{\tan^{\frac{3}{2}}(c+dx)}\,dx\\
&= \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\left(\frac{a^2}{\tan^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}+\frac{a^2\sec^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\right)dx\\
&= \left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx\\
&\quad+\left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\frac{\sec^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx\\
&\quad+\left(2a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\frac{\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx\\
&= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d}-\frac{2a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d}\\
&\quad-\left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\sqrt{\tan(c+dx)}\,dx-\left(4a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int \text{c}\\
&= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d}-\frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d}\\
&\quad-\frac{\left(4a^2(e\cot(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)\right)\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{\cos^{\frac{3}{2}}(c+dx)}\\
&\quad-\frac{\left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{\sqrt{x}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\
&= -\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d}-\frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d}\\
&\quad-\frac{\left(4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)\tan(c+dx)\right)\int\sqrt{\sin(2c+2dx)}\,dx}{\sqrt{\sin(2c+2dx)}}\\
&\quad-\frac{\left(2a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d}
\end{aligned}$$

$$=-\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d}-\frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d}$$
$$-\frac{4a^2(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{d\sqrt{\sin(2c+2dx)}}$$
$$+\frac{\left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d}$$
$$-\frac{\left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d}$$

$$=-\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d}-\frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d}$$
$$-\frac{4a^2(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{d\sqrt{\sin(2c+2dx)}}$$
$$-\frac{\left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d}$$
$$-\frac{\left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d}$$
$$-\frac{\left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d}$$
$$-\frac{\left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d}$$

$$=-\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d}-\frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d}$$
$$-\frac{4a^2(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{d\sqrt{\sin(2c+2dx)}}$$
$$-\frac{a^2(e\cot(c+dx))^{3/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d}$$
$$+\frac{a^2(e\cot(c+dx))^{3/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d}$$
$$-\frac{\left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}$$
$$+\frac{\left(a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}$$

$$=-\frac{4a^2(e\cot(c+dx))^{3/2}\sin(c+dx)}{d}-\frac{4a^2(e\cot(c+dx))^{3/2}\tan(c+dx)}{d}$$
$$-\frac{4a^2(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{d\sqrt{\sin(2c+2dx)}}$$
$$+\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d}$$
$$-\frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d}$$
$$-\frac{a^2(e\cot(c+dx))^{3/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d}$$
$$+\frac{a^2(e\cot(c+dx))^{3/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 14.44 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.64

$$\int(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx=$$
$$-\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)(e\cot(c+dx))^{3/2}\left(2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(c-\right.\right.}{}$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*(a^2*Cos[(c + d*x)/2]^4*(e*Cot[c + d*x])^(3/2)*(2*Sqrt[2]*ArcTan[1 - S
qrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]
]] + 16*Sqrt[Cot[c + d*x]] + 16*Sqrt[Cot[c + d*x]]*Hypergeometric2F1[-1/4,
1/2, 3/4, -Tan[c + d*x]^2] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*
Sec[ArcCot[Cot[c + d*x]]/2]^4)/(d*Cot[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 10.33 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.53

method	result
parts	$-\frac{2a^2e\left(\sqrt{e\cot(dx+c)}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8}}{d}$
default	Expression too large to display

```
[In] int((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^2/d*e*((e*cot(d*x+c))^(1/2)-1/8*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)+1)))-2*a^2/d*e*(e*cot(d*x+c))^(1/2)-2*a^2/d*2^(1/2)*e*(e*cot(d*x+c))^(1/2
)*(-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(
d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1
/2))+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d
*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/
2))-2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(
d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1
/2))*sec(d*x+c)+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(
1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2
),1/2*2^(1/2))*sec(d*x+c)+2^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx=\text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))**(3/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \int (e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \int (e\cot(c+d\,x))^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\,dx$$

```
[In] int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2, x)
```

3.240 $\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 311

$$\begin{aligned}
\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx &= \frac{2a^2\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} \\
&\quad - \frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}d} \\
&\quad + \frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}d} \\
&\quad - \frac{a^2\sqrt{e\cot(c+dx)}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d} \\
&\quad + \frac{a^2\sqrt{e\cot(c+dx)}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d} \\
&\quad + \frac{2a^2\sqrt{e\cot(c+dx)}\tan(c+dx)}{d}
\end{aligned}$$

```
[Out] -2*a^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi
+d*x),2^(1/2))*sec(d*x+c)*(e*cot(d*x+c))^(1/2)*sin(2*d*x+2*c)^(1/2)/d+1/2*a
^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2
)/d*2^(1/2)+1/2*a^2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)
*tan(d*x+c)^(1/2)/d*2^(1/2)-1/4*a^2*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c
))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)+1/4*a^2*ln(1+2^(1/2)*tan
(d*x+c)^(1/2)+tan(d*x+c))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/d*2^(1/2)+2
*a^2*(e*cot(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3985, 3971, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720, 2687, 30}

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx$$
$$= -\frac{a^2\sqrt{\tan(c+dx)}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}d}$$
$$+\frac{a^2\sqrt{\tan(c+dx)}\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}d}$$
$$+\frac{2a^2\tan(c+dx)\sqrt{e\cot(c+dx)}}{d}$$
$$-\frac{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d}$$
$$+\frac{a^2\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d}$$
$$+\frac{2a^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)\sqrt{e\cot(c+dx)}}{d}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sqrt[e*Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[
Sin[2*c + 2*d*x]])/d - (a^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*C
ot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) + (a^2*ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*d) - (a^2
*Sqrt[e*Cot[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sq
rt[Tan[c + d*x]])/(2*Sqrt[2]*d) + (a^2*Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*d) + (2*
a^2*Sqrt[e*Cot[c + d*x]]*Tan[c + d*x])/d
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
```

```
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int\frac{(a+a\sec(c+dx))^2}{\sqrt{\tan(c+dx)}}\,dx \\
&= \left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int\left(\frac{a^2}{\sqrt{\tan(c+dx)}}+\frac{2a^2\sec(c+dx)}{\sqrt{\tan(c+dx)}}+\frac{a^2\sec^2(c+dx)}{\sqrt{\tan(c+dx)}}\right)\,dx
\end{aligned}$$

$$= \left(a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx + \left(a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\sec^2(c+dx)}{\sqrt{\tan(c+dx)}}\,dx + \left(2a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{\left(2a^2\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{\left(a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\tan(c+dx)}{d} + \left(2a^2\sqrt{e\cot(c+dx)}\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx + \frac{\left(2a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\,\mathrm{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} + \frac{2a^2\sqrt{e\cot(c+dx)}\tan(c+dx)}{d} + \frac{\left(a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d} + \frac{\left(a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d}$$

$$=\frac{2a^2\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d}$$
$$+\frac{2a^2\sqrt{e\cot(c+dx)}\tan(c+dx)}{d}$$
$$+\frac{\left(a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d}$$
$$+\frac{\left(a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d}$$
$$-\frac{\left(a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d}$$
$$-\frac{\left(a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d}$$

$$=\frac{2a^2\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d}$$
$$-\frac{a^2\sqrt{e\cot(c+dx)}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d}$$
$$+\frac{a^2\sqrt{e\cot(c+dx)}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d}$$
$$+\frac{2a^2\sqrt{e\cot(c+dx)}\tan(c+dx)}{d}$$
$$+\frac{\left(a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}$$
$$-\frac{\left(a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}$$

$$= \frac{2a^2\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{d} - \frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}d} + \frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}d} - \frac{a^2\sqrt{e\cot(c+dx)}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d} + \frac{a^2\sqrt{e\cot(c+dx)}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}d} + \frac{2a^2\sqrt{e\cot(c+dx)}\tan(c+dx)}{d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.01 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.38

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx = \frac{a^2e(1+\cos(c+dx))^2\left(3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\cot^2(c+dx)\right)+6\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\right.\right.}{6d\sqrt{e\cot}}$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*e*(1 + Cos[c + d*x])^2*(3*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x
]^2] + 6*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2] - 2*Cot[c + d*x]
^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*Sec[ArcCot[Cot[c + d*x]
]/2]^4)/(6*d*Sqrt[e*Cot[c + d*x]])
```

Maple [A] (verified)

Time = 11.14 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.85

method	result
parts	$-\frac{a^2 e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)\right)}{4d\left(e^2\right)^{\frac{1}{4}}}+$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^2/d*e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+2*a^2/d*e/(e*cot(d
*x+c))^(1/2)+2*a^2/d*2^(1/2)*(e*cot(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)
^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellipt
icF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*(1+sec(d*x+c))
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sqrt{e\cot(c+dx)}\,dx + \int 2\sqrt{e\cot(c+dx)}\sec(c+dx)\,dx + \int \sqrt{e\cot(c+dx)}\sec^2(c+dx)\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2*(e*cot(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(2*sqrt(e*cot(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*cot(c + d*x))*sec(c + d*x)**2, x))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx = \int \sqrt{e\cot(dx+c)}(a\sec(dx+c)+a)^2\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2*(e*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2\,dx = \int \sqrt{e\cot(c+d\,x)}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\,dx$$

```
[In] int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2,x)
[Out] int((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2, x)
```

3.241 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 339

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx &= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad + \frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad + \frac{a^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad - \frac{a^2\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad + \frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}
\end{aligned}
$$

```
[Out] 4*a^2*sin(d*x+c)/d/(e*cot(d*x+c))^(1/2)+4*a^2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)
^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/d/(e*cot(d
*x+c))^(1/2)/sin(2*d*x+2*c)^(1/2)+1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2
))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/2*a^2*arctan(1+2^(1/2)
*tan(d*x+c)^(1/2))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/4*a^2*
ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/ta
n(d*x+c)^(1/2)-1/4*a^2*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d*2^(1/2)/
(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+2/3*a^2*tan(d*x+c)/d/(e*cot(d*x+c))^(
1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3985, 3971, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719, 2687, 30}

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = -\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}} + \frac{a^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}} + \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} + \frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}} + \frac{a^2\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}} - \frac{a^2\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{d\sqrt{\sin(2c+2dx)}\sqrt{e\cot(c+dx)}}$$

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (4*a^2*Sin[c + d*x])/(d*Sqrt[e*Cot[c + d*x]]) - (4*a^2*Cos[c + d*x]*Ellipti
cE[c - Pi/4 + d*x, 2])/(d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - (a
^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*
Sqrt[Tan[c + d*x]]) + (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]
*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c +
d*x]]) - (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2
]*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + (2*a^2*Tan[c + d*x])/(3*d*Sq
rt[e*Cot[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
```

```
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

`&& !IntegerQ[m]`

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (a+a\sec(c+dx))^2\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{\int \left(a^2\sqrt{\tan(c+dx)}+2a^2\sec(c+dx)\sqrt{\tan(c+dx)}+a^2\sec^2(c+dx)\sqrt{\tan(c+dx)}\right)\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{a^2\int \sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{a^2\int \sec^2(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&\quad+\frac{(2a^2)\int \sec(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}-\frac{(4a^2)\int \cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&\quad+\frac{a^2\text{Subst}\left(\int \sqrt{x}\,dx,x,\tan(c+dx)\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{a^2\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}+\frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}\\
&\quad-\frac{\left(4a^2\sqrt{\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}}\\
&\quad+\frac{(2a^2)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}}+\frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}-\frac{(4a^2\cos(c+dx))\int \sqrt{\sin(2c+2dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}\\
&\quad-\frac{a^2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{a^2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}} + \frac{a^2\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{a^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad - \frac{a^2\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}} \\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4a^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}} - \frac{4a^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad + \frac{a^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad - \frac{a^2\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{2a^2\tan(c+dx)}{3d\sqrt{e\cot(c+dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 25.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.65

$$\int \frac{(a + a\sec(c + dx))^2}{\sqrt{e\cot(c + dx)}}\,dx$$
$$= \frac{a^2\cos^5\left(\frac{1}{2}(c + dx)\right)\left(4\,\text{Hypergeometric2F1}\left(-\frac{3}{4}, 1, \frac{1}{4}, -\cot^2(c + dx)\right) + 8\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -\text{ta}\right.\right.}{}$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^5*(4*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2]
 + 8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*Cot[c +
d*x]^(3/2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]
*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] -
 Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))*Sec[c + d*x]*Sec[ArcC
ot[Cot[c + d*x]]/2]^4*Sin[(c + d*x)/2])/(3*d*Sqrt[e*Cot[c + d*x]])
```

Maple [A] (verified)

Time = 11.51 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.57

method	result
parts	$-\frac{a^2(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)\right)}{4de}$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^2/d/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+2/3*a^2/d*e/(e*cot
(d*x+c))^(3/2)+2*a^2/d*2^(1/2)*(2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-
cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)
*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((c
sc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)+2*(csc(d*x+c)-cot(d*x
+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-(csc(d*x+c)-cot(d*x+
c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*E
```

```
llipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2^(1/2)*cos(d*x+c)+2^
(1/2))/(e*cot(d*x+c))^(1/2)*csc(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = a^2\left(\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx + \int \frac{2\sec(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx + \int \frac{\sec^2(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
[Out] a**2*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
cot(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*cot(c + d*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\cot(dx+c)}}\,dx$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*cot(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{e\cot(c+d\,x)}}\,dx$$

```
[In] int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(1/2), x)
```

3.242 $\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 375

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx &= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}}+\frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} \\
&\quad -\frac{2a^2\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}} \\
&\quad +\frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}-\frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad +\frac{a^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad -\frac{a^2\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}+\frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}}
\end{aligned}$$

```
[Out] 2*a^2*cot(d*x+c)/d/(e*cot(d*x+c))^(3/2)+4/3*a^2*csc(d*x+c)/d/(e*cot(d*x+c))
^(3/2)+2/3*a^2*cot(d*x+c)*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*
Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/d/(e*cot(
d*x+c))^(3/2)-1/2*a^2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/d/(e*cot(d*x+c))^
(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/2*a^2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/d
/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)+1/4*a^2*ln(1-2^(1/2)*tan(d*x
+c)^(1/2)+tan(d*x+c))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/4*a
^2*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/d/(e*cot(d*x+c))^(3/2)*2^(1/2)
/tan(d*x+c)^(3/2)+2/5*a^2*tan(d*x+c)/d/(e*cot(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3985, 3971, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2691, 2694, 2653, 2720, 2687, 30}

$$\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}} - \frac{a^2\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}} + \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} + \frac{a^2\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}} - \frac{a^2\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}} - \frac{2a^2\sqrt{\sin(2c+2dx)}\cot(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3d(e\cot(c+dx))^{3/2}}$$

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*Cot[c + d*x])/(d*(e*Cot[c + d*x])^(3/2)) + (4*a^2*Csc[c + d*x])/(3*d
*(e*Cot[c + d*x])^(3/2)) - (2*a^2*Cot[c + d*x]*Csc[c + d*x]*EllipticF[c - P
i/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*d*(e*Cot[c + d*x])^(3/2)) + (a^2*A
rcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Ta
n[c + d*x]^(3/2)) - (a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/(Sqrt[2]*d
*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + (a^2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]])/(2*Sqrt[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x
]^(3/2)) - (a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/(2*Sqrt
[2]*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + (2*a^2*Tan[c + d*x])/(5*
d*(e*Cot[c + d*x])^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
```

```
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\int (a+a\sec(c+dx))^2\tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\int \left(a^2\tan^{\frac{3}{2}}(c+dx)+2a^2\sec(c+dx)\tan^{\frac{3}{2}}(c+dx)+a^2\sec^2(c+dx)\tan^{\frac{3}{2}}(c+dx)\right)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^2\int \tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}+\frac{a^2\int \sec^2(c+dx)\tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}+\frac{(2a^2)\int \sec(c+dx)\tan^{\frac{3}{2}}(c+dx)\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}}+\frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}}-\frac{(2a^2)\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}-\frac{a^2\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}+\frac{a^2\text{Subst}\left(\int x^{3/2}\,dx,x,\tan(c+dx)\right)}{d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}}+\frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}}+\frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}}-\frac{\left(2a^2\cos^{\frac{3}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3(e\cot(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)}-\frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx,x,\tan(c+dx)\right)}{d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}}+\frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}}+\frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}}-\frac{\left(2a^2\cot(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3(e\cot(c+dx))^{3/2}}-\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$
\begin{aligned}
&= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{2a^2\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}} - \frac{a^2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{2a^2\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}} - \frac{a^2\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{2a^2\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{a^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{a^2\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{a^2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2a^2\cot(c+dx)}{d(e\cot(c+dx))^{3/2}} + \frac{4a^2\csc(c+dx)}{3d(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{2a^2\cot(c+dx)\csc(c+dx)\,\mathrm{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3d(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{a^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{a^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{a^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{a^2\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{2a^2\tan(c+dx)}{5d(e\cot(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.34

$$
\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \frac{a^2\left(\mathrm{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\cot^2(c+dx)\right)+2\left(5\cot^2(c+dx)\,\mathrm{Hyperg}\right.\right.}{}
$$

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2] + 2*(5*Cot[c + d*x]
^2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Hypergeometric2F1[1/2
, 5/4, 9/4, -Tan[c + d*x]^2]))*(1 + Sec[c + d*x])^2*Sec[ArcCot[Cot[c + d*x]
]/2]^4*Sin[c + d*x]^2)/(10*d*e*Sqrt[e*Cot[c + d*x]])
```

Maple [A] (verified)

Time = 11.11 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.08

method	result
parts	$-\frac{2a^2e\left(-\frac{1}{e^2\sqrt{e\cot(dx+c)}}-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(d}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e^2(e^2)^{\frac{1}{4}}}\right.}{d}$
default	Expression too large to display

```
[In] int((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^2/d*e*(-1/e^2/(e*cot(d*x+c))^(1/2)-1/8/e^2/(e^2)^(1/4)*2^(1/2)*(ln((e*
cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x
+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)
/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)+1)))+2/5*a^2*e/d/(e*cot(d*x+c))^(5/2)-2/3*a^2/d*2^(1/2)*(-(csc
(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc
(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d
*x+c)^2-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(co
t(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^
(1/2))*cos(d*x+c)+2^(1/2)*sin(d*x+c))/e/(e*cot(d*x+c))^(1/2)/(cos(d*x+c)^2-
1)*tan(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = a^2\left(\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{2\sec(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\sec^2(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx\right)$$

```
[In] integrate((a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*co
t(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*cot(c + d*x))**(3/2),
x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e

Giac [F]

$$\int \frac{(a + a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

[In] integrate((a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^2/(e*cot(d*x + c))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{(e\cot(c+d\,x))^{3/2}}\,dx$$

[In] int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(3/2),x)

[Out] int((a + a/cos(c + d*x))^2/(e*cot(c + d*x))^(3/2), x)

3.243 $\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 405

$$
\begin{aligned}
\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx &= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} \\
&- \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+dx)}{5ad} \\
&+ \frac{6(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{5ad\sqrt{\sin(2c+2dx)}} \\
&+ \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}ad} \\
&- \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}ad} \\
&- \frac{(e\cot(c+dx))^{3/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}ad} \\
&+ \frac{(e\cot(c+dx))^{3/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}ad} \\
&- \frac{6(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{5ad}
\end{aligned}
$$

```
[Out] 2/5*cot(d*x+c)*(e*cot(d*x+c))^(3/2)*(1-sec(d*x+c))/a/d-2/5*(e*cot(d*x+c))^(
3/2)*(5-3*sec(d*x+c))*tan(d*x+c)/a/d-6/5*(e*cot(d*x+c))^(3/2)*(sin(c+1/4*Pi
+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*sin(d
*x+c)*tan(d*x+c)/a/d/sin(2*d*x+2*c)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^
(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/a/d*2^(1/2)-1/2*arctan(1+2^(1/
2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(3/2)*tan(d*x+c)^(3/2)/a/d*2^(1/2)-1/4*
```

```
(e*cot(d*x+c))^(3/2)*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(
3/2)/a/d*2^(1/2)+1/4*(e*cot(d*x+c))^(3/2)*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan
(d*x+c))*tan(d*x+c)^(3/2)/a/d*2^(1/2)-6/5*(e*cot(d*x+c))^(3/2)*sin(d*x+c)*t
an(d*x+c)^2/a/d
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3985, 3973, 3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$
\begin{aligned}
\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx &= \frac{\tan^{\frac{3}{2}}(c+dx)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}}{\sqrt{2}ad} \\
&- \frac{\tan^{\frac{3}{2}}(c+dx)\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)(e\cot(c+dx))^{3/2}}{\sqrt{2}ad} \\
&+ \frac{2\cot(c+dx)(1-\sec(c+dx))(e\cot(c+dx))^{3/2}}{5ad} \\
&- \frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad} \\
&+ \frac{\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad} \\
&- \frac{6\sin(c+dx)\tan^2(c+dx)(e\cot(c+dx))^{3/2}}{5ad} \\
&- \frac{2\tan(c+dx)(5-3\sec(c+dx))(e\cot(c+dx))^{3/2}}{5ad} \\
&+ \frac{6\sin(c+dx)\tan(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)(e\cot(c+dx))^{3/2}}{5ad\sqrt{\sin(2c+2dx)}}
\end{aligned}
$$

```
[In] Int[(e*Cot[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(e*Cot[c + d*x])^(3/2)*(1 - Sec[c + d*x]))/(5*a*d) - (2*(e*
Cot[c + d*x])^(3/2)*(5 - 3*Sec[c + d*x])*Tan[c + d*x])/(5*a*d) + (6*(e*Cot[
c + d*x])^(3/2)*EllipticE[c - Pi/4 + d*x, 2]*Sin[c + d*x]*Tan[c + d*x])/(5*
a*d*Sqrt[Sin[2*c + 2*d*x]]) + (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*(e*Co
t[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*a*d) - (ArcTan[1 + Sqrt[2]*S
qrt[Tan[c + d*x]]]*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))/(Sqrt[2]*a*d)
 - ((e*Cot[c + d*x])^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*a*d) + ((e*Cot[c + d*x])^(3/2)*Log[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*a*d
) - (6*(e*Cot[c + d*x])^(3/2)*Sin[c + d*x]*Tan[c + d*x]^2)/(5*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
```

```
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\frac{1}{(a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)}\,dx\\
&= \frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\frac{-a+a\sec(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^2}\\
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad}\\
&\quad+\frac{\left(2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\frac{\frac{5a}{2}-\frac{3}{2}a\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{5a^2}\\
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad}\\
&\quad-\frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+dx)}{5ad}\\
&\quad+\frac{\left(4(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\left(-\frac{5a}{4}-\frac{3}{4}a\sec(c+dx)\right)\sqrt{\tan(c+dx)}\,dx}{5a^2}\\
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad}\\
&\quad-\frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+dx)}{5ad}\\
&\quad-\frac{\left(3(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\sec(c+dx)\sqrt{\tan(c+dx)}\,dx}{5a}\\
&\quad-\frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\sqrt{\tan(c+dx)}\,dx}{a}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} \\
&\quad - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+dx)}{5ad} \\
&\quad - \frac{6(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{5ad} \\
&\quad + \frac{\left(6(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\int\cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{5a} \\
&\quad - \frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{ad}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} \\
&\quad - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+dx)}{5ad} \\
&\quad - \frac{6(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{5ad} \\
&\quad + \frac{\left(6(e\cot(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)\right)\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{5a\cos^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{\left(2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} \\
&\quad - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+dx)}{5ad} \\
&\quad - \frac{6(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{5ad} \\
&\quad + \frac{(6(e\cot(c+dx))^{3/2}\sin(c+dx)\tan(c+dx))\int\sqrt{\sin(2c+2dx)}\,dx}{5a\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad} \\
&\quad - \frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad}
\end{aligned}
$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+dx)}{5ad} + \frac{6(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{5ad\sqrt{\sin(2c+2dx)}} - \frac{6(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{5ad} - \frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2ad} - \frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2ad} - \frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad}$$

$$= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+dx)}{5ad} + \frac{6(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sin(c+dx)\tan(c+dx)}{5ad\sqrt{\sin(2c+2dx)}} - \frac{(e\cot(c+dx))^{3/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}ad} + \frac{(e\cot(c+dx))^{3/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}ad} - \frac{6(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{5ad} - \frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad} + \frac{\left((e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad}$$

$$
\begin{aligned}
&= \frac{2\cot(c+dx)(e\cot(c+dx))^{3/2}(1-\sec(c+dx))}{5ad} \\
&\quad - \frac{2(e\cot(c+dx))^{3/2}(5-3\sec(c+dx))\tan(c+dx)}{5ad} \\
&\quad + \frac{6(e\cot(c+dx))^{3/2}E\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sin(c+dx)\tan(c+dx)}{5ad\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}ad} \\
&\quad - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}ad} \\
&\quad - \frac{(e\cot(c+dx))^{3/2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}ad} \\
&\quad + \frac{(e\cot(c+dx))^{3/2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}ad} \\
&\quad - \frac{6(e\cot(c+dx))^{3/2}\sin(c+dx)\tan^2(c+dx)}{5ad}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 14.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.78

$$
\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = -\frac{e\sqrt{e\cot(c+dx)}\left(30\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot^{\frac{3}{2}}(c+dx)-30\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right.\right.}{}
$$

```
[In] Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/30*(e*Sqrt[e*Cot[c + d*x]]*(30*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d
*x]]]*Cot[c + d*x]^(3/2) - 30*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
]*Cot[c + d*x]^(3/2) + 120*Cot[c + d*x]^2 - 24*Cot[c + d*x]^4 + 24*Cot[c +
d*x]^4*Hypergeometric2F1[-5/4, -1/2, -1/4, -Tan[c + d*x]^2] - 120*Cot[c + d
*x]^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -Tan[c + d*x]^2] - 40*Hypergeometr
ic2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 15*Sqrt[2]*Cot[c + d*x]^(3/2)*Log[1
 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 15*Sqrt[2]*Cot[c + d*x]^(3/
2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sec[c + d*x]*(1 + Sq
rt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(a*d)
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.23 (sec) , antiderivative size = 1116, normalized size of antiderivative = 2.76

method	result	size
default	Expression too large to display	1116

```
[In] int((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/a/d*2^(1/2)*(-e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)-sin(d*x+c
)))^(3/2)*(1-cos(d*x+c))*(5*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+
c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)
-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*((1-cos(d*x+c))
^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-5*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2
-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi(
(csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*((1-
cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)+12*(csc(d*x+c)-cot(d*x+c)+1
)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*E
llipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*((1-c
os(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-6*(csc(d*x+c)-cot(d*x+c)+1)^
(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ell
ipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*((1-cos
(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-5*(csc(d*x+c)-cot(d*x+c)+1)^(1
/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellip
ticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(d*x+c)
)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)-5*(csc(d*x+c)-cot(d*x
+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1
/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-c
os(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)+((1-cos(d*x+
c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(1-cos(d*x+c))^4*cs
c(d*x+c)^4-((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1
/2)*(1-cos(d*x+c))^2*csc(d*x+c)^2-5*(1-cos(d*x+c))^2*((1-cos(d*x+c))^3*csc(
d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)*csc(d*x+c)^2+5*((1-cos(d*x+c))^3*csc(
d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2))/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2/
((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)*csc(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{(e\cot(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate((e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
[Out] Integral((e*cot(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*cot(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx=\int \frac{\cos\left(c+d\,x\right)\,\left(e\cot(c+d\,x)\right)^{3/2}}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

```
[In] int((e*cot(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*cot(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

3.244 $\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 325

$$\begin{aligned}
\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx &= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} \\
&\quad - \frac{\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad} \\
&\quad - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}ad} \\
&\quad + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}ad} \\
&\quad - \frac{\sqrt{e\cot(c+dx)}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}ad} \\
&\quad + \frac{\sqrt{e\cot(c+dx)}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}ad}
\end{aligned}$$

```
[Out] 2/3*cot(d*x+c)*(1-sec(d*x+c))*(e*cot(d*x+c))^(1/2)/a/d+1/3*(sin(c+1/4*Pi+d*
x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+
c)*(e*cot(d*x+c))^(1/2)*sin(2*d*x+2*c)^(1/2)/a/d+1/2*arctan(-1+2^(1/2)*tan(
d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/a/d*2^(1/2)+1/2*arctan(
1+2^(1/2)*tan(d*x+c)^(1/2))*(e*cot(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/a/d*2^(1/
2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(e*cot(d*x+c))^(1/2)*tan(d
*x+c)^(1/2)/a/d*2^(1/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(e*co
t(d*x+c))^(1/2)*tan(d*x+c)^(1/2)/a/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3985, 3973, 3967, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx$$
$$= -\frac{\sqrt{\tan(c+dx)}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}ad} + \frac{\sqrt{\tan(c+dx)}\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{e\cot(c+dx)}}{\sqrt{2}ad} + \frac{2\cot(c+dx)(1-\sec(c+dx))\sqrt{e\cot(c+dx)}}{3ad} - \frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad} + \frac{\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad} - \frac{\sqrt{\sin(2c+2dx)}\sec(c+dx)\operatorname{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)\sqrt{e\cot(c+dx)}}{3ad}$$

```
[In] Int[Sqrt[e*Cot[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*Sqrt[e*Cot[c + d*x]]*(1 - Sec[c + d*x]))/(3*a*d) - (Sqrt[e*
Cot[c + d*x]]*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*
x]])/(3*a*d) - (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[e*Cot[c + d*x]]
*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a*d) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
]*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[2]*a*d) - (Sqrt[e*Cot[c +
d*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]]
)/(2*Sqrt[2]*a*d) + (Sqrt[e*Cot[c + d*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{-a+a\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} + \frac{\left(2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\frac{3a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3a^2}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} - \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3a} + \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{a}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} - \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3a\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} - \frac{\left(\sqrt{e\cot(c+dx)}\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3a} + \frac{\left(2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad} + \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad} + \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad} + \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2ad} + \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2ad} - \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad}$$

$$= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} - \frac{\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad} - \frac{\sqrt{e\cot(c+dx)}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}ad} + \frac{\sqrt{e\cot(c+dx)}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}ad} + \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad} - \frac{\left(\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad}$$

$$
\begin{aligned}
&= \frac{2\cot(c+dx)\sqrt{e\cot(c+dx)}(1-\sec(c+dx))}{3ad} \\
&- \frac{\sqrt{e\cot(c+dx)}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3ad} \\
&- \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}ad} \\
&+ \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{2}ad} \\
&- \frac{\sqrt{e\cot(c+dx)}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}ad} \\
&+ \frac{\sqrt{e\cot(c+dx)}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}ad}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.42

$$
\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx = -\frac{4\sqrt{e\cot(c+dx)}\csc(c+dx)\left(\cot^2(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\tan^2(c+dx)\right)+3\,\text{Hyperg}\right.}{}
$$

```
[In] Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*Sqrt[e*Cot[c + d*x]]*Csc[c + d*x]*(Cot[c + d*x]^2*Hypergeometric2F1[-3/
4, -1/2, 1/4, -Tan[c + d*x]^2] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c
+ d*x]^2] + Cot[c + d*x]^2*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*
x]^2]))*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.16 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.88

method	result
default	$-\frac{\sqrt{2}\sqrt{-\frac{e\left((1-\cos(dx+c))^2\csc(dx+c)-\sin(dx+c)\right)}{1-\cos(dx+c)}}\,(1-\cos(dx+c))\left(3i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\,\sqrt{\cot\right.}{}$

```
[In] int((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/d*2^(1/2)*(-e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)-sin(d*x+c)
))^(1/2)*(1-cos(d*x+c))*(3*I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c
)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-
cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*(csc(d*x+c)-cot(d*x+c)+1)^(1
/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellip
ticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*(csc(d*x+c)-
cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x
+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*(csc(d*
x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-cs
c(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^
(1/2))+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2
)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2+1/2*I,1/2*2^(1/2))+2*(1-cos(d*x+c))^3*csc(d*x+c)^3-2*csc(d*x+c)+2*cot(d
*x+c))/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/
((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)*csc(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{e\cot(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

```
[In] integrate((e*cot(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*cot(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{\sqrt{e \cot(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\cot\left(dx+c\right)}}{a\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(e*cot(d*x + c))/(a*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{e \cot(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{e\cot\left(dx+c\right)}}{a\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*cot(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(e*cot(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e \cot(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\sqrt{e\cot\left(c+d\,x\right)}}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

```
[In] int((e*cot(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*cot(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

3.245 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 347

$$\begin{aligned}\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx &= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}}\\ &\quad+\frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}}\\ &\quad-\frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}\\ &\quad-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\ &\quad+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\ &\quad+\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\ &\quad-\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\end{aligned}$$

```
[Out] 2*cot(d*x+c)*(1-sec(d*x+c))/a/d/(e*cot(d*x+c))^(1/2)+2*sin(d*x+c)/a/d/(e*co
t(d*x+c))^(1/2)+2*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*
EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a/d/(e*cot(d*x+c))^(1/2)/sin(2*d*x+2*c
)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a/d*2^(1/2)/(e*cot(d*x+c))^
(1/2)/tan(d*x+c)^(1/2)+1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d*2^(1/2)/(
e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(
d*x+c))/a/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)-1/4*ln(1+2^(1/2)*
```

```
tan(d*x+c)^(1/2)+tan(d*x+c))/a/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1
/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3985, 3973, 3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\begin{aligned}\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx = &-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}} \\ &+\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}} \\ &+\frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} \\ &+\frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} \\ &+\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}} \\ &-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}} \\ &-\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{ad\sqrt{\sin(2c+2dx)}\sqrt{e\cot(c+dx)}}\end{aligned}$$

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (2*Cot[c + d*x]*(1 - Sec[c + d*x]))/(a*d*Sqrt[e*Cot[c + d*x]]) + (2*Sin[c +
 d*x])/(a*d*Sqrt[e*Cot[c + d*x]]) - (2*Cos[c + d*x]*EllipticE[c - Pi/4 + d*
x, 2])/(a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]
) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]
]*Sqrt[Tan[c + d*x]]) + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/
(2*Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - Log[1 + Sqrt[2]*S
qrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*Sqrt[e*Cot[c + d*x]]*Sqrt[
Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{\tan(c+dx)}}{a+a\sec(c+dx)}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{\int \frac{-a+a\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\int \left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)\sqrt{\tan(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{\int \sqrt{\tan(c+dx)}\,dx}{a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&\quad + \frac{\int \sec(c+dx)\sqrt{\tan(c+dx)}\,dx}{a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}}\\
&\quad - \frac{2\int \cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{a\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}}\\
&\quad - \frac{\left(2\sqrt{\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{a\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}}\\
&\quad + \frac{2\operatorname{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{(2\cos(c+dx))\int\sqrt{\sin(2c+2dx)}\,dx}{a\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad + \frac{\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} \\
&\quad - \frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad + \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad - \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&= \frac{2\cot(c+dx)(1-\sec(c+dx))}{ad\sqrt{e\cot(c+dx)}} + \frac{2\sin(c+dx)}{ad\sqrt{e\cot(c+dx)}} - \frac{2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.94 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx = -\frac{\csc(c+dx)\left(24\cot^2(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\tan^2(c+dx)\right)+8\,\text{Hypergeometric2F1}\left(\right.\right.}{}$$

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] -1/6*(Csc[c + d*x]*(24*Cot[c + d*x]^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -T
an[c + d*x]^2] + 8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 3*Co
t[c + d*x]^(3/2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt
[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]
*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[
2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c +
 d*x)/2]^2)/(a*d*Sqrt[e*Cot[c + d*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.73

method	result
default	$\frac{\left(-\frac{1}{2}-\frac{i}{2}\right)\sqrt{2}\left(\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)+2i\,\text{EllipticE}\left(\right.\right.}{}$

```
[In] int(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2-1/2*I)/a/d*2^(1/2)*(EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/
2*I,1/2*2^(1/2))-I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2
*2^(1/2))+2*I*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-I*Elli
pticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*EllipticE((csc(d*x+c)-
cot(d*x+c)+1)^(1/2),1/2*2^(1/2))+EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2*2^(1/2)))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/
2)*(cot(d*x+c)-csc(d*x+c))^(1/2)/(e*cot(d*x+c))^(1/2)*(cot(d*x+c)+csc(d*x+c
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e \cot(c+dx)}(a+a\sec(c+dx))} \, dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{e \cot(c+dx)}(a+a\sec(c+dx))} \, dx = \frac{\int \frac{1}{\sqrt{e \cot(c+dx)}\sec(c+dx)+\sqrt{e \cot(c+dx)}} \, dx}{a}$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*cot(c + d*x))*sec(c + d*x) + sqrt(e*cot(c + d*x))), x)/a
```

Maxima [F]

$$\int \frac{1}{\sqrt{e \cot(c+dx)}(a+a\sec(c+dx))} \, dx = \int \frac{1}{\sqrt{e \cot(dx+c)}(a\sec(dx+c)+a)} \, dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{e \cot(c+dx)}(a+a\sec(c+dx))} \, dx = \int \frac{1}{\sqrt{e \cot(dx+c)}(a\sec(dx+c)+a)} \, dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+d\,x)}{a\,\sqrt{e\cot(c+d\,x)}\,\left(\cos(c+d\,x)+1\right)}\,dx$$

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

3.246 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 290

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2d}}{ad(e\cot(c+dx))^{3/2}}$$
$$+\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$
$$+\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}-\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

```
[Out] -cot(d*x+c)*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ellipt
icF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/a/d/(e*cot(d*x+c))^(3/2
)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/
tan(d*x+c)^(3/2)-1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^
(3/2)*2^(1/2)/tan(d*x+c)^(3/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c)
)/a/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/4*ln(1+2^(1/2)*tan(d*
x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules

used = {3985, 3973, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}} - \frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}} + \frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}} - \frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}} + \frac{\sqrt{\sin(2c+2dx)}\cot(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{ad(e\cot(c+dx))^{3/2}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cot[c + d*x]*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*
x]])/(a*d*(e*Cot[c + d*x])^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/
(Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - ArcTan[1 + Sqrt[2
]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2
)) + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*C
ot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}\\
&= \frac{\int \frac{-a+a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}\\
&= -\frac{\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}+\frac{\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}\\
&= \frac{\cos^{\frac{3}{2}}(c+dx)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{a(e\cot(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx,x,\tan(c+dx)\right)}{ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}\\
&= \frac{\left(\cot(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{a(e\cot(c+dx))^{3/2}}\\
&\quad -\frac{2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\cot(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{\cot(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{\cot(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

$$= \frac{\cot(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{ad(e\cot(c+dx))^{3/2}} + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.39

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{4\cot^2(c+dx)\csc(c+dx)\left(3\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan\right.\right.}{}$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (4*Cot[c + d*x]^2*Csc[c + d*x]*(3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c +
 d*x]^2] + Cot[c + d*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*
(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d*(e*Cot[c + d*x])^(3/2
))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.97 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.71

method	result
default	$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{2}\left(i\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)+\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)-2\,\text{EllipticF}\left(\sqrt{\text{cs}}\right.\right.}{}$

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2-1/2*I)/a/d*2^(1/2)*(I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1
/2*I,1/2*2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*
2^(1/2))-2*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*I*Ellip
ticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2)))*(csc(d*x+c)-cot(d*x+c)+1
)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)/(cos(
d*x+c)-1)/e/(e*cot(d*x+c))^(1/2)*sin(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*sec(c + d*x) + (e*cot(c + d*x))**(3/2))
, x)/a
```

Maxima [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \int \frac{\cos\left(c+d\,x\right)}{a\,\left(e\cot\left(c+d\,x\right)\right)^{3/2}\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

3.247 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 325

$$\begin{aligned}
\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx &= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} \\
&\quad - \frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&\quad - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}
\end{aligned}$$

```
[Out] 2*cos(d*x+c)*cot(d*x+c)/a/d/(e*cot(d*x+c))^(5/2)+2*cos(d*x+c)*cot(d*x+c)^2*
(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2
^(1/2))/a/d/(e*cot(d*x+c))^(5/2)/sin(2*d*x+2*c)^(1/2)-1/2*arctan(-1+2^(1/2)
*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)-1/2*ar
ctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c
)^(5/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot(d*x+c))^(5
/2)*2^(1/2)/tan(d*x+c)^(5/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/
a/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3985, 3973, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}} - \frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}} + \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}} + \frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}} - \frac{2\cos(c+dx)\cot^2(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{ad\sqrt{\sin(2c+2dx)}(e\cot(c+dx))^{5/2}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (2*Cos[c + d*x]*Cot[c + d*x])/(a*d*(e*Cot[c + d*x])^(5/2)) - (2*Cos[c + d*x
]*Cot[c + d*x]^2*EllipticE[c - Pi/4 + d*x, 2])/(a*d*(e*Cot[c + d*x])^(5/2)*
Sqrt[Sin[2*c + 2*d*x]]) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a
*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan
[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - Log[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x
])^(5/2)*Tan[c + d*x]^(5/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
```

```
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&= \frac{\int(-a+a\sec(c+dx))\sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}
\end{aligned}
$$

$$= -\frac{\int \sqrt{\tan(c+dx)}\,dx}{a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} + \frac{\int \sec(c+dx)\sqrt{\tan(c+dx)}\,dx}{a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\int \cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{a(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{\left(2\cos^{\frac{5}{2}}(c+dx)\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{a(e\cot(c+dx))^{5/2}\sin^{\frac{5}{2}}(c+dx)} - \frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{\left(2\cos(c+dx)\cot^2(c+dx)\right)\int \sqrt{\sin(2c+2dx)}\,dx}{a(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}} - \frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$=\frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}}-\frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}}$$
$$-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$
$$+\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$
$$-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$
$$+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

$$=\frac{2\cos(c+dx)\cot(c+dx)}{ad(e\cot(c+dx))^{5/2}}-\frac{2\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{ad(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}}$$
$$+\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$
$$-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$
$$+\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 23.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.60

$$\int\frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx=$$
$$-\frac{\sqrt{e\cot(c+dx)}\left(-8\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2(c+dx)\right)+3\sqrt{2}\cot^{\frac{3}{2}}(c+dx)\left(2\arctan\left(1-\sqrt{2}\sqrt{\right.\right.\right.}{}$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] -1/6*(Sqrt[e*Cot[c + d*x]]*(-8*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*
x]^2] + 3*Sqrt[2]*Cot[c + d*x]^(3/2)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x
]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c
 + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x
]]))*Sec[c + d*x]*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(a*d*e^3)
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.77 (sec) , antiderivative size = 1100, normalized size of antiderivative = 3.38

method	result	size
default	Expression too large to display	1100

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/d*2^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^3*(I*(csc(d*x+c)-cot(d*x
+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1
/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-c
os(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))
^(1/2)-I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2
)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+
c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)-4*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*c
sc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(
d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*
x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)+2*(csc(d*x+c)-cot(d*x+c
)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2
)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*(-
cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)-(csc(d
*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-c
sc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2
^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1
)*csc(d*x+c))^(1/2)-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d
*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c
)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1
)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)-4*(1-cos(d*x+c))^2*((1-cos(d*
x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)*csc(d*x+c)^2)/(-e/(1-cos(
d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)-sin(d*x+c)))^(5/2)/(1-cos(d*x+c))^2*si
n(d*x+c)^2/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1
/2)/((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*cs
c(d*x+c))^(1/2)/((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\sec(c+dx)+(e\cot(c+dx))^{\frac{5}{2}}}\,dx}{a}$$

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
[Out] Integral(1/((e*cot(c + d*x))**(5/2)*sec(c + d*x) + (e*cot(c + d*x))**(5/2))
, x)/a
```

Maxima [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+d\,x)}{a\,(e\cot(c+d\,x))^{5/2}\,(\cos(c+d\,x)+1)}\,dx$$

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

3.248 $\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 335

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3ad(e\cot(c+dx))^{7/2}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

```
[Out] -2/3*cot(d*x+c)^3*(3-sec(d*x+c))/a/d/(e*cot(d*x+c))^(7/2)+1/3*cot(d*x+c)^3*
csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/
4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/a/d/(e*cot(d*x+c))^(7/2)+1/2*arctan
(-1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(
7/2)+1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(7/2)*2^(1/2
)/tan(d*x+c)^(7/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot
(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+t
an(d*x+c))/a/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3985, 3973, 3966, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720}

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}+\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}} -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}} +\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}} -\frac{\sqrt{\sin(2c+2dx)}\cot^3(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3ad(e\cot(c+dx))^{7/2}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-2*Cot[c + d*x]^3*(3 - Sec[c + d*x]))/(3*a*d*(e*Cot[c + d*x])^(7/2)) - (Co
t[c + d*x]^3*Csc[c + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x
]])/(3*a*d*(e*Cot[c + d*x])^(7/2)) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + ArcTan[1 + Sqrt[
2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/
2)) - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*
Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{\tan^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}\\ &= \frac{\int(-a+a\sec(c+dx))\tan^{\frac{3}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}\end{aligned}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3a(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{a(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cos^{\frac{7}{2}}(c+dx)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3a(e\cot(c+dx))^{7/2}\sin^{\frac{7}{2}}(c+dx)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(c+dx)\right)}{ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\left(\cot^3(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3a(e\cot(c+dx))^{7/2}} + \frac{2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sqrt{\sin(2c+2dx)}}{3ad(e\cot(c+dx))^{7/2}} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$
\begin{aligned}
&= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} \\
&\quad - \frac{\cot^3(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3ad(e\cot(c+dx))^{7/2}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} \\
&\quad - \frac{\cot^3(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3ad(e\cot(c+dx))^{7/2}} \\
&\quad - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&\quad + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}
\end{aligned}
$$

$$= -\frac{2\cot^3(c+dx)(3-\sec(c+dx))}{3ad(e\cot(c+dx))^{7/2}} - \frac{\cot^3(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3ad(e\cot(c+dx))^{7/2}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.94 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.39

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = -\frac{4\sqrt{e\cot(c+dx)}\csc(c+dx)\left(3-3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\tan^2(c+dx)\right)+3\,\text{Hypergeometric2F1}\right.}{}$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-4*Sqrt[e*Cot[c + d*x]]*Csc[c + d*x]*(3 - 3*Hypergeometric2F1[-1/2, 1/4, 5
/4, -Tan[c + d*x]^2] + 3*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[c + d*x]^2]
+ Cot[c + d*x]^2*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])*(1 + Sqrt
[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(3*a*d*e^4)
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.80 (sec) , antiderivative size = 976, normalized size of antiderivative = 2.91

method	result	size
default	Expression too large to display	976

```
[In] int(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/d*2^(1/2)*(3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c
))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+
```

```
1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-3*I*(cot(d*x+c)-csc(d*x+c)+1)^
(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*Ellipti
cPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+3*I
*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-
cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/
2*2^(1/2))*cos(d*x+c)-3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d
*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x
+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)+8*(csc(d*x+c)-cot(d*x+c)+1)^
(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellipti
cF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-3*(cot(d*x+c)-
csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)
^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*co
s(d*x+c)^2-3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+8*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(
d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)-3*(cot(d*x+c)-csc(d*x+c)+1)
^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(
1/2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)-3*(c
ot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot
(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2
^(1/2))*cos(d*x+c)+6*cos(d*x+c)*sin(d*x+c)*2^(1/2)-2*2^(1/2)*sin(d*x+c))/(c
os(d*x+c)-1)^2/(cos(d*x+c)+1)^2/e^3/(e*cot(d*x+c))^(1/2)*sin(d*x+c)^2*tan(d
*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx=\text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx=\int \frac{1}{\left(e\cot\left(dx+c\right)\right)^{\frac{7}{2}}\left(a\sec\left(dx+c\right)+a\right)}\,dx$$

[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx=\int \frac{1}{\left(e\cot\left(dx+c\right)\right)^{\frac{7}{2}}\left(a\sec\left(dx+c\right)+a\right)}\,dx$$

[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx=\int \frac{\cos\left(c+d\,x\right)}{a\,{\left(e\cot\left(c+d\,x\right)\right)}^{7/2}\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

[In] int(1/((e*cot(c + d*x))^(7/2)*(a + a/cos(c + d*x))),x)

[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(7/2)*(cos(c + d*x) + 1)), x)

3.249 $\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 371

$$\begin{aligned}
\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx &= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} \\
&\quad - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{6\cos(c+dx)\cot^4(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{5ad(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}
\end{aligned}$$

```
[Out] -6/5*cos(d*x+c)*cot(d*x+c)^3/a/d/(e*cot(d*x+c))^(9/2)-2/15*cot(d*x+c)^3*(5-
3*sec(d*x+c))/a/d/(e*cot(d*x+c))^(9/2)-6/5*cos(d*x+c)*cot(d*x+c)^4*(sin(c+1
/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/
a/d/(e*cot(d*x+c))^(9/2)/sin(2*d*x+2*c)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x
+c)^(1/2))/a/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+1/2*arctan(1+2
^(1/2)*tan(d*x+c)^(1/2))/a/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+
1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*cot(d*x+c))^(9/2)*2^(1
/2)/tan(d*x+c)^(9/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a/d/(e*c
ot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3985, 3973, 3966, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719}

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}} + \frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}ad\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}} - \frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} + \frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}} - \frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}ad\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}} + \frac{6\cos(c+dx)\cot^4(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{5ad\sqrt{\sin(2c+2dx)}(e\cot(c+dx))^{9/2}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-6*Cos[c + d*x]*Cot[c + d*x]^3)/(5*a*d*(e*Cot[c + d*x])^(9/2)) - (2*Cot[c
+ d*x]^3*(5 - 3*Sec[c + d*x]))/(15*a*d*(e*Cot[c + d*x])^(9/2)) + (6*Cos[c +
 d*x]*Cot[c + d*x]^4*EllipticE[c - Pi/4 + d*x, 2])/(5*a*d*(e*Cot[c + d*x])^
(9/2)*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqr
t[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) + ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]]/(Sqrt[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) +
 Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c
 + d*x])^(9/2)*Tan[c + d*x]^(9/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]]/(2*Sqrt[2]*a*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
```

```
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
```

```
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{\tan^{\frac{9}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&= \frac{\int(-a+a\sec(c+dx))\tan^{\frac{5}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&= -\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} - \frac{2\int\left(-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)\right)\sqrt{\tan(c+dx)}\,dx}{5a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&= -\frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} - \frac{3\int\sec(c+dx)\sqrt{\tan(c+dx)}\,dx}{5a(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad + \frac{\int\sqrt{\tan(c+dx)}\,dx}{a(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} \\
&\quad + \frac{6\int\cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{5a(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + \frac{\text{Subst}\left(\int\frac{\sqrt{x}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} \\
&\quad + \frac{\left(6\cos^{\frac{9}{2}}(c+dx)\right)\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{5a(e\cot(c+dx))^{9/2}\sin^{\frac{9}{2}}(c+dx)} \\
&\quad + \frac{2\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} \\
&\quad + \frac{(6\cos(c+dx)\cot^4(c+dx))\int\sqrt{\sin(2c+2dx)}\,dx}{5a(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + \frac{\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} \\
&\quad + \frac{6\cos(c+dx)\cot^4(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{5ad(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}} \\
&\quad + \frac{6\cos(c+dx)\cot^4(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{5ad(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}
\end{aligned}
$$

$$= -\frac{6\cos(c+dx)\cot^3(c+dx)}{5ad(e\cot(c+dx))^{9/2}} - \frac{2\cot^3(c+dx)(5-3\sec(c+dx))}{15ad(e\cot(c+dx))^{9/2}}$$
$$+ \frac{6\cos(c+dx)\cot^4(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{5ad(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}}$$
$$- \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$
$$+ \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$
$$- \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}ad(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 14.81 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.70

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \frac{\sqrt{e\cot(c+dx)}\left(-8+6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot\right.}{}$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Sqrt[e*Cot[c + d*x]]*(-8 + 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
]*Cot[c + d*x]^(3/2) - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cot
[c + d*x]^(3/2) + 8*Hypergeometric2F1[-1/2, 3/4, 7/4, -Tan[c + d*x]^2] - 8*
Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[c + d*x]^2] + 3*Sqrt[2]*Cot[c + d*x]^
(3/2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 3*Sqrt[2]*Cot[c
+ d*x]^(3/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sec[c + d*
x]*(1 + Sqrt[Sec[c + d*x]^2])*Sin[(c + d*x)/2]^2)/(6*a*d*e^5)
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.23 (sec) , antiderivative size = 1166, normalized size of antiderivative = 3.14

method	result	size
default	Expression too large to display	1166

```
[In] int(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/a/d*2^(1/2)*(-15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x
+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c
)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-15*I*(cot(d*x+c)-csc(d*x+c)+
1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*Elli
pticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^3+
15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x
+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*
I,1/2*2^(1/2))*cos(d*x+c)^3+15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c
)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-
cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+18*(csc(d*x+c)-cot(
d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/
2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3-15*(
cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-co
t(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*
2^(1/2))*cos(d*x+c)^3-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*
x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2
))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^3-36*(csc(d*x+c)-cot(d*x+c)+1
)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellip
ticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+18*(csc(d*x+
c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+
c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)
^2-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*
x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2
*I,1/2*2^(1/2))*cos(d*x+c)^2-15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)
-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-c
ot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-36*(csc(d*x+c)-cot(d
*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2
)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2+28*co
s(d*x+c)^3*2^(1/2)-24*2^(1/2)*cos(d*x+c)^2-10*2^(1/2)*cos(d*x+c)+6*2^(1/2))
/e^4/(e*cot(d*x+c))^(1/2)/(cos(d*x+c)^2-1)^2*sin(d*x+c)*tan(d*x+c)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))**(9/2)/(a+a*sec(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{9}{2}}(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)), x)
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{9}{2}}(a\sec(dx+c)+a)}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))}\,dx = \int \frac{\cos(c+d\,x)}{a\,(e\cot(c+d\,x))^{9/2}\,(\cos(c+d\,x)+1)}\,dx$$

```
[In] int(1/((e*cot(c + d*x))^(9/2)*(a + a/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/(a*(e*cot(c + d*x))^(9/2)*(cos(c + d*x) + 1)), x)
```

3.250 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 413

$$
\begin{aligned}
\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx &= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} \\
&\quad - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} \\
&\quad + \frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} \\
&\quad - \frac{12\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{5a^2d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
\end{aligned}
$$

```
[Out] 2*cot(d*x+c)/a^2/d/(e*cot(d*x+c))^(1/2)-12/5*cos(d*x+c)*cot(d*x+c)/a^2/d/(e
*cot(d*x+c))^(1/2)-4/5*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(1/2)+4/5*cot(d*x+
c)^2*csc(d*x+c)/a^2/d/(e*cot(d*x+c))^(1/2)+12/5*cos(d*x+c)*(sin(c+1/4*Pi+d*
x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a^2/d/(e
*cot(d*x+c))^(1/2)/sin(2*d*x+2*c)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1
```

```
/2))/a^2/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/2*arctan(1+2^(1/
2)*tan(d*x+c)^(1/2))/a^2/d*2^(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)+1/
4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d*2^(1/2)/(e*cot(d*x+c))^(1
/2)/tan(d*x+c)^(1/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d*2^
(1/2)/(e*cot(d*x+c))^(1/2)/tan(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3985, 3973, 3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2689, 2688, 2695, 2652, 2719, 2687, 30}

$$\begin{aligned}\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}dx = & -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}} \\ & +\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}a^2d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}} \\ & -\frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}+\frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} \\ & -\frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} \\ & +\frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} \\ & +\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}} \\ & -\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\sqrt{\tan(c+dx)}\sqrt{e\cot(c+dx)}} \\ & -\frac{12\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{5a^2d\sqrt{\sin(2c+2dx)}\sqrt{e\cot(c+dx)}}\end{aligned}$$

```
[In] Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (2*Cot[c + d*x])/(a^2*d*Sqrt[e*Cot[c + d*x]]) - (12*Cos[c + d*x]*Cot[c + d*
x])/(5*a^2*d*Sqrt[e*Cot[c + d*x]]) - (4*Cot[c + d*x]^3)/(5*a^2*d*Sqrt[e*Cot
[c + d*x]]) + (4*Cot[c + d*x]^2*Csc[c + d*x])/(5*a^2*d*Sqrt[e*Cot[c + d*x]]
) - (12*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2])/(5*a^2*d*Sqrt[e*Cot[c +
d*x]]*Sqrt[Sin[2*c + 2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqr
t[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) + ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) +
 Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*Sqrt[e
```

```
*Cot[c + d*x]]*Sqrt[Tan[c + d*x]]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]]/(2*Sqrt[2]*a^2*d*Sqrt[e*Cot[c + d*x]]*Sqrt[Tan[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{\tan(c+dx)}}{(a+a\sec(c+dx))^2}\,dx}{\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^4\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}\\
&= \frac{\int \left(\frac{a^2}{\tan^{\frac{7}{2}}(c+dx)}-\frac{2a^2\sec(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}+\frac{a^2\sec^2(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\right)dx}{a^4\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
\end{aligned}
$$

$$= \frac{\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{\int \frac{\sec^2(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} - \frac{2\int \frac{\sec(c+dx)}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= -\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{6\int \frac{\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{5a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{\mathrm{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x, \tan(c+dx)\right)}{a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{\int \sqrt{\tan(c+dx)}\,dx}{a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} - \frac{12\int \cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{5a^2\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{\left(12\sqrt{\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{5a^2\sqrt{e\cot(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{(12\cos(c+dx))\int \sqrt{\sin(2c+2dx)}\,dx}{5a^2\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} + \frac{2\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$=\frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}}-\frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}-\frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}-\frac{12\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{5a^2d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}-\frac{\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$=\frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}}-\frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}-\frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}-\frac{12\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{5a^2d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}+\frac{\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$=\frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}}-\frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}-\frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}}-\frac{12\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{5a^2d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}}+\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}-\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}+\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}-\frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}$$

$$
\begin{aligned}
&= \frac{2\cot(c+dx)}{a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)\cot(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{4\cot^3(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} \\
&\quad + \frac{4\cot^2(c+dx)\csc(c+dx)}{5a^2d\sqrt{e\cot(c+dx)}} - \frac{12\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{5a^2d\sqrt{e\cot(c+dx)}\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}} \\
&\quad - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d\sqrt{e\cot(c+dx)}\sqrt{\tan(c+dx)}}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx
$$

```
[In] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.55 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.71

method	result
default	$\frac{\sqrt{2}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)\left(5i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\,\text{Ellipt}\right.}{}$

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/10/a^2/d*2^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(5*I*(csc(d*x+c)-cot(d
*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^
(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*I
*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d
*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*
I,1/2*2^(1/2))+24*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x
+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1
)^(1/2),1/2*2^(1/2))-12*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*c
```

```
ot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*
x+c)+1)^(1/2),1/2*2^(1/2))-5*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c
)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-
cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*(csc(d*x+c)-cot(d*x+c)+1)^(1/2
)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellipti
cPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*(1-cos(d*x+c))
^4*csc(d*x+c)^4-2*(1-cos(d*x+c))^2*csc(d*x+c)^2)/(-e/(1-cos(d*x+c))*((1-cos
(d*x+c))^2*csc(d*x+c)-sin(d*x+c)))^(1/2)/((1-cos(d*x+c))*((1-cos(d*x+c))^2*
csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)
-csc(d*x+c))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\sqrt{e\cot(c+dx)}\sec^2(c+dx)+2\sqrt{e\cot(c+dx)}\sec(c+dx)+\sqrt{e\cot(c+dx)}}\,dx}{a^2}$$

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*cot(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*cot(c + d*x))*s
ec(c + d*x) + sqrt(e*cot(c + d*x))), x)/a**2
```

Maxima [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\cot(dx+c)}(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\cot(dx+c)}(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*cot(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2}{a^2\,\sqrt{e\cot(c+d\,x)}\,(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int(1/((e*cot(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

3.251 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 359

$$
\begin{aligned}
\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx =& -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} \\
&+\frac{2\cot(c+dx)\csc(c+dx)\,\mathrm{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3a^2d(e\cot(c+dx))^{3/2}} \\
&+\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&+\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&-\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

```
[Out] -4/3*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(3/2)+4/3*cot(d*x+c)^2*csc(d*x+c)/a^
2/d/(e*cot(d*x+c))^(3/2)-2/3*cot(d*x+c)*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1
/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(
1/2)/a^2/d/(e*cot(d*x+c))^(3/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2
/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)-1/2*arctan(1+2^(1/2)*tan(d
*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)+1/4*ln(1-2
^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(3/2)*2^(1/2)/tan(
d*x+c)^(3/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x
+c))^(3/2)*2^(1/2)/tan(d*x+c)^(3/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3985, 3973, 3971, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2689, 2694, 2653, 2720, 2687, 30}

$$\begin{aligned}&\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}dx=\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}\\&-\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}a^2d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}-\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}\\&+\frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}\\&-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{3}{2}}(c+dx)(e\cot(c+dx))^{3/2}}\\&+\frac{2\sqrt{\sin(2c+2dx)}\cot(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3a^2d(e\cot(c+dx))^{3/2}}\end{aligned}$$

```
[In] Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-4*Cot[c + d*x]^3)/(3*a^2*d*(e*Cot[c + d*x])^(3/2)) + (4*Cot[c + d*x]^2*Cs
c[c + d*x])/(3*a^2*d*(e*Cot[c + d*x])^(3/2)) + (2*Cot[c + d*x]*Csc[c + d*x]
*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*(e*Cot[c + d
*x])^(3/2)) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[
c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2)) + Log[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/
2)*Tan[c + d*x]^(3/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
```

```
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^4(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{\int \left(\frac{a^2}{\tan^{\frac{5}{2}}(c+dx)}-\frac{2a^2\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}+\frac{a^2\sec^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\right)dx}{a^4(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&= \frac{\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}+\frac{\int \frac{\sec^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad -\frac{2\int \frac{\sec(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&= -\frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} \\
&\quad +\frac{2\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad +\frac{\operatorname{Subst}\left(\int \frac{1}{x^{5/2}}\,dx,x,\tan(c+dx)\right)}{a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}}+\frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} \\
&\quad +\frac{\left(2\cos^{\frac{3}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3a^2(e\cot(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)} \\
&\quad -\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx,x,\tan(c+dx)\right)}{a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{\left(2\cot(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3a^2(e\cot(c+dx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sqrt{\sin(2c+2dx)}}{3a^2d(e\cot(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{2\cot(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sqrt{\sin(2c+2dx)}}{3a^2d(e\cot(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}$$

$$
\begin{aligned}
&= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{2\cot(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3a^2d(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&= -\frac{4\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} + \frac{4\cot^2(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{2\cot(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3a^2d(e\cot(c+dx))^{3/2}} \\
&\quad + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{3/2}\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx
$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.01 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.78

method	result
default	$\frac{\sqrt{2}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^2\left(3i\sqrt{\csc(dx+c)-\cot(dx+c)+1}\,\sqrt{2-2\csc(dx+c)+2\cot(dx+c)}\,\sqrt{\cot(dx+c)-\csc(dx+c)}\ \mathrm{Ellip}\right.}{}$

```
[In] int(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^2/d*2^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2*(3*I*(csc(d*x+c)-cot(
d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))
^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*
I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(
d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2
*I,1/2*2^(1/2))-10*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*
x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+
1)^(1/2),1/2*2^(1/2))+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*c
ot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d
*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-
2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((
csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*(1-cos(d*x+c))^3*cs
c(d*x+c)^3-4*csc(d*x+c)+4*cot(d*x+c))/(-e/(1-cos(d*x+c))*((1-cos(d*x+c))^2*
csc(d*x+c)-sin(d*x+c)))^(3/2)/(1-cos(d*x+c))*sin(d*x+c)/((1-cos(d*x+c))*((1
-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))^3*csc(d*x+
c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\sec^2(c+dx)+2(e\cot(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}\,dx}{a^2}$$

```
[In] integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
[Out] Integral(1/((e*cot(c + d*x))**(3/2)*sec(c + d*x)**2 + 2*(e*cot(c + d*x))**(
3/2)*sec(c + d*x) + (e*cot(c + d*x))**(3/2)), x)/a**2
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2}{a^2\,(e\cot(c+d\,x))^{3/2}\,(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int(1/((e*cot(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(3/2)*(cos(c + d*x) + 1)^2), x)
```

3.252 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 355

$$\begin{aligned}
\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx &= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} \\
&+ \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}} \\
&+ \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&- \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&+ \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}
\end{aligned}$$

```
[Out] -4*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(5/2)+4*cos(d*x+c)*cot(d*x+c)^3/a^2/d/
(e*cot(d*x+c))^(5/2)-4*cos(d*x+c)*cot(d*x+c)^2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/
sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a^2/d/(e*cot(d*x+c))
^(5/2)/sin(2*d*x+2*c)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(
e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)-1/2*arctan(1+2^(1/2)*tan(d*x+c
)^(1/2))/a^2/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)-1/4*ln(1-2^(1/
2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(5/2)*2^(1/2)/tan(d*x+
c)^(5/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))
^(5/2)*2^(1/2)/tan(d*x+c)^(5/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3985, 3973, 3971, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2688, 2695, 2652, 2719, 2687, 30}

$$
\begin{aligned}
\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx &= \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}} \\
&\quad-\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}}-\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} \\
&\quad+\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}} \\
&\quad+\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{5}{2}}(c+dx)(e\cot(c+dx))^{5/2}} \\
&\quad+\frac{4\cos(c+dx)\cot^2(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{a^2d\sqrt{\sin(2c+2dx)}(e\cot(c+dx))^{5/2}}
\end{aligned}
$$

```
[In] Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-4*Cot[c + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(5/2)) + (4*Cos[c + d*x]*Cot[c
+ d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(5/2)) + (4*Cos[c + d*x]*Cot[c + d*x]^2*E
llipticE[c - Pi/4 + d*x, 2])/(a^2*d*(e*Cot[c + d*x])^(5/2)*Sqrt[Sin[2*c + 2
*d*x]]) + ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c +
d*x])^(5/2)*Tan[c + d*x]^(5/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(S
qrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2)) - Log[1 - Sqrt[2]*S
qrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*T
an[c + d*x]^(5/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*
Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(5/2)*Tan[c + d*x]^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}\\
&= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^4(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}\\
&= \frac{\int \left(\frac{a^2}{\tan^{\frac{3}{2}}(c+dx)}-\frac{2a^2\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}+\frac{a^2\sec^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\right)dx}{a^4(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}\\
&= \frac{\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}+\frac{\int \frac{\sec^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}\\
&\quad-\frac{2\int \frac{\sec(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}\\
&= -\frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}\\
&\quad-\frac{\int \sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}+\frac{4\int \cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx,x,\tan(c+dx)\right)}{a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}\\
&= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}\\
&\quad+\frac{\left(4\cos^{\frac{5}{2}}(c+dx)\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\sin^{\frac{5}{2}}(c+dx)}\\
&\quad-\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} \\
&\quad +\frac{(4\cos(c+dx)\cot^2(c+dx))\int\sqrt{\sin(2c+2dx)}\,dx}{a^2(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}} \\
&\quad -\frac{2\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} \\
&\quad +\frac{4\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|\,2\right)}{a^2d(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}} \\
&\quad +\frac{\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}-\frac{\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}}+\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} \\
&\quad +\frac{4\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|\,2\right)}{a^2d(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}} \\
&\quad -\frac{\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&\quad -\frac{\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&\quad -\frac{\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&\quad -\frac{\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} \\
&\quad + \frac{4\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} + \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{5/2}} \\
&\quad + \frac{4\cos(c+dx)\cot^2(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{5/2}\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&\quad - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{5/2}\tan^{\frac{5}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx
$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.10 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.74

method	result
default	$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{2}\left(i\,\text{EllipticPi}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-4i\,\text{EllipticE}\left(\sqrt{\csc(dx+c)-\cot(dx+c)+1},\frac{\sqrt{2}}{2}\right)+2i\,\text{EllipticF}\left(\sqrt{\csc(d\right.\right.}{}$

```
[In] int(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2+1/2*I)/a^2/d*2^(1/2)*(I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2
-1/2*I,1/2*2^(1/2))-4*I*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/
2))+2*I*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticPi((
csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*EllipticE((csc(d*x+
c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(
1/2),1/2*2^(1/2)))*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1
)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)/(cos(d*x+c)-1)/e^2/(e*cot(d*x+c))^(1/
2)*sin(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\left(e\cot\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\left(e\cot\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{a^2\left(e\cot\left(c+d\,x\right)\right)^{5/2}\left(\cos\left(c+d\,x\right)+1\right)^2}\,dx$$

```
[In] int(1/((e*cot(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(5/2)*(cos(c + d*x) + 1)^2), x)
```

3.253 $\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 321

$$\begin{aligned}\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx &= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} \\ &\quad - \frac{2\cot^3(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{a^2d(e\cot(c+dx))^{7/2}} \\ &\quad - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\ &\quad - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\ &\quad + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}\end{aligned}$$

```
[Out] 2*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(7/2)+2*cot(d*x+c)^3*csc(d*x+c)*(sin(c+
1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))
*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*cot(d*x+c))^(7/2)+1/2*arctan(-1+2^(1/2)*tan(
d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)+1/2*arcta
n(1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)
^(7/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(
7/2)*2^(1/2)/tan(d*x+c)^(7/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))
/a^2/d/(e*cot(d*x+c))^(7/2)*2^(1/2)/tan(d*x+c)^(7/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3985, 3973, 3971, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720, 2687, 30}

$$\begin{aligned}
&\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \\
&-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}}+\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}} \\
&+\frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}} \\
&+\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{7}{2}}(c+dx)(e\cot(c+dx))^{7/2}} \\
&-\frac{2\sqrt{\sin(2c+2dx)}\cot^3(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{a^2d(e\cot(c+dx))^{7/2}}
\end{aligned}$$

```
[In] Int[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (2*Cot[c + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(7/2)) - (2*Cot[c + d*x]^3*Csc[c
 + d*x]*EllipticF[c - Pi/4 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(a^2*d*(e*Cot[
c + d*x])^(7/2)) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e
*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2)) - Log[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x]
)^(7/2)*Tan[c + d*x]^(7/2)) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(7/2)*Tan[c + d*x]^(7/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\tan^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}$$

$$
\begin{aligned}
&= \frac{\int \frac{(-a+a\sec(c+dx))^2}{\sqrt{\tan(c+dx)}}\,dx}{a^4(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&= \frac{\int \left(\frac{a^2}{\sqrt{\tan(c+dx)}}-\frac{2a^2\sec(c+dx)}{\sqrt{\tan(c+dx)}}+\frac{a^2\sec^2(c+dx)}{\sqrt{\tan(c+dx)}}\right)dx}{a^4(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&= \frac{\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}+\frac{\int \frac{\sec^2(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&\quad -\frac{2\int \frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&= -\frac{\left(2\cos^{\frac{7}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{7/2}\sin^{\frac{7}{2}}(c+dx)} \\
&\quad +\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx,x,\tan(c+dx)\right)}{a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx,x,\tan(c+dx)\right)}{a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} \\
&\quad -\frac{\left(2\cot^3(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{a^2(e\cot(c+dx))^{7/2}} \\
&\quad +\frac{2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}}-\frac{2\cot^3(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{a^2d(e\cot(c+dx))^{7/2}} \\
&\quad +\frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}+\frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}}-\frac{2\cot^3(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{a^2d(e\cot(c+dx))^{7/2}} \\
&\quad +\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}+\frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&\quad -\frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}-\frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} \\
&\quad - \frac{2\cot^3(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{a^2d(e\cot(c+dx))^{7/2}} \\
&\quad - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&\quad + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{7/2}} - \frac{2\cot^3(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{a^2d(e\cot(c+dx))^{7/2}} \\
&\quad - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} \\
&\quad - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)} + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{7/2}\tan^{\frac{7}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx
$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/((e*Cot[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 961, normalized size of antiderivative = 2.99

method	result	size
default	Expression too large to display	961

```
[In] int(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/d*2^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^4*(I*(csc(d*x+c)-cot(d
*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^
(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1
-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c
))^(1/2)-I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1
/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2
),1/2+1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*
x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)-6*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2
*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((cs
c(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(
d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)+(csc(d*x+c)-cot(d*x+c
)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2
)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos
(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(
1/2)+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(c
ot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+
1/2*I,1/2*2^(1/2))*((1-cos(d*x+c))*(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-c
ot(d*x+c)+1)*csc(d*x+c))^(1/2)+4*((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-
csc(d*x+c))^(1/2)*(-cot(d*x+c)+csc(d*x+c)))/(-e/(1-cos(d*x+c))*((1-cos(d*x+
c))^2*csc(d*x+c)-sin(d*x+c)))^(7/2)/(1-cos(d*x+c))^3*sin(d*x+c)^3/((1-cos(d
*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)/((1-cos(d*x+c))*
(-cot(d*x+c)+csc(d*x+c)-1)*(csc(d*x+c)-cot(d*x+c)+1)*csc(d*x+c))^(1/2)/((1-
cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(7/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2}{a^2\,(e\cot(c+d\,x))^{7/2}\,(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int(1/((e*cot(c + d*x))^(7/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(7/2)*(cos(c + d*x) + 1)^2), x)
```

3.254 $\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 357

$$
\begin{aligned}
\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx &= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} \\
&- \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{4\cos(c+dx)\cot^4(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}} \\
&- \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&+ \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&- \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}
\end{aligned}
$$

```
[Out] 2/3*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(9/2)-4*cos(d*x+c)*cot(d*x+c)^3/a^2/d
/(e*cot(d*x+c))^(9/2)-4*cos(d*x+c)*cot(d*x+c)^4*(sin(c+1/4*Pi+d*x)^2)^(1/2)
/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))/a^2/d/(e*cot(d*x+c)
)^(9/2)/sin(2*d*x+2*c)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/
(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+1/2*arctan(1+2^(1/2)*tan(d*x+
c)^(1/2))/a^2/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)+1/4*ln(1-2^(1
/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(9/2)*2^(1/2)/tan(d*x
+c)^(9/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c)
)^(9/2)*2^(1/2)/tan(d*x+c)^(9/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3985, 3973, 3971, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719, 2687, 30}

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}}+\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}} + \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}}-\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} + \frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}} - \frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{9}{2}}(c+dx)(e\cot(c+dx))^{9/2}} + \frac{4\cos(c+dx)\cot^4(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)}{a^2d\sqrt{\sin(2c+2dx)}(e\cot(c+dx))^{9/2}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (2*Cot[c + d*x]^3)/(3*a^2*d*(e*Cot[c + d*x])^(9/2)) - (4*Cos[c + d*x]*Cot[c
 + d*x]^3)/(a^2*d*(e*Cot[c + d*x])^(9/2)) + (4*Cos[c + d*x]*Cot[c + d*x]^4*
EllipticE[c - Pi/4 + d*x, 2])/(a^2*d*(e*Cot[c + d*x])^(9/2)*Sqrt[Sin[2*c +
2*d*x]]) - ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c +
 d*x])^(9/2)*Tan[c + d*x]^(9/2)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(
Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2)) + Log[1 - Sqrt[2]*
Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*
Tan[c + d*x]^(9/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2
*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(9/2)*Tan[c + d*x]^(9/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
```

```
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
```

```
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\tan^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}\\
&= \frac{\int (-a+a\sec(c+dx))^2\sqrt{\tan(c+dx)}\,dx}{a^4(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}\\
&= \frac{\int \left(a^2\sqrt{\tan(c+dx)}-2a^2\sec(c+dx)\sqrt{\tan(c+dx)}+a^2\sec^2(c+dx)\sqrt{\tan(c+dx)}\right)\,dx}{a^4(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}\\
&= \frac{\int \sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}+\frac{\int \sec^2(c+dx)\sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}\\
&\quad-\frac{2\int \sec(c+dx)\sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}\\
&= -\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}}+\frac{4\int \cos(c+dx)\sqrt{\tan(c+dx)}\,dx}{a^2(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}\\
&\quad+\frac{\text{Subst}\left(\int \sqrt{x}\,dx,x,\tan(c+dx)\right)}{a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}+\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}\\
&= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}}-\frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}}\\
&\quad+\frac{\left(4\cos^{\frac{9}{2}}(c+dx)\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{a^2(e\cot(c+dx))^{9/2}\sin^{\frac{9}{2}}(c+dx)}\\
&\quad+\frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} \\
&\quad + \frac{(4\cos(c+dx)\cot^4(c+dx))\int\sqrt{\sin(2c+2dx)}\,dx}{a^2(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + \frac{\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} \\
&\quad + \frac{4\cos(c+dx)\cot^4(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad + \frac{\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad + \frac{\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad + \frac{\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} \\
&\quad + \frac{4\cos(c+dx)\cot^4(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad + \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad - \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\cot^3(c+dx)}{3a^2d(e\cot(c+dx))^{9/2}} - \frac{4\cos(c+dx)\cot^3(c+dx)}{a^2d(e\cot(c+dx))^{9/2}} \\
&\quad + \frac{4\cos(c+dx)\cot^4(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)}{a^2d(e\cot(c+dx))^{9/2}\sqrt{\sin(2c+2dx)}} \\
&\quad - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)} \\
&\quad - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{9/2}\tan^{\frac{9}{2}}(c+dx)}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx
$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/((e*Cot[c + d*x])^(9/2)*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.91 (sec) , antiderivative size = 1141, normalized size of antiderivative = 3.20

method	result	size
default	Expression too large to display	1141

```
[In] int(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/6/a^2/d*2^(1/2)*(3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x
+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c
)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2-3*I*(cot(d*x+c)-csc(d*x+c)+1
)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*Ellip
ticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+3
*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c
)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,
1/2*2^(1/2))*cos(d*x+c)-3*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc
(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d
*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)+24*(csc(d*x+c)-cot(d*x+c)+
```

```
1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elli
pticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-12*(csc(d*x
+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x
+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c
)^2+3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*
x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2
*I,1/2*2^(1/2))*cos(d*x+c)^2+3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-
csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-co
t(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+24*(csc(d*x+c)-cot(d*
x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)
*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)-12*(csc(
d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(
d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*
x+c)+3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellipt
icPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*x+c)-cot
(d*x+c)+1)^(1/2)*cos(d*x+c)+3*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-c
sc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot
(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)-10*2^(1/2)*cos(d*x+c)^2+
12*2^(1/2)*cos(d*x+c)-2*2^(1/2))/e^4/(e*cot(d*x+c))^(1/2)/(cos(d*x+c)^2-1)^
2*sin(d*x+c)^2*tan(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))**(9/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{9}{2}}(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{9}{2}}(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(9/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{9/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2}{a^2\,(e\cot(c+d\,x))^{9/2}\,(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int(1/((e*cot(c + d*x))^(9/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(9/2)*(cos(c + d*x) + 1)^2), x)
```

3.255 $\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 389

$$\begin{aligned}
\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx &= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} \\
&+ \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}} \\
&+ \frac{2\cot^5(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3a^2d(e\cot(c+dx))^{11/2}} \\
&+ \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} \\
&+ \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} \\
&- \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}
\end{aligned}$$

```
[Out] 2/5*cot(d*x+c)^3/a^2/d/(e*cot(d*x+c))^(11/2)+2*cot(d*x+c)^5/a^2/d/(e*cot(d*
x+c))^(11/2)-4/3*cot(d*x+c)^4*csc(d*x+c)/a^2/d/(e*cot(d*x+c))^(11/2)-2/3*co
t(d*x+c)^5*csc(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*Ellipti
cF(cos(c+1/4*Pi+d*x),2^(1/2))*sin(2*d*x+2*c)^(1/2)/a^2/d/(e*cot(d*x+c))^(11
/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(e*cot(d*x+c))^(11/2)*2^(
1/2)/tan(d*x+c)^(11/2)-1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))/a^2/d/(e*cot(
d*x+c))^(11/2)*2^(1/2)/tan(d*x+c)^(11/2)+1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+
tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(11/2)*2^(1/2)/tan(d*x+c)^(11/2)-1/4*ln(1+
2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))/a^2/d/(e*cot(d*x+c))^(11/2)*2^(1/2)/ta
n(d*x+c)^(11/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3985, 3973, 3971, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2691, 2694, 2653, 2720, 2687, 30}

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}} - \frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}a^2d\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}} + \frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}} - \frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}a^2d\tan^{\frac{11}{2}}(c+dx)(e\cot(c+dx))^{11/2}} + \frac{2\sqrt{\sin(2c+2dx)}\cot^5(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{3a^2d(e\cot(c+dx))^{11/2}}$$

```
[In] Int[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (2*Cot[c + d*x]^3)/(5*a^2*d*(e*Cot[c + d*x])^(11/2)) + (2*Cot[c + d*x]^5)/(
a^2*d*(e*Cot[c + d*x])^(11/2)) - (4*Cot[c + d*x]^4*Csc[c + d*x])/(3*a^2*d*(
e*Cot[c + d*x])^(11/2)) + (2*Cot[c + d*x]^5*Csc[c + d*x]*EllipticF[c - Pi/4
 + d*x, 2]*Sqrt[Sin[2*c + 2*d*x]])/(3*a^2*d*(e*Cot[c + d*x])^(11/2)) + ArcT
an[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(11/2)*T
an[c + d*x]^(11/2)) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/(Sqrt[2]*a^2*d
*(e*Cot[c + d*x])^(11/2)*Tan[c + d*x]^(11/2)) + Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*a^2*d*(e*Cot[c + d*x])^(11/2)*Tan[c + d*
x]^(11/2)) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]*
a^2*d*(e*Cot[c + d*x])^(11/2)*Tan[c + d*x]^(11/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
```

```
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rule 3985

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*((a_) + (b_.)*sec[(c_.) + (d_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(e*Cot[c + d*x])^m*Tan[c + d*x]^m, Int[(a + b
*Sec[c + d*x])^n/Tan[c + d*x]^m, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x]
&&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\tan^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}\\
&= \frac{\int(-a+a\sec(c+dx))^2\tan^{\frac{3}{2}}(c+dx)\,dx}{a^4(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}\\
&= \frac{\int\left(a^2\tan^{\frac{3}{2}}(c+dx)-2a^2\sec(c+dx)\tan^{\frac{3}{2}}(c+dx)+a^2\sec^2(c+dx)\tan^{\frac{3}{2}}(c+dx)\right)dx}{a^4(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}\\
&= \frac{\int\tan^{\frac{3}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}+\frac{\int\sec^2(c+dx)\tan^{\frac{3}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}\\
&\quad-\frac{2\int\sec(c+dx)\tan^{\frac{3}{2}}(c+dx)\,dx}{a^2(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}\\
&= \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}}-\frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}}+\frac{2\int\frac{\sec(c+dx)}{\sqrt{\tan(c+dx)}}\,dx}{3a^2(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}\\
&\quad-\frac{\int\frac{1}{\sqrt{\tan(c+dx)}}\,dx}{a^2(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}+\frac{\operatorname{Subst}\left(\int x^{3/2}\,dx,x,\tan(c+dx)\right)}{a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}\\
&= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}}+\frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}}-\frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}}\\
&\quad+\frac{\left(2\cos^{\frac{11}{2}}(c+dx)\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3a^2(e\cot(c+dx))^{11/2}\sin^{\frac{11}{2}}(c+dx)}-\frac{\operatorname{Subst}\left(\int\frac{1}{\sqrt{x}(1+x^2)}\,dx,x,\tan(c+dx)\right)}{a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}\\
&= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}}+\frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}}-\frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}}\\
&\quad+\frac{\left(2\cot^5(c+dx)\csc(c+dx)\sqrt{\sin(2c+2dx)}\right)\int\frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3a^2(e\cot(c+dx))^{11/2}}\\
&\quad-\frac{2\operatorname{Subst}\left(\int\frac{1}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}} \\
&+ \frac{2\cot^5(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3a^2d(e\cot(c+dx))^{11/2}} \\
&- \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\sqrt{\tan(c+dx)}\right)}{a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} \\
&= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}} \\
&+ \frac{2\cot^5(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3a^2d(e\cot(c+dx))^{11/2}} \\
&- \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} \\
&- \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} \\
&+ \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} \\
&+ \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\sqrt{\tan(c+dx)}\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} \\
&= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}} \\
&+ \frac{2\cot^5(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3a^2d(e\cot(c+dx))^{11/2}} \\
&+ \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} \\
&- \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} \\
&- \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)} \\
&+ \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}
\end{aligned}
$$

$$= \frac{2\cot^3(c+dx)}{5a^2d(e\cot(c+dx))^{11/2}} + \frac{2\cot^5(c+dx)}{a^2d(e\cot(c+dx))^{11/2}} - \frac{4\cot^4(c+dx)\csc(c+dx)}{3a^2d(e\cot(c+dx))^{11/2}}$$
$$+ \frac{2\cot^5(c+dx)\csc(c+dx)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sqrt{\sin(2c+2dx)}}{3a^2d(e\cot(c+dx))^{11/2}}$$
$$+ \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$
$$- \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$
$$+ \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$
$$- \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}a^2d(e\cot(c+dx))^{11/2}\tan^{\frac{11}{2}}(c+dx)}$$

Mathematica [F]

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx$$

```
[In] Integrate[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2),x]
[Out] Integrate[1/((e*Cot[c + d*x])^(11/2)*(a + a*Sec[c + d*x])^2), x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.69 (sec) , antiderivative size = 1007, normalized size of antiderivative = 2.59

method	result	size
default	Expression too large to display	1007

```
[In] int(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/30/a^2/d*2^(1/2)*(15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*
x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+
c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^3-15*I*(cot(d*x+c)-csc(d*x+c)
+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*Ell
ipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^3
+15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*
x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2
```

```
*I,1/2*2^(1/2))*cos(d*x+c)^2-15*I*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+
c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)
-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+15*(cot(d*x+c)-csc
(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1
/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d
*x+c)^3+15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(csc(d*x+c)
-cot(d*x+c)+1)^(1/2)*cos(d*x+c)^3-50*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d
*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+
c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^3+15*(cot(d*x+c)-csc(d*x+c)+
1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*Elli
pticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2+
15*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(csc(d*x+c
)-cot(d*x+c)+1)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,
1/2*2^(1/2))*cos(d*x+c)^2-50*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-cs
c(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d
*x+c)+1)^(1/2),1/2*2^(1/2))*cos(d*x+c)^2-24*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)
+20*cos(d*x+c)*sin(d*x+c)*2^(1/2)-6*2^(1/2)*sin(d*x+c))/(cos(d*x+c)-1)^3/(c
os(d*x+c)+1)^3/e^5/(e*cot(d*x+c))^(1/2)*sin(d*x+c)^3*tan(d*x+c)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))**(11/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(e\cot(dx+c))^{\frac{11}{2}}(a\sec(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(e*cot(d*x+c))^(11/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cot(d*x + c))^(11/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{11/2}(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+d\,x)^2}{a^2\,(e\cot(c+d\,x))^{11/2}\,(\cos(c+d\,x)+1)^2}\,dx$$

```
[In] int(1/((e*cot(c + d*x))^(11/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*cot(c + d*x))^(11/2)*(cos(c + d*x) + 1)^2), x)
```

3.256 $\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$
\begin{aligned}
\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx = {} & \frac{a\log(\cos(c+dx))}{d}-\frac{16b\sec(c+dx)}{35d} \\
& +\frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d} \\
& -\frac{(35a+24b\sec(c+dx))\tan^4(c+dx)}{140d} \\
& +\frac{(7a+6b\sec(c+dx))\tan^6(c+dx)}{42d}
\end{aligned}
$$

```
[Out] a*ln(cos(d*x+c))/d-16/35*b*sec(d*x+c)/d+1/70*(35*a+16*b*sec(d*x+c))*tan(d*x
+c)^2/d-1/140*(35*a+24*b*sec(d*x+c))*tan(d*x+c)^4/d+1/42*(7*a+6*b*sec(d*x+c
))*tan(d*x+c)^6/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3966, 3969, 3556, 2686, 8}

$$
\begin{aligned}
\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx = {} & \frac{\tan^6(c+dx)(7a+6b\sec(c+dx))}{42d} \\
& -\frac{\tan^4(c+dx)(35a+24b\sec(c+dx))}{140d} \\
& +\frac{\tan^2(c+dx)(35a+16b\sec(c+dx))}{70d} \\
& +\frac{a\log(\cos(c+dx))}{d}-\frac{16b\sec(c+dx)}{35d}
\end{aligned}
$$

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (16*b*Sec[c + d*x])/(35*d) + ((35*a + 16*b*Sec[c
+ d*x])*Tan[c + d*x]^2)/(70*d) - ((35*a + 24*b*Sec[c + d*x])*Tan[c + d*x]^4
)/(140*d) + ((7*a + 6*b*Sec[c + d*x])*Tan[c + d*x]^6)/(42*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(7a+6b\sec(c+dx))\tan^6(c+dx)}{42d} - \frac{1}{7}\int (7a+6b\sec(c+dx))\tan^5(c+dx)\,dx \\
&= -\frac{(35a+24b\sec(c+dx))\tan^4(c+dx)}{140d} + \frac{(7a+6b\sec(c+dx))\tan^6(c+dx)}{42d} \\
&\quad + \frac{1}{35}\int (35a+24b\sec(c+dx))\tan^3(c+dx)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d} - \frac{(35a+24b\sec(c+dx))\tan^4(c+dx)}{140d} \\
&\quad + \frac{(7a+6b\sec(c+dx))\tan^6(c+dx)}{42d} - \frac{1}{105}\int(105a+48b\sec(c+dx))\tan(c \\
&\quad + dx)\,dx \\
&= \frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d} - \frac{(35a+24b\sec(c+dx))\tan^4(c+dx)}{140d} \\
&\quad + \frac{(7a+6b\sec(c+dx))\tan^6(c+dx)}{42d} - a\int\tan(c+dx)\,dx \\
&\quad - \frac{1}{35}(16b)\int\sec(c+dx)\tan(c+dx)\,dx \\
&= \frac{a\log(\cos(c+dx))}{d} + \frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d} \\
&\quad - \frac{(35a+24b\sec(c+dx))\tan^4(c+dx)}{140d} \\
&\quad + \frac{(7a+6b\sec(c+dx))\tan^6(c+dx)}{42d} - \frac{(16b)\mathrm{Subst}(\int 1\,dx, x, \sec(c+dx))}{35d} \\
&= \frac{a\log(\cos(c+dx))}{d} - \frac{16b\sec(c+dx)}{35d} + \frac{(35a+16b\sec(c+dx))\tan^2(c+dx)}{70d} \\
&\quad - \frac{(35a+24b\sec(c+dx))\tan^4(c+dx)}{140d} + \frac{(7a+6b\sec(c+dx))\tan^6(c+dx)}{42d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$
\begin{aligned}
&\int(a+b\sec(c+dx))\tan^7(c+dx)\,dx \\
&= -\frac{b\sec(c+dx)}{d} + \frac{b\sec^3(c+dx)}{d} - \frac{3b\sec^5(c+dx)}{5d} + \frac{b\sec^7(c+dx)}{7d} \\
&\quad + \frac{a(12\log(\cos(c+dx)) + 6\tan^2(c+dx) - 3\tan^4(c+dx) + 2\tan^6(c+dx))}{12d}
\end{aligned}
$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^7,x]
```

```
[Out] -((b*Sec[c + d*x])/d) + (b*Sec[c + d*x]^3)/d - (3*b*Sec[c + d*x]^5)/(5*d) +
 (b*Sec[c + d*x]^7)/(7*d) + (a*(12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 - 3
*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{b\sec(dx+c)^7}{7}+\frac{a\sec(dx+c)^6}{6}-\frac{3b\sec(dx+c)^5}{5}-\frac{3a\sec(dx+c)^4}{4}+b\sec(dx+c)^3+\frac{3a\sec(dx+c)^2}{2}-b\sec(dx+c)-a\ln(\sec(dx+c))}{d}$
default	$\frac{\frac{b\sec(dx+c)^7}{7}+\frac{a\sec(dx+c)^6}{6}-\frac{3b\sec(dx+c)^5}{5}-\frac{3a\sec(dx+c)^4}{4}+b\sec(dx+c)^3+\frac{3a\sec(dx+c)^2}{2}-b\sec(dx+c)-a\ln(\sec(dx+c))}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^6}{6}-\frac{\tan(dx+c)^4}{4}+\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b\left(\frac{\sec(dx+c)^7}{7}-\frac{3\sec(dx+c)^5}{5}+\sec(dx+c)^3-\sec(dx+c)\right.}{d}$
risch	$-iax-\frac{2iac}{d}-\frac{2(105b\,e^{13i(dx+c)}-315a\,e^{12i(dx+c)}+210b\,e^{11i(dx+c)}-945a\,e^{10i(dx+c)}+903b\,e^{9i(dx+c)}-1820a\,e^{8i(dx}}{}$

[In] int((a+b*sec(d*x+c))*tan(d*x+c)^7,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/7*b*sec(d*x+c)^7+1/6*a*sec(d*x+c)^6-3/5*b*sec(d*x+c)^5-3/4*a*sec(d*x+c)^4+b*sec(d*x+c)^3+3/2*a*sec(d*x+c)^2-b*sec(d*x+c)-a*ln(sec(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx$$
$$=\frac{420\,a\cos(dx+c)^7\log(-\cos(dx+c))-420\,b\cos(dx+c)^6+630\,a\cos(dx+c)^5+420\,b\cos(dx+c)^4-}{420\,d\cos(dx+c)^7}$$

[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="fricas")

[Out] 1/420*(420*a*cos(d*x + c)^7*log(-cos(d*x + c)) - 420*b*cos(d*x + c)^6 + 630*a*cos(d*x + c)^5 + 420*b*cos(d*x + c)^4 - 315*a*cos(d*x + c)^3 - 252*b*cos(d*x + c)^2 + 70*a*cos(d*x + c) + 60*b)/(d*cos(d*x + c)^7)

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx$$
$$=\begin{cases}-\frac{a\log(\tan^2(c+dx)+1)}{2d}+\frac{a\tan^6(c+dx)}{6d}-\frac{a\tan^4(c+dx)}{4d}+\frac{a\tan^2(c+dx)}{2d}+\frac{b\tan^6(c+dx)\sec(c+dx)}{7d}-\frac{6b\tan^4(c+dx)\sec(c+dx)}{35d} \\ x(a+b\sec(c))\tan^7(c)\end{cases}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**6/(6*d) - a*
tan(c + d*x)**4/(4*d) + a*tan(c + d*x)**2/(2*d) + b*tan(c + d*x)**6*sec(c +
 d*x)/(7*d) - 6*b*tan(c + d*x)**4*sec(c + d*x)/(35*d) + 8*b*tan(c + d*x)**2
*sec(c + d*x)/(35*d) - 16*b*sec(c + d*x)/(35*d), Ne(d, 0)), (x*(a + b*sec(c
))*tan(c)**7, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx = \frac{420\,a\log\left(\cos\left(dx+c\right)\right) - \frac{420\,b\cos(dx+c)^6-630\,a\cos(dx+c)^5-420\,b\cos(dx+c)^4+315\,a\cos(dx+c)^3+252\,b\cos(dx+c)^2-70\,a\cos(dx+c)-6}{\cos(dx+c)^7}}{420\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(420*a*log(cos(d*x + c)) - (420*b*cos(d*x + c)^6 - 630*a*cos(d*x + c)
^5 - 420*b*cos(d*x + c)^4 + 315*a*cos(d*x + c)^3 + 252*b*cos(d*x + c)^2 - 7
0*a*cos(d*x + c) - 60*b)/cos(d*x + c)^7)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(103) = 206.

Time = 3.38 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.86

$$\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx =$$

$$-\frac{420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{1089\,a+384\,b+\frac{8463\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2688\,b(\cos(dx+c)-}{\cos(dx+c)+1}}{}}{}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1089*a + 384*b + 84
63*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2688*b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 28749*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 8064*b
*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 56035*a*(cos(d*x + c) - 1)^3/(
cos(d*x + c) + 1)^3 + 13440*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 5
6035*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a*(cos(d*x + c) -
1)^5/(cos(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^
6 + 1089*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 18.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.99

$$\int (a+b\sec(c+dx))\tan^7(c+dx)\,dx$$
$$= \frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{128\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3}+\left(-\frac{128\,a}{3}-32\,b\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\left(14\,a+\frac{96\,b}{5}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\right.}$$
$$-\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\right)}{d}$$

```
[In] int(tan(c + d*x)^7*(a + b/cos(c + d*x)),x)
```

```
[Out] ((32*b)/35 - tan(c/2 + (d*x)/2)^2*(2*a + (32*b)/5) + tan(c/2 + (d*x)/2)^4*(
14*a + (96*b)/5) - tan(c/2 + (d*x)/2)^6*((128*a)/3 + 32*b) + (128*a*tan(c/2
 + (d*x)/2)^8)/3 - 14*a*tan(c/2 + (d*x)/2)^10 + 2*a*tan(c/2 + (d*x)/2)^12)/
(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2
)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x
)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1)) - (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.257 $\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx = -\frac{a\log(\cos(c+dx))}{d}+\frac{8b\sec(c+dx)}{15d} - \frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d} + \frac{(5a+4b\sec(c+dx))\tan^4(c+dx)}{20d}$$

```
[Out] -a*ln(cos(d*x+c))/d+8/15*b*sec(d*x+c)/d-1/30*(15*a+8*b*sec(d*x+c))*tan(d*x+
c)^2/d+1/20*(5*a+4*b*sec(d*x+c))*tan(d*x+c)^4/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3966, 3969, 3556, 2686, 8}

$$\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx = \frac{\tan^4(c+dx)(5a+4b\sec(c+dx))}{20d} - \frac{\tan^2(c+dx)(15a+8b\sec(c+dx))}{30d} - \frac{a\log(\cos(c+dx))}{d}+\frac{8b\sec(c+dx)}{15d}$$

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (8*b*Sec[c + d*x])/(15*d) - ((15*a + 8*b*Sec[c
 + d*x])*Tan[c + d*x]^2)/(30*d) + ((5*a + 4*b*Sec[c + d*x])*Tan[c + d*x]^4)
/(20*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(5a+4b\sec(c+dx))\tan^4(c+dx)}{20d} - \frac{1}{5}\int(5a+4b\sec(c+dx))\tan^3(c+dx)\,dx \\
&= -\frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d} + \frac{(5a+4b\sec(c+dx))\tan^4(c+dx)}{20d} \\
&\quad + \frac{1}{15}\int(15a+8b\sec(c+dx))\tan(c+dx)\,dx \\
&= -\frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d} + \frac{(5a+4b\sec(c+dx))\tan^4(c+dx)}{20d} \\
&\quad + a\int\tan(c+dx)\,dx + \frac{1}{15}(8b)\int\sec(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\log(\cos(c+dx))}{d} - \frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d} \\
&\quad + \frac{(5a+4b\sec(c+dx))\tan^4(c+dx)}{20d} + \frac{(8b)\text{Subst}(\int 1\,dx, x, \sec(c+dx))}{15d}
\end{aligned}
$$

$$
= -\frac{a\log(\cos(c+dx))}{d} + \frac{8b\sec(c+dx)}{15d} - \frac{(15a+8b\sec(c+dx))\tan^2(c+dx)}{30d} + \frac{(5a+4b\sec(c+dx))\tan^4(c+dx)}{20d}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$
\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx = \frac{b\sec(c+dx)}{d} - \frac{2b\sec^3(c+dx)}{3d} + \frac{b\sec^5(c+dx)}{5d} - \frac{a(4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx))}{4d}
$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (b*Sec[c + d*x])/d - (2*b*Sec[c + d*x]^3)/(3*d) + (b*Sec[c + d*x]^5)/(5*d)
- (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{b\sec(dx+c)^5}{5}+\frac{a\sec(dx+c)^4}{4}-\frac{2b\sec(dx+c)^3}{3}-a\sec(dx+c)^2+b\sec(dx+c)+a\ln(\sec(dx+c))}{d}$
default	$\frac{\frac{b\sec(dx+c)^5}{5}+\frac{a\sec(dx+c)^4}{4}-\frac{2b\sec(dx+c)^3}{3}-a\sec(dx+c)^2+b\sec(dx+c)+a\ln(\sec(dx+c))}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b\left(\frac{\sec(dx+c)^5}{5}-\frac{2\sec(dx+c)^3}{3}+\sec(dx+c)\right)}{d}$
risch	$iax+\frac{2iac}{d}+\frac{2b\,\mathrm{e}^{9i(dx+c)}-4a\,\mathrm{e}^{8i(dx+c)}+\frac{8b\,\mathrm{e}^{7i(dx+c)}}{3}-8a\,\mathrm{e}^{6i(dx+c)}+\frac{116b\,\mathrm{e}^{5i(dx+c)}}{15}-8a\,\mathrm{e}^{4i(dx+c)}+\frac{8b\,\mathrm{e}^{3i(dx+c)}}{3}-4a\,\mathrm{e}}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*b*sec(d*x+c)^5+1/4*a*sec(d*x+c)^4-2/3*b*sec(d*x+c)^3-a*sec(d*x+c)^
2+b*sec(d*x+c)+a*ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx = -\frac{60\,a\cos\left(dx+c\right)^5\log\left(-\cos\left(dx+c\right)\right)-60\,b\cos\left(dx+c\right)^4+60\,a\cos\left(dx+c\right)^3+40\,b\cos\left(dx+c\right)^2-15}{60\,d\cos\left(dx+c\right)^5}$$

[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")

[Out] -1/60*(60*a*cos(d*x + c)^5*log(-cos(d*x + c)) - 60*b*cos(d*x + c)^4 + 60*a*cos(d*x + c)^3 + 40*b*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 12*b)/(d*cos(d*x + c)^5)

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.33

$$\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx = \begin{cases} \frac{a\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a\tan^4\left(c+dx\right)}{4d}-\frac{a\tan^2\left(c+dx\right)}{2d}+\frac{b\tan^4\left(c+dx\right)\sec\left(c+dx\right)}{5d}-\frac{4b\tan^2\left(c+dx\right)\sec\left(c+dx\right)}{15d}+\frac{8b\sec\left(c+dx\right)}{15d} & \text{f} \\ x(a+b\sec\left(c\right))\tan^5\left(c\right) & \text{c} \end{cases}$$

[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**5,x)

[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**4/(4*d) - a*tan(c + d*x)**2/(2*d) + b*tan(c + d*x)**4*sec(c + d*x)/(5*d) - 4*b*tan(c + d*x)**2*sec(c + d*x)/(15*d) + 8*b*sec(c + d*x)/(15*d), Ne(d, 0)), (x*(a + b*sec(c))*tan(c)**5, True))

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int (a+b\sec(c+dx))\tan^5(c+dx)\,dx = -\frac{60\,a\log\left(\cos\left(dx+c\right)\right)-\frac{60\,b\cos(dx+c)^4-60\,a\cos(dx+c)^3-40\,b\cos(dx+c)^2+15\,a\cos(dx+c)+12\,b}{\cos(dx+c)^5}}{60\,d}$$

[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")

[Out] -1/60*(60*a*log(cos(d*x + c)) - (60*b*cos(d*x + c)^4 - 60*a*cos(d*x + c)^3 - 40*b*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 12*b)/cos(d*x + c)^5)/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(78) = 156$.

Time = 1.53 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.95

$$\int (a + b\sec(c + dx))\tan^5(c + dx)\,dx = \frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{137\,a+64\,b+\frac{805\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{320\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1970\,a(\cdots}{(\cos\cdots}}{60\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (137*a + 64*b + 805*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 320*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1970*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 640*b*(cos(d*x
+ c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 805*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a*(cos(d*x
 + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 19.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.93

$$\int (a + b\sec(c + dx))\tan^5(c + dx)\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d} - \frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + \left(10\,a+\frac{32\,b}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + \left(-2\,a-\frac{16\,b}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + \frac{16\,b}{15}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + 10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1\right)}$$

```
[In] int(tan(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - ((16*b)/15 - tan(c/2 + (d*x)/2)^2*(2*
a + (16*b)/3) + tan(c/2 + (d*x)/2)^4*(10*a + (32*b)/3) - 10*a*tan(c/2 + (d*
x)/2)^6 + 2*a*tan(c/2 + (d*x)/2)^8)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2
 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2
+ (d*x)/2)^10 - 1))
```

3.258 $\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx = \frac{a\log(\cos(c+dx))}{d} - \frac{2b\sec(c+dx)}{3d} + \frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d}$$

```
[Out] a*ln(cos(d*x+c))/d-2/3*b*sec(d*x+c)/d+1/6*(3*a+2*b*sec(d*x+c))*tan(d*x+c)^2
/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3966, 3969, 3556, 2686, 8}

$$\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx = \frac{\tan^2(c+dx)(3a+2b\sec(c+dx))}{6d} + \frac{a\log(\cos(c+dx))}{d} - \frac{2b\sec(c+dx)}{3d}$$

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (a*Log[Cos[c + d*x]])/d - (2*b*Sec[c + d*x])/(3*d) + ((3*a + 2*b*Sec[c + d*
x])*Tan[c + d*x]^2)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d}-\frac{1}{3}\int(3a+2b\sec(c+dx))\tan(c+dx)\,dx\\
&= \frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d}-a\int\tan(c+dx)\,dx-\frac{1}{3}(2b)\int\sec(c+dx)\tan(c+dx)\,dx\\
&= \frac{a\log(\cos(c+dx))}{d}+\frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d}-\frac{(2b)\text{Subst}\left(\int 1\,dx,x,\sec(c+dx)\right)}{3d}\\
&= \frac{a\log(\cos(c+dx))}{d}-\frac{2b\sec(c+dx)}{3d}+\frac{(3a+2b\sec(c+dx))\tan^2(c+dx)}{6d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx = -\frac{b\sec(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d} + \frac{a(2\log(\cos(c+dx))+\tan^2(c+dx))}{2d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] -((b*Sec[c + d*x])/d) + (b*Sec[c + d*x]^3)/(3*d) + (a*(2*Log[Cos[c + d*x]]
+ Tan[c + d*x]^2))/(2*d)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

method	result	siz
derivativedivides	$\frac{\frac{b\sec(dx+c)^3}{3}+\frac{a\sec(dx+c)^2}{2}-b\sec(dx+c)-a\ln(\sec(dx+c))}{d}$	47
default	$\frac{\frac{b\sec(dx+c)^3}{3}+\frac{a\sec(dx+c)^2}{2}-b\sec(dx+c)-a\ln(\sec(dx+c))}{d}$	47
parts	$\frac{a\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b\left(\frac{\sec(dx+c)^3}{3}-\sec(dx+c)\right)}{d}$	55
risch	$-iax-\frac{2iac}{d}-\frac{2\left(3b\,e^{5i(dx+c)}-3a\,e^{4i(dx+c)}+2b\,e^{3i(dx+c)}-3a\,e^{2i(dx+c)}+3b\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	11

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b*sec(d*x+c)^3+1/2*a*sec(d*x+c)^2-b*sec(d*x+c)-a*ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx = \frac{6\,a\cos(dx+c)^3\log(-\cos(dx+c))-6\,b\cos(dx+c)^2+3\,a\cos(dx+c)+2\,b}{6\,d\cos(dx+c)^3}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*cos(d*x + c)^3*log(-cos(d*x + c)) - 6*b*cos(d*x + c)^2 + 3*a*cos(d
*x + c) + 2*b)/(d*cos(d*x + c)^3)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

$$\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx$$
$$= \begin{cases} -\frac{a\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{a\tan^2(c+dx)}{2d}+\frac{b\tan^2(c+dx)\sec(c+dx)}{3d}-\frac{2b\sec(c+dx)}{3d} & \text{for } d\neq 0 \\ x(a+b\sec(c))\tan^3(c) & \text{otherwise} \end{cases}$$

[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**3,x)

[Out] Piecewise((-a*log(tan(c + d*x)**2 + 1)/(2*d) + a*tan(c + d*x)**2/(2*d) + b*tan(c + d*x)**2*sec(c + d*x)/(3*d) - 2*b*sec(c + d*x)/(3*d), Ne(d, 0)), (x*(a + b*sec(c))*tan(c)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx = \frac{6\,a\log\left(\cos\left(dx+c\right)\right)-\frac{6\,b\cos\left(dx+c\right)^2-3\,a\cos\left(dx+c\right)-2\,b}{\cos\left(dx+c\right)^3}}{6\,d}$$

[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")

[Out] 1/6*(6*a*log(cos(d*x + c)) - (6*b*cos(d*x + c)^2 - 3*a*cos(d*x + c) - 2*b)/cos(d*x + c)^3)/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(51) = 102$.

Time = 0.62 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.25

$$\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx =$$
$$-\frac{6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-6\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{11\,a+8\,b+\frac{45\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{24\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a(\cos(dx-}{(\cos(dx+c}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")

[Out] -1/6*(6*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 6*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (11*a + 8*b + 45*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 24*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3)/d

Mupad [B] (verification not implemented)

Time = 15.54 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.85

$$\int (a+b\sec(c+dx))\tan^3(c+dx)\,dx$$
$$=\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+(-2\,a-4\,b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{4\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

```
[In] int(tan(c + d*x)^3*(a + b/cos(c + d*x)),x)
[Out] ((4*b)/3 - tan(c/2 + (d*x)/2)^2*(2*a + 4*b) + 2*a*tan(c/2 + (d*x)/2)^4)/(d*
(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1
)) - (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.259 $\int (a+b\sec(c+dx))\tan(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int (a+b\sec(c+dx))\tan(c+dx)\,dx=-\frac{a\log(\cos(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] -a*ln(cos(d*x+c))/d+b*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3969, 3556, 2686, 8}

$$\int (a+b\sec(c+dx))\tan(c+dx)\,dx=\frac{b\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= a\int \tan(c+dx)\,dx + b\int \sec(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\log(\cos(c+dx))}{d} + \frac{b\text{Subst}(\int 1\,dx, x, \sec(c+dx))}{d} \\
&= -\frac{a\log(\cos(c+dx))}{d} + \frac{b\sec(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$
\int (a+b\sec(c+dx))\tan(c+dx)\,dx = -\frac{a\log(\cos(c+dx))}{d} + \frac{b\sec(c+dx)}{d}
$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x],x]
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{b\sec(dx+c)+a\ln(\sec(dx+c))}{d}$	23
default	$\frac{b\sec(dx+c)+a\ln(\sec(dx+c))}{d}$	23
parts	$\frac{a\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{b\sec(dx+c)}{d}$	30
risch	$iax+\frac{2iac}{d}+\frac{2b\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	61

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*sec(d*x+c)+a*ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int (a+b\sec(c+dx))\tan(c+dx)\,dx = -\frac{a\cos(dx+c)\log(-\cos(dx+c))-b}{d\cos(dx+c)}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c)*log(-cos(d*x + c)) - b)/(d*cos(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int (a+b\sec(c+dx))\tan(c+dx)\,dx = \begin{cases} \frac{a\log(\tan^2(c+dx)+1)}{2d} + \frac{b\sec(c+dx)}{d} & \text{for } d \neq 0 \\ x(a+b\sec(c))\tan(c) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x)
```

```
[Out] Piecewise((a*log(tan(c + d*x)**2 + 1)/(2*d) + b*sec(c + d*x)/d, Ne(d, 0)),
(x*(a + b*sec(c))*tan(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int (a+b\sec(c+dx))\tan(c+dx)\,dx = -\frac{a\log(\cos(dx+c)) - \frac{b}{\cos(dx+c)}}{d}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*log(cos(d*x + c)) - b/cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(25) = 50$.

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.28

$$\int (a + b\sec(c + dx))\tan(c + dx)\,dx = \frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{a+2\,b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

```
[Out] (a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + 2*b + a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int (a + b\sec(c + dx))\tan(c + dx)\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2\right)}{d} - \frac{2\,b}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 - 1\right)}$$

```
[In] int(tan(c + d*x)*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.260 $\int \cot(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx=\frac{(a+b)\log(1-\cos(c+dx))}{2d}+\frac{(a-b)\log(1+\cos(c+dx))}{2d}$$

```
[Out] 1/2*(a+b)*ln(1-cos(d*x+c))/d+1/2*(a-b)*ln(1+cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3968, 2747, 647, 31}

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx=\frac{(a+b)\log(1-\cos(c+dx))}{2d}+\frac{(a-b)\log(\cos(c+dx)+1)}{2d}$$

```
[In] Int[Cot[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a + b)*Log[1 - Cos[c + d*x]])/(2*d) + ((a - b)*Log[1 + Cos[c + d*x]])/(2*
d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3968

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (b + a\cos(c+dx))\csc(c+dx)\,dx \\
&= -\frac{a\text{Subst}\left(\int \frac{b+x}{a^2-x^2}\,dx, x, a\cos(c+dx)\right)}{d} \\
&= -\frac{(a-b)\text{Subst}\left(\int \frac{1}{-a-x}\,dx, x, a\cos(c+dx)\right)}{2d} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{a-x}\,dx, x, a\cos(c+dx)\right)}{2d} \\
&= \frac{(a+b)\log(1-\cos(c+dx))}{2d} + \frac{(a-b)\log(1+\cos(c+dx))}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = -\frac{b\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{a\log(\cos(c+dx))}{d} + \frac{b\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{a\log(\tan(c+dx))}{d}$$

```
[In] Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x]),x]
[Out] -((b*Log[Cos[c/2 + (d*x)/2]])/d) + (a*Log[Cos[c + d*x]])/d + (b*Log[Sin[c/2
 + (d*x)/2]])/d + (a*Log[Tan[c + d*x]])/d
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{a\ln(\sin(dx+c))+b\ln(-\cot(dx+c)+\csc(dx+c))}{d}$	33
default	$\frac{a\ln(\sin(dx+c))+b\ln(-\cot(dx+c)+\csc(dx+c))}{d}$	33
risch	$-iax-\frac{2iac}{d}+\frac{a\ln(\mathrm{e}^{i(dx+c)}-1)}{d}+\frac{\ln(\mathrm{e}^{i(dx+c)}-1)b}{d}+\frac{a\ln(\mathrm{e}^{i(dx+c)}+1)}{d}-\frac{\ln(\mathrm{e}^{i(dx+c)}+1)b}{d}$	84

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*ln(sin(d*x+c))+b*ln(-cot(d*x+c)+csc(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = \frac{(a-b)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+(a+b)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,d}$$

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] 1/2*((a - b)*log(1/2*cos(d*x + c) + 1/2) + (a + b)*log(-1/2*cos(d*x + c) +
1/2))/d
```

Sympy [F]

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec\left(c+dx\right))\cot\left(c+dx\right)\,dx$$

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x)
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = \frac{(a-b)\log(\cos(dx+c)+1)+(a+b)\log(\cos(dx+c)-1)}{2\,d}$$

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/2*((a - b)*log(cos(d*x + c) + 1) + (a + b)*log(cos(d*x + c) - 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = \frac{(a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{2\,d}$$

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] 1/2*((a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \cot(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}+\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

```
[In] int(cot(c + d*x)*(a + b/cos(c + d*x)),x)
[Out] (a*log(tan(c/2 + (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (b*lo
g(tan(c/2 + (d*x)/2)))/d
```

3.261 $\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{(2a+b)\log(1-\cos(c+dx))}{4d} - \frac{(2a-b)\log(1+\cos(c+dx))}{4d} - \frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

```
[Out] -1/4*(2*a+b)*ln(1-cos(d*x+c))/d-1/4*(2*a-b)*ln(1+cos(d*x+c))/d-1/2*cot(d*x+
c)^2*(a+b*sec(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3967, 3968, 2747, 647, 31}

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{(2a+b)\log(1-\cos(c+dx))}{4d} - \frac{(2a-b)\log(\cos(c+dx)+1)}{4d} - \frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

```
[In] Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/4*((2*a + b)*Log[1 - Cos[c + d*x]])/d - ((2*a - b)*Log[1 + Cos[c + d*x]]
)/(4*d) - (Cot[c + d*x]^2*(a + b*Sec[c + d*x]))/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3968

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{1}{2}\int\cot(c+dx)(-2a-b\sec(c+dx))\,dx\\
&= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{1}{2}\int(-b-2a\cos(c+dx))\csc(c+dx)\,dx\\
&= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{a\text{Subst}\left(\int\frac{-b+x}{4a^2-x^2}\,dx,x,-2a\cos(c+dx)\right)}{d}\\
&= -\frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}+\frac{(2a-b)\text{Subst}\left(\int\frac{1}{2a-x}\,dx,x,-2a\cos(c+dx)\right)}{4d}\\
&\quad+\frac{(2a+b)\text{Subst}\left(\int\frac{1}{-2a-x}\,dx,x,-2a\cos(c+dx)\right)}{4d}
\end{aligned}$$

$$= -\frac{(2a+b)\log(1-\cos(c+dx))}{4d} - \frac{(2a-b)\log(1+\cos(c+dx))}{4d} - \frac{\cot^2(c+dx)(a+b\sec(c+dx))}{2d}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.58

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{2d} + \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/8*(b*Csc[(c + d*x)/2]^2)/d + (b*Log[Cos[(c + d*x)/2]])/(2*d) - (b*Log[Si
n[(c + d*x)/2]])/(2*d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[T
an[c + d*x]]))/(2*d) + (b*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+b\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)}{d}$
default	$\frac{a\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+b\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)}{d}$
risch	$iax+\frac{2iac}{d}+\frac{b\,e^{3i(dx+c)}+2a\,e^{2i(dx+c)}+b\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{\ln\left(e^{i(dx+c)}-1\right)b}{2d}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{d}+$

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+b*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3
-1/2*cos(d*x+c)-1/2*ln(-cot(d*x+c)+csc(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{2\,b\cos\left(dx+c\right) - \left((2\,a-b)\cos\left(dx+c\right)^2 - 2\,a+b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right) - \left((2\,a+b)\cos\left(dx+c\right)^2 - 2\,a-b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right) + 2\,a}{4\left(d\cos\left(dx+c\right)^2 - d\right)}$$

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] 1/4*(2*b*cos(d*x + c) - ((2*a - b)*cos(d*x + c)^2 - 2*a + b)*log(1/2*cos(d*
x + c) + 1/2) - ((2*a + b)*cos(d*x + c)^2 - 2*a - b)*log(-1/2*cos(d*x + c)
+ 1/2) + 2*a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F]

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx = \int \left(a+b\sec\left(c+dx\right)\right)\cot^3\left(c+dx\right)dx$$

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c)),x)
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$= -\frac{(2\,a-b)\log\left(\cos\left(dx+c\right)+1\right) + (2\,a+b)\log\left(\cos\left(dx+c\right)-1\right) - \frac{2\,(b\cos(dx+c)+a)}{\cos(dx+c)^2-1}}{4\,d}$$

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/4*((2*a - b)*log(cos(d*x + c) + 1) + (2*a + b)*log(cos(d*x + c) - 1) - 2
*(b*cos(d*x + c) + a)/(cos(d*x + c)^2 - 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(66) = 132$.

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.36

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{2\,(2\,a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 8\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - \frac{\left(a+b+\frac{4\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+}{\cos(dx+c)-1}}{8\,d}$$

[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] -1/8*(2*(2*a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*a*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 4*a*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*
(cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/d

Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

$$\int \cot^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a}{8}-\frac{b}{8}\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a}{8}+\frac{b}{8}\right)}{d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a+\frac{b}{2}\right)}{d}$$

[In] int(cot(c + d*x)^3*(a + b/cos(c + d*x)),x)

[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (tan(c/2 + (d*x)/2)^2*(a/8 - b/8))/d
- (cot(c/2 + (d*x)/2)^2*(a/8 + b/8))/d - (log(tan(c/2 + (d*x)/2))*(a + b/2)
)/d

3.262 $\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx = \frac{(8a+3b)\log(1-\cos(c+dx))}{16d} + \frac{(8a-3b)\log(1+\cos(c+dx))}{16d} - \frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d}$$

```
[Out] 1/16*(8*a+3*b)*ln(1-cos(d*x+c))/d+1/16*(8*a-3*b)*ln(1+cos(d*x+c))/d-1/4*cot
(d*x+c)^4*(a+b*sec(d*x+c))/d+1/8*cot(d*x+c)^2*(4*a+3*b*sec(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3967, 3968, 2747, 647, 31}

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx = \frac{(8a+3b)\log(1-\cos(c+dx))}{16d} + \frac{(8a-3b)\log(\cos(c+dx)+1)}{16d} - \frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d}$$

```
[In] Int[Cot[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] ((8*a + 3*b)*Log[1 - Cos[c + d*x]])/(16*d) + ((8*a - 3*b)*Log[1 + Cos[c + d
*x]])/(16*d) - (Cot[c + d*x]^4*(a + b*Sec[c + d*x]))/(4*d) + (Cot[c + d*x]^
2*(4*a + 3*b*Sec[c + d*x]))/(8*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3968

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}+\frac{1}{4}\int \cot^3(c+dx)(-4a-3b\sec(c+dx))\,dx\\
&= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d}+\frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d}\\
&\quad+\frac{1}{8}\int \cot(c+dx)(8a+3b\sec(c+dx))\,dx
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d} \\
&\quad + \frac{1}{8}\int (3b+8a\cos(c+dx))\csc(c+dx)\,dx \\
&= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d} \\
&\quad - \frac{a\text{Subst}\left(\int \frac{3b+x}{64a^2-x^2}\,dx, x, 8a\cos(c+dx)\right)}{d} \\
&= -\frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d} \\
&\quad - \frac{(8a-3b)\text{Subst}\left(\int \frac{1}{-8a-x}\,dx, x, 8a\cos(c+dx)\right)}{16d} \\
&\quad - \frac{(8a+3b)\text{Subst}\left(\int \frac{1}{8a-x}\,dx, x, 8a\cos(c+dx)\right)}{16d} \\
&= \frac{(8a+3b)\log(1-\cos(c+dx))}{16d} + \frac{(8a-3b)\log(1+\cos(c+dx))}{16d} \\
&\quad - \frac{\cot^4(c+dx)(a+b\sec(c+dx))}{4d} + \frac{\cot^2(c+dx)(4a+3b\sec(c+dx))}{8d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.72

$$
\begin{aligned}
\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx &= \frac{a\cot^2(c+dx)}{2d} - \frac{a\cot^4(c+dx)}{4d} \\
&\quad + \frac{5b\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} \\
&\quad - \frac{3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{a\log(\cos(c+dx))}{d} \\
&\quad + \frac{3b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{a\log(\tan(c+dx))}{d} \\
&\quad - \frac{5b\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{b\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}
\end{aligned}
$$

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^4)/(4*d) + (5*b*Csc[(c + d*x)/2]
^2)/(32*d) - (b*Csc[(c + d*x)/2]^4)/(64*d) - (3*b*Log[Cos[(c + d*x)/2]])/(8
*d) + (a*Log[Cos[c + d*x]])/d + (3*b*Log[Sin[(c + d*x)/2]])/(8*d) + (a*Log[
Tan[c + d*x]])/d - (5*b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)
/(64*d)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\ln(\sin(dx+c))\right)+b\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(dx+c)}{8}\right.}{d}$
default	$\frac{a\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\ln(\sin(dx+c))\right)+b\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(dx+c)}{8}\right.}{d}$
risch	$-iax-\frac{2iac}{d}-\frac{5b\,e^{7i(dx+c)}+16a\,e^{6i(dx+c)}+3b\,e^{5i(dx+c)}-16a\,e^{4i(dx+c)}+3b\,e^{3i(dx+c)}+16a\,e^{2i(dx+c)}+5b\,e^{i(dx+c)}}{4d\left(e^{2i(dx+c)}-1\right)^4}+$

```
[In] int(cot(d*x+c)^5*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+b*(-1/4/sin(d*x+
c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(d*
x+c)+3/8*ln(-cot(d*x+c)+csc(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.65

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx = -\frac{10\,b\cos(dx+c)^3+16\,a\cos(dx+c)^2-6\,b\cos(dx+c)-\left((8\,a-3\,b)\cos(dx+c)^4-2\,(8\,a-3\,b)\cos(dx\right.}{1}$$

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(10*b*cos(d*x + c)^3 + 16*a*cos(d*x + c)^2 - 6*b*cos(d*x + c) - ((8*a
 - 3*b)*cos(d*x + c)^4 - 2*(8*a - 3*b)*cos(d*x + c)^2 + 8*a - 3*b)*log(1/2*
cos(d*x + c) + 1/2) - ((8*a + 3*b)*cos(d*x + c)^4 - 2*(8*a + 3*b)*cos(d*x +
 c)^2 + 8*a + 3*b)*log(-1/2*cos(d*x + c) + 1/2) - 12*a)/(d*cos(d*x + c)^4 -
 2*d*cos(d*x + c)^2 + d)
```

Sympy [F]

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec{(c+dx)})\cot^5{(c+dx)}\,dx$$

[In] integrate(cot(d*x+c)**5*(a+b*sec(d*x+c)),x)

[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**5, x)

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{(8a-3b)\log{(\cos{(dx+c)}+1)}+(8a+3b)\log{(\cos{(dx+c)}-1)}-\frac{2\left(5b\cos(dx+c)^3+8a\cos(dx+c)^2-3b\cos(dx+c}{\cos(dx+c)^4-2\cos(dx+c)^2+1}}{16d}$$

[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/16*((8*a - 3*b)*log(cos(d*x + c) + 1) + (8*a + 3*b)*log(cos(d*x + c) - 1) - 2*(5*b*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 - 3*b*cos(d*x + c) - 6*a)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(94) = 188$.

Time = 0.34 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.61

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{4(8a+3b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-64a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+b+\frac{12a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48a(\cos(d}{(\cos(dx}}{(\cos(dx+c)}}{64d}$$

[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] 1/64*(4*(8*a + 3*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 64*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 12*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 - 12*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/d

Mupad [B] (verification not implemented)

Time = 14.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int \cot^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3a}{16}-\frac{b}{8}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a}{64}-\frac{b}{64}\right)}{d} - \frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left((-3a-2b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{4}+\frac{b}{4}\right)}{16d} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a+\frac{3b}{8}\right)}{d}$$

```
[In] int(cot(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*((3*a)/16 - b/8))/d - (tan(c/2 + (d*x)/2)^4*(a/64 - b
/64))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (cot(c/2 + (d*x)/2)^4*(a/4
+ b/4 - tan(c/2 + (d*x)/2)^2*(3*a + 2*b)))/(16*d) + (log(tan(c/2 + (d*x)/2)
)*(a + (3*b)/8))/d
```

3.263 $\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx = -\frac{(16a+5b)\log(1-\cos(c+dx))}{32d} - \frac{(16a-5b)\log(1+\cos(c+dx))}{32d} - \frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d} - \frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{16d}$$

```
[Out] -1/32*(16*a+5*b)*ln(1-cos(d*x+c))/d-1/32*(16*a-5*b)*ln(1+cos(d*x+c))/d-1/6*
cot(d*x+c)^6*(a+b*sec(d*x+c))/d+1/24*cot(d*x+c)^4*(6*a+5*b*sec(d*x+c))/d-1/
16*cot(d*x+c)^2*(8*a+5*b*sec(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used

$= \{3967, 3968, 2747, 647, 31\}$

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx = -\frac{(16a+5b)\log(1-\cos(c+dx))}{32d} - \frac{(16a-5b)\log(\cos(c+dx)+1)}{32d} - \frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d} + \frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d} - \frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{16d}$$

```
[In] Int[Cot[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/32*((16*a + 5*b)*Log[1 - Cos[c + d*x]])/d - ((16*a - 5*b)*Log[1 + Cos[c
+ d*x]])/(32*d) - (Cot[c + d*x]^6*(a + b*Sec[c + d*x]))/(6*d) + (Cot[c + d*
x]^4*(6*a + 5*b*Sec[c + d*x]))/(24*d) - (Cot[c + d*x]^2*(8*a + 5*b*Sec[c +
d*x]))/(16*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3968

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))/cot[(c_.) + (d_.)*(x_)], x_Symbo
l] :> Int[(b + a*Sin[c + d*x])/Cos[c + d*x], x] /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{1}{6}\int\cot^5(c+dx)(-6a-5b\sec(c+dx))\,dx\\
&= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d}\\
&\quad+\frac{1}{24}\int\cot^3(c+dx)(24a+15b\sec(c+dx))\,dx\\
&= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d}\\
&\quad-\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{16d}+\frac{1}{48}\int\cot(c+dx)(-48a-15b\sec(c+dx))\,dx\\
&= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d}\\
&\quad-\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{16d}+\frac{1}{48}\int(-15b-48a\cos(c+dx))\csc(c+dx)\,dx\\
&= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d}\\
&\quad-\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{16d}+\frac{a\text{Subst}\left(\int\frac{-15b+x}{2304a^2-x^2}\,dx,x,-48a\cos(c+dx)\right)}{d}\\
&= -\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d}\\
&\quad-\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{16d}\\
&\quad+\frac{(16a-5b)\text{Subst}\left(\int\frac{1}{48a-x}\,dx,x,-48a\cos(c+dx)\right)}{32d}\\
&\quad+\frac{(16a+5b)\text{Subst}\left(\int\frac{1}{-48a-x}\,dx,x,-48a\cos(c+dx)\right)}{32d}\\
&= -\frac{(16a+5b)\log(1-\cos(c+dx))}{32d}-\frac{(16a-5b)\log(1+\cos(c+dx))}{32d}\\
&\quad-\frac{\cot^6(c+dx)(a+b\sec(c+dx))}{6d}+\frac{\cot^4(c+dx)(6a+5b\sec(c+dx))}{24d}\\
&\quad-\frac{\cot^2(c+dx)(8a+5b\sec(c+dx))}{16d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.66

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx = -\frac{11b\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{b\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d}+\frac{5b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{5b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d} - \frac{a(6\cot^2(c+dx)-3\cot^4(c+dx)+2\cot^6(c+dx)+12\log(\cos(c+dx))+12\log(\tan(c+dx)))}{12d} + \frac{11b\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{b\sec^4\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{b\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

```
[In] Integrate[Cot[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-11*b*Csc[(c + d*x)/2]^2)/(64*d) + (b*Csc[(c + d*x)/2]^4)/(32*d) - (b*Csc[
(c + d*x)/2]^6)/(384*d) + (5*b*Log[Cos[(c + d*x)/2]])/(16*d) - (5*b*Log[Sin
[(c + d*x)/2]])/(16*d) - (a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c
+ d*x]^6 + 12*Log[Cos[c + d*x]] + 12*Log[Tan[c + d*x]]))/(12*d) + (11*b*Sec
[(c + d*x)/2]^2)/(64*d) - (b*Sec[(c + d*x)/2]^4)/(32*d) + (b*Sec[(c + d*x)/
2]^6)/(384*d)
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)^6}{6}+\frac{\cot(dx+c)^4}{4}-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+b\left(-\frac{\cos(dx+c)^7}{6\sin(dx+c)^6}+\frac{\cos(dx+c)^7}{24\sin(dx+c)^4}-\frac{\cos(dx+c)^7}{16\sin(dx+c)^2}-\frac{\cos(dx+c)^5}{16}-\right.}{d}$
default	$\frac{a\left(-\frac{\cot(dx+c)^6}{6}+\frac{\cot(dx+c)^4}{4}-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+b\left(-\frac{\cos(dx+c)^7}{6\sin(dx+c)^6}+\frac{\cos(dx+c)^7}{24\sin(dx+c)^4}-\frac{\cos(dx+c)^7}{16\sin(dx+c)^2}-\frac{\cos(dx+c)^5}{16}-\right.}{d}$
risch	$iax+\frac{2iac}{d}+\frac{e^{i(dx+c)}\left(33b\,e^{10i(dx+c)}+144a\,e^{9i(dx+c)}+5b\,e^{8i(dx+c)}-288a\,e^{7i(dx+c)}+90b\,e^{6i(dx+c)}+544a\,e^{5i(dx+c)}+9\right.}{24d\left(e^{2i(dx+c)}-1\right)^6}$

```
[In] int(cot(d*x+c)^7*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/6*cot(d*x+c)^6+1/4*cot(d*x+c)^4-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))
+b*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/24/sin(d*x+c)^4*cos(d*x+c)^7-1/16/sin(
d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^5-5/48*cos(d*x+c)^3-5/16*cos(d*x+c)-5
/16*ln(-cot(d*x+c)+csc(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.82

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx = \frac{66\,b\cos(dx+c)^5+144\,a\cos(dx+c)^4-80\,b\cos(dx+c)^3-216\,a\cos(dx+c)^2+30\,b\cos(dx+c)-3\,((}{}$$

[In] integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/96*(66*b*cos(d*x + c)^5 + 144*a*cos(d*x + c)^4 - 80*b*cos(d*x + c)^3 - 21
6*a*cos(d*x + c)^2 + 30*b*cos(d*x + c) - 3*((16*a - 5*b)*cos(d*x + c)^6 - 3
*(16*a - 5*b)*cos(d*x + c)^4 + 3*(16*a - 5*b)*cos(d*x + c)^2 - 16*a + 5*b)*
log(1/2*cos(d*x + c) + 1/2) - 3*((16*a + 5*b)*cos(d*x + c)^6 - 3*(16*a + 5*
b)*cos(d*x + c)^4 + 3*(16*a + 5*b)*cos(d*x + c)^2 - 16*a - 5*b)*log(-1/2*co
s(d*x + c) + 1/2) + 88*a)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(
d*x + c)^2 - d)

Sympy [F]

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cot^7(c+dx)\,dx$$

[In] integrate(cot(d*x+c)**7*(a+b*sec(d*x+c)),x)

[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**7, x)

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx =$$

$$-\frac{3\,(16\,a-5\,b)\log(\cos(dx+c)+1)+3\,(16\,a+5\,b)\log(\cos(dx+c)-1)-\frac{2\left(33\,b\cos(dx+c)^5+72\,a\cos(dx+c)^4-\right.}{\cos(dx+c)^6}}{96\,d}$$

[In] integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/96*(3*(16*a - 5*b)*log(cos(d*x + c) + 1) + 3*(16*a + 5*b)*log(cos(d*x +
c) - 1) - 2*(33*b*cos(d*x + c)^5 + 72*a*cos(d*x + c)^4 - 40*b*cos(d*x + c)^
3 - 108*a*cos(d*x + c)^2 + 15*b*cos(d*x + c) + 44*a)/(cos(d*x + c)^6 - 3*co
s(d*x + c)^4 + 3*cos(d*x + c)^2 - 1))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(120) = 240.

Time = 0.36 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.75

$$\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx =$$

$$-\frac{12\,(16\,a+5\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-\frac{\left(a+b+\frac{12\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{87}{(}\right.}{}}{}$$

[In] integrate(cot(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] -1/384*(12*(16*a + 5*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 384*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - (a + b + 12*a *(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 87*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 45*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 352*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 110*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1) ^3/(cos(d*x + c) - 1)^3 - 87*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 45*b *(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 9*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1) ^3)/d

Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \cot^7(c+dx)(a+b\sec(c+dx))\,dx &= \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{a}{32}-\frac{3\,b}{128}\right)}{d} \\ &\quad -\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\left(\frac{29\,a}{2}+\frac{15\,b}{2}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(-2\,a-\frac{3\,b}{2}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a}{6}+\frac{b}{6}\right)}{64\,d} \\ &\quad -\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{29\,a}{128}-\frac{15\,b}{128}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{a}{384}-\frac{b}{384}\right)}{d} \\ &\quad +\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a+\frac{5\,b}{16}\right)}{d}\end{aligned}$$

[In] int(cot(c + d*x)^7*(a + b/cos(c + d*x)),x)

[Out] (tan(c/2 + (d*x)/2)^4*(a/32 - (3*b)/128))/d - (cot(c/2 + (d*x)/2)^6*(a/6 + b/6 - tan(c/2 + (d*x)/2)^2*(2*a + (3*b)/2) + tan(c/2 + (d*x)/2)^4*((29*a)/2

```
 + (15*b)/2)))/(64*d) - (tan(c/2 + (d*x)/2)^2*((29*a)/128 - (15*b)/128))/d
- (tan(c/2 + (d*x)/2)^6*(a/384 - b/384))/d + (a*log(tan(c/2 + (d*x)/2)^2 +
1))/d - (log(tan(c/2 + (d*x)/2))*(a + (5*b)/16))/d
```

3.264 $\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx = -ax-\frac{5b\text{arctanh}(\sin(c+dx))}{16d}+\frac{(16a+5b\sec(c+dx))\tan(c+dx)}{16d}-\frac{(8a+5b\sec(c+dx))\tan^3(c+dx)}{24d}+\frac{(6a+5b\sec(c+dx))\tan^5(c+dx)}{30d}$$

```
[Out] -a*x-5/16*b*arctanh(sin(d*x+c))/d+1/16*(16*a+5*b*sec(d*x+c))*tan(d*x+c)/d-1
/24*(8*a+5*b*sec(d*x+c))*tan(d*x+c)^3/d+1/30*(6*a+5*b*sec(d*x+c))*tan(d*x+c
)^5/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3966, 3855}

$$\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx = \frac{\tan^5(c+dx)(6a+5b\sec(c+dx))}{30d}-\frac{\tan^3(c+dx)(8a+5b\sec(c+dx))}{24d}+\frac{\tan(c+dx)(16a+5b\sec(c+dx))}{16d}-ax-\frac{5b\text{arctanh}(\sin(c+dx))}{16d}$$

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -(a*x) - (5*b*ArcTanh[Sin[c + d*x]])/(16*d) + ((16*a + 5*b*Sec[c + d*x])*Ta
n[c + d*x])/(16*d) - ((8*a + 5*b*Sec[c + d*x])*Tan[c + d*x]^3)/(24*d) + ((6
*a + 5*b*Sec[c + d*x])*Tan[c + d*x]^5)/(30*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(6a+5b\sec(c+dx))\tan^5(c+dx)}{30d} - \frac{1}{6}\int (6a+5b\sec(c+dx))\tan^4(c+dx)\,dx \\
&= -\frac{(8a+5b\sec(c+dx))\tan^3(c+dx)}{24d} + \frac{(6a+5b\sec(c+dx))\tan^5(c+dx)}{30d} \\
&\quad + \frac{1}{24}\int (24a+15b\sec(c+dx))\tan^2(c+dx)\,dx \\
&= \frac{(16a+5b\sec(c+dx))\tan(c+dx)}{16d} - \frac{(8a+5b\sec(c+dx))\tan^3(c+dx)}{24d} \\
&\quad + \frac{(6a+5b\sec(c+dx))\tan^5(c+dx)}{30d} - \frac{1}{48}\int (48a+15b\sec(c+dx))\,dx \\
&= -ax + \frac{(16a+5b\sec(c+dx))\tan(c+dx)}{16d} - \frac{(8a+5b\sec(c+dx))\tan^3(c+dx)}{24d} \\
&\quad + \frac{(6a+5b\sec(c+dx))\tan^5(c+dx)}{30d} - \frac{1}{16}(5b)\int \sec(c+dx)\,dx \\
&= -ax - \frac{5b\text{arctanh}(\sin(c+dx))}{16d} + \frac{(16a+5b\sec(c+dx))\tan(c+dx)}{16d} \\
&\quad - \frac{(8a+5b\sec(c+dx))\tan^3(c+dx)}{24d} + \frac{(6a+5b\sec(c+dx))\tan^5(c+dx)}{30d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.75

$$\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx = -\frac{a\arctan(\tan(c+dx))}{d} - \frac{5b\text{arctanh}(\sin(c+dx))}{16d} + \frac{a\tan(c+dx)}{d} - \frac{5b\sec(c+dx)\tan(c+dx)}{16d} - \frac{5b\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{5b\sec^5(c+dx)\tan(c+dx)}{6d} - \frac{a\tan^3(c+dx)}{3d} - \frac{5b\sec^3(c+dx)\tan^3(c+dx)}{3d} + \frac{a\tan^5(c+dx)}{5d} + \frac{b\sec(c+dx)\tan^5(c+dx)}{d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) - (5*b*ArcTanh[Sin[c + d*x]])/(16*d) + (a*Tan
[c + d*x])/d - (5*b*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (5*b*Sec[c + d*x]^3
*Tan[c + d*x])/(24*d) + (5*b*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) - (a*Tan[c
+ d*x]^3)/(3*d) - (5*b*Sec[c + d*x]^3*Tan[c + d*x]^3)/(3*d) + (a*Tan[c + d*
x]^5)/(5*d) + (b*Sec[c + d*x]*Tan[c + d*x]^5)/d
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{a\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-dx-c\right)+b\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(dx+c)^3}{48}\right.}{d}$
default	$\frac{a\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-dx-c\right)+b\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(dx+c)^3}{48}\right.}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{b\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+}{16}\right.}{}$
risch	$-ax-\frac{i\left(165b\,e^{11i(dx+c)}-720a\,e^{10i(dx+c)}-25b\,e^{9i(dx+c)}-2160a\,e^{8i(dx+c)}+450b\,e^{7i(dx+c)}-3680a\,e^{6i(dx+c)}-450b\,e^{5i(}\right.}{120d\left(e^{2i(dx+c)}+1\right)^6}$

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c)+b*(1/6*sin(d*x+
c)^7/cos(d*x+c)^6-1/24*sin(d*x+c)^7/cos(d*x+c)^4+1/16*sin(d*x+c)^7/cos(d*x+
c)^2+1/16*sin(d*x+c)^5+5/48*sin(d*x+c)^3+5/16*sin(d*x+c)-5/16*ln(sec(d*x+c)
+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.31

$$\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx =$$
$$-\frac{480\,adx\cos\left(dx+c\right)^{6}+75\,b\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)-75\,b\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+\cdots\right.}{\cdots}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/480*(480*a*d*x*cos(d*x + c)^6 + 75*b*cos(d*x + c)^6*log(sin(d*x + c) + 1
) - 75*b*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 2*(368*a*cos(d*x + c)^5 +
165*b*cos(d*x + c)^4 - 176*a*cos(d*x + c)^3 - 130*b*cos(d*x + c)^2 + 48*a*c
os(d*x + c) + 40*b)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]

$$\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx = \int (a+b\sec{(c+dx)})\tan^{6}{(c+dx)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**6,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*tan(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.31

$$\int (a+b\sec(c+dx))\tan^6(c+dx)\,dx$$
$$=\frac{32\left(3\tan\left(dx+c\right)^{5}-5\tan\left(dx+c\right)^{3}-15\,dx-15\,c+15\tan\left(dx+c\right)\right)a-5\,b\left(\frac{2\left(33\sin(dx+c)^{5}-40\sin(dx+c)^{3}\cdots\right.}{\sin(dx+c)^{6}-3\sin(dx+c)^{4}+3\sin\cdots}\cdots\right.}{480\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x
 + c))*a - 5*b*(2*(33*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 15*sin(d*x + c))
/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 15*log(sin(d*
x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(94) = 188$.

Time = 2.13 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.24

$$\int (a + b\sec(c + dx))\tan^6(c + dx)\,dx =$$
$$-\frac{240\,(dx+c)a + 75\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 75\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}\right.}{}}{}$$

[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")

```
[Out] -1/240*(240*(d*x + c)*a + 75*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 75*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(240*a*tan(1/2*d*x + 1/2*c)^11 - 75*b*
tan(1/2*d*x + 1/2*c)^11 - 1520*a*tan(1/2*d*x + 1/2*c)^9 + 425*b*tan(1/2*d*x
 + 1/2*c)^9 + 4128*a*tan(1/2*d*x + 1/2*c)^7 - 990*b*tan(1/2*d*x + 1/2*c)^7
- 4128*a*tan(1/2*d*x + 1/2*c)^5 - 990*b*tan(1/2*d*x + 1/2*c)^5 + 1520*a*tan
(1/2*d*x + 1/2*c)^3 + 425*b*tan(1/2*d*x + 1/2*c)^3 - 240*a*tan(1/2*d*x + 1/
2*c) - 75*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 15.02 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.25

$$\int (a + b\sec(c + dx))\tan^6(c + dx)\,dx$$
$$= \frac{\left(\frac{5b}{8} - 2a\right)\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11} + \left(\frac{38a}{3} - \frac{85b}{24}\right)\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{9} + \left(\frac{33b}{4} - \frac{172a}{5}\right)\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{7} + \left(\frac{172a}{5} + \frac{33b}{4}\right)\tan\left(\frac{c}{2}\right.}{d\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{12} - 6\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 15\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{8} - 20\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{6} + 1\right.}$$
$$- \frac{5\,b\,\mathrm{atanh}\left(\frac{125\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\left(20\,a^2\,b+\frac{125\,b^3}{64}\right)} + \frac{20\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{20\,a^2\,b+\frac{125\,b^3}{64}}\right)}{8\,d}$$
$$- \frac{2\,a\,\mathrm{atan}\left(\frac{64\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^3+\frac{25\,a\,b^2}{4}} + \frac{25\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\left(64\,a^3+\frac{25\,a\,b^2}{4}\right)}\right)}{d}$$

[In] int(tan(c + d*x)^6*(a + b/cos(c + d*x)),x)

```
[Out] (tan(c/2 + (d*x)/2)*(2*a + (5*b)/8) - tan(c/2 + (d*x)/2)^11*(2*a - (5*b)/8)
 - tan(c/2 + (d*x)/2)^3*((38*a)/3 + (85*b)/24) + tan(c/2 + (d*x)/2)^9*((38*
a)/3 - (85*b)/24) + tan(c/2 + (d*x)/2)^5*((172*a)/5 + (33*b)/4) - tan(c/2 +
 (d*x)/2)^7*((172*a)/5 - (33*b)/4))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2
 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c
```

```
/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - (5*b*atanh((125*b^3*tan(c/
2 + (d*x)/2))/(64*(20*a^2*b + (125*b^3)/64)) + (20*a^2*b*tan(c/2 + (d*x)/2)
)/(20*a^2*b + (125*b^3)/64)))/(8*d) - (2*a*atan((64*a^3*tan(c/2 + (d*x)/2))
/((25*a*b^2)/4 + 64*a^3) + (25*a*b^2*tan(c/2 + (d*x)/2))/(4*((25*a*b^2)/4 +
 64*a^3))))/d
```

3.265 $\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx = ax+\frac{3b\operatorname{arctanh}(\sin(c+dx))}{8d} - \frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d} + \frac{(4a+3b\sec(c+dx))\tan^3(c+dx)}{12d}$$

```
[Out] a*x+3/8*b*arctanh(sin(d*x+c))/d-1/8*(8*a+3*b*sec(d*x+c))*tan(d*x+c)/d+1/12*
(4*a+3*b*sec(d*x+c))*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3966, 3855}

$$\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx = \frac{\tan^3(c+dx)(4a+3b\sec(c+dx))}{12d} - \frac{\tan(c+dx)(8a+3b\sec(c+dx))}{8d} + ax+\frac{3b\operatorname{arctanh}(\sin(c+dx))}{8d}$$

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x + (3*b*ArcTanh[Sin[c + d*x]])/(8*d) - ((8*a + 3*b*Sec[c + d*x])*Tan[c +
 d*x])/(8*d) + ((4*a + 3*b*Sec[c + d*x])*Tan[c + d*x]^3)/(12*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(4a+3b\sec(c+dx))\tan^3(c+dx)}{12d} - \frac{1}{4}\int(4a+3b\sec(c+dx))\tan^2(c+dx)\,dx \\
&= -\frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d} \\
&\quad + \frac{(4a+3b\sec(c+dx))\tan^3(c+dx)}{12d} + \frac{1}{8}\int(8a+3b\sec(c+dx))\,dx \\
&= ax - \frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d} \\
&\quad + \frac{(4a+3b\sec(c+dx))\tan^3(c+dx)}{12d} + \frac{1}{8}(3b)\int\sec(c+dx)\,dx \\
&= ax + \frac{3b\text{arctanh}(\sin(c+dx))}{8d} - \frac{(8a+3b\sec(c+dx))\tan(c+dx)}{8d} \\
&\quad + \frac{(4a+3b\sec(c+dx))\tan^3(c+dx)}{12d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$
\begin{aligned}
\int(a+b\sec(c+dx))\tan^4(c+dx)\,dx &= \frac{a\arctan(\tan(c+dx))}{d} + \frac{3b\text{arctanh}(\sin(c+dx))}{8d} \\
&\quad - \frac{a\tan(c+dx)}{d} + \frac{3b\sec(c+dx)\tan(c+dx)}{8d} \\
&\quad - \frac{3b\sec^3(c+dx)\tan(c+dx)}{4d} \\
&\quad + \frac{a\tan^3(c+dx)}{3d} + \frac{b\sec(c+dx)\tan^3(c+dx)}{d}
\end{aligned}
$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d + (3*b*ArcTanh[Sin[c + d*x]])/(8*d) - (a*Tan[c +
 d*x])/d + (3*b*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*b*Sec[c + d*x]^3*Tan[
c + d*x])/(4*d) + (a*Tan[c + d*x]^3)/(3*d) + (b*Sec[c + d*x]*Tan[c + d*x]^3
)/d
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{a\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+dx+c\right)+b\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{a\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+dx+c\right)+b\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
parts	$\frac{a\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{b\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+}{d}$
risch	$ax+\frac{i\left(15b\,e^{7i(dx+c)}-48a\,e^{6i(dx+c)}-9b\,e^{5i(dx+c)}-96a\,e^{4i(dx+c)}+9b\,e^{3i(dx+c)}-80a\,e^{2i(dx+c)}-15b\,e^{i(dx+c)}-32a\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}+\frac{3}{}$

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+b*(1/4*sin(d*x+c)^5/cos(d*x+c)^4
-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d
*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.53

$$\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx = \frac{48\,adx\cos(dx+c)^4+9\,b\cos(dx+c)^4\log(\sin(dx+c)+1)-9\,b\cos(dx+c)^4\log(-\sin(dx+c)+1)-2}{48\,d\cos(dx+c)^4}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a*d*x*cos(d*x + c)^4 + 9*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
9*b*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 2*(32*a*cos(d*x + c)^3 + 15*b*c
os(d*x + c)^2 - 8*a*cos(d*x + c) - 6*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx = \int (a+b\sec(c+dx))\tan^4(c+dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**4,x)
[Out] Integral((a + b*sec(c + d*x))*tan(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx$$
$$= \frac{16\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a+3\,b\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log(\sin(dx+c)+1)\right.}{48\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a + 3*b*(2*(5*sin(
d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*lo
g(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(67) = 134.

Time = 0.93 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.36

$$\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx$$
$$= \frac{24\,(dx+c)a+9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-9\,b\tan\right.}{}}{2}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
[Out] 1/24*(24*(d*x + c)*a + 9*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*b*log(abs
(tan(1/2*d*x + 1/2*c) - 1)) + 2*(24*a*tan(1/2*d*x + 1/2*c)^7 - 9*b*tan(1/2*
d*x + 1/2*c)^7 - 104*a*tan(1/2*d*x + 1/2*c)^5 + 33*b*tan(1/2*d*x + 1/2*c)^5
 + 104*a*tan(1/2*d*x + 1/2*c)^3 + 33*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1/
2*d*x + 1/2*c) - 9*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/
d
```

Mupad [B] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.66

$$\int (a+b\sec(c+dx))\tan^4(c+dx)\,dx$$
$$=\frac{2\,a\,\mathrm{atan}\left(\frac{64\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^3+9\,a\,b^2}+\frac{9\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^3+9\,a\,b^2}\right)}{d}+\frac{3\,b\,\mathrm{atanh}\left(\frac{27\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,\left(24\,a^2\,b+\frac{27\,b^3}{8}\right)}+\frac{24\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{24\,a^2\,b+\frac{27\,b^3}{8}}\right)}{4\,d}$$
$$-\frac{\left(\frac{3\,b}{4}-2\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{26\,a}{3}-\frac{11\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{26\,a}{3}-\frac{11\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a+\frac{3\,b}{4}\right)\,\tan\left(\frac{c}{2}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

```
[In] int(tan(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*a*atan((64*a^3*tan(c/2 + (d*x)/2))/(9*a*b^2 + 64*a^3) + (9*a*b^2*tan(c/2
 + (d*x)/2))/(9*a*b^2 + 64*a^3)))/d + (3*b*atanh((27*b^3*tan(c/2 + (d*x)/2)
)/(8*(24*a^2*b + (27*b^3)/8)) + (24*a^2*b*tan(c/2 + (d*x)/2))/(24*a^2*b + (
27*b^3)/8)))/(4*d) - (tan(c/2 + (d*x)/2)*(2*a + (3*b)/4) - tan(c/2 + (d*x)/
2)^7*(2*a - (3*b)/4) - tan(c/2 + (d*x)/2)^3*((26*a)/3 + (11*b)/4) + tan(c/2
 + (d*x)/2)^5*((26*a)/3 - (11*b)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2
 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.266 $\int (a+b\sec(c+dx))\tan^2(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int (a+b\sec(c+dx))\tan^2(c+dx)\,dx = -ax-\frac{b\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d}$$

```
[Out] -a*x-1/2*b*arctanh(sin(d*x+c))/d+1/2*(2*a+b*sec(d*x+c))*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3966, 3855}

$$\int (a+b\sec(c+dx))\tan^2(c+dx)\,dx = \frac{\tan(c+dx)(2a+b\sec(c+dx))}{2d} - ax-\frac{b\operatorname{arctanh}(\sin(c+dx))}{2d}$$

```
[In] Int[(a + b*Sec[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) - (b*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a + b*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d}-\frac{1}{2}\int(2a+b\sec(c+dx))\,dx\\
&= -ax+\frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d}-\frac{1}{2}b\int\sec(c+dx)\,dx\\
&= -ax-\frac{b\text{arctanh}(\sin(c+dx))}{2d}+\frac{(2a+b\sec(c+dx))\tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int(a+b\sec(c+dx))\tan^2(c+dx)\,dx=-\frac{a\arctan(\tan(c+dx))}{d}-\frac{b\text{arctanh}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])*Tan[c + d*x]^2,x]
[Out] -((a*ArcTan[Tan[c + d*x]])/d) - (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c
+ d*x])/d + (b*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.49

method	result	size
derivativedivides	$\frac{a(\tan(dx+c)-dx-c)+b\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	67
default	$\frac{a(\tan(dx+c)-dx-c)+b\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	67
parts	$\frac{a(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{b\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	71
risch	$-ax-\frac{i\left(b\,e^{3i(dx+c)}-2a\,e^{2i(dx+c)}-b\,e^{i(dx+c)}-2a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{b\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{b\ln\left(e^{i(dx+c)}-i\right)}{2d}$	102

```
[In] int((a+b*sec(d*x+c))*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(tan(d*x+c)-d*x-c)+b*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1
/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(41) = 82$.

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.93

$$\int (a + b\sec(c + dx))\tan^2(c + dx)\,dx = -\frac{4\,adx\cos(dx+c)^2 + b\cos(dx+c)^2\log(\sin(dx+c)+1) - b\cos(dx+c)^2\log(-\sin(dx+c)+1) - 2}{4\,d\cos(dx+c)^2}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*d*x*cos(d*x + c)^2 + b*cos(d*x + c)^2*log(sin(d*x + c) + 1) - b*c
os(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a*cos(d*x + c) + b)*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a + b\sec(c + dx))\tan^2(c + dx)\,dx = \int (a + b\sec(c + dx))\tan^2(c + dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*tan(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int (a + b\sec(c + dx))\tan^2(c + dx)\,dx = -\frac{4\,(dx + c - \tan(dx+c))a + b\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\right)}{4\,d}$$

```
[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c - tan(d*x + c))*a + b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
 + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(41) = 82$.

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.56

$$\int (a + b\sec(c + dx))\tan^2(c + dx)\,dx =$$

$$-\frac{2\,(dx + c)a + b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(2\,a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - b\tan\left(\frac{1}{2}\,dx +\right.\right.}{\left(\tan\left(\frac{1}{2}\right.\right.}}{2\,d}$$

[In] integrate((a+b*sec(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")

[Out] -1/2*(2*(d*x + c)*a + b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(2*a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d

Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.13

$$\int (a + b\sec(c + dx))\tan^2(c + dx)\,dx = \frac{a\,\sin\left(c + d\,x\right)}{d\,\cos\left(c + d\,x\right)} - \frac{b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} - \frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d\,x}{2}\right)}\right)}{d} + \frac{b\,\sin\left(c + d\,x\right)}{2\,d\cos\left(c + d\,x\right)^2}$$

[In] int(tan(c + d*x)^2*(a + b/cos(c + d*x)),x)

[Out] (a*sin(c + d*x))/(d*cos(c + d*x)) - (b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b*sin(c + d*x))/(2*d*cos(c + d*x)^2)

3.267 $\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx = -ax - \frac{\cot(c+dx)(a+b\sec(c+dx))}{d}$$

```
[Out] -a*x-cot(d*x+c)*(a+b*sec(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3967, 8}

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\cot(c+dx)(a+b\sec(c+dx))}{d} - ax$$

```
[In] Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x]),x]
[Out] -(a*x) - (Cot[c + d*x]*(a + b*Sec[c + d*x]))/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\cot(c+dx)(a+b\sec(c+dx))}{d} - \int a\,dx$$
$$= -ax - \frac{\cot(c+dx)(a+b\sec(c+dx))}{d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx$$
$$= -\frac{b\csc(c+dx)}{d} - \frac{a\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d}$$

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x]),x]
[Out] -((b*Csc[c + d*x])/d) - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Ta
n[c + d*x]^2])/d
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$\frac{a(-\cot(dx+c)-dx-c)-\frac{b}{\sin(dx+c)}}{d}$	35
default	$\frac{a(-\cot(dx+c)-dx-c)-\frac{b}{\sin(dx+c)}}{d}$	35
risch	$-ax-\frac{2i\left(b\,\mathrm{e}^{i(dx+c)}+a\right)}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)}$	38

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-cot(d*x+c)-d*x-c)-b/sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx = -\frac{adx\sin(dx+c)+a\cos(dx+c)+b}{d\sin(dx+c)}$$

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] -(a*d*x*sin(d*x + c) + a*cos(d*x + c) + b)/(d*sin(d*x + c))
```

Sympy [F]

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cot^2(c+dx)\,dx$$

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c)),x)
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a+\frac{b}{\sin(dx+c)}}{d}$$

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] -((d*x + c + 1/tan(d*x + c))*a + b/sin(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx = -\frac{2(dx+c)a-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{a+b}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] -1/2*(2*(d*x + c)*a - a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*c) + (a
+ b)/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \cot^2(c+dx)(a+b\sec(c+dx))\,dx=\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{d}-\frac{\frac{a}{2}+\frac{b}{2}}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-a\,x$$

```
[In] int(cot(c + d*x)^2*(a + b/cos(c + d*x)),x)
[Out] (tan(c/2 + (d*x)/2)*(a/2 - b/2))/d - (a/2 + b/2)/(d*tan(c/2 + (d*x)/2)) - a
*x
```

3.268 $\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx = ax - \frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d} + \frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d}$$

```
[Out] a*x-1/3*cot(d*x+c)^3*(a+b*sec(d*x+c))/d+1/3*cot(d*x+c)*(3*a+2*b*sec(d*x+c))
/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3967, 8}

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d} + \frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d} + ax$$

```
[In] Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^3*(a + b*Sec[c + d*x]))/(3*d) + (Cot[c + d*x]*(3*a + 2*
b*Sec[c + d*x]))/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{1}{3}\int\cot^2(c+dx)(-3a-2b\sec(c+dx))\,dx\\
&= -\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d}+\frac{1}{3}\int 3a\,dx\\
&= ax-\frac{\cot^3(c+dx)(a+b\sec(c+dx))}{3d}+\frac{\cot(c+dx)(3a+2b\sec(c+dx))}{3d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\begin{aligned}
&\int\cot^4(c+dx)(a+b\sec(c+dx))\,dx\\
&=\frac{b\csc(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}\\
&\quad-\frac{a\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d}
\end{aligned}$$

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x]),x]
[Out] (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d) - (a*Cot[c + d*x]^3*Hypergeom
etric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d)
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)}{d}$	86
default	$\frac{a\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)}{d}$	86
risch	$ax+\frac{2i\left(3b\,e^{5i(dx+c)}+6a\,e^{4i(dx+c)}-2b\,e^{3i(dx+c)}-6a\,e^{2i(dx+c)}+3b\,e^{i(dx+c)}+4a\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}$	88

```
[In] int(cot(d*x+c)^4*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+b*(-1/3/sin(d*x+c)^3*cos(d*x+c)
^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.58

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{4\,a\cos\left(dx+c\right)^3+3\,b\cos\left(dx+c\right)^2-3\,a\cos\left(dx+c\right)+3\,\left(adx\cos\left(dx+c\right)^2-adx\right)\sin\left(dx+c\right)-2\,b}{3\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(4*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 3*a*cos(d*x + c) + 3*(a*d*x*
cos(d*x + c)^2 - a*d*x)*sin(d*x + c) - 2*b)/((d*cos(d*x + c)^2 - d)*sin(d*x
 + c))
```

Sympy [F]

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec{(c+dx)})\cot^4{(c+dx)}\,dx$$

```
[In] integrate(cot(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+\frac{\left(3\,\sin(dx+c)^2-1\right)b}{\sin(dx+c)^3}}{3\,d}$$

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + (3*sin(d*x +
 c)^2 - 1)*b/sin(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(51) = 102$.

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.04

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,(dx+c)a-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{15\,a}{}}{24\,d}$$

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 + 24*(d*x + c)*a
- 15*a*tan(1/2*d*x + 1/2*c) + 9*b*tan(1/2*d*x + 1/2*c) + (15*a*tan(1/2*d*x
+ 1/2*c)^2 + 9*b*tan(1/2*d*x + 1/2*c)^2 - a - b)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.64

$$\int \cot^4(c+dx)(a+b\sec(c+dx))\,dx$$
$$= a\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{a}{24}-\frac{b}{24}\right)}{d}$$
$$-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\left(-5\,a-3\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a}{3}+\frac{b}{3}\right)}{8\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a}{8}-\frac{3\,b}{8}\right)}{d}$$

```
[In] int(cot(c + d*x)^4*(a + b/cos(c + d*x)),x)
[Out] a*x + (tan(c/2 + (d*x)/2)^3*(a/24 - b/24))/d - (cot(c/2 + (d*x)/2)^3*(a/3 +
 b/3 - tan(c/2 + (d*x)/2)^2*(5*a + 3*b)))/(8*d) - (tan(c/2 + (d*x)/2)*((5*a
)/8 - (3*b)/8))/d
```

3.269 $\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx = -ax-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d}-\frac{\cot(c+dx)(15a+8b\sec(c+dx))}{15d}$$

```
[Out] -a*x-1/5*cot(d*x+c)^5*(a+b*sec(d*x+c))/d+1/15*cot(d*x+c)^3*(5*a+4*b*sec(d*x
+c))/d-1/15*cot(d*x+c)*(15*a+8*b*sec(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3967, 8}

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d}-\frac{\cot(c+dx)(15a+8b\sec(c+dx))}{15d}-ax$$

```
[In] Int[Cot[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] -(a*x) - (Cot[c + d*x]^5*(a + b*Sec[c + d*x]))/(5*d) + (Cot[c + d*x]^3*(5*a
 + 4*b*Sec[c + d*x]))/(15*d) - (Cot[c + d*x]*(15*a + 8*b*Sec[c + d*x]))/(15
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{1}{5}\int\cot^4(c+dx)(-5a-4b\sec(c+dx))\,dx\\
&= -\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d}\\
&\quad+\frac{1}{15}\int\cot^2(c+dx)(15a+8b\sec(c+dx))\,dx\\
&= -\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d}\\
&\quad-\frac{\cot(c+dx)(15a+8b\sec(c+dx))}{15d}+\frac{1}{15}\int-15a\,dx\\
&= -ax-\frac{\cot^5(c+dx)(a+b\sec(c+dx))}{5d}\\
&\quad+\frac{\cot^3(c+dx)(5a+4b\sec(c+dx))}{15d}-\frac{\cot(c+dx)(15a+8b\sec(c+dx))}{15d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\begin{aligned}
&\int\cot^6(c+dx)(a+b\sec(c+dx))\,dx\\
&= -\frac{b\csc(c+dx)}{d}+\frac{2b\csc^3(c+dx)}{3d}-\frac{b\csc^5(c+dx)}{5d}\\
&\quad-\frac{a\cot^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5d}
\end{aligned}$$

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) + (2*b*Csc[c + d*x]^3)/(3*d) - (b*Csc[c + d*x]^5)/(5*
d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(
5*d)
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)^5}{5}+\frac{\cot(dx+c)^3}{3}-\cot(dx+c)-dx-c\right)+b\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos}{}\right.}{5}\right.}{d}$
default	$\frac{a\left(-\frac{\cot(dx+c)^5}{5}+\frac{\cot(dx+c)^3}{3}-\cot(dx+c)-dx-c\right)+b\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos}{}\right.}{5}\right.}{d}$
risch	$-ax-\frac{2i\left(15b\,e^{9i(dx+c)}+45a\,e^{8i(dx+c)}-20b\,e^{7i(dx+c)}-90a\,e^{6i(dx+c)}+58b\,e^{5i(dx+c)}+140a\,e^{4i(dx+c)}-20b\,e^{3i(dx+c)}-\right.}{15d\left(e^{2i(dx+c)}-1\right)^5}$

[In] int(cot(d*x+c)^6*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+b*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.55

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx =$$
$$-\frac{23\,a\cos{(dx+c)}^5+15\,b\cos{(dx+c)}^4-35\,a\cos{(dx+c)}^3-20\,b\cos{(dx+c)}^2+15\,a\cos{(dx+c)}+15}{15\left(d\cos{(dx+c)}^4-2\,d\cos{(dx+c)}^2+d\right)\sin}$$

[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/15*(23*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 35*a*cos(d*x + c)^3 - 20*b*cos(d*x + c)^2 + 15*a*cos(d*x + c) + 15*(a*d*x*cos(d*x + c)^4 - 2*a*d*x*cos(d*x + c)^2 + a*d*x)*sin(d*x + c) + 8*b)/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))

Sympy [F]

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cot^6(c+dx)\,dx$$

```
[In] integrate(cot(d*x+c)**6*(a+b*sec(d*x+c)),x)
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx$$
$$= -\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a+\frac{\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)b}{\sin(dx+c)^5}}{15\,d}$$

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
 + c)^5)*a + (15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*b/sin(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(78) = 156.

Time = 0.33 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.02

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+25\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-480\,(dx+c)}{}$$

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] 1/480*(3*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 - 35*a*tan(1
/2*d*x + 1/2*c)^3 + 25*b*tan(1/2*d*x + 1/2*c)^3 - 480*(d*x + c)*a + 330*a*t
an(1/2*d*x + 1/2*c) - 150*b*tan(1/2*d*x + 1/2*c) - (330*a*tan(1/2*d*x + 1/2
*c)^4 + 150*b*tan(1/2*d*x + 1/2*c)^4 - 35*a*tan(1/2*d*x + 1/2*c)^2 - 25*b*t
an(1/2*d*x + 1/2*c)^2 + 3*a + 3*b)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 14.83 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.57

$$\int \cot^6(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{a}{160}-\frac{b}{160}\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left((22a+10b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(-\frac{7a}{3}-\frac{5b}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{5}+\frac{b}{5}\right)}{32d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{7a}{96}-\frac{5b}{96}\right)}{d} - ax + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{11a}{16}-\frac{5b}{16}\right)}{d}$$

```
[In] int(cot(c + d*x)^6*(a + b/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(a/160 - b/160))/d - (cot(c/2 + (d*x)/2)^5*(a/5 + b/5
 - tan(c/2 + (d*x)/2)^2*((7*a)/3 + (5*b)/3) + tan(c/2 + (d*x)/2)^4*(22*a +
10*b)))/(32*d) - (tan(c/2 + (d*x)/2)^3*((7*a)/96 - (5*b)/96))/d - a*x + (ta
n(c/2 + (d*x)/2)*((11*a)/16 - (5*b)/16))/d
```

3.270 $\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx = ax - \frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d} + \frac{\cot(c+dx)(35a+16b\sec(c+dx))}{35d} - \frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{105d}$$

```
[Out] a*x-1/7*cot(d*x+c)^7*(a+b*sec(d*x+c))/d+1/35*cot(d*x+c)^5*(7*a+6*b*sec(d*x+
c))/d+1/35*cot(d*x+c)*(35*a+16*b*sec(d*x+c))/d-1/105*cot(d*x+c)^3*(35*a+24*
b*sec(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3967, 8}

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx = -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d} - \frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{105d} + \frac{\cot(c+dx)(35a+16b\sec(c+dx))}{35d} + ax$$

```
[In] Int[Cot[c + d*x]^8*(a + b*Sec[c + d*x]),x]
```

```
[Out] a*x - (Cot[c + d*x]^7*(a + b*Sec[c + d*x]))/(7*d) + (Cot[c + d*x]^5*(7*a +
6*b*Sec[c + d*x]))/(35*d) + (Cot[c + d*x]*(35*a + 16*b*Sec[c + d*x]))/(35*d
) - (Cot[c + d*x]^3*(35*a + 24*b*Sec[c + d*x]))/(105*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{1}{7}\int \cot^6(c+dx)(-7a-6b\sec(c+dx))\,dx \\
&= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d} \\
&\quad + \frac{1}{35}\int \cot^4(c+dx)(35a+24b\sec(c+dx))\,dx \\
&= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d} \\
&\quad - \frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{105d} \\
&\quad + \frac{1}{105}\int \cot^2(c+dx)(-105a-48b\sec(c+dx))\,dx \\
&= -\frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d} \\
&\quad + \frac{\cot(c+dx)(35a+16b\sec(c+dx))}{35d} \\
&\quad - \frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{105d} + \frac{1}{105}\int 105a\,dx \\
&= ax - \frac{\cot^7(c+dx)(a+b\sec(c+dx))}{7d} + \frac{\cot^5(c+dx)(7a+6b\sec(c+dx))}{35d} \\
&\quad + \frac{\cot(c+dx)(35a+16b\sec(c+dx))}{35d} - \frac{\cot^3(c+dx)(35a+24b\sec(c+dx))}{105d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.83

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{b\csc(c+dx)}{d}-\frac{b\csc^3(c+dx)}{d}+\frac{3b\csc^5(c+dx)}{5d}-\frac{b\csc^7(c+dx)}{7d}-\frac{a\cot^7(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7d}$$

```
[In] Integrate[Cot[c + d*x]^8*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/d + (3*b*Csc[c + d*x]^5)/(5*d) - (b
*Csc[c + d*x]^7)/(7*d) - (a*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2,
 -Tan[c + d*x]^2])/(7*d)
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(dx+c)^7}{7}+\frac{\cot(dx+c)^5}{5}-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(dx+c}{7\sin(dx+}\right.}{d}$
default	$\frac{a\left(-\frac{\cot(dx+c)^7}{7}+\frac{\cot(dx+c)^5}{5}-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(dx+c}{7\sin(dx+}\right.}{d}$
risch	$ax+\frac{2i(105b\,e^{13i(dx+c)}+420a\,e^{12i(dx+c)}-210b\,e^{11i(dx+c)}-1260a\,e^{10i(dx+c)}+903b\,e^{9i(dx+c)}+3080a\,e^{8i(dx+c)}-636b\,e^{7}}{105d(e^{2i(d}}$

```
[In] int(cot(d*x+c)^8*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+
c)+b*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/si
n(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^6+
6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.61

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx = \frac{176\,a\cos\left(dx+c\right)^7+105\,b\cos\left(dx+c\right)^6-406\,a\cos\left(dx+c\right)^5-210\,b\cos\left(dx+c\right)^4+350\,a\cos\left(dx+c\right)^3+}{105\,\left(d\cos\left(dx+c\right)^6-\right.}$$

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/105*(176*a*cos(d*x + c)^7 + 105*b*cos(d*x + c)^6 - 406*a*cos(d*x + c)^5 -
 210*b*cos(d*x + c)^4 + 350*a*cos(d*x + c)^3 + 168*b*cos(d*x + c)^2 - 105*a
*cos(d*x + c) + 105*(a*d*x*cos(d*x + c)^6 - 3*a*d*x*cos(d*x + c)^4 + 3*a*d*
x*cos(d*x + c)^2 - a*d*x)*sin(d*x + c) - 48*b)/((d*cos(d*x + c)^6 - 3*d*cos
(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F]

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cot^8(c+dx)\,dx$$

```
[In] integrate(cot(d*x+c)**8*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cot(c + d*x)**8, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx = \frac{\left(105\,dx+105\,c+\frac{105\,\tan\left(dx+c\right)^6-35\,\tan\left(dx+c\right)^4+21\,\tan\left(dx+c\right)^2-15}{\tan\left(dx+c\right)^7}\right)a+\frac{3\,\left(35\,\sin\left(dx+c\right)^6-35\,\sin\left(dx+c\right)^4+21\,\sin\left(dx+c\right)^2-5\right)b}{\sin\left(dx+c\right)^7}}{105\,d}$$

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a + 3*(35*sin(d*x + c)^6 - 35*sin(d*x + c)
^4 + 21*sin(d*x + c)^2 - 5)*b/sin(d*x + c)^7)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(103) = 206$.

Time = 0.37 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.03

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 189\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 147\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 1295\,a\,\mathrm{t}}{}$$

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*a*tan(1/2*d*x + 1/2*c)^7 - 15*b*tan(1/2*d*x + 1/2*c)^7 - 189*a*
tan(1/2*d*x + 1/2*c)^5 + 147*b*tan(1/2*d*x + 1/2*c)^5 + 1295*a*tan(1/2*d*x
+ 1/2*c)^3 - 735*b*tan(1/2*d*x + 1/2*c)^3 + 13440*(d*x + c)*a - 9765*a*tan(
1/2*d*x + 1/2*c) + 3675*b*tan(1/2*d*x + 1/2*c) + (9765*a*tan(1/2*d*x + 1/2*
c)^6 + 3675*b*tan(1/2*d*x + 1/2*c)^6 - 1295*a*tan(1/2*d*x + 1/2*c)^4 - 735*
b*tan(1/2*d*x + 1/2*c)^4 + 189*a*tan(1/2*d*x + 1/2*c)^2 + 147*b*tan(1/2*d*x
 + 1/2*c)^2 - 15*a - 15*b)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.57

$$\int \cot^8(c+dx)(a+b\sec(c+dx))\,dx$$
$$= a\,x + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{37\,a}{384}-\frac{7\,b}{128}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\frac{9\,a}{640}-\frac{7\,b}{640}\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(\frac{a}{896}-\frac{b}{896}\right)}{d}$$
$$- \frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(\left(-93\,a-35\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{37\,a}{3}+7\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(-\frac{9\,a}{5}-\frac{7\,b}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\right.}{128\,d}$$
$$- \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{93\,a}{128}-\frac{35\,b}{128}\right)}{d}$$

```
[In] int(cot(c + d*x)^8*(a + b/cos(c + d*x)),x)
```

```
[Out] a*x + (tan(c/2 + (d*x)/2)^3*((37*a)/384 - (7*b)/128))/d - (tan(c/2 + (d*x)/
2)^5*((9*a)/640 - (7*b)/640))/d + (tan(c/2 + (d*x)/2)^7*(a/896 - b/896))/d
- (cot(c/2 + (d*x)/2)^7*(a/7 + b/7 - tan(c/2 + (d*x)/2)^2*((9*a)/5 + (7*b)/
5) + tan(c/2 + (d*x)/2)^4*((37*a)/3 + 7*b) - tan(c/2 + (d*x)/2)^6*(93*a + 3
5*b)))/(128*d) - (tan(c/2 + (d*x)/2)*((93*a)/128 - (35*b)/128))/d
```

3.271 $\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 185

$$\begin{aligned}\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx = &-\frac{a^2\log(\cos(c+dx))}{d}\\ &+\frac{2ab\sec(c+dx)}{d}-\frac{2a^2\sec^2(c+dx)}{d}\\ &-\frac{8ab\sec^3(c+dx)}{3d}+\frac{3a^2\sec^4(c+dx)}{2d}\\ &+\frac{12ab\sec^5(c+dx)}{5d}-\frac{2a^2\sec^6(c+dx)}{3d}\\ &-\frac{8ab\sec^7(c+dx)}{7d}+\frac{a^2\sec^8(c+dx)}{8d}\\ &+\frac{2ab\sec^9(c+dx)}{9d}+\frac{b^2\tan^{10}(c+dx)}{10d}\end{aligned}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a*b*sec(d*x+c)/d-2*a^2*sec(d*x+c)^2/d-8/3*a*b*sec(d
*x+c)^3/d+3/2*a^2*sec(d*x+c)^4/d+12/5*a*b*sec(d*x+c)^5/d-2/3*a^2*sec(d*x+c)
^6/d-8/7*a*b*sec(d*x+c)^7/d+1/8*a^2*sec(d*x+c)^8/d+2/9*a*b*sec(d*x+c)^9/d+1
/10*b^2*tan(d*x+c)^10/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used

$= \{3970, 962\}$

$$\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx = \frac{(a^2-4b^2)\sec^8(c+dx)}{8d}-\frac{(2a^2-3b^2)\sec^6(c+dx)}{3d}+\frac{(3a^2-2b^2)\sec^4(c+dx)}{2d}-\frac{(4a^2-b^2)\sec^2(c+dx)}{2d}-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^9(c+dx)}{9d}-\frac{8ab\sec^7(c+dx)}{7d}+\frac{12ab\sec^5(c+dx)}{5d}-\frac{8ab\sec^3(c+dx)}{3d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^{10}(c+dx)}{10d}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d - ((4*a^2 - b^2)*Sec[
c + d*x]^2)/(2*d) - (8*a*b*Sec[c + d*x]^3)/(3*d) + ((3*a^2 - 2*b^2)*Sec[c +
 d*x]^4)/(2*d) + (12*a*b*Sec[c + d*x]^5)/(5*d) - ((2*a^2 - 3*b^2)*Sec[c + d
*x]^6)/(3*d) - (8*a*b*Sec[c + d*x]^7)/(7*d) + ((a^2 - 4*b^2)*Sec[c + d*x]^8
)/(8*d) + (2*a*b*Sec[c + d*x]^9)/(9*d) + (b^2*Sec[c + d*x]^10)/(10*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^4}{x}\,dx, x, b\sec(c+dx)\right)}{b^8 d}$$

$$= \frac{\text{Subst}\Big(\int \Big(2ab^8+\frac{a^2b^8}{x}-b^6(4a^2-b^2)\,x-8ab^6x^2+2b^4(3a^2-2b^2)\,x^3+12ab^4x^4-2b^2(2a^2-3b^2)\,x^5}{b^8 d}$$

$$= -\frac{a^2 \log(\cos(c+dx))}{d} + \frac{2ab\sec(c+dx)}{d} - \frac{(4a^2-b^2)\sec^2(c+dx)}{2d} - \frac{8ab\sec^3(c+dx)}{3d} + \frac{(3a^2-2b^2)\sec^4(c+dx)}{2d} + \frac{12ab\sec^5(c+dx)}{5d} - \frac{(2a^2-3b^2)\sec^6(c+dx)}{3d} - \frac{8ab\sec^7(c+dx)}{7d} + \frac{(a^2-4b^2)\sec^8(c+dx)}{8d} + \frac{2ab\sec^9(c+dx)}{9d} + \frac{b^2\sec^{10}(c+dx)}{10d}$$

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.94

$$\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx = \frac{-2520a^2\log(\cos(c+dx)) + 5040ab\sec(c+dx) - 1260(4a^2-b^2)\sec^2(c+dx) - 6720ab\sec^3(c+dx) + 12$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^9,x]
```

```
[Out] (-2520*a^2*Log[Cos[c + d*x]] + 5040*a*b*Sec[c + d*x] - 1260*(4*a^2 - b^2)*S
ec[c + d*x]^2 - 6720*a*b*Sec[c + d*x]^3 + 1260*(3*a^2 - 2*b^2)*Sec[c + d*x]
^4 + 6048*a*b*Sec[c + d*x]^5 - 840*(2*a^2 - 3*b^2)*Sec[c + d*x]^6 - 2880*a*
b*Sec[c + d*x]^7 + 315*(a^2 - 4*b^2)*Sec[c + d*x]^8 + 560*a*b*Sec[c + d*x]^
9 + 252*b^2*Sec[c + d*x]^10)/(2520*d)
```

Maple [A] (verified)

Time = 6.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.72

method	result
parts	$\frac{a^2\left(\frac{\tan(dx+c)^8}{8}-\frac{\tan(dx+c)^6}{6}+\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b^2\tan(dx+c)^{10}}{10d}+\frac{2ab\left(\frac{\sec(dx+c)^9}{9}-4\right.}{}$
derivativedivides	$\frac{\frac{b^2\sec(dx+c)^{10}}{10}+\frac{2ab\sec(dx+c)^9}{9}+\frac{a^2\sec(dx+c)^8}{8}-\frac{b^2\sec(dx+c)^8}{2}-\frac{8ab\sec(dx+c)^7}{7}-\frac{2a^2\sec(dx+c)^6}{3}+b^2\sec(dx+c)^6+\frac{12ab\sec(c}{5}}{}$
default	$\frac{\frac{b^2\sec(dx+c)^{10}}{10}+\frac{2ab\sec(dx+c)^9}{9}+\frac{a^2\sec(dx+c)^8}{8}-\frac{b^2\sec(dx+c)^8}{2}-\frac{8ab\sec(dx+c)^7}{7}-\frac{2a^2\sec(dx+c)^6}{3}+b^2\sec(dx+c)^6+\frac{12ab\sec(c}{5}}{}$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{4ab\,e^{19i(dx+c)}-8a^2e^{18i(dx+c)}+2b^2e^{18i(dx+c)}+\frac{44ab\,e^{17i(dx+c)}}{3}-40a^2e^{16i(dx+c)}+\frac{1072ab\,e^{15i(dx+c)}}{15}}{}$

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(1/8*tan(d*x+c)^8-1/6*tan(d*x+c)^6+1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2+
1/2*ln(1+tan(d*x+c)^2))+1/10*b^2*tan(d*x+c)^10/d+2*a*b/d*(1/9*sec(d*x+c)^9-
4/7*sec(d*x+c)^7+6/5*sec(d*x+c)^5-4/3*sec(d*x+c)^3+sec(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.98

$$\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx = -\frac{2520\,a^2\cos\left(dx+c\right)^{10}\log\left(-\cos\left(dx+c\right)\right)-5040\,ab\cos\left(dx+c\right)^9+6720\,ab\cos\left(dx+c\right)^7+1260\,(4\,a^2-$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(2520*a^2*cos(d*x + c)^10*log(-cos(d*x + c)) - 5040*a*b*cos(d*x + c
)^9 + 6720*a*b*cos(d*x + c)^7 + 1260*(4*a^2 - b^2)*cos(d*x + c)^8 - 6048*a*
b*cos(d*x + c)^5 - 1260*(3*a^2 - 2*b^2)*cos(d*x + c)^6 + 2880*a*b*cos(d*x +
 c)^3 + 840*(2*a^2 - 3*b^2)*cos(d*x + c)^4 - 560*a*b*cos(d*x + c) - 315*(a^
2 - 4*b^2)*cos(d*x + c)^2 - 252*b^2)/(d*cos(d*x + c)^10)
```

Sympy [A] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.70

$$\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx = \begin{cases} \frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{a^2\tan^8\left(c+dx\right)}{8d}-\frac{a^2\tan^6\left(c+dx\right)}{6d}+\frac{a^2\tan^4\left(c+dx\right)}{4d}-\frac{a^2\tan^2\left(c+dx\right)}{2d}+\frac{2ab\tan^8\left(c+dx\right)\sec\left(c+dx\right)}{9d}-\frac{16ab}{} \\ x(a+b\sec\left(c\right))^2\tan^9\left(c\right) \end{cases}$$

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**9,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**8/(8*d)
 - a**2*tan(c + d*x)**6/(6*d) + a**2*tan(c + d*x)**4/(4*d) - a**2*tan(c + d
*x)**2/(2*d) + 2*a*b*tan(c + d*x)**8*sec(c + d*x)/(9*d) - 16*a*b*tan(c + d*
x)**6*sec(c + d*x)/(63*d) + 32*a*b*tan(c + d*x)**4*sec(c + d*x)/(105*d) - 1
28*a*b*tan(c + d*x)**2*sec(c + d*x)/(315*d) + 256*a*b*sec(c + d*x)/(315*d)
+ b**2*tan(c + d*x)**8*sec(c + d*x)**2/(10*d) - b**2*tan(c + d*x)**6*sec(c
+ d*x)**2/(10*d) + b**2*tan(c + d*x)**4*sec(c + d*x)**2/(10*d) - b**2*tan(c
 + d*x)**2*sec(c + d*x)**2/(10*d) + b**2*sec(c + d*x)**2/(10*d), Ne(d, 0)),
 (x*(a + b*sec(c))**2*tan(c)**9, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.94

$$\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx =$$

$$-\frac{2520\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{5040\,ab\cos(dx+c)^9-6720\,ab\cos(dx+c)^7-1260\,(4\,a^2-b^2)\cos(dx+c)^8+6048\,ab\cos(dx+c)^5+1260\,(3\,a^2}{}}{2520\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(2520*a^2*log(cos(d*x + c)) - (5040*a*b*cos(d*x + c)^9 - 6720*a*b*c
os(d*x + c)^7 - 1260*(4*a^2 - b^2)*cos(d*x + c)^8 + 6048*a*b*cos(d*x + c)^5
 + 1260*(3*a^2 - 2*b^2)*cos(d*x + c)^6 - 2880*a*b*cos(d*x + c)^3 - 840*(2*a
^2 - 3*b^2)*cos(d*x + c)^4 + 560*a*b*cos(d*x + c) + 315*(a^2 - 4*b^2)*cos(d
*x + c)^2 + 252*b^2)/cos(d*x + c)^10)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(169) = 338$.

Time = 5.92 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.64

$$\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx$$

$$=\frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{7381\,a^2+4096\,ab+\frac{78850\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{40960\,a}{}}{}}{}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 252
0*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (7381*a^2 + 40
96*a*b + 78850*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 40960*a*b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 382545*a^2*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 + 184320*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 111420
0*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 491520*a*b*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3 + 2171610*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c
) + 1)^4 + 860160*a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 2736972*a
^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 516096*a*b*(cos(d*x + c) - 1
)^5/(cos(d*x + c) + 1)^5 - 258048*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) +
1)^5 + 2171610*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1114200*a^2*
(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 382545*a^2*(cos(d*x + c) - 1)^8
/(cos(d*x + c) + 1)^8 + 78850*a^2*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9
 + 7381*a^2*(cos(d*x + c) - 1)^10/(cos(d*x + c) + 1)^10)/((cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 1)^10)/d
```

Mupad [B] (verification not implemented)

Time = 17.87 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.86

$$\int (a+b\sec(c+dx))^2\tan^9(c+dx)\,dx$$

$$= \frac{\frac{512\,a\,b}{315} + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(20\,a^2+\frac{512\,b\,a}{7}\right) - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a^2+\frac{1024\,b\,a}{63}\right) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(\frac{740\,a^2}{3}+\frac{1024\,b\,a}{3}\right) - \mathrm{t}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20} - 10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18} + 45\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16} - 120\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}\right.}$$

$$+ \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

```
[In] int(tan(c + d*x)^9*(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((512*a*b)/315 + tan(c/2 + (d*x)/2)^4*((512*a*b)/7 + 20*a^2) - tan(c/2 + (d
*x)/2)^2*((1024*a*b)/63 + 2*a^2) + tan(c/2 + (d*x)/2)^8*((1024*a*b)/3 + (74
0*a^2)/3) - tan(c/2 + (d*x)/2)^6*((4096*a*b)/21 + (272*a^2)/3) + (740*a^2*t
an(c/2 + (d*x)/2)^12)/3 - (272*a^2*tan(c/2 + (d*x)/2)^14)/3 + 20*a^2*tan(c/
2 + (d*x)/2)^16 - 2*a^2*tan(c/2 + (d*x)/2)^18 - tan(c/2 + (d*x)/2)^10*((102
4*a*b)/5 + 348*a^2 - (512*b^2)/5))/(d*(45*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2
 + (d*x)/2)^2 - 120*tan(c/2 + (d*x)/2)^6 + 210*tan(c/2 + (d*x)/2)^8 - 252*t
an(c/2 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)^12 - 120*tan(c/2 + (d*x)/2)^1
4 + 45*tan(c/2 + (d*x)/2)^16 - 10*tan(c/2 + (d*x)/2)^18 + tan(c/2 + (d*x)/2
)^20 + 1)) + (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.272 $\int (a+b\sec(c+dx))^2 \tan^7(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\begin{aligned}\int (a+b\sec(c+dx))^2 \tan^7(c+dx)\,dx = {} & \frac{a^2\log(\cos(c+dx))}{d} \\ & - \frac{2ab\sec(c+dx)}{d} + \frac{3a^2\sec^2(c+dx)}{2d} \\ & + \frac{2ab\sec^3(c+dx)}{d} - \frac{3a^2\sec^4(c+dx)}{4d} \\ & - \frac{6ab\sec^5(c+dx)}{5d} + \frac{a^2\sec^6(c+dx)}{6d} \\ & + \frac{2ab\sec^7(c+dx)}{7d} + \frac{b^2\tan^8(c+dx)}{8d}\end{aligned}$$

```
[Out] a^2*ln(cos(d*x+c))/d-2*a*b*sec(d*x+c)/d+3/2*a^2*sec(d*x+c)^2/d+2*a*b*sec(d*
x+c)^3/d-3/4*a^2*sec(d*x+c)^4/d-6/5*a*b*sec(d*x+c)^5/d+1/6*a^2*sec(d*x+c)^6
/d+2/7*a*b*sec(d*x+c)^7/d+1/8*b^2*tan(d*x+c)^8/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used

$= \{3970, 962\}$

$$\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx = \frac{(a^2-3b^2)\sec^6(c+dx)}{6d} - \frac{3(a^2-b^2)\sec^4(c+dx)}{4d} + \frac{(3a^2-b^2)\sec^2(c+dx)}{2d} + \frac{a^2\log(\cos(c+dx))}{d} + \frac{2ab\sec^7(c+dx)}{7d} - \frac{6ab\sec^5(c+dx)}{5d} + \frac{2ab\sec^3(c+dx)}{d} - \frac{2ab\sec(c+dx)}{d} + \frac{b^2\sec^8(c+dx)}{8d}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a*b*Sec[c + d*x])/d + ((3*a^2 - b^2)*Sec[c +
 d*x]^2)/(2*d) + (2*a*b*Sec[c + d*x]^3)/d - (3*(a^2 - b^2)*Sec[c + d*x]^4)/
(4*d) - (6*a*b*Sec[c + d*x]^5)/(5*d) + ((a^2 - 3*b^2)*Sec[c + d*x]^6)/(6*d)
 + (2*a*b*Sec[c + d*x]^7)/(7*d) + (b^2*Sec[c + d*x]^8)/(8*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^3}{x}\,dx, x, b\sec(c+dx)\right)}{b^6 d}$$

$$= -\frac{\text{Subst}\left(\int \left(2ab^6 + \frac{a^2b^6}{x} - b^4(3a^2-b^2)\,x - 6ab^4x^2 + 3b^2(a^2-b^2)\,x^3 + 6ab^2x^4 - (a^2-3b^2)\,x^5 - 2a\right.\right.}{b^6 d}$$

$$= \frac{a^2 \log(\cos(c+dx))}{d} - \frac{2ab\sec(c+dx)}{d} + \frac{(3a^2-b^2)\sec^2(c+dx)}{2d} + \frac{2ab\sec^3(c+dx)}{d} - \frac{3(a^2-b^2)\sec^4(c+dx)}{4d} - \frac{6ab\sec^5(c+dx)}{5d} + \frac{(a^2-3b^2)\sec^6(c+dx)}{6d} + \frac{2ab\sec^7(c+dx)}{7d} + \frac{b^2\sec^8(c+dx)}{8d}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93

$$\int (a+b\sec(c+dx))^2 \tan^7(c+dx)\,dx = \frac{840a^2\log(\cos(c+dx)) - 1680ab\sec(c+dx) + 420(3a^2-b^2)\sec^2(c+dx) + 1680ab\sec^3(c+dx) - 630(a^2}{}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^7,x]
```

```
[Out] (840*a^2*Log[Cos[c + d*x]] - 1680*a*b*Sec[c + d*x] + 420*(3*a^2 - b^2)*Sec[
c + d*x]^2 + 1680*a*b*Sec[c + d*x]^3 - 630*(a^2 - b^2)*Sec[c + d*x]^4 - 100
8*a*b*Sec[c + d*x]^5 + 140*(a^2 - 3*b^2)*Sec[c + d*x]^6 + 240*a*b*Sec[c + d
*x]^7 + 105*b^2*Sec[c + d*x]^8)/(840*d)
```

Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.76

method	result
parts	$\frac{a^2\left(\frac{\tan(dx+c)^6}{6}-\frac{\tan(dx+c)^4}{4}+\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b^2\tan(dx+c)^8}{8d}+\frac{2ab\left(\frac{\sec(dx+c)^7}{7}-\frac{3\sec(dx+c)^5}{5}+\mathrm{s}\right.}{d}$
derivativedivides	$\frac{\frac{b^2\sec(dx+c)^8}{8}+\frac{2ab\sec(dx+c)^7}{7}+\frac{a^2\sec(dx+c)^6}{6}-\frac{b^2\sec(dx+c)^6}{2}-\frac{6ab\sec(dx+c)^5}{5}-\frac{3a^2\sec(dx+c)^4}{4}+\frac{3b^2\sec(dx+c)^4}{4}+2ab\sec(d}{d}$
default	$\frac{\frac{b^2\sec(dx+c)^8}{8}+\frac{2ab\sec(dx+c)^7}{7}+\frac{a^2\sec(dx+c)^6}{6}-\frac{b^2\sec(dx+c)^6}{2}-\frac{6ab\sec(dx+c)^5}{5}-\frac{3a^2\sec(dx+c)^4}{4}+\frac{3b^2\sec(dx+c)^4}{4}+2ab\sec(d}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{2\left(210ab\,\mathrm{e}^{15i(dx+c)}-315a^2\mathrm{e}^{14i(dx+c)}+105b^2\mathrm{e}^{14i(dx+c)}+630ab\,\mathrm{e}^{13i(dx+c)}-1260a^2\mathrm{e}^{12i(dx+c)}+22\right.}{}$

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(1/6*tan(d*x+c)^6-1/4*tan(d*x+c)^4+1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+
c)^2))+1/8*b^2*tan(d*x+c)^8/d+2*a*b/d*(1/7*sec(d*x+c)^7-3/5*sec(d*x+c)^5+se
c(d*x+c)^3-sec(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.98

$$\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx$$
$$= \frac{840\,a^2\cos(dx+c)^8\log(-\cos(dx+c)) - 1680\,ab\cos(dx+c)^7 + 1680\,ab\cos(dx+c)^5 + 420\,(3\,a^2-b^2)\cos\cdots}{8\cdots}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/840*(840*a^2*cos(d*x + c)^8*log(-cos(d*x + c)) - 1680*a*b*cos(d*x + c)^7
+ 1680*a*b*cos(d*x + c)^5 + 420*(3*a^2 - b^2)*cos(d*x + c)^6 - 1008*a*b*cos
(d*x + c)^3 - 630*(a^2 - b^2)*cos(d*x + c)^4 + 240*a*b*cos(d*x + c) + 140*(
a^2 - 3*b^2)*cos(d*x + c)^2 + 105*b^2)/(d*cos(d*x + c)^8)
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.69

$$\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx$$
$$= \begin{cases} -\frac{a^2\log(\tan^2(c+dx)+1)}{2d} + \frac{a^2\tan^6(c+dx)}{6d} - \frac{a^2\tan^4(c+dx)}{4d} + \frac{a^2\tan^2(c+dx)}{2d} + \frac{2ab\tan^6(c+dx)\sec(c+dx)}{7d} - \frac{12ab\tan^4(c+dx)\sec(\cdots}{35d} \cdots \\ x(a+b\sec(c))^2\tan^7(c) \end{cases}$$

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**6/(6*d
) - a**2*tan(c + d*x)**4/(4*d) + a**2*tan(c + d*x)**2/(2*d) + 2*a*b*tan(c +
 d*x)**6*sec(c + d*x)/(7*d) - 12*a*b*tan(c + d*x)**4*sec(c + d*x)/(35*d) +
16*a*b*tan(c + d*x)**2*sec(c + d*x)/(35*d) - 32*a*b*sec(c + d*x)/(35*d) + b
**2*tan(c + d*x)**6*sec(c + d*x)**2/(8*d) - b**2*tan(c + d*x)**4*sec(c + d*
x)**2/(8*d) + b**2*tan(c + d*x)**2*sec(c + d*x)**2/(8*d) - b**2*sec(c + d*x
)**2/(8*d), Ne(d, 0)), (x*(a + b*sec(c))**2*tan(c)**7, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx$$
$$=\frac{840\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{1680\,ab\cos(dx+c)^7-1680\,ab\cos(dx+c)^5-420\,(3\,a^2-b^2)\cos(dx+c)^6+1008\,ab\cos(dx+c)^3+630\,(a^2-b^2)\cos(}{\cos(dx+c)^8}}{840\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/840*(840*a^2*log(cos(d*x + c)) - (1680*a*b*cos(d*x + c)^7 - 1680*a*b*cos(
d*x + c)^5 - 420*(3*a^2 - b^2)*cos(d*x + c)^6 + 1008*a*b*cos(d*x + c)^3 + 6
30*(a^2 - b^2)*cos(d*x + c)^4 - 240*a*b*cos(d*x + c) - 140*(a^2 - 3*b^2)*co
s(d*x + c)^2 - 105*b^2)/cos(d*x + c)^8)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(137) = 274$.

Time = 3.31 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.79

$$\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx =$$
$$-\frac{840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-840\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2283\,a^2+1536\,ab+\frac{19944\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12288\,a}{\mathrm{co}}}{}}{}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] -1/840*(840*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 840*
a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2283*a^2 + 1536
*a*b + 19944*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12288*a*b*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 77364*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 + 43008*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 175448*a^2
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 86016*a*b*(cos(d*x + c) - 1)^3
/(cos(d*x + c) + 1)^3 + 231490*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^
4 + 53760*a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 26880*b^2*(cos(d*
x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 175448*a^2*(cos(d*x + c) - 1)^5/(cos(d
*x + c) + 1)^5 + 77364*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1994
4*a^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 2283*a^2*(cos(d*x + c) -
1)^8/(cos(d*x + c) + 1)^8)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 17.89 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.88

$$\int (a+b\sec(c+dx))^2\tan^7(c+dx)\,dx =$$

$$-\frac{\frac{64\,a\,b}{35}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(16\,a^2+\frac{256\,b\,a}{5}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a^2+\frac{512\,b\,a}{35}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{170\,a^2}{3}+\frac{512\,b\,a}{5}\right)-}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-56\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+7\right.}$$
$$-\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

```
[In] int(tan(c + d*x)^7*(a + b/cos(c + d*x))^2,x)
```

```
[Out] - ((64*a*b)/35 + tan(c/2 + (d*x)/2)^4*((256*a*b)/5 + 16*a^2) - tan(c/2 + (d
*x)/2)^2*((512*a*b)/35 + 2*a^2) - tan(c/2 + (d*x)/2)^6*((512*a*b)/5 + (170*
a^2)/3) - (170*a^2*tan(c/2 + (d*x)/2)^10)/3 + 16*a^2*tan(c/2 + (d*x)/2)^12
- 2*a^2*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^8*(64*a*b + (256*a^2)/3
- 32*b^2))/(d*(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/
2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*ta
n(c/2 + (d*x)/2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1))
 - (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.273 $\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx=-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{a^2\sec^2(c+dx)}{d}-\frac{4ab\sec^3(c+dx)}{3d}+\frac{a^2\sec^4(c+dx)}{4d}+\frac{2ab\sec^5(c+dx)}{5d}+\frac{b^2\tan^6(c+dx)}{6d}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a*b*sec(d*x+c)/d-a^2*sec(d*x+c)^2/d-4/3*a*b*sec(d*x
+c)^3/d+1/4*a^2*sec(d*x+c)^4/d+2/5*a*b*sec(d*x+c)^5/d+1/6*b^2*tan(d*x+c)^6/
d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3970, 962}

$$\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx=\frac{(a^2-2b^2)\sec^4(c+dx)}{4d}-\frac{(2a^2-b^2)\sec^2(c+dx)}{2d}-\frac{a^2\log(\cos(c+dx))}{d}+\frac{2ab\sec^5(c+dx)}{5d}-\frac{4ab\sec^3(c+dx)}{3d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec^6(c+dx)}{6d}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d - ((2*a^2 - b^2)*Sec[
c + d*x]^2)/(2*d) - (4*a*b*Sec[c + d*x]^3)/(3*d) + ((a^2 - 2*b^2)*Sec[c + d
*x]^4)/(4*d) + (2*a*b*Sec[c + d*x]^5)/(5*d) + (b^2*Sec[c + d*x]^6)/(6*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x}\,dx, x, b\sec(c+dx)\right)}{b^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(2ab^4 + \frac{a^2b^4}{x} - b^2(2a^2-b^2)\,x - 4ab^2x^2 + (a^2-2b^2)\,x^3 + 2ax^4 + x^5\right)\,dx, x, b\sec(c+dx)\right)}{b^4 d}$$

$$= -\frac{a^2\log(\cos(c+dx))}{d} + \frac{2ab\sec(c+dx)}{d} - \frac{(2a^2-b^2)\sec^2(c+dx)}{2d} - \frac{4ab\sec^3(c+dx)}{3d} + \frac{(a^2-2b^2)\sec^4(c+dx)}{4d} + \frac{2ab\sec^5(c+dx)}{5d} + \frac{b^2\sec^6(c+dx)}{6d}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx = \frac{-60a^2\log(\cos(c+dx)) + 120ab\sec(c+dx) + 30(-2a^2+b^2)\sec^2(c+dx) - 80ab\sec^3(c+dx) + 15(a^2-2}{60d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-60*a^2*Log[Cos[c + d*x]] + 120*a*b*Sec[c + d*x] + 30*(-2*a^2 + b^2)*Sec[c
 + d*x]^2 - 80*a*b*Sec[c + d*x]^3 + 15*(a^2 - 2*b^2)*Sec[c + d*x]^4 + 24*a*
b*Sec[c + d*x]^5 + 10*b^2*Sec[c + d*x]^6)/(60*d)
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.81

method	result
parts	$\frac{a^2\left(\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b^2\tan(dx+c)^6}{6d}+\frac{2ab\left(\frac{\sec(dx+c)^5}{5}-\frac{2\sec(dx+c)^3}{3}+\sec(dx+c)\right)}{d}$
derivativedivides	$\frac{\frac{b^2\sec(dx+c)^6}{6}+\frac{2ab\sec(dx+c)^5}{5}+\frac{a^2\sec(dx+c)^4}{4}-\frac{b^2\sec(dx+c)^4}{2}-\frac{4ab\sec(dx+c)^3}{3}-a^2\sec(dx+c)^2+\frac{\sec(dx+c)^2b^2}{2}+2ab\sec(dx-}{d}$
default	$\frac{\frac{b^2\sec(dx+c)^6}{6}+\frac{2ab\sec(dx+c)^5}{5}+\frac{a^2\sec(dx+c)^4}{4}-\frac{b^2\sec(dx+c)^4}{2}-\frac{4ab\sec(dx+c)^3}{3}-a^2\sec(dx+c)^2+\frac{\sec(dx+c)^2b^2}{2}+2ab\sec(dx-}{d}$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{4ab\,e^{11i(dx+c)}-4a^2e^{10i(dx+c)}+2b^2e^{10i(dx+c)}+\frac{28ab\,e^{9i(dx+c)}}{3}-12a^2e^{8i(dx+c)}+\frac{104ab\,e^{7i(dx+c)}}{5}-16a}{}$

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2+1/2*ln(1+tan(d*x+c)^2))+1/6*b^2*ta
n(d*x+c)^6/d+2*a*b/d*(1/5*sec(d*x+c)^5-2/3*sec(d*x+c)^3+sec(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx =$$
$$-\frac{60\,a^2\cos(dx+c)^6\log(-\cos(dx+c))-120\,ab\cos(dx+c)^5+80\,ab\cos(dx+c)^3+30\,(2\,a^2-b^2)\cos(}{60\,d\cos(dx+c)^6}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(60*a^2*cos(d*x + c)^6*log(-cos(d*x + c)) - 120*a*b*cos(d*x + c)^5 +
80*a*b*cos(d*x + c)^3 + 30*(2*a^2 - b^2)*cos(d*x + c)^4 - 24*a*b*cos(d*x +
c) - 15*(a^2 - 2*b^2)*cos(d*x + c)^2 - 10*b^2)/(d*cos(d*x + c)^6)
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.64

$$\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx$$
$$=\begin{cases}\frac{a^2\log(\tan^2(c+dx)+1)}{2d}+\frac{a^2\tan^4(c+dx)}{4d}-\frac{a^2\tan^2(c+dx)}{2d}+\frac{2ab\tan^4(c+dx)\sec(c+dx)}{5d}-\frac{8ab\tan^2(c+dx)\sec(c+dx)}{15d}+\frac{16ab\sec(c-}{15d}\\ x(a+b\sec(c))^2\tan^5(c)\end{cases}$$

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**5,x)
```

```
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**4/(4*d)
 - a**2*tan(c + d*x)**2/(2*d) + 2*a*b*tan(c + d*x)**4*sec(c + d*x)/(5*d) -
8*a*b*tan(c + d*x)**2*sec(c + d*x)/(15*d) + 16*a*b*sec(c + d*x)/(15*d) + b*
*2*tan(c + d*x)**4*sec(c + d*x)**2/(6*d) - b**2*tan(c + d*x)**2*sec(c + d*x
)**2/(6*d) + b**2*sec(c + d*x)**2/(6*d), Ne(d, 0)), (x*(a + b*sec(c))**2*ta
n(c)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx = -\frac{60\,a^2\log\left(\cos\left(dx+c\right)\right) - \frac{120\,ab\cos(dx+c)^5-80\,ab\cos(dx+c)^3-30\,(2\,a^2-b^2)\cos(dx+c)^4+24\,ab\cos(dx+c)+15\,(a^2-2\,b^2)\cos(dx+c)^2}{\cos(dx+c)^6}}{60\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(60*a^2*log(cos(d*x + c)) - (120*a*b*cos(d*x + c)^5 - 80*a*b*cos(d*x
+ c)^3 - 30*(2*a^2 - b^2)*cos(d*x + c)^4 + 24*a*b*cos(d*x + c) + 15*(a^2 -
2*b^2)*cos(d*x + c)^2 + 10*b^2)/cos(d*x + c)^6)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(107) = 214$.

Time = 1.69 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.97

$$\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx = \frac{60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{147\,a^2+128\,ab+\frac{1002\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{768\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}}{}}{}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (147*a^2 + 128*a*b +
 1002*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 768*a*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 2925*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 1920*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4140*a^2*(cos(d*x + c
) - 1)^3/(cos(d*x + c) + 1)^3 + 1280*a*b*(cos(d*x + c) - 1)^3/(cos(d*x + c)
```

```
 + 1)^3 - 640*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*a^2*(cos
(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^2*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 + 147*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/((cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.87

$$\int (a+b\sec(c+dx))^2\tan^5(c+dx)\,dx = \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$
$$+\frac{\frac{32\,a\,b}{15}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(12\,a^2+32\,b\,a\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a^2+\frac{64\,b\,a}{5}\right)+12\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\,a^2\tan\left(\frac{c}{2}+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+15\tan\left(\frac{c}{2}\right.\right.}$$

```
[In] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d + ((32*a*b)/15 + tan(c/2 + (d*x)/2)^4
*(32*a*b + 12*a^2) - tan(c/2 + (d*x)/2)^2*((64*a*b)/5 + 2*a^2) + 12*a^2*tan
(c/2 + (d*x)/2)^8 - 2*a^2*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^6*((64
*a*b)/3 + 20*a^2 - (32*b^2)/3))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (
d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 +
 (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.274 $\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx = \frac{a^2\log(\cos(c+dx))}{d} - \frac{2ab\sec(c+dx)}{d} + \frac{(a^2-b^2)\sec^2(c+dx)}{2d} + \frac{2ab\sec^3(c+dx)}{3d} + \frac{b^2\sec^4(c+dx)}{4d}$$

```
[Out] a^2*ln(cos(d*x+c))/d-2*a*b*sec(d*x+c)/d+1/2*(a^2-b^2)*sec(d*x+c)^2/d+2/3*a*
b*sec(d*x+c)^3/d+1/4*b^2*sec(d*x+c)^4/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3970, 908}

$$\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx = \frac{(a^2-b^2)\sec^2(c+dx)}{2d} + \frac{a^2\log(\cos(c+dx))}{d} + \frac{2ab\sec^3(c+dx)}{3d} - \frac{2ab\sec(c+dx)}{d} + \frac{b^2\sec^4(c+dx)}{4d}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d - (2*a*b*Sec[c + d*x])/d + ((a^2 - b^2)*Sec[c + d
*x]^2)/(2*d) + (2*a*b*Sec[c + d*x]^3)/(3*d) + (b^2*Sec[c + d*x]^4)/(4*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)}{x}\,dx, x, b\sec(c+dx)\right)}{b^2d}\\
&= -\frac{\text{Subst}\left(\int \left(2ab^2+\frac{a^2b^2}{x}-(a^2-b^2)\,x-2ax^2-x^3\right)\,dx, x, b\sec(c+dx)\right)}{b^2d}\\
&= \frac{a^2\log(\cos(c+dx))}{d}-\frac{2ab\sec(c+dx)}{d}\\
&\quad+\frac{(a^2-b^2)\sec^2(c+dx)}{2d}+\frac{2ab\sec^3(c+dx)}{3d}+\frac{b^2\sec^4(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx = \frac{12a^2\log(\cos(c+dx))-24ab\sec(c+dx)+6(a^2-b^2)\sec^2(c+dx)+8ab\sec^3(c+dx)+3b^2\sec^4(c+dx)}{12d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^3,x]
[Out] (12*a^2*Log[Cos[c + d*x]] - 24*a*b*Sec[c + d*x] + 6*(a^2 - b^2)*Sec[c + d*x
]^2 + 8*a*b*Sec[c + d*x]^3 + 3*b^2*Sec[c + d*x]^4)/(12*d)
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

method	result
parts	$\frac{a^2\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{b^2\tan(dx+c)^4}{4d}+\frac{2ab\left(\frac{\sec(dx+c)^3}{3}-\sec(dx+c)\right)}{d}$
derivativedivides	$\frac{\frac{b^2\sec(dx+c)^4}{4}+\frac{2ab\sec(dx+c)^3}{3}+\frac{a^2\sec(dx+c)^2}{2}-\frac{\sec(dx+c)^2b^2}{2}-2ab\sec(dx+c)-a^2\ln(\sec(dx+c))}{d}$
default	$\frac{\frac{b^2\sec(dx+c)^4}{4}+\frac{2ab\sec(dx+c)^3}{3}+\frac{a^2\sec(dx+c)^2}{2}-\frac{\sec(dx+c)^2b^2}{2}-2ab\sec(dx+c)-a^2\ln(\sec(dx+c))}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{2\left(6ab\,e^{7i(dx+c)}-3a^2e^{6i(dx+c)}+3b^2e^{6i(dx+c)}+10ab\,e^{5i(dx+c)}-6a^2e^{4i(dx+c)}+10ab\,e^{3i(dx+c)}-3a^2e^{2}\right.}{3d\left(e^{2i(dx+c)}+1\right)^4}$

[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] a^2/d*(1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c)^2))+1/4*b^2*tan(d*x+c)^4/d+2*a*b/d*(1/3*sec(d*x+c)^3-sec(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx$$
$$=\frac{12\,a^2\cos{(dx+c)}^4\log{(-\cos{(dx+c)})}-24\,ab\cos{(dx+c)}^3+8\,ab\cos{(dx+c)}+6\,(a^2-b^2)\cos{(dx+c)}^2+}{12\,d\cos{(dx+c)}^4}$$

[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")

[Out] 1/12*(12*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) - 24*a*b*cos(d*x + c)^3 + 8*a*b*cos(d*x + c) + 6*(a^2 - b^2)*cos(d*x + c)^2 + 3*b^2)/(d*cos(d*x + c)^4)

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx$$
$$=\begin{cases}-\frac{a^2\log{(\tan^2{(c+dx)}+1)}}{2d}+\frac{a^2\tan^2{(c+dx)}}{2d}+\frac{2ab\tan^2{(c+dx)}\sec{(c+dx)}}{3d}-\frac{4ab\sec{(c+dx)}}{3d}+\frac{b^2\tan^2{(c+dx)}\sec^2{(c+dx)}}{4d}-\frac{b^2\sec^2{(c+}}{4d}\\ x(a+b\sec{(c)})^2\tan^3{(c)}\end{cases}$$

[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**3,x)

```
[Out] Piecewise((-a**2*log(tan(c + d*x)**2 + 1)/(2*d) + a**2*tan(c + d*x)**2/(2*d
) + 2*a*b*tan(c + d*x)**2*sec(c + d*x)/(3*d) - 4*a*b*sec(c + d*x)/(3*d) + b
**2*tan(c + d*x)**2*sec(c + d*x)**2/(4*d) - b**2*sec(c + d*x)**2/(4*d), Ne(
d, 0)), (x*(a + b*sec(c))**2*tan(c)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx = \frac{12\,a^2\log\left(\cos\left(dx+c\right)\right) - \frac{24\,ab\cos(dx+c)^3-8\,ab\cos(dx+c)-6\,(a^2-b^2)\cos(dx+c)^2-3\,b^2}{\cos(dx+c)^4}}{12\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*a^2*log(cos(d*x + c)) - (24*a*b*cos(d*x + c)^3 - 8*a*b*cos(d*x + c
) - 6*(a^2 - b^2)*cos(d*x + c)^2 - 3*b^2)/cos(d*x + c)^4)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(81) = 162$.

Time = 0.74 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.07

$$\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx = -\frac{12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{25\,a^2+32\,ab+\frac{124\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{128\,ab(\cos(dx+c)-}{\cos(dx+c)+1}}{}}{12\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^2
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (25*a^2 + 32*a*b +
124*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 128*a*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) + 198*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 96
*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 48*b^2*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
+ 25*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 16.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.74

$$\int (a+b\sec(c+dx))^2\tan^3(c+dx)\,dx = -\frac{\frac{8\,a\,b}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a^2+\frac{32\,b\,a}{3}\right)-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(4\,a^2+8\,a\,b-4\,b^2\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)} - \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

```
[In] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

```
[Out] - ((8*a*b)/3 - tan(c/2 + (d*x)/2)^2*((32*a*b)/3 + 2*a^2) - 2*a^2*tan(c/2 +
(d*x)/2)^6 + tan(c/2 + (d*x)/2)^4*(8*a*b + 4*a^2 - 4*b^2))/(d*(6*tan(c/2 +
(d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d
*x)/2)^8 + 1)) - (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.275 $\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx = -\frac{a^2\log(\cos(c+dx))}{d} + \frac{2ab\sec(c+dx)}{d} + \frac{b^2\sec^2(c+dx)}{2d}$$

```
[Out] -a^2*ln(cos(d*x+c))/d+2*a*b*sec(d*x+c)/d+1/2*b^2*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3970, 45}

$$\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx = -\frac{a^2\log(\cos(c+dx))}{d} + \frac{2ab\sec(c+dx)}{d} + \frac{b^2\sec^2(c+dx)}{2d}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d + (b^2*Sec[c + d*x]^2
)/(2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+x)^2}{x} \, dx, x, b\sec(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(2a + \frac{a^2}{x} + x\right) \, dx, x, b\sec(c+dx)\right)}{d} \\
&= -\frac{a^2 \log(\cos(c+dx))}{d} + \frac{2ab\sec(c+dx)}{d} + \frac{b^2\sec^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$
\int (a+b\sec(c+dx))^2 \tan(c+dx) \, dx = \frac{-2a^2\log(\cos(c+dx)) + 4ab\sec(c+dx) + b^2\sec^2(c+dx)}{2d}
$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x],x]
[Out] (-2*a^2*Log[Cos[c + d*x]] + 4*a*b*Sec[c + d*x] + b^2*Sec[c + d*x]^2)/(2*d)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\sec(dx+c)^2 b^2}{2}+2ab\sec(dx+c)+a^2\ln(\sec(dx+c))}{d}$	40
default	$\frac{\frac{\sec(dx+c)^2 b^2}{2}+2ab\sec(dx+c)+a^2\ln(\sec(dx+c))}{d}$	40
parts	$\frac{a^2\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{b^2\sec(dx+c)^2}{2d}+\frac{2ab\sec(dx+c)}{d}$	50
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2b\left(2a\,\mathrm{e}^{3i(dx+c)}+b\,\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{i(dx+c)}a\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{a^2\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	94

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/2*sec(d*x+c)^2*b^2+2*a*b*sec(d*x+c)+a^2*ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx = -\frac{2\,a^2\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-4\,ab\cos\left(dx+c\right)-b^2}{2\,d\cos\left(dx+c\right)^2}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
[Out] -1/2*(2*a^2*cos(d*x + c)^2*log(-cos(d*x + c)) - 4*a*b*cos(d*x + c) - b^2)/(
d*cos(d*x + c)^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx = \begin{cases} \frac{a^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{2ab\sec\left(c+dx\right)}{d}+\frac{b^2\sec^2\left(c+dx\right)}{2d} & \text{for}\ d\neq 0 \\ x(a+b\sec\left(c\right))^2\tan\left(c\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c),x)
[Out] Piecewise((a**2*log(tan(c + d*x)**2 + 1)/(2*d) + 2*a*b*sec(c + d*x)/d + b**
2*sec(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*sec(c))**2*tan(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx = -\frac{2\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{4\,ab\cos(dx+c)+b^2}{\cos(dx+c)^2}}{2\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^2*log(cos(d*x + c)) - (4*a*b*cos(d*x + c) + b^2)/cos(d*x + c)^2)/
d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(45) = 90$.

Time = 0.36 (sec) , antiderivative size = 191, normalized size of antiderivative = 4.06

$$\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx$$

$$= \frac{2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{3\,a^2+8\,ab+\frac{6\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b^2(\cos(dx}{\cos(dx+}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (3*a^2 + 8*a*b + 6*a^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^
2)/d
```

Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.72

$$\int (a+b\sec(c+dx))^2\tan(c+dx)\,dx = \frac{4\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(4\,a\,b-2\,b^2\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)} + \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$

```
[In] int(tan(c + d*x)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (4*a*b - tan(c/2 + (d*x)/2)^2*(4*a*b - 2*b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2
*tan(c/2 + (d*x)/2)^2 + 1)) + (2*a^2*atanh(tan(c/2 + (d*x)/2)^2))/d
```

3.276 $\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{a^2\log(\cos(c+dx))}{d} + \frac{(a+b)^2\log(1-\sec(c+dx))}{2d} + \frac{(a-b)^2\log(1+\sec(c+dx))}{2d}$$

```
[Out] a^2*ln(cos(d*x+c))/d+1/2*(a+b)^2*ln(1-sec(d*x+c))/d+1/2*(a-b)^2*ln(1+sec(d*
x+c))/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3970, 1816}

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{a^2\log(\cos(c+dx))}{d} + \frac{(a+b)^2\log(1-\sec(c+dx))}{2d} + \frac{(a-b)^2\log(\sec(c+dx)+1)}{2d}$$

```
[In] Int[Cot[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d + ((a + b)^2*Log[1 - Sec[c + d*x]])/(2*d) + ((a -
 b)^2*Log[1 + Sec[c + d*x]])/(2*d)
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{b^2\text{Subst}\left(\int \frac{(a+x)^2}{x(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d}\\ &= -\frac{b^2\text{Subst}\left(\int \left(\frac{(a+b)^2}{2b^2(b-x)}+\frac{a^2}{b^2x}-\frac{(a-b)^2}{2b^2(b+x)}\right)\,dx, x, b\sec(c+dx)\right)}{d}\\ &= \frac{a^2\log(\cos(c+dx))}{d}+\frac{(a+b)^2\log(1-\sec(c+dx))}{2d}+\frac{(a-b)^2\log(1+\sec(c+dx))}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2a^2\log(\cos(c+dx))+(a+b)^2\log(1-\sec(c+dx))+(a-b)^2\log(1+\sec(c+dx))}{2d}$$

```
[In] Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x])^2,x]
[Out] (2*a^2*Log[Cos[c + d*x]] + (a + b)^2*Log[1 - Sec[c + d*x]] + (a - b)^2*Log[
1 + Sec[c + d*x]])/(2*d)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{a^2\ln(\sin(dx+c))+2ab\ln(-\cot(dx+c)+\csc(dx+c))+b^2\ln(\tan(dx+c))}{d}$
default	$\frac{a^2\ln(\sin(dx+c))+2ab\ln(-\cot(dx+c)+\csc(dx+c))+b^2\ln(\tan(dx+c))}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d}+\frac{2\ln\left(\mathrm{e}^{i(dx+c)}-1\right)ab}{d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)b^2}{d}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d}-$ 2 ln(e

[In] int(cot(d*x+c)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*ln(sin(d*x+c))+2*a*b*ln(-cot(d*x+c)+csc(d*x+c))+b^2*ln(tan(d*x+c))
)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{2\,b^2\log\left(-\cos\left(dx+c\right)\right)-\left(a^2-2\,ab+b^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2+2\,ab+b^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+\right.\right.}{2\,d}$$

[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] -1/2*(2*b^2*log(-cos(d*x + c)) - (a^2 - 2*a*b + b^2)*log(1/2*cos(d*x + c) +
 1/2) - (a^2 + 2*a*b + b^2)*log(-1/2*cos(d*x + c) + 1/2))/d

Sympy [F]

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^2\cot\left(c+dx\right)\,dx$$

[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))**2,x)

[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{2\,b^2\log\left(\cos\left(dx+c\right)\right)-\left(a^2-2\,ab+b^2\right)\log\left(\cos\left(dx+c\right)+1\right)-\left(a^2+2\,ab+b^2\right)\log\left(\cos\left(dx+c\right)-1\right)}{2\,d}$$

[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] -1/2*(2*b^2*log(cos(d*x + c)) - (a^2 - 2*a*b + b^2)*log(cos(d*x + c) + 1) -
 (a^2 + 2*a*b + b^2)*log(cos(d*x + c) - 1))/d

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.66

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{2\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)+2\,b^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-(a^2+2\,ab+b^2)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{2\,d}$$

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + 2*b^2*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 + 2*a*b + b^2)*lo
g(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int \cot(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{b^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{b^2\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}{d}+\frac{2\,a\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

```
[In] int(cot(c + d*x)*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (b^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*lo
g(tan(c/2 + (d*x)/2)^2 + 1))/d - (b^2*log(tan(c/2 + (d*x)/2)^2 - 1))/d + (2
*a*b*log(tan(c/2 + (d*x)/2)))/d
```

3.277 $\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{a^2\log(\cos(c+dx))}{d} - \frac{a(a+b)\log(1-\sec(c+dx))}{2d} - \frac{a(a-b)\log(1+\sec(c+dx))}{2d} - \frac{\cot^2(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{2d}$$

```
[Out] -a^2*ln(cos(d*x+c))/d-1/2*a*(a+b)*ln(1-sec(d*x+c))/d-1/2*a*(a-b)*ln(1+sec(d
*x+c))/d-1/2*cot(d*x+c)^2*(a^2+b^2+2*a*b*sec(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3970, 1819, 815}

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{\cot^2(c+dx)\left(a^2+2ab\sec(c+dx)+b^2\right)}{2d} - \frac{a^2\log(\cos(c+dx))}{d} - \frac{a(a+b)\log(1-\sec(c+dx))}{2d} - \frac{a(a-b)\log(\sec(c+dx)+1)}{2d}$$

```
[In] Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/d) - (a*(a + b)*Log[1 - Sec[c + d*x]])/(2*d) - (a
*(a - b)*Log[1 + Sec[c + d*x]])/(2*d) - (Cot[c + d*x]^2*(a^2 + b^2 + 2*a*b*
Sec[c + d*x]))/(2*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b^4\text{Subst}\left(\int \frac{(a+x)^2}{x(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= -\frac{\cot^2(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{2d} - \frac{b^2\text{Subst}\left(\int \frac{-2a^2-2ax}{x(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{2d}\\
&= -\frac{\cot^2(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{2d}\\
&\quad - \frac{b^2\text{Subst}\left(\int \left(-\frac{a(a+b)}{b^2(b-x)} - \frac{2a^2}{b^2x} + \frac{a(a-b)}{b^2(b+x)}\right)\,dx, x, b\sec(c+dx)\right)}{2d}\\
&= -\frac{a^2\log(\cos(c+dx))}{d} - \frac{a(a+b)\log(1-\sec(c+dx))}{2d}\\
&\quad - \frac{a(a-b)\log(1+\sec(c+dx))}{2d} - \frac{\cot^2(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{4a^2\log(\cos(c+dx))+2a(a+b)\log(1-\sec(c+dx))+2a(a-b)\log(1+\sec(c+dx))+\frac{(a+b)^2}{-1+\sec(c+dx)}-}{4d}$$

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*(4*a^2*Log[Cos[c + d*x]] + 2*a*(a + b)*Log[1 - Sec[c + d*x]] + 2*a*(a
- b)*Log[1 + Sec[c + d*x]] + (a + b)^2/(-1 + Sec[c + d*x]) - (a - b)^2/(1 +
 Sec[c + d*x]))/d
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)-\frac{b^2}{2\sin(dx+c)^2}}{d}$
default	$\frac{a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos(dx+c)^3}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(-\cot(dx+c)+\csc(dx+c))}{2}\right)-\frac{b^2}{2\sin(dx+c)^2}}{d}$
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{2ab\,e^{3i(dx+c)}+2a^2e^{2i(dx+c)}+2b^2e^{2i(dx+c)}+2ab\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a^2\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{\ln\left(e^{i(dx+c)}-1\right)}{d}$

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+2*a*b*(-1/2/sin(d*x+c)^2*cos(d*
x+c)^3-1/2*cos(d*x+c)-1/2*ln(-cot(d*x+c)+csc(d*x+c)))-1/2*b^2/sin(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.23

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\,ab\cos(dx+c)+a^2+b^2-\left((a^2-ab)\cos(dx+c)^2-a^2+ab\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left((a^2+ab)\cos\right.}{2\left(d\cos(dx+c)^2-d\right)}$$

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*cos(d*x + c) + a^2 + b^2 - ((a^2 - a*b)*cos(d*x + c)^2 - a^2 + a
*b)*log(1/2*cos(d*x + c) + 1/2) - ((a^2 + a*b)*cos(d*x + c)^2 - a^2 - a*b)*
log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cot^3(c+dx)\,dx$$

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.78

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= -\frac{(a^2-ab)\log(\cos(dx+c)+1)+(a^2+ab)\log(\cos(dx+c)-1)-\frac{2\,ab\cos(dx+c)+a^2+b^2}{\cos(dx+c)^2-1}}{2\,d}$$

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -1/2*((a^2 - a*b)*log(cos(d*x + c) + 1) + (a^2 + a*b)*log(cos(d*x + c) - 1)
 - (2*a*b*cos(d*x + c) + a^2 + b^2)/(cos(d*x + c)^2 - 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(86) = 172.

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.27

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{8\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-4\,(a^2+ab)\log\left(\frac{|-\cos(dx+c)}{|\cos(dx+c)+}\right.}{8\,d}$$

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] 1/8*(8*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + a^2*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*(a^2 + a*b)*log(abs(-co
s(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a^2 + 2*a*b + b^2 + 4*a^2*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 4*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 14.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2(a-b)^2}{8\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2+b\,a\right)}{d} - \frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{a^2}{8}+\frac{a\,b}{4}+\frac{b^2}{8}\right)}{d}$$

```
[In] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (tan(c/2 + (d*x)/2)^2*(a - b)^2)/(8
*d) - (log(tan(c/2 + (d*x)/2))*(a*b + a^2))/d - (cot(c/2 + (d*x)/2)^2*((a*b
)/4 + a^2/8 + b^2/8))/d
```

3.278 $\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{a^2\log(\cos(c+dx))}{d} + \frac{a(4a+3b)\log(1-\sec(c+dx))}{8d} + \frac{a(4a-3b)\log(1+\sec(c+dx))}{8d} + \frac{a\cot^2(c+dx)(2a+3b\sec(c+dx))}{4d} - \frac{\cot^4(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{4d}$$

```
[Out] a^2*ln(cos(d*x+c))/d+1/8*a*(4*a+3*b)*ln(1-sec(d*x+c))/d+1/8*a*(4*a-3*b)*ln(
1+sec(d*x+c))/d+1/4*a*cot(d*x+c)^2*(2*a+3*b*sec(d*x+c))/d-1/4*cot(d*x+c)^4*
(a^2+b^2+2*a*b*sec(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used

$= \{3970, 1819, 837, 815\}$

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{\cot^4(c+dx)\left(a^2+2ab\sec(c+dx)+b^2\right)}{4d} + \frac{a^2\log(\cos(c+dx))}{d} + \frac{a(4a+3b)\log(1-\sec(c+dx))}{8d} + \frac{a(4a-3b)\log(\sec(c+dx)+1)}{8d} + \frac{a\cot^2(c+dx)(2a+3b\sec(c+dx))}{4d}$$

```
[In] Int[Cot[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Cos[c + d*x]])/d + (a*(4*a + 3*b)*Log[1 - Sec[c + d*x]])/(8*d) + (
a*(4*a - 3*b)*Log[1 + Sec[c + d*x]])/(8*d) + (a*Cot[c + d*x]^2*(2*a + 3*b*S
ec[c + d*x]))/(4*d) - (Cot[c + d*x]^4*(a^2 + b^2 + 2*a*b*Sec[c + d*x]))/(4*
d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b^6\text{Subst}\left(\int \frac{(a+x)^2}{x(b^2-x^2)^3}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= -\frac{\cot^4(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{4d} + \frac{b^4\text{Subst}\left(\int \frac{-4a^2-6ax}{x(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{4d}\\
&= \frac{a\cot^2(c+dx)(2a+3b\sec(c+dx))}{4d} - \frac{\cot^4(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{4d}\\
&\quad + \frac{\text{Subst}\left(\int \frac{-8a^2b^2-6ab^2x}{x(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{8d}\\
&= \frac{a\cot^2(c+dx)(2a+3b\sec(c+dx))}{4d} - \frac{\cot^4(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{4d}\\
&\quad + \frac{\text{Subst}\left(\int \left(-\frac{a(4a+3b)}{b-x} - \frac{8a^2}{x} + \frac{a(4a-3b)}{b+x}\right)dx, x, b\sec(c+dx)\right)}{8d}\\
&= \frac{a^2\log(\cos(c+dx))}{d} + \frac{a(4a+3b)\log(1-\sec(c+dx))}{8d} + \frac{a(4a-3b)\log(1+\sec(c+dx))}{8d}\\
&\quad + \frac{a\cot^2(c+dx)(2a+3b\sec(c+dx))}{4d} - \frac{\cot^4(c+dx)\left(a^2+b^2+2ab\sec(c+dx)\right)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.10

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{16a^2\log(\cos(c+dx)) + 2a(4a+3b)\log(1-\sec(c+dx)) + 2a(4a-3b)\log(1+\sec(c+dx)) - \frac{(a+b)^2}{(-1+\sec(c+dx}}{16d}$$

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (16*a^2*Log[Cos[c + d*x]] + 2*a*(4*a + 3*b)*Log[1 - Sec[c + d*x]] + 2*a*(4*
a - 3*b)*Log[1 + Sec[c + d*x]] - (a + b)^2/(-1 + Sec[c + d*x])^2 + ((a + b)
*(5*a + b))/(-1 + Sec[c + d*x]) - (a - b)^2/(1 + Sec[c + d*x])^2 - ((a - b)
*(5*a - b))/(1 + Sec[c + d*x]))/(16*d)
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos(dx+c)^5}{4\sin(dx+c)^4}+\frac{\cos(dx+c)^5}{8\sin(dx+c)^2}+\frac{\cos(dx+c)^3}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(-\cot(}{}\right.}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{5ab\,e^{7i(dx+c)}+8a^2e^{6i(dx+c)}+4b^2e^{6i(dx+c)}+3ab\,e^{5i(dx+c)}-8a^2e^{4i(dx+c)}+3ab\,e^{3i(dx+c)}+8a^2e^{2i(}}{2d\left(e^{2i(dx+c)}-1\right)^4}$

```
[In] int(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+2*a*b*(-1/4/si
n(d*x+c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*
cos(d*x+c)+3/8*ln(-cot(d*x+c)+csc(d*x+c)))-1/4*b^2/sin(d*x+c)^4*cos(d*x+c)^
4)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.61

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx =$$

$$-\frac{10\,ab\cos{(dx+c)}^3-6\,ab\cos{(dx+c)}+4\,(2\,a^2+b^2)\cos{(dx+c)}^2-6\,a^2-2\,b^2-\left((4\,a^2-3\,ab)\cos{(dx\right.}{}$$

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(10*a*b*cos(d*x + c)^3 - 6*a*b*cos(d*x + c) + 4*(2*a^2 + b^2)*cos(d*x
+ c)^2 - 6*a^2 - 2*b^2 - ((4*a^2 - 3*a*b)*cos(d*x + c)^4 - 2*(4*a^2 - 3*a*b
)*cos(d*x + c)^2 + 4*a^2 - 3*a*b)*log(1/2*cos(d*x + c) + 1/2) - ((4*a^2 + 3
*a*b)*cos(d*x + c)^4 - 2*(4*a^2 + 3*a*b)*cos(d*x + c)^2 + 4*a^2 + 3*a*b)*lo
g(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F]

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx=\int (a+b\sec{(c+dx)})^2\cot^5{(c+dx)}\,dx$$

```
[In] integrate(cot(d*x+c)**5*(a+b*sec(d*x+c))**2,x)
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{(4a^2-3ab)\log{(\cos{(dx+c)}+1)}+(4a^2+3ab)\log{(\cos{(dx+c)}-1)}-\frac{2\left(5ab\cos(dx+c)^3-3ab\cos(dx+c)+2(2a^2+\right.}{\cos(dx+c)^4-2\cos(dx+}}{8d}$$

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] 1/8*((4*a^2 - 3*a*b)*log(cos(d*x + c) + 1) + (4*a^2 + 3*a*b)*log(cos(d*x +
c) - 1) - 2*(5*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c) + 2*(2*a^2 + b^2)*co
s(d*x + c)^2 - 3*a^2 - b^2)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(118) = 236$.

Time = 0.37 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.86

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx=$$
$$-\frac{64a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)+\frac{12a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{16ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-2}{}$$

```
[In] integrate(cot(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] -1/64*(64*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + 12*a^2
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*a*b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 8*(4*a^2 + 3*a*b)*lo
```

```
g(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a^2 + 2*a*b + b^2 + 12*a
^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16*a*b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(d*
x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 36*a*b*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.30

$$\int \cot^5(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{5a^2}{32}-\frac{3ab}{16}+\frac{b^2}{32}+\frac{(a-b)^2}{32}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(a-b)^2}{64\,d}$$
$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2+\frac{3ba}{4}\right)}{d}-\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$
$$-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{ab}{2}+\frac{a^2}{4}+\frac{b^2}{4}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(3a^2+4ab+b^2)\right)}{16\,d}$$

```
[In] int(cot(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*((5*a^2)/32 - (3*a*b)/16 + b^2/32 + (a - b)^2/32))/d
- (tan(c/2 + (d*x)/2)^4*(a - b)^2)/(64*d) + (log(tan(c/2 + (d*x)/2))*((3*a*
b)/4 + a^2))/d - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (cot(c/2 + (d*x)/2
)^4*((a*b)/2 + a^2/4 + b^2/4 - tan(c/2 + (d*x)/2)^2*(4*a*b + 3*a^2 + b^2)))
/(16*d)
```

3.279 $\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 157

$$\begin{aligned}\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx = &-a^2x-\frac{5ab\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)}{d}\\&+\frac{5ab\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^2\tan^3(c+dx)}{3d}\\&-\frac{5ab\sec(c+dx)\tan^3(c+dx)}{12d}+\frac{a^2\tan^5(c+dx)}{5d}\\&+\frac{ab\sec(c+dx)\tan^5(c+dx)}{3d}+\frac{b^2\tan^7(c+dx)}{7d}\end{aligned}$$

```
[Out] -a^2*x-5/8*a*b*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+5/8*a*b*sec(d*x+c)*ta
n(d*x+c)/d-1/3*a^2*tan(d*x+c)^3/d-5/12*a*b*sec(d*x+c)*tan(d*x+c)^3/d+1/5*a^
2*tan(d*x+c)^5/d+1/3*a*b*sec(d*x+c)*tan(d*x+c)^5/d+1/7*b^2*tan(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30}

$$\begin{aligned}\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx = &\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\tan^3(c+dx)}{3d}\\&+\frac{a^2\tan(c+dx)}{d}-a^2x-\frac{5ab\operatorname{arctanh}(\sin(c+dx))}{8d}\\&+\frac{ab\tan^5(c+dx)\sec(c+dx)}{3d}\\&-\frac{5ab\tan^3(c+dx)\sec(c+dx)}{12d}\\&+\frac{5ab\tan(c+dx)\sec(c+dx)}{8d}+\frac{b^2\tan^7(c+dx)}{7d}\end{aligned}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] -(a^2*x) - (5*a*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*Tan[c + d*x])/d + (5*
a*b*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Tan[c + d*x]^3)/(3*d) - (5*a*b*
Sec[c + d*x]*Tan[c + d*x]^3)/(12*d) + (a^2*Tan[c + d*x]^5)/(5*d) + (a*b*Sec
[c + d*x]*Tan[c + d*x]^5)/(3*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2 \tan^6(c+dx) + 2ab\sec(c+dx)\tan^6(c+dx) + b^2\sec^2(c+dx)\tan^6(c+dx)\right) dx$$

$$= a^2 \int \tan^6(c+dx)\,dx + (2ab)\int \sec(c+dx)\tan^6(c+dx)\,dx + b^2 \int \sec^2(c+dx)\tan^6(c+dx)\,dx$$

$$= \frac{a^2\tan^5(c+dx)}{5d} + \frac{ab\sec(c+dx)\tan^5(c+dx)}{3d} - a^2\int \tan^4(c+dx)\,dx - \frac{1}{3}(5ab)\int \sec(c+dx)\tan^4(c+dx)\,dx + \frac{b^2\text{Subst}\left(\int x^6\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{a^2\tan^3(c+dx)}{3d} - \frac{5ab\sec(c+dx)\tan^3(c+dx)}{12d} + \frac{a^2\tan^5(c+dx)}{5d} + \frac{ab\sec(c+dx)\tan^5(c+dx)}{3d} + \frac{b^2\tan^7(c+dx)}{7d} + a^2\int \tan^2(c+dx)\,dx + \frac{1}{4}(5ab)\int \sec(c+dx)\tan^2(c+dx)\,dx$$

$$= \frac{a^2\tan(c+dx)}{d} + \frac{5ab\sec(c+dx)\tan(c+dx)}{8d} - \frac{a^2\tan^3(c+dx)}{3d} - \frac{5ab\sec(c+dx)\tan^3(c+dx)}{12d} + \frac{a^2\tan^5(c+dx)}{5d} + \frac{ab\sec(c+dx)\tan^5(c+dx)}{3d} + \frac{b^2\tan^7(c+dx)}{7d} - a^2\int 1\,dx - \frac{1}{8}(5ab)\int \sec(c+dx)\,dx$$

$$= -a^2x - \frac{5ab\text{arctanh}(\sin(c+dx))}{8d} + \frac{a^2\tan(c+dx)}{d} + \frac{5ab\sec(c+dx)\tan(c+dx)}{8d} - \frac{a^2\tan^3(c+dx)}{3d} - \frac{5ab\sec(c+dx)\tan^3(c+dx)}{12d} + \frac{a^2\tan^5(c+dx)}{5d} + \frac{ab\sec(c+dx)\tan^5(c+dx)}{3d} + \frac{b^2\tan^7(c+dx)}{7d}$$

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx = \frac{-840a^2\arctan(\tan(c+dx)) - 525ab\text{arctanh}(\sin(c+dx)) + \tan(c+dx)\left(175ab(-1+7\cos(2(c+dx)))\sec\right.}{}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^6,x]
[Out] (-840*a^2*ArcTan[Tan[c + d*x]] - 525*a*b*ArcTanh[Sin[c + d*x]] + Tan[c + d*
x]*(175*a*b*(-1 + 7*Cos[2*(c + d*x)])*Sec[c + d*x]^5 + 105*a*b*Sec[c + d*x]
*(-5 + 16*Tan[c + d*x]^4) + 8*(105*a^2 - 35*a^2*Tan[c + d*x]^2 + 21*a^2*Tan
[c + d*x]^4 + 15*b^2*Tan[c + d*x]^6)))/(840*d)
```

Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

method	result
parts	$\frac{a^2\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{b^2\tan(dx+c)^7}{7d}+\frac{2ab\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}\right.}{}$
derivativedivides	$\frac{a^2\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-dx-c\right)+2ab\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(}{4}\right.}{d}$
default	$\frac{a^2\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-dx-c\right)+2ab\left(\frac{\sin(dx+c)^7}{6\cos(dx+c)^6}-\frac{\sin(dx+c)^7}{24\cos(dx+c)^4}+\frac{\sin(dx+c)^7}{16\cos(dx+c)^2}+\frac{\sin(dx+c)^5}{16}+\frac{5\sin(}{4}\right.}{d}$
risch	$-a^2x-\frac{i(1155ab\,e^{13i(dx+c)}-2520a^2e^{12i(dx+c)}+840b^2e^{12i(dx+c)}+980ab\,e^{11i(dx+c)}-10080a^2e^{10i(dx+c)}+2975ab\,e^{9i}}{}$

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-arctan(tan(d*x+c)))+1/7
*b^2*tan(d*x+c)^7/d+2*a*b/d*(1/6*sin(d*x+c)^7/cos(d*x+c)^6-1/24*sin(d*x+c)^
7/cos(d*x+c)^4+1/16*sin(d*x+c)^7/cos(d*x+c)^2+1/16*sin(d*x+c)^5+5/48*sin(d*
x+c)^3+5/16*sin(d*x+c)-5/16*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.17

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx =$$
$$-\frac{1680\,a^2dx\cos(dx+c)^7+525\,ab\cos(dx+c)^7\log(\sin(dx+c)+1)-525\,ab\cos(dx+c)^7\log(-\sin(dx}{}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/1680*(1680*a^2*d*x*cos(d*x + c)^7 + 525*a*b*cos(d*x + c)^7*log(sin(d*x +
 c) + 1) - 525*a*b*cos(d*x + c)^7*log(-sin(d*x + c) + 1) - 2*(1155*a*b*cos(
d*x + c)^5 + 8*(161*a^2 - 15*b^2)*cos(d*x + c)^6 - 910*a*b*cos(d*x + c)^3 -
 8*(77*a^2 - 45*b^2)*cos(d*x + c)^4 + 280*a*b*cos(d*x + c) + 24*(7*a^2 - 15
*b^2)*cos(d*x + c)^2 + 120*b^2)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx = \int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**6,x)
[Out] Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$$
$$= \frac{240\,b^2\tan(dx+c)^7 + 112\,(3\,\tan(dx+c)^5 - 5\,\tan(dx+c)^3 - 15\,dx - 15\,c + 15\,\tan(dx+c))a^2 - 35\,ab\Big(}{1680\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
[Out] 1/1680*(240*b^2*tan(d*x + c)^7 + 112*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 -
 15*d*x - 15*c + 15*tan(d*x + c))*a^2 - 35*a*b*(2*(33*sin(d*x + c)^5 - 40*s
in(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin
(d*x + c)^2 - 1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 2.30 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.80

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx =$$
$$-\frac{840\,(dx+c)a^2 + 525\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 525\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(840\,a^2\tan\left(\frac{1}{2}\,d\right.\right.}{}}{}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
[Out] -1/840*(840*(d*x + c)*a^2 + 525*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 52
5*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(840*a^2*tan(1/2*d*x + 1/2*c)^
13 - 525*a*b*tan(1/2*d*x + 1/2*c)^13 - 6160*a^2*tan(1/2*d*x + 1/2*c)^11 + 3
```

```
500*a*b*tan(1/2*d*x + 1/2*c)^11 + 19768*a^2*tan(1/2*d*x + 1/2*c)^9 - 9905*a
*b*tan(1/2*d*x + 1/2*c)^9 - 28896*a^2*tan(1/2*d*x + 1/2*c)^7 + 7680*b^2*tan
(1/2*d*x + 1/2*c)^7 + 19768*a^2*tan(1/2*d*x + 1/2*c)^5 + 9905*a*b*tan(1/2*d
*x + 1/2*c)^5 - 6160*a^2*tan(1/2*d*x + 1/2*c)^3 - 3500*a*b*tan(1/2*d*x + 1/
2*c)^3 + 840*a^2*tan(1/2*d*x + 1/2*c) + 525*a*b*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.57

$$\int (a+b\sec(c+dx))^2\tan^6(c+dx)\,dx$$
$$= \frac{\left(\frac{5\,a\,b}{4}-2\,a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\left(\frac{44\,a^2}{3}-\frac{25\,a\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{283\,a\,b}{12}-\frac{706\,a^2}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(\frac{344\,a^2}{5}-\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\right.}$$
$$-\frac{2\,a^2\,\mathrm{atan}\left(\frac{64\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^6+25\,a^4\,b^2}+\frac{25\,a^4\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^6+25\,a^4\,b^2}\right)}{d}$$
$$-\frac{5\,a\,b\,\mathrm{atanh}\left(\frac{40\,a^5\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{40\,a^5\,b+\frac{125\,a^3\,b^3}{8}}+\frac{125\,a^3\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\left(40\,a^5\,b+\frac{125\,a^3\,b^3}{8}\right)}\right)}{4\,d}$$

```
[In] int(tan(c + d*x)^6*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^7*((344*a^2)/5 - (128*b^2)/7) + tan(c/2 + (d*x)/2)^13*(
(5*a*b)/4 - 2*a^2) + tan(c/2 + (d*x)/2)^3*((25*a*b)/3 + (44*a^2)/3) - tan(c
/2 + (d*x)/2)^11*((25*a*b)/3 - (44*a^2)/3) - tan(c/2 + (d*x)/2)^5*((283*a*b
)/12 + (706*a^2)/15) + tan(c/2 + (d*x)/2)^9*((283*a*b)/12 - (706*a^2)/15) -
 tan(c/2 + (d*x)/2)*((5*a*b)/4 + 2*a^2))/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*ta
n(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21
*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 -
1)) - (2*a^2*atan((64*a^6*tan(c/2 + (d*x)/2))/(64*a^6 + 25*a^4*b^2) + (25*a
^4*b^2*tan(c/2 + (d*x)/2))/(64*a^6 + 25*a^4*b^2)))/d - (5*a*b*atanh((40*a^5
*b*tan(c/2 + (d*x)/2))/(40*a^5*b + (125*a^3*b^3)/8) + (125*a^3*b^3*tan(c/2
+ (d*x)/2))/(8*(40*a^5*b + (125*a^3*b^3)/8))))/(4*d)
```

3.280 $\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx = a^2x+\frac{3ab\operatorname{arctanh}(\sin(c+dx))}{4d}-\frac{a^2\tan(c+dx)}{d}-\frac{3ab\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\tan^3(c+dx)}{3d}+\frac{ab\sec(c+dx)\tan^3(c+dx)}{2d}+\frac{b^2\tan^5(c+dx)}{5d}$$

```
[Out] a^2*x+3/4*a*b*arctanh(sin(d*x+c))/d-a^2*tan(d*x+c)/d-3/4*a*b*sec(d*x+c)*tan
(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d+1/2*a*b*sec(d*x+c)*tan(d*x+c)^3/d+1/5*b^2*
tan(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30}

$$\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx = \frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x+\frac{3ab\operatorname{arctanh}(\sin(c+dx))}{4d}+\frac{ab\tan^3(c+dx)\sec(c+dx)}{2d}-\frac{3ab\tan(c+dx)\sec(c+dx)}{4d}+\frac{b^2\tan^5(c+dx)}{5d}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] a^2*x + (3*a*b*ArcTanh[Sin[c + d*x]])/(4*d) - (a^2*Tan[c + d*x])/d - (3*a*b
*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (a*b*Sec[c
 + d*x]*Tan[c + d*x]^3)/(2*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2\tan^4(c+dx)+2ab\sec(c+dx)\tan^4(c+dx)+b^2\sec^2(c+dx)\tan^4(c+dx)\right)\,dx$$
$$= a^2\int \tan^4(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan^4(c+dx)\,dx+b^2\int \sec^2(c+dx)\tan^4(c+dx)\,dx$$
$$= \frac{a^2\tan^3(c+dx)}{3d}+\frac{ab\sec(c+dx)\tan^3(c+dx)}{2d}-a^2\int \tan^2(c+dx)\,dx - \frac{1}{2}(3ab)\int \sec(c+dx)\tan^2(c+dx)\,dx+\frac{b^2\mathrm{Subst}\left(\int x^4\,dx,x,\tan(c+dx)\right)}{d}$$
$$= -\frac{a^2\tan(c+dx)}{d}-\frac{3ab\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\tan^3(c+dx)}{3d}+\frac{ab\sec(c+dx)\tan^3(c+dx)}{2d}+\frac{b^2\tan^5(c+dx)}{5d}+a^2\int 1\,dx+\frac{1}{4}(3ab)\int \sec(c+dx)\,dx$$
$$= a^2x+\frac{3ab\operatorname{arctanh}(\sin(c+dx))}{4d}-\frac{a^2\tan(c+dx)}{d}-\frac{3ab\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\tan^3(c+dx)}{3d}+\frac{ab\sec(c+dx)\tan^3(c+dx)}{2d}+\frac{b^2\tan^5(c+dx)}{5d}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.92

$$\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx = \frac{60a^2\arctan(\tan(c+dx))+45ab\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(-90ab\sec^3(c+dx)+15ab\sec(c+dx\right.}{60d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (60*a^2*ArcTan[Tan[c + d*x]] + 45*a*b*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*
(-90*a*b*Sec[c + d*x]^3 + 15*a*b*Sec[c + d*x]*(3 + 8*Tan[c + d*x]^2) + 4*(-
15*a^2 + 5*a^2*Tan[c + d*x]^2 + 3*b^2*Tan[c + d*x]^4)))/(60*d)
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{a^{2}\left(\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+dx+c\right)+2ab\left(\frac{\sin(dx+c)^{5}}{4\cos(dx+c)^{4}}-\frac{\sin(dx+c)^{5}}{8\cos(dx+c)^{2}}-\frac{\sin(dx+c)^{3}}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+}{8}\right.}{d}$
default	$\frac{a^{2}\left(\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+dx+c\right)+2ab\left(\frac{\sin(dx+c)^{5}}{4\cos(dx+c)^{4}}-\frac{\sin(dx+c)^{5}}{8\cos(dx+c)^{2}}-\frac{\sin(dx+c)^{3}}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+}{8}\right.}{d}$
parts	$\frac{a^{2}\left(\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{b^{2}\tan(dx+c)^{5}}{5d}+\frac{2ab\left(\frac{\sin(dx+c)^{5}}{4\cos(dx+c)^{4}}-\frac{\sin(dx+c)^{5}}{8\cos(dx+c)^{2}}-\frac{\sin(dx+c)^{3}}{8}-\right.}{d}$
risch	$a^{2}x+\frac{i\left(75ab\,e^{9i(dx+c)}-120a^{2}e^{8i(dx+c)}+60b^{2}e^{8i(dx+c)}+30ab\,e^{7i(dx+c)}-360a^{2}e^{6i(dx+c)}-440a^{2}e^{4i(dx+c)}+120b^{2}e^{4i(\right.}{30d\left(e^{2i(dx+c)}+1\right)^{5}}$

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+2*a*b*(1/4*sin(d*x+c)^5/cos(d*
x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln
(sec(d*x+c)+tan(d*x+c)))+1/5*b^2*sin(d*x+c)^5/cos(d*x+c)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.30

$$\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx$$
$$=\frac{120\,a^2dx\cos{(dx+c)}^5+45\,ab\cos{(dx+c)}^5\log{(\sin{(dx+c)}+1)}-45\,ab\cos{(dx+c)}^5\log{(-\sin{(dx+c)}}}{}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/120*(120*a^2*d*x*cos(d*x + c)^5 + 45*a*b*cos(d*x + c)^5*log(sin(d*x + c)
+ 1) - 45*a*b*cos(d*x + c)^5*log(-sin(d*x + c) + 1) - 2*(75*a*b*cos(d*x + c
)^3 + 4*(20*a^2 - 3*b^2)*cos(d*x + c)^4 - 30*a*b*cos(d*x + c) - 4*(5*a^2 -
6*b^2)*cos(d*x + c)^2 - 12*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx=\int \left(a+b\sec{\left(c+dx\right)}\right)^2\tan^4{\left(c+dx\right)}\,dx$$

[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**4,x)

[Out] Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

$$\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx$$
$$=\frac{24\,b^2\tan\left(dx+c\right)^5+40\,\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^2+15\,ab\left(\frac{2\,\left(5\,\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^4-2\,\sin\left(dx+c\right)^2+1}+\right.}{120\,d}$$

[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")

[Out] 1/120*(24*b^2*tan(d*x + c)^5 + 40*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2 + 15*a*b*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(106) = 212.

Time = 1.04 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.90

$$\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx$$
$$=\frac{60\,(dx+c)a^2+45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\,\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-\right.}{}}{}$$

[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")

[Out] 1/60*(60*(d*x + c)*a^2 + 45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 45*a*b*tan(1/2*d*x + 1/2*c)^9 - 320*a^2*tan(1/2*d*x + 1/2*c)^7 + 210*a*b*tan(1/2*d*x + 1/2*c)^7 + 520*a^2*tan(1/2*d*x + 1/2*c)^5 - 192*b^2*tan(1/2*d*x + 1/2*c)^5 - 320*a^2*tan(1/2*d*x + 1/2*c)^3 - 210*a*b*tan(1/2*d*x + 1/2*c)^3 + 60*a^2*tan(1/2*d*x + 1/2*c) + 45*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d

Mupad [B] (verification not implemented)

Time = 15.41 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.86

$$\int (a+b\sec(c+dx))^2\tan^4(c+dx)\,dx$$
$$=\frac{\left(2a^2-\frac{3ab}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(7ab-\frac{32a^2}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{52a^2}{3}-\frac{32b^2}{5}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\frac{32a^2}{3}-7b\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\right.}$$
$$+\frac{2a^2\,\mathrm{atan}\left(\frac{64a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64a^6+36a^4b^2}+\frac{36a^4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64a^6+36a^4b^2}\right)}{d}$$
$$+\frac{3ab\,\mathrm{atanh}\left(\frac{48a^5b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{48a^5b+27a^3b^3}+\frac{27a^3b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{48a^5b+27a^3b^3}\right)}{2d}$$

```
[In] int(tan(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*((52*a^2)/3 - (32*b^2)/5) - tan(c/2 + (d*x)/2)^9*((3*
a*b)/2 - 2*a^2) - tan(c/2 + (d*x)/2)^3*(7*a*b + (32*a^2)/3) + tan(c/2 + (d*
x)/2)^7*(7*a*b - (32*a^2)/3) + tan(c/2 + (d*x)/2)*((3*a*b)/2 + 2*a^2))/(d*(
5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6
- 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) + (2*a^2*atan((64*a^
6*tan(c/2 + (d*x)/2))/(64*a^6 + 36*a^4*b^2) + (36*a^4*b^2*tan(c/2 + (d*x)/2
))/(64*a^6 + 36*a^4*b^2)))/d + (3*a*b*atanh((48*a^5*b*tan(c/2 + (d*x)/2))/(
48*a^5*b + 27*a^3*b^3) + (27*a^3*b^3*tan(c/2 + (d*x)/2))/(48*a^5*b + 27*a^3
*b^3)))/(2*d)
```

3.281 $\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx = -a^2x - \frac{ab\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{ab\sec(c+dx)\tan(c+dx)}{d} + \frac{b^2\tan^3(c+dx)}{3d}$$

```
[Out] -a^2*x-a*b*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d+a*b*sec(d*x+c)*tan(d*x+c)
/d+1/3*b^2*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.13 (sec), antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3971, 3554, 8, 2691, 3855, 2687, 30}

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx = \frac{a^2\tan(c+dx)}{d} - a^2x - \frac{ab\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{ab\tan(c+dx)\sec(c+dx)}{d} + \frac{b^2\tan^3(c+dx)}{3d}$$

```
[In] Int[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*x) - (a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a*b*Sec[
c + d*x]*Tan[c + d*x])/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2\tan^2(c+dx)+2ab\sec(c+dx)\tan^2(c+dx)+b^2\sec^2(c+dx)\tan^2(c+dx)\right)\,dx \\
&= a^2\int \tan^2(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan^2(c+dx)\,dx+b^2\int \sec^2(c+dx)\tan^2(c+dx)\,dx \\
&= \frac{a^2\tan(c+dx)}{d}+\frac{ab\sec(c+dx)\tan(c+dx)}{d}-a^2\int 1\,dx \\
&\quad -(ab)\int \sec(c+dx)\,dx+\frac{b^2\text{Subst}\left(\int x^2\,dx,x,\tan(c+dx)\right)}{d}
\end{aligned}
$$

$$= -a^2x - \frac{ab\text{arctanh}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{ab\sec(c+dx)\tan(c+dx)}{d} + \frac{b^2\tan^3(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx = \frac{-3a^2\arctan(\tan(c+dx)) - 3ab\text{arctanh}(\sin(c+dx)) + \tan(c+dx)\left(3a^2+3ab\sec(c+dx)+b^2\tan^2(c+dx)\right)}{3d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Tan[c + d*x]^2,x]
[Out] (-3*a^2*ArcTan[Tan[c + d*x]] - 3*a*b*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(
3*a^2 + 3*a*b*Sec[c + d*x] + b^2*Tan[c + d*x]^2))/(3*d)
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

method	result
parts	$\frac{a^2(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{b^2\tan(dx+c)^3}{3d}+\frac{2ab\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
derivativedivides	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b^2\sin(dx+c)^3}{3\cos(dx+c)^3}}{d}$
default	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b^2\sin(dx+c)^3}{3\cos(dx+c)^3}}{d}$
risch	$-a^2x-\frac{2i\left(3ab\,e^{5i(dx+c)}-3a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}-6a^2e^{2i(dx+c)}-3ab\,e^{i(dx+c)}-3a^2+b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{ab\ln\left(e^{i(dx+c)}+i\right)}{d}+$

```
[In] int((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] a^2/d*(tan(d*x+c)-arctan(tan(d*x+c)))+1/3*b^2*tan(d*x+c)^3/d+2*a*b/d*(1/2*s
in(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.64

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx = -\frac{6\,a^2dx\cos\left(dx+c\right)^3+3\,ab\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,ab\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+\right.}{6\,d\cos\left(dx+c\right)^3}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*a*b*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(3*a*b*cos(d*x + c) + (3
*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^2\tan^2\left(c+dx\right)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**2*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*tan(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx = \frac{2\,b^2\tan\left(dx+c\right)^3-6\left(dx+c-\tan\left(dx+c\right)\right)a^2-3\,ab\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx\right.\right.\right.}{6\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a*b*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(68) = 136$.

Time = 0.57 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.26

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx =$$

$$-\frac{3\,(dx+c)a^2 + 3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3}{}}{3\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)*a^2 + 3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*
tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x +
 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.24

$$\int (a+b\sec(c+dx))^2\tan^2(c+dx)\,dx =$$

$$-\frac{\frac{b^2\,\sin(3\,c+3\,d\,x)}{12}-\frac{b^2\,\sin(c+d\,x)}{4}-\frac{a^2\,\sin(3\,c+3\,d\,x)}{4}-\frac{a^2\,\sin(c+d\,x)}{4}+\frac{3\,a^2\,\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}}{d\,\left(\frac{3\,\cos(c+d\,x)}{4}+\frac{\cos(3\,c}{}\right)}$$

```
[In] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

```
[Out] -((b^2*sin(3*c + 3*d*x))/12 - (b^2*sin(c + d*x))/4 - (a^2*sin(3*c + 3*d*x))
/4 - (a^2*sin(c + d*x))/4 + (3*a^2*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/2 + (a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(
3*c + 3*d*x))/2 - (a*b*sin(2*c + 2*d*x))/2 + (3*a*b*cos(c + d*x)*atanh(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (a*b*atanh(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2))*cos(3*c + 3*d*x))/2)/(d*((3*cos(c + d*x))/4 + cos(3*c + 3*d*
x)/4))
```

3.282 $\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx = -a^2x-\frac{a^2\cot(c+dx)}{d}-\frac{b^2\cot(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}$$

```
[Out] -a^2*x-a^2*cot(d*x+c)/d-b^2*cot(d*x+c)/d-2*a*b*csc(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3971, 3554, 8, 2686, 3852}

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{a^2\cot(c+dx)}{d}+a^2(-x)-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot(c+dx)}{d}$$

```
[In] Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d - (b^2*Cot[c + d*x])/d - (2*a*b*Csc[c + d*x
])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2\cot^2(c+dx)+2ab\cot(c+dx)\csc(c+dx)+b^2\csc^2(c+dx)\right)\,dx \\
&= a^2\int \cot^2(c+dx)\,dx+(2ab)\int \cot(c+dx)\csc(c+dx)\,dx+b^2\int \csc^2(c+dx)\,dx \\
&= -\frac{a^2\cot(c+dx)}{d}-a^2\int 1\,dx-\frac{(2ab)\text{Subst}(\int 1\,dx,x,\csc(c+dx))}{d} \\
&\quad -\frac{b^2\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{d} \\
&= -a^2x-\frac{a^2\cot(c+dx)}{d}-\frac{b^2\cot(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx=-\frac{(a^2+b^2)\cot(c+dx)+a(a(c+dx)+2b\csc(c+dx))}{d}$$

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
[Out] -(((a^2 + b^2)*Cot[c + d*x] + a*(a*(c + d*x) + 2*b*Csc[c + d*x]))/d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

method	result	size
risch	$-a^2x-\frac{2i\left(2ab\,\mathrm{e}^{i(dx+c)}+a^2+b^2\right)}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)}$	47
derivativedivides	$\frac{a^2(-\cot(dx+c)-dx-c)-\frac{2ab}{\sin(dx+c)}-b^2\cot(dx+c)}{d}$	49
default	$\frac{a^2(-\cot(dx+c)-dx-c)-\frac{2ab}{\sin(dx+c)}-b^2\cot(dx+c)}{d}$	49

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -a^2*x-2*I*(2*a*b*exp(I*(d*x+c))+a^2+b^2)/d/(exp(2*I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx=-\frac{a^2dx\sin(dx+c)+2\,ab+(a^2+b^2)\cos(dx+c)}{d\sin(dx+c)}$$

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] -(a^2*d*x*sin(d*x + c) + 2*a*b + (a^2 + b^2)*cos(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cot^2(c+dx)\,dx$$

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2+\frac{2\,ab}{\sin(dx+c)}+\frac{b^2}{\tan(dx+c)}}{d}$$

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -((d*x + c + 1/tan(d*x + c))*a^2 + 2*a*b/sin(d*x + c) + b^2/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{2\,(dx+c)a^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{a^2+2\,ab+b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] -1/2*(2*(d*x + c)*a^2 - a^2*tan(1/2*d*x + 1/2*c) + 2*a*b*tan(1/2*d*x + 1/2*
c) - b^2*tan(1/2*d*x + 1/2*c) + (a^2 + 2*a*b + b^2)/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(a-b)^2}{2\,d} - \frac{\frac{a^2}{2}+a\,b+\frac{b^2}{2}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} - a^2\,x$$

```
[In] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(a - b)^2)/(2*d) - (a*b + a^2/2 + b^2/2)/(d*tan(c/2 + (
d*x)/2)) - a^2*x
```

3.283 $\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx = a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{b^2\cot^3(c+dx)}{3d}+\frac{2ab\csc(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}$$

```
[Out] a^2*x+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d-1/3*b^2*cot(d*x+c)^3/d+2*a*b*
csc(d*x+c)/d-2/3*a*b*csc(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3971, 3554, 8, 2686, 2687, 30}

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^3(c+dx)}{3d}+\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot^3(c+dx)}{3d}$$

```
[In] Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) - (b^2*Cot[c + d*
x]^3)/(3*d) + (2*a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2\cot^4(c+dx)+2ab\cot^3(c+dx)\csc(c+dx)+b^2\cot^2(c+dx)\csc^2(c+dx)\right)\,dx\\
&= a^2\int \cot^4(c+dx)\,dx+(2ab)\int \cot^3(c+dx)\csc(c+dx)\,dx+b^2\int \cot^2(c+dx)\csc^2(c\\
&\quad +dx)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a^2\cot^3(c+dx)}{3d} - a^2\int \cot^2(c+dx)\,dx \\
&\quad - \frac{(2ab)\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \csc(c+dx)\right)}{d} \\
&\quad + \frac{b^2\text{Subst}\left(\int x^2\,dx, x, -\cot(c+dx)\right)}{d} \\
&= \frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} - \frac{b^2\cot^3(c+dx)}{3d} \\
&\quad + \frac{2ab\csc(c+dx)}{d} - \frac{2ab\csc^3(c+dx)}{3d} + a^2\int 1\,dx \\
&= a^2x + \frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} \\
&\quad - \frac{b^2\cot^3(c+dx)}{3d} + \frac{2ab\csc(c+dx)}{d} - \frac{2ab\csc^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$
\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx =
$$
$$
-\frac{b\left(b\cot^3(c+dx)+2a\csc(c+dx)\left(-3+\csc^2(c+dx)\right)\right)+a^2\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2}\right.}{3d}
$$

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*(b*(b*Cot[c + d*x]^3 + 2*a*Csc[c + d*x]*(-3 + Csc[c + d*x]^2)) + a^2*C
ot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/d
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.31

method	result	si
derivativedivides	$\frac{a^2\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)-\frac{b^2\cos(dx+c)^3}{3\sin(dx+c)^3}}{d}$	1
default	$\frac{a^2\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos(dx+c)^4}{3\sin(dx+c)^3}+\frac{\cos(dx+c)^4}{3\sin(dx+c)}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}\right)-\frac{b^2\cos(dx+c)^3}{3\sin(dx+c)^3}}{d}$	1
risch	$a^2x+\frac{2i\left(6ab\,e^{5i(dx+c)}+6a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}-4ab\,e^{3i(dx+c)}-6a^2e^{2i(dx+c)}+6ab\,e^{i(dx+c)}+4a^2+b^2\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}$	1

```
[In] int(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*a*b*(-1/3/sin(d*x+c)^3*cos(
d*x+c)^4+1/3/sin(d*x+c)*cos(d*x+c)^4+1/3*(2+cos(d*x+c)^2)*sin(d*x+c))-1/3*b
^2/sin(d*x+c)^3*cos(d*x+c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.20

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{6\,ab\cos\left(dx+c\right)^2+\left(4\,a^2+b^2\right)\cos\left(dx+c\right)^3-3\,a^2\cos\left(dx+c\right)-4\,ab+3\,\left(a^2dx\cos\left(dx+c\right)^2-a^2dx\right)\sin}{3\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*a*b*cos(d*x + c)^2 + (4*a^2 + b^2)*cos(d*x + c)^3 - 3*a^2*cos(d*x +
c) - 4*a*b + 3*(a^2*d*x*cos(d*x + c)^2 - a^2*d*x)*sin(d*x + c))/((d*cos(d*x
 + c)^2 - d)*sin(d*x + c))
```

Sympy [F]

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^2\cot^4\left(c+dx\right)\,dx$$

```
[In] integrate(cot(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^2-1}{\tan\left(dx+c\right)^3}\right)a^2+\frac{2\,\left(3\,\sin\left(dx+c\right)^2-1\right)ab}{\sin\left(dx+c\right)^3}-\frac{b^2}{\tan\left(dx+c\right)^3}}{3\,d}$$

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 + 2*(3*sin(d
*x + c)^2 - 1)*a*b/sin(d*x + c)^3 - b^2/tan(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(79) = 158$.

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.07

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 24\,(dx+c)a^2 - 15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2}$$

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(1
/2*d*x + 1/2*c)^3 + 24*(d*x + c)*a^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + 18*a*b
*tan(1/2*d*x + 1/2*c) - 3*b^2*tan(1/2*d*x + 1/2*c) + (15*a^2*tan(1/2*d*x +
1/2*c)^2 + 18*a*b*tan(1/2*d*x + 1/2*c)^2 + 3*b^2*tan(1/2*d*x + 1/2*c)^2 - a
^2 - 2*a*b - b^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 14.00 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= a^2\,x + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(a-b)^2}{24\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a\,(a-b)}{2}+\frac{(a-b)^2}{8}\right)}{d}$$
$$- \frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{2\,a\,b}{3}+\frac{a^2}{3}+\frac{b^2}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(5\,a^2+6\,a\,b+b^2)\right)}{8\,d}$$

```
[In] int(cot(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (tan(c/2 + (d*x)/2)^3*(a - b)^2)/(24*d) - (tan(c/2 + (d*x)/2)*((a*(
a - b))/2 + (a - b)^2/8))/d - (cot(c/2 + (d*x)/2)^3*((2*a*b)/3 + a^2/3 + b^
2/3 - tan(c/2 + (d*x)/2)^2*(6*a*b + 5*a^2 + b^2)))/(8*d)
```

3.284 $\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx = -a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\cot^5(c+dx)}{5d}-\frac{2ab\csc(c+dx)}{d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc^5(c+dx)}{5d}$$

```
[Out] -a^2*x-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/d-1/5*a^2*cot(d*x+c)^5/d-1/5*b
^2*cot(d*x+c)^5/d-2*a*b*csc(d*x+c)/d+4/3*a*b*csc(d*x+c)^3/d-2/5*a*b*csc(d*x
+c)^5/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3971, 3554, 8, 2686, 200, 2687, 30}

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x-\frac{2ab\csc^5(c+dx)}{5d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot^5(c+dx)}{5d}$$

```
[In] Int[Cot[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[c + d*x])/d + (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c +
 d*x]^5)/(5*d) - (b^2*Cot[c + d*x]^5)/(5*d) - (2*a*b*Csc[c + d*x])/d + (4*a
*b*Csc[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2\cot^6(c+dx)+2ab\cot^5(c+dx)\csc(c+dx)+b^2\cot^4(c+dx)\csc^2(c+dx)\right)\,dx\\
&= a^2\int \cot^6(c+dx)\,dx+(2ab)\int \cot^5(c+dx)\csc(c+dx)\,dx+b^2\int \cot^4(c+dx)\csc^2(c+dx)\,dx\\
&= -\frac{a^2\cot^5(c+dx)}{5d}-a^2\int \cot^4(c+dx)\,dx-\frac{(2ab)\mathrm{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{d}+\frac{b^2\mathrm{Subst}\left(\int x^4\,dx,x,-\cot(c+dx)\right)}{d}\\
&= \frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\cot^5(c+dx)}{5d}+a^2\int \cot^2(c+dx)\,dx-\frac{(2ab)\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&= -\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\cot^5(c+dx)}{5d}-\frac{2ab\csc(c+dx)}{d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc^5(c+dx)}{5d}-a^2\int 1\,dx\\
&= -a^2x-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{b^2\cot^5(c+dx)}{5d}-\frac{2ab\csc(c+dx)}{d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc^5(c+dx)}{5d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.69

$$
\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{b\left(3b\cot^5(c+dx)+2a\csc(c+dx)\left(15-10\csc^2(c+dx)+3\csc^4(c+dx)\right)\right)+3a^2\cot^5(c+dx)\,\mathrm{Hyperge}}{15d}
$$

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/15*(b*(3*b*Cot[c + d*x]^5 + 2*a*Csc[c + d*x]*(15 - 10*Csc[c + d*x]^2 + 3
*Csc[c + d*x]^4)) + 3*a^2*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -
Tan[c + d*x]^2])/d
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(dx+c)^5}{5}+\frac{\cot(dx+c)^3}{3}-\cot(dx+c)-dx-c\right)+2ab\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos\cdots}{}\right.}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cot(dx+c)^5}{5}+\frac{\cot(dx+c)^3}{3}-\cot(dx+c)-dx-c\right)+2ab\left(-\frac{\cos(dx+c)^6}{5\sin(dx+c)^5}+\frac{\cos(dx+c)^6}{15\sin(dx+c)^3}-\frac{\cos(dx+c)^6}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos\cdots}{}\right.}{}\right.}{d}$
risch	$-a^2x-\frac{2i\left(30ab\,e^{9i(dx+c)}+45a^2e^{8i(dx+c)}+15b^2e^{8i(dx+c)}-40ab\,e^{7i(dx+c)}-90a^2e^{6i(dx+c)}+116ab\,e^{5i(dx+c)}+140a^2e^{4i(\cdots}{15d\left(e^{2i(dx+c)}-1\right)^5}$

[In] int(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+2*a*b*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/5*b^2/sin(d*x+c)^5*cos(d*x+c)^5)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.25

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{30\,ab\cos\left(dx+c\right)^4+\left(23\,a^2+3\,b^2\right)\cos\left(dx+c\right)^5-35\,a^2\cos\left(dx+c\right)^3-40\,ab\cos\left(dx+c\right)^2+15\,a^2\cos\left(d\cdots\right.}{15\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2\cdots\right.}$$

[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] -1/15*(30*a*b*cos(d*x + c)^4 + (23*a^2 + 3*b^2)*cos(d*x + c)^5 - 35*a^2*cos(d*x + c)^3 - 40*a*b*cos(d*x + c)^2 + 15*a^2*cos(d*x + c) + 16*a*b + 15*(a^2*d*x*cos(d*x + c)^4 - 2*a^2*d*x*cos(d*x + c)^2 + a^2*d*x)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))

Sympy [F]

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cot^6(c+dx)\,dx$$

```
[In] integrate(cot(d*x+c)**6*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cot(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2+\frac{2\left(15\,\sin(dx+c)^4-10\,\sin(dx+c)^2+3\right)ab}{\sin(dx+c)^5}+\frac{3\,b^2}{\tan(dx+c)^5}}{15\,d}$$

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*((15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a^2 + 2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 + 3)*a*b/sin(d*x + c
)^5 + 3*b^2/tan(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(112) = 224.

Time = 0.36 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.24

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+50\,ab\tan$$

```
[In] integrate(cot(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^2*
tan(1/2*d*x + 1/2*c)^5 - 35*a^2*tan(1/2*d*x + 1/2*c)^3 + 50*a*b*tan(1/2*d*x
 + 1/2*c)^3 - 15*b^2*tan(1/2*d*x + 1/2*c)^3 - 480*(d*x + c)*a^2 + 330*a^2*t
an(1/2*d*x + 1/2*c) - 300*a*b*tan(1/2*d*x + 1/2*c) + 30*b^2*tan(1/2*d*x + 1
/2*c) - (330*a^2*tan(1/2*d*x + 1/2*c)^4 + 300*a*b*tan(1/2*d*x + 1/2*c)^4 +
30*b^2*tan(1/2*d*x + 1/2*c)^4 - 35*a^2*tan(1/2*d*x + 1/2*c)^2 - 50*a*b*tan(
1/2*d*x + 1/2*c)^2 - 15*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b + 3*b^2)
/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.57

$$\int \cot^6(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 (a-b)^2}{160\,d} - a^2\,x - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 \left(\frac{a^2}{16}-\frac{a\,b}{12}+\frac{b^2}{48}+\frac{(a-b)^2}{96}\right)}{d}$$
$$- \frac{\frac{2\,a\,b}{5}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 (22\,a^2+20\,a\,b+2\,b^2)+\frac{a^2}{5}+\frac{b^2}{5}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 \left(\frac{7\,a^2}{3}+\frac{10\,a\,b}{3}+b^2\right)}{32\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$
$$+ \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{21\,a^2}{32}-\frac{9\,a\,b}{16}+\frac{b^2}{32}+\frac{(a-b)^2}{32}\right)}{d}$$

```
[In] int(cot(c + d*x)^6*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(a - b)^2)/(160*d) - a^2*x - (tan(c/2 + (d*x)/2)^3*(a
^2/16 - (a*b)/12 + b^2/48 + (a - b)^2/96))/d - ((2*a*b)/5 + tan(c/2 + (d*x)
/2)^4*(20*a*b + 22*a^2 + 2*b^2) + a^2/5 + b^2/5 - tan(c/2 + (d*x)/2)^2*((10
*a*b)/3 + (7*a^2)/3 + b^2))/(32*d*tan(c/2 + (d*x)/2)^5) + (tan(c/2 + (d*x)/
2)*((21*a^2)/32 - (9*a*b)/16 + b^2/32 + (a - b)^2/32))/d
```

3.285 $\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 153

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx = a^2x+\frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d} - \frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot^7(c+dx)}{7d} + \frac{2ab\csc(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{d} + \frac{6ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^7(c+dx)}{7d}$$

```
[Out] a^2*x+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d+1/5*a^2*cot(d*x+c)^5/d-1/7*a^
2*cot(d*x+c)^7/d-1/7*b^2*cot(d*x+c)^7/d+2*a*b*csc(d*x+c)/d-2*a*b*csc(d*x+c)
^3/d+6/5*a*b*csc(d*x+c)^5/d-2/7*a*b*csc(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{3971, 3554, 8, 2686, 200, 2687, 30\}$

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{a^2\cot^7(c+dx)}{7d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{2ab\csc^7(c+dx)}{7d}+\frac{6ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^3(c+dx)}{d}+\frac{2ab\csc(c+dx)}{d}-\frac{b^2\cot^7(c+dx)}{7d}$$

```
[In] Int[Cot[c + d*x]^8*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*
x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)/(7*d) - (b^2*Cot[c + d*x]^7)/(7*d) + (2*
a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/d + (6*a*b*Csc[c + d*x]^5)/(5*
d) - (2*a*b*Csc[c + d*x]^7)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2\cot^8(c+dx)+2ab\cot^7(c+dx)\csc(c+dx)+b^2\cot^6(c+dx)\csc^2(c+dx)\right)\,dx\\
&= a^2\int \cot^8(c+dx)\,dx+(2ab)\int \cot^7(c+dx)\csc(c+dx)\,dx+b^2\int \cot^6(c+dx)\csc^2(c+dx)\,dx\\
&= -\frac{a^2\cot^7(c+dx)}{7d}-a^2\int \cot^6(c+dx)\,dx-\frac{(2ab)\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\csc(c+dx)\right)}{d}+\frac{b^2\text{Subst}\left(\int x^6\,dx,x,-\cot(c+dx)\right)}{d}\\
&= \frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot^7(c+dx)}{7d}+a^2\int \cot^4(c+dx)\,dx-\frac{(2ab)\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&= -\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot^7(c+dx)}{7d}+\frac{2ab\csc(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{d}+\frac{6ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^7(c+dx)}{7d}-a^2\int \cot^2(c+dx)\,dx\\
&= \frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot^7(c+dx)}{7d}+\frac{2ab\csc(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{d}+\frac{6ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^7(c+dx)}{7d}+a^2\int 1\,dx\\
&= a^2x+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{b^2\cot^7(c+dx)}{7d}+\frac{2ab\csc(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{d}+\frac{6ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.61

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{b(-5b\cot^7(c+dx)+2a\csc(c+dx)\left(35-35\csc^2(c+dx)+21\csc^4(c+dx)-5\csc^6(c+dx)\right))-5a^2\cot^7(c}{35d}$$

[In] Integrate[Cot[c + d*x]^8*(a + b*Sec[c + d*x])^2,x]

[Out] (b*(-5*b*Cot[c + d*x]^7 + 2*a*Csc[c + d*x]*(35 - 35*Csc[c + d*x]^2 + 21*Csc[c + d*x]^4 - 5*Csc[c + d*x]^6)) - 5*a^2*Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[c + d*x]^2])/(35*d)

Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(dx+c)^7}{7}+\frac{\cot(dx+c)^5}{5}-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(d}{7\sin(}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cot(dx+c)^7}{7}+\frac{\cot(dx+c)^5}{5}-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos(dx+c)^8}{7\sin(dx+c)^7}+\frac{\cos(dx+c)^8}{35\sin(dx+c)^5}-\frac{\cos(dx+c)^8}{35\sin(dx+c)^3}+\frac{\cos(d}{7\sin(}\right.}{d}$
risch	$a^2x+\frac{2i\left(210ab\,e^{13i(dx+c)}+420a^2e^{12i(dx+c)}+105b^2e^{12i(dx+c)}-420ab\,e^{11i(dx+c)}-1260a^2e^{10i(dx+c)}+1806ab\,e^{9i(dx+c)}-\right.}{}$

[In] int(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-1/7*cot(d*x+c)^7+1/5*cot(d*x+c)^5-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*a*b*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/7*b^2/sin(d*x+c)^7*cos(d*x+c)^7)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.35

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{210\,ab\cos(dx+c)^6+(176\,a^2+15\,b^2)\cos(dx+c)^7-406\,a^2\cos(dx+c)^5-420\,ab\cos(dx+c)^4+350\,a^2}{105\,(d\cos(}$$

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(210*a*b*cos(d*x + c)^6 + (176*a^2 + 15*b^2)*cos(d*x + c)^7 - 406*a^2
*cos(d*x + c)^5 - 420*a*b*cos(d*x + c)^4 + 350*a^2*cos(d*x + c)^3 + 336*a*b
*cos(d*x + c)^2 - 105*a^2*cos(d*x + c) - 96*a*b + 105*(a^2*d*x*cos(d*x + c)
^6 - 3*a^2*d*x*cos(d*x + c)^4 + 3*a^2*d*x*cos(d*x + c)^2 - a^2*d*x)*sin(d*x
 + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*si
n(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)**8*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.76

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\left(105\,dx+105\,c+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{\tan(dx+c)^7}\right)a^2+\frac{6\left(35\,\sin(dx+c)^6-35\,\sin(dx+c)^4+21\,\sin(dx+c)^2-5\right)a}{\sin(dx+c)^7}}{105\,d}$$

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*((105*d*x + 105*c + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan(
d*x + c)^2 - 15)/tan(d*x + c)^7)*a^2 + 6*(35*sin(d*x + c)^6 - 35*sin(d*x +
c)^4 + 21*sin(d*x + c)^2 - 5)*a*b/sin(d*x + c)^7 - 15*b^2/tan(d*x + c)^7)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(141) = 282$.

Time = 0.40 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.39

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 189\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 294\,a}{}$$

```
[In] integrate(cot(d*x+c)^8*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 - 30*a*b*tan(1/2*d*x + 1/2*c)^7 + 15
*b^2*tan(1/2*d*x + 1/2*c)^7 - 189*a^2*tan(1/2*d*x + 1/2*c)^5 + 294*a*b*tan(
1/2*d*x + 1/2*c)^5 - 105*b^2*tan(1/2*d*x + 1/2*c)^5 + 1295*a^2*tan(1/2*d*x
+ 1/2*c)^3 - 1470*a*b*tan(1/2*d*x + 1/2*c)^3 + 315*b^2*tan(1/2*d*x + 1/2*c)
^3 + 13440*(d*x + c)*a^2 - 9765*a^2*tan(1/2*d*x + 1/2*c) + 7350*a*b*tan(1/2
*d*x + 1/2*c) - 525*b^2*tan(1/2*d*x + 1/2*c) + (9765*a^2*tan(1/2*d*x + 1/2*
c)^6 + 7350*a*b*tan(1/2*d*x + 1/2*c)^6 + 525*b^2*tan(1/2*d*x + 1/2*c)^6 - 1
295*a^2*tan(1/2*d*x + 1/2*c)^4 - 1470*a*b*tan(1/2*d*x + 1/2*c)^4 - 315*b^2*
tan(1/2*d*x + 1/2*c)^4 + 189*a^2*tan(1/2*d*x + 1/2*c)^2 + 294*a*b*tan(1/2*d
*x + 1/2*c)^2 + 105*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2 - 30*a*b - 15*b^2)/
tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.69

$$\int \cot^8(c+dx)(a+b\sec(c+dx))^2\,dx = a^2\,x + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(a-b)^2}{896\,d}$$
$$+ \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{3\,a^2}{32}-\frac{5\,a\,b}{48}+\frac{b^2}{48}+\frac{(a-b)^2}{384}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{a^2}{80}-\frac{3\,a\,b}{160}+\frac{b^2}{160}+\frac{(a-b)^2}{640}\right)}{d}$$
$$- \frac{\frac{2\,a\,b}{7}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{37\,a^2}{3}+14\,a\,b+3\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,(93\,a^2+70\,a\,b+5\,b^2)+\frac{a^2}{7}+\frac{b^2}{7}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{128\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$
$$- \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{23\,a^2}{32}-\frac{17\,a\,b}{32}+\frac{b^2}{32}+\frac{(a-b)^2}{128}\right)}{d}$$

```
[In] int(cot(c + d*x)^8*(a + b/cos(c + d*x))^2,x)
```

```
[Out] a^2*x + (tan(c/2 + (d*x)/2)^7*(a - b)^2)/(896*d) + (tan(c/2 + (d*x)/2)^3*((
3*a^2)/32 - (5*a*b)/48 + b^2/48 + (a - b)^2/384))/d - (tan(c/2 + (d*x)/2)^5
*(a^2/80 - (3*a*b)/160 + b^2/160 + (a - b)^2/640))/d - ((2*a*b)/7 + tan(c/2
```

```
 + (d*x)/2)^4*(14*a*b + (37*a^2)/3 + 3*b^2) - tan(c/2 + (d*x)/2)^6*(70*a*b
+ 93*a^2 + 5*b^2) + a^2/7 + b^2/7 - tan(c/2 + (d*x)/2)^2*((14*a*b)/5 + (9*a
^2)/5 + b^2))/(128*d*tan(c/2 + (d*x)/2)^7) - (tan(c/2 + (d*x)/2)*((23*a^2)/
32 - (17*a*b)/32 + b^2/32 + (a - b)^2/128))/d
```

3.286 $\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 250

$$\begin{aligned}\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx = &-\frac{\log(\cos(c+dx))}{ad}-\frac{(a^2-b^2)^4\log(a+b\sec(c+dx))}{ab^8d}\\&+\frac{(a^6-4a^4b^2+6a^2b^4-4b^6)\sec(c+dx)}{b^7d}\\&-\frac{a(a^4-4a^2b^2+6b^4)\sec^2(c+dx)}{2b^6d}\\&+\frac{(a^4-4a^2b^2+6b^4)\sec^3(c+dx)}{3b^5d}-\frac{a(a^2-4b^2)\sec^4(c+dx)}{4b^4d}\\&+\frac{(a^2-4b^2)\sec^5(c+dx)}{5b^3d}-\frac{a\sec^6(c+dx)}{6b^2d}+\frac{\sec^7(c+dx)}{7bd}\end{aligned}$$

```
[Out] -ln(cos(d*x+c))/a/d-(a^2-b^2)^4*ln(a+b*sec(d*x+c))/a/b^8/d+(a^6-4*a^4*b^2+6
*a^2*b^4-4*b^6)*sec(d*x+c)/b^7/d-1/2*a*(a^4-4*a^2*b^2+6*b^4)*sec(d*x+c)^2/b
^6/d+1/3*(a^4-4*a^2*b^2+6*b^4)*sec(d*x+c)^3/b^5/d-1/4*a*(a^2-4*b^2)*sec(d*x
+c)^4/b^4/d+1/5*(a^2-4*b^2)*sec(d*x+c)^5/b^3/d-1/6*a*sec(d*x+c)^6/b^2/d+1/7
*sec(d*x+c)^7/b/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used

$= \{3970, 908\}$

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{(a^2-b^2)^4\log(a+b\sec(c+dx))}{ab^8d} - \frac{a(a^2-4b^2)\sec^4(c+dx)}{4b^4d} + \frac{(a^2-4b^2)\sec^5(c+dx)}{5b^3d} - \frac{a(a^4-4a^2b^2+6b^4)\sec^2(c+dx)}{2b^6d} + \frac{(a^4-4a^2b^2+6b^4)\sec^3(c+dx)}{3b^5d} + \frac{(a^6-4a^4b^2+6a^2b^4-4b^6)\sec(c+dx)}{b^7d} - \frac{a\sec^6(c+dx)}{6b^2d} - \frac{\log(\cos(c+dx))}{ad} + \frac{\sec^7(c+dx)}{7bd}$$

```
[In] Int[Tan[c + d*x]^9/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((a^2 - b^2)^4*Log[a + b*Sec[c + d*x]])/(a*b^8
*d) + ((a^6 - 4*a^4*b^2 + 6*a^2*b^4 - 4*b^6)*Sec[c + d*x])/(b^7*d) - (a*(a^
4 - 4*a^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/(2*b^6*d) + ((a^4 - 4*a^2*b^2 + 6*b^
4)*Sec[c + d*x]^3)/(3*b^5*d) - (a*(a^2 - 4*b^2)*Sec[c + d*x]^4)/(4*b^4*d) +
 ((a^2 - 4*b^2)*Sec[c + d*x]^5)/(5*b^3*d) - (a*Sec[c + d*x]^6)/(6*b^2*d) +
Sec[c + d*x]^7/(7*b*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^4}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{b^8d}$$

$$= \frac{\text{Subst}\left(\int \left(a^6\left(1+\frac{-4a^4b^2+6a^2b^4-4b^6}{a^6}\right)+\frac{b^8}{ax}-a(a^4-4a^2b^2+6b^4)\,x+(a^4-4a^2b^2+6b^4)\,x^2-a(a^2 -\right.\right.}{b^8d}$$

$$
\begin{aligned}
&= -\frac{\log(\cos(c+dx))}{ad} - \frac{(a^2-b^2)^4 \log(a+b\sec(c+dx))}{ab^8d} \\
&\quad + \frac{(a^6-4a^4b^2+6a^2b^4-4b^6)\sec(c+dx)}{b^7d} - \frac{a(a^4-4a^2b^2+6b^4)\sec^2(c+dx)}{2b^6d} \\
&\quad + \frac{(a^4-4a^2b^2+6b^4)\sec^3(c+dx)}{3b^5d} - \frac{a(a^2-4b^2)\sec^4(c+dx)}{4b^4d} \\
&\quad + \frac{(a^2-4b^2)\sec^5(c+dx)}{5b^3d} - \frac{a\sec^6(c+dx)}{6b^2d} + \frac{\sec^7(c+dx)}{7bd}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.91 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.91

$$
\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx
$$

$$
= \frac{-\frac{b^8\log(\cos(c+dx))}{a} - \frac{(a^2-b^2)^4\log(a+b\sec(c+dx))}{a} + b(a^6-4a^4b^2+6a^2b^4-4b^6)\sec(c+dx) - \frac{1}{2}ab^2(a^4-4a^2b^2+6b}{}
$$

```
[In] Integrate[Tan[c + d*x]^9/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-((b^8*Log[Cos[c + d*x]])/a) - ((a^2 - b^2)^4*Log[a + b*Sec[c + d*x]])/a +
 b*(a^6 - 4*a^4*b^2 + 6*a^2*b^4 - 4*b^6)*Sec[c + d*x] - (a*b^2*(a^4 - 4*a^2
*b^2 + 6*b^4)*Sec[c + d*x]^2)/2 + (b^3*(a^4 - 4*a^2*b^2 + 6*b^4)*Sec[c + d*
x]^3)/3 - (a*b^4*(a^2 - 4*b^2)*Sec[c + d*x]^4)/4 + (b^5*(a^2 - 4*b^2)*Sec[c
 + d*x]^5)/5 - (a*b^6*Sec[c + d*x]^6)/6 + (b^7*Sec[c + d*x]^7)/7)/(b^8*d)
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{\left(-a^8+4a^6b^2-6a^4b^4+4a^2b^6-b^8\right)\ln(b+a\cos(dx+c))}{b^8a}-\frac{a}{6b^2\cos(dx+c)^6}-\frac{-a^2+4b^2}{5b^3\cos(dx+c)^5}-\frac{-a^4+4a^2b^2-6b^4}{3b^5\cos(dx+c)^3}-\frac{-a^6+4a^4b^2-6a^2b^4+4}{b^7\cos(dx+c)}}{d}$
default	$\frac{\frac{\left(-a^8+4a^6b^2-6a^4b^4+4a^2b^6-b^8\right)\ln(b+a\cos(dx+c))}{b^8a}-\frac{a}{6b^2\cos(dx+c)^6}-\frac{-a^2+4b^2}{5b^3\cos(dx+c)^5}-\frac{-a^4+4a^2b^2-6b^4}{3b^5\cos(dx+c)^3}-\frac{-a^6+4a^4b^2-6a^2b^4+4}{b^7\cos(dx+c)}}{d}$
risch	Expression too large to display

```
[In] int(tan(d*x+c)^9/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-a^8+4*a^6*b^2-6*a^4*b^4+4*a^2*b^6-b^8)/b^8/a*ln(b+a*cos(d*x+c))-1/6/
b^2*a/cos(d*x+c)^6-1/5*(-a^2+4*b^2)/b^3/cos(d*x+c)^5-1/3*(-a^4+4*a^2*b^2-6*
b^4)/b^5/cos(d*x+c)^3-(-a^6+4*a^4*b^2-6*a^2*b^4+4*b^6)/b^7/cos(d*x+c)-1/4*(
a^2-4*b^2)/b^4*a/cos(d*x+c)^4-1/2*(a^4-4*a^2*b^2+6*b^4)/b^6*a/cos(d*x+c)^2+
(a^6-4*a^4*b^2+6*a^2*b^4-4*b^6)/b^8*a*ln(cos(d*x+c))+1/7/b/cos(d*x+c)^7)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.17

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{70\,a^2b^6\cos\left(dx+c\right)+420\left(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8\right)\cos\left(dx+c\right)^7\log\left(a\cos\left(dx+c\right)+b\right)-42}{}$$

[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/420*(70*a^2*b^6*cos(d*x + c) + 420*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cos(d*x + c)^7*log(a*cos(d*x + c) + b) - 420*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^7*log(-cos(d*x + c)) - 60*a*b^7 - 420*(a^7*b - 4*a^5*b^3 + 6*a^3*b^5 - 4*a*b^7)*cos(d*x + c)^6 + 210*(a^6*b^2 - 4*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c)^5 - 140*(a^5*b^3 - 4*a^3*b^5 + 6*a*b^7)*cos(d*x + c)^4 + 105*(a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^3 - 84*(a^3*b^5 - 4*a*b^7)*cos(d*x + c)^2)/(a*b^8*d*cos(d*x + c)^7)

Sympy [F]

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\tan^9\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

[In] integrate(tan(d*x+c)**9/(a+b*sec(d*x+c)),x)

[Out] Integral(tan(c + d*x)**9/(a + b*sec(c + d*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{420\,(a^7-4\,a^5b^2+6\,a^3b^4-4\,ab^6)\log(\cos(dx+c))}{b^8}-\frac{420\,(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\log(a\cos(dx+c)+b)}{ab^8}-\frac{70\,ab^5\cos(dx+c)-420\,(a^6-4\,a^4}{}}{}$$

[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/420*(420*(a^7 - 4*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(cos(d*x + c))/b^8 - 420*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*log(a*cos(d*x + c) + b)

```
/(a*b^8) - (70*a*b^5*cos(d*x + c) - 420*(a^6 - 4*a^4*b^2 + 6*a^2*b^4 - 4*b^
6)*cos(d*x + c)^6 - 60*b^6 + 210*(a^5*b - 4*a^3*b^3 + 6*a*b^5)*cos(d*x + c)
^5 - 140*(a^4*b^2 - 4*a^2*b^4 + 6*b^6)*cos(d*x + c)^4 + 105*(a^3*b^3 - 4*a*
b^5)*cos(d*x + c)^3 - 84*(a^2*b^4 - 4*b^6)*cos(d*x + c)^2)/(b^7*cos(d*x + c
)^7))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1768 vs. $2(238) = 476$.

Time = 5.76 (sec) , antiderivative size = 1768, normalized size of antiderivative = 7.07

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(210*(a^7 - 4*a^5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(abs(a + b - 2*b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2))/b^8 - 420*(a^7 - 4*a^
5*b^2 + 6*a^3*b^4 - 4*a*b^6)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 1))/b^8 - 210*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + 2*b^8)*log(abs(
2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/(b^8*abs(a))
 + (1089*a^7 - 840*a^6*b - 4356*a^5*b^2 + 3080*a^4*b^3 + 6534*a^3*b^4 - 408
8*a^2*b^5 - 4356*a*b^6 + 2232*b^7 + 7623*a^7*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - 5040*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 31332*a^5*b^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 19040*a^4*b^3*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) + 48258*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 26
096*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 33012*a*b^6*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 14784*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 22869*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12600*a^6*b*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 95676*a^5*b^2*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 47880*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 151494*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 67368*a^2*b
^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 107436*a*b^6*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 40152*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 38115*a^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 16800*a^6*b*(
cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 160860*a^5*b^2*(cos(d*x + c) - 1
)^3/(cos(d*x + c) + 1)^3 + 62720*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c)
 + 1)^3 + 258930*a^3*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 86240*
a^2*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 192220*a*b^6*(cos(d*x +
 c) - 1)^3/(cos(d*x + c) + 1)^3 + 53760*b^7*(cos(d*x + c) - 1)^3/(cos(d*x +
 c) + 1)^3 + 38115*a^7*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 12600*a^
6*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 160860*a^5*b^2*(cos(d*x + c
```

```
) - 1)^4/(cos(d*x + c) + 1)^4 + 45080*a^4*b^3*(cos(d*x + c) - 1)^4/(cos(d*x
 + c) + 1)^4 + 258930*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 5
6840*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 192220*a*b^6*(cos(
d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 24360*b^7*(cos(d*x + c) - 1)^4/(cos(
d*x + c) + 1)^4 + 22869*a^7*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 504
0*a^6*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 95676*a^5*b^2*(cos(d*x
+ c) - 1)^5/(cos(d*x + c) + 1)^5 + 16800*a^4*b^3*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 + 151494*a^3*b^4*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5
- 18480*a^2*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 107436*a*b^6*(c
os(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 6720*b^7*(cos(d*x + c) - 1)^5/(co
s(d*x + c) + 1)^5 + 7623*a^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 84
0*a^6*b*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 31332*a^5*b^2*(cos(d*x
+ c) - 1)^6/(cos(d*x + c) + 1)^6 + 2520*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d
*x + c) + 1)^6 + 48258*a^3*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 -
2520*a^2*b^5*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 33012*a*b^6*(cos(d
*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 840*b^7*(cos(d*x + c) - 1)^6/(cos(d*x
 + c) + 1)^6 + 1089*a^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 4356*a^
5*b^2*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 6534*a^3*b^4*(cos(d*x + c
) - 1)^7/(cos(d*x + c) + 1)^7 - 4356*a*b^6*(cos(d*x + c) - 1)^7/(cos(d*x +
c) + 1)^7)/(b^8*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^7))/d
```

Mupad [B] (verification not implemented)

Time = 15.15 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.52

$$\int \frac{\tan^9(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d} - \frac{\frac{2\,(105\,a^6-385\,a^4\,b^2+511\,a^2\,b^4-279\,b^6)}{105\,b^7}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}(a^6+a^5\,b-3\,a^4\,b^2-3\,a^3\,b^3+3\,a^2\,b^4+3\,a\,b^5-b^6)}{b^7}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}(6\,a^6+5\,a^5\,b\cdots}{\cdots}}{\cdots}$$

$$- \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\,(-a^7+4\,a^5\,b^2-6\,a^3\,b^4+4\,a\,b^6)}{b^8\,d} - \frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\,(a^2-b^2)^4}{a\,b^8\,d}$$

```
[In] int(tan(c + d*x)^9/(a + b/cos(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - ((2*(105*a^6 - 279*b^6 + 511*a^2*b^4
- 385*a^4*b^2))/(105*b^7) + (2*tan(c/2 + (d*x)/2)^12*(3*a*b^5 + a^5*b + a^6
 - b^6 + 3*a^2*b^4 - 3*a^3*b^3 - 3*a^4*b^2))/b^7 - (2*tan(c/2 + (d*x)/2)^10
*(19*a*b^5 + 5*a^5*b + 6*a^6 - 8*b^6 + 22*a^2*b^4 - 17*a^3*b^3 - 20*a^4*b^2
))/b^7 - (4*tan(c/2 + (d*x)/2)^6*(71*a*b^5 + 15*a^5*b + 30*a^6 - 96*b^6 + 1
54*a^2*b^4 - 54*a^3*b^3 - 112*a^4*b^2))/(3*b^7) + (2*tan(c/2 + (d*x)/2)^8*(
```

```
142*a*b^5 + 30*a^5*b + 45*a^6 - 87*b^6 + 203*a^2*b^4 - 108*a^3*b^3 - 161*a^
4*b^2))/(3*b^7) + (2*tan(c/2 + (d*x)/2)^4*(95*a*b^5 + 25*a^5*b + 75*a^6 - 2
39*b^6 + 401*a^2*b^4 - 85*a^3*b^3 - 285*a^4*b^2))/(5*b^7) - (2*tan(c/2 + (d
*x)/2)^2*(45*a*b^5 + 15*a^5*b + 90*a^6 - 264*b^6 + 466*a^2*b^4 - 45*a^3*b^3
 - 340*a^4*b^2))/(15*b^7))/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/
2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*
x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1)) - (log(tan
(c/2 + (d*x)/2)^2 - 1)*(4*a*b^6 - a^7 - 6*a^3*b^4 + 4*a^5*b^2))/(b^8*d) - (
log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a^2 - b^2)^4)
/(a*b^8*d)
```

3.287 $\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\log(\cos(c+dx))}{ad} - \frac{(a^2-b^2)^3\log(a+b\sec(c+dx))}{ab^6d} + \frac{(a^4-3a^2b^2+3b^4)\sec(c+dx)}{b^5d} - \frac{a(a^2-3b^2)\sec^2(c+dx)}{2b^4d} + \frac{(a^2-3b^2)\sec^3(c+dx)}{3b^3d} - \frac{a\sec^4(c+dx)}{4b^2d} + \frac{\sec^5(c+dx)}{5bd}$$

```
[Out] ln(cos(d*x+c))/a/d-(a^2-b^2)^3*ln(a+b*sec(d*x+c))/a/b^6/d+(a^4-3*a^2*b^2+3*
b^4)*sec(d*x+c)/b^5/d-1/2*a*(a^2-3*b^2)*sec(d*x+c)^2/b^4/d+1/3*(a^2-3*b^2)*
sec(d*x+c)^3/b^3/d-1/4*a*sec(d*x+c)^4/b^2/d+1/5*sec(d*x+c)^5/b/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3970, 908}

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{(a^2-b^2)^3\log(a+b\sec(c+dx))}{ab^6d} - \frac{a(a^2-3b^2)\sec^2(c+dx)}{2b^4d} + \frac{(a^2-3b^2)\sec^3(c+dx)}{3b^3d} + \frac{(a^4-3a^2b^2+3b^4)\sec(c+dx)}{b^5d} - \frac{a\sec^4(c+dx)}{4b^2d} + \frac{\log(\cos(c+dx))}{ad} + \frac{\sec^5(c+dx)}{5bd}$$

```
[In] Int[Tan[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) - ((a^2 - b^2)^3*Log[a + b*Sec[c + d*x]])/(a*b^6*d)
 + ((a^4 - 3*a^2*b^2 + 3*b^4)*Sec[c + d*x])/(b^5*d) - (a*(a^2 - 3*b^2)*Sec[
c + d*x]^2)/(2*b^4*d) + ((a^2 - 3*b^2)*Sec[c + d*x]^3)/(3*b^3*d) - (a*Sec[c
 + d*x]^4)/(4*b^2*d) + Sec[c + d*x]^5/(5*b*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{b^6 d}$$

$$= -\frac{\text{Subst}\left(\int \left(-a^4\left(1+\frac{3b^2\left(-a^2+b^2\right)}{a^4}\right)+\frac{b^6}{ax}+a\left(a^2-3b^2\right)x-\left(a^2-3b^2\right)x^2+ax^3-x^4+\frac{\left(a^2-b^2\right)^3}{a(a+x)}\right)\,dx}{b^6 d}$$

$$= \frac{\log(\cos(c+dx))}{ad}-\frac{\left(a^2-b^2\right)^3\log(a+b\sec(c+dx))}{ab^6 d}+\frac{\left(a^4-3a^2b^2+3b^4\right)\sec(c+dx)}{b^5 d} -\frac{a\left(a^2-3b^2\right)\sec^2(c+dx)}{2b^4 d}+\frac{\left(a^2-3b^2\right)\sec^3(c+dx)}{3b^3 d}-\frac{a\sec^4(c+dx)}{4b^2 d}+\frac{\sec^5(c+dx)}{5bd}$$

Mathematica [A] (verified)

Time = 6.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{-\frac{b^6\log(\cos(c+dx))}{a}+\frac{\left(a^2-b^2\right)^3\log(a+b\sec(c+dx))}{a}-b\left(a^4-3a^2b^2+3b^4\right)\sec(c+dx)+\frac{1}{2}ab^2\left(a^2-3b^2\right)\sec^2(c+dx)}{b^6 d}$$

```
[In] Integrate[Tan[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((-((b^6*Log[Cos[c + d*x]])/a) + ((a^2 - b^2)^3*Log[a + b*Sec[c + d*x]])/a
 - b*(a^4 - 3*a^2*b^2 + 3*b^4)*Sec[c + d*x] + (a*b^2*(a^2 - 3*b^2)*Sec[c +
d*x]^2)/2 - (b^3*(a^2 - 3*b^2)*Sec[c + d*x]^3)/3 + (a*b^4*Sec[c + d*x]^4)/4
 - (b^5*Sec[c + d*x]^5)/5)/(b^6*d))
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-\frac{a}{4b^2\cos(dx+c)^4}-\frac{-a^2+3b^2}{3b^3\cos(dx+c)^3}-\frac{-a^4+3a^2b^2-3b^4}{b^5\cos(dx+c)}-\frac{(a^2-3b^2)a}{2b^4\cos(dx+c)^2}+\frac{(a^4-3a^2b^2+3b^4)a\ln(\cos(dx+c))}{b^6}+\frac{1}{5b\cos(dx+c)^5}+\frac{(-}{}}{d}$
default	$\frac{-\frac{a}{4b^2\cos(dx+c)^4}-\frac{-a^2+3b^2}{3b^3\cos(dx+c)^3}-\frac{-a^4+3a^2b^2-3b^4}{b^5\cos(dx+c)}-\frac{(a^2-3b^2)a}{2b^4\cos(dx+c)^2}+\frac{(a^4-3a^2b^2+3b^4)a\ln(\cos(dx+c))}{b^6}+\frac{1}{5b\cos(dx+c)^5}+\frac{(-}{}}{d}$
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{2a^4e^{9i(dx+c)}-6a^2b^2e^{9i(dx+c)}+6b^4e^{9i(dx+c)}-2a^3b\,e^{8i(dx+c)}+6a\,b^3e^{8i(dx+c)}+8a^4e^{7i(dx+c)}-\frac{64a^2b^2e^{7i}}{3}}{}$

```
[In] int(tan(d*x+c)^7/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/b^2*a/cos(d*x+c)^4-1/3*(-a^2+3*b^2)/b^3/cos(d*x+c)^3-(-a^4+3*a^2*
b^2-3*b^4)/b^5/cos(d*x+c)-1/2*(a^2-3*b^2)/b^4*a/cos(d*x+c)^2+(a^4-3*a^2*b^2
+3*b^4)/b^6*a*ln(cos(d*x+c))+1/5/b/cos(d*x+c)^5+(-a^6+3*a^4*b^2-3*a^2*b^4+b
^6)/b^6/a*ln(b+a*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.21

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{15\,a^2b^4\cos(dx+c)+60\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\cos(dx+c)^5\log(a\cos(dx+c)+b)-60\,(a^6-3\,a^4}{}$$

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(15*a^2*b^4*cos(d*x + c) + 60*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos
(d*x + c)^5*log(a*cos(d*x + c) + b) - 60*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cos(
d*x + c)^5*log(-cos(d*x + c)) - 12*a*b^5 - 60*(a^5*b - 3*a^3*b^3 + 3*a*b^5)
*cos(d*x + c)^4 + 30*(a^4*b^2 - 3*a^2*b^4)*cos(d*x + c)^3 - 20*(a^3*b^3 - 3
*a*b^5)*cos(d*x + c)^2)/(a*b^6*d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx$$

```
[In] integrate(tan(d*x+c)**7/(a+b*sec(d*x+c)),x)
[Out] Integral(tan(c + d*x)**7/(a + b*sec(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{60\,(a^5-3\,a^3b^2+3\,ab^4)\log(\cos(dx+c))}{b^6} - \frac{60\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\log(a\cos(dx+c)+b)}{ab^6} - \frac{15\,ab^3\cos(dx+c)-60\,(a^4-3\,a^2b^2+3\,b^4)\cos(dx+c)^4}{b}}{60\,d}$$

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] 1/60*(60*(a^5 - 3*a^3*b^2 + 3*a*b^4)*log(cos(d*x + c))/b^6 - 60*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*log(a*cos(d*x + c) + b)/(a*b^6) - (15*a*b^3*cos(d*
x + c) - 60*(a^4 - 3*a^2*b^2 + 3*b^4)*cos(d*x + c)^4 - 12*b^4 + 30*(a^3*b -
 3*a*b^3)*cos(d*x + c)^3 - 20*(a^2*b^2 - 3*b^4)*cos(d*x + c)^2)/(b^5*cos(d*
x + c)^5))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. 2(162) = 324.

Time = 3.41 (sec) , antiderivative size = 1052, normalized size of antiderivative = 6.19

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] -1/60*(30*(a^5 - 3*a^3*b^2 + 3*a*b^4)*log(abs(a + b - 2*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2))/b^6 - 60*(a^5 - 3*a^3*b^2 + 3*a*b^4
)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^6 - 30*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - 2*b^6)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2
```

```
*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 2*abs(a)))/(b^6*abs(a)) + (137*a^5 - 120*a^4*b - 411*a^3*b^2 +
320*a^2*b^3 + 411*a*b^4 - 264*b^5 + 685*a^5*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - 480*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2175*a^3*b^2*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 1360*a^2*b^3*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 2295*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1200*b^5*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1370*a^5*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 720*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 447
0*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2000*a^2*b^3*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5070*a*b^4*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 - 1920*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1370*a
^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 480*a^4*b*(cos(d*x + c) - 1)
^3/(cos(d*x + c) + 1)^3 - 4470*a^3*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 + 1200*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 5070*a*b^4
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 720*b^5*(cos(d*x + c) - 1)^3/(
cos(d*x + c) + 1)^3 + 685*a^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 1
20*a^4*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 2175*a^3*b^2*(cos(d*x
+ c) - 1)^4/(cos(d*x + c) + 1)^4 + 240*a^2*b^3*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 + 2295*a*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 120*
b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^5*(cos(d*x + c) - 1)^
5/(cos(d*x + c) + 1)^5 - 411*a^3*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1
)^5 + 411*a*b^4*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/(b^6*((cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 15.60 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.32

$$\int \frac{\tan^7(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(a^4-3\,a^2\,b^2+3\,b^4\right)}{b^6\,d}$$
$$-\frac{\frac{2\,\left(15\,a^4-40\,a^2\,b^2+33\,b^4\right)}{15\,b^5}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(a^4+a^3\,b-2\,a^2\,b^2-2\,a\,b^3+b^4\right)}{b^5}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(4\,a^4+3\,a^3\,b-10\,a^2\,b^2-8\,a\,b^3+6\,b^4\right)}{b^5}-\frac{2\tan}{}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\right.}$$
$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}-\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(a^2-b^2\right)^3}{a\,b^6\,d}$$

```
[In] int(tan(c + d*x)^7/(a + b/cos(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 - 1)*(a^4 + 3*b^4 - 3*a^2*b^2))/(b^6*d) - ((2*(
15*a^4 + 33*b^4 - 40*a^2*b^2))/(15*b^5) + (2*tan(c/2 + (d*x)/2)^8*(a^3*b -
2*a*b^3 + a^4 + b^4 - 2*a^2*b^2))/b^5 - (2*tan(c/2 + (d*x)/2)^6*(3*a^3*b -
8*a*b^3 + 4*a^4 + 6*b^4 - 10*a^2*b^2))/b^5 - (2*tan(c/2 + (d*x)/2)^2*(3*a^3
*b - 6*a*b^3 + 12*a^4 + 30*b^4 - 34*a^2*b^2))/(3*b^5) + (2*tan(c/2 + (d*x)/
```

```
2)^4*(9*a^3*b - 24*a*b^3 + 18*a^4 + 48*b^4 - 50*a^2*b^2))/(3*b^5))/(d*(5*ta
n(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*
tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - log(tan(c/2 + (d*x)/2)
^2 + 1)/(a*d) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^
2)*(a^2 - b^2)^3)/(a*b^6*d)
```

3.288 $\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\log(\cos(c+dx))}{ad} - \frac{(a^2-b^2)^2\log(a+b\sec(c+dx))}{ab^4d} + \frac{(a^2-2b^2)\sec(c+dx)}{b^3d} - \frac{a\sec^2(c+dx)}{2b^2d} + \frac{\sec^3(c+dx)}{3bd}$$

```
[Out] -ln(cos(d*x+c))/a/d-(a^2-b^2)^2*ln(a+b*sec(d*x+c))/a/b^4/d+(a^2-2*b^2)*sec(
d*x+c)/b^3/d-1/2*a*sec(d*x+c)^2/b^2/d+1/3*sec(d*x+c)^3/b/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3970, 908}

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{(a^2-b^2)^2\log(a+b\sec(c+dx))}{ab^4d} + \frac{(a^2-2b^2)\sec(c+dx)}{b^3d} - \frac{a\sec^2(c+dx)}{2b^2d} - \frac{\log(\cos(c+dx))}{ad} + \frac{\sec^3(c+dx)}{3bd}$$

```
[In] Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((a^2 - b^2)^2*Log[a + b*Sec[c + d*x]])/(a*b^4
*d) + ((a^2 - 2*b^2)*Sec[c + d*x])/(b^3*d) - (a*Sec[c + d*x]^2)/(2*b^2*d) +
 Sec[c + d*x]^3/(3*b*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{b^4 d}\\ &= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{(a^2-b^2)^2}{a(a+x)}\right)dx, x, b\sec(c+dx)\right)}{b^4 d}\\ &= -\frac{\log(\cos(c+dx))}{ad}-\frac{(a^2-b^2)^2\log(a+b\sec(c+dx))}{ab^4 d}\\ &\quad+\frac{(a^2-2b^2)\sec(c+dx)}{b^3 d}-\frac{a\sec^2(c+dx)}{2b^2 d}+\frac{\sec^3(c+dx)}{3bd}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.91

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-\frac{b^4\log(\cos(c+dx))}{a}-\frac{(a^2-b^2)^2\log(a+b\sec(c+dx))}{a}+b(a^2-2b^2)\sec(c+dx)-\frac{1}{2}ab^2\sec^2(c+dx)+\frac{1}{3}b^3\sec^3(c+dx)}{b^4 d}$$

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x]),x]
[Out] (-((b^4*Log[Cos[c + d*x]])/a) - ((a^2 - b^2)^2*Log[a + b*Sec[c + d*x]])/a +
 b*(a^2 - 2*b^2)*Sec[c + d*x] - (a*b^2*Sec[c + d*x]^2)/2 + (b^3*Sec[c + d*x
]^3)/3)/(b^4*d)
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\frac{\left(-a^4+2a^2b^2-b^4\right)\ln(b+a\cos(dx+c))}{b^4a}-\frac{a}{2b^2\cos(dx+c)^2}-\frac{-a^2+2b^2}{b^3\cos(dx+c)}+\frac{\left(a^2-2b^2\right)a\ln(\cos(dx+c))}{b^4}+\frac{1}{3b\cos(dx+c)^3}}{d}$
default	$\frac{\frac{\left(-a^4+2a^2b^2-b^4\right)\ln(b+a\cos(dx+c))}{b^4a}-\frac{a}{2b^2\cos(dx+c)^2}-\frac{-a^2+2b^2}{b^3\cos(dx+c)}+\frac{\left(a^2-2b^2\right)a\ln(\cos(dx+c))}{b^4}+\frac{1}{3b\cos(dx+c)^3}}{d}$
risch	$\frac{ix}{a}+\frac{2ic}{ad}+\frac{2a^2\mathrm{e}^{5i(dx+c)}-4b^2\mathrm{e}^{5i(dx+c)}-2ab\,\mathrm{e}^{4i(dx+c)}+4a^2\mathrm{e}^{3i(dx+c)}-\frac{16b^2\mathrm{e}^{3i(dx+c)}}{3}-2ab\,\mathrm{e}^{2i(dx+c)}+2a^2\mathrm{e}^{i(dx+c)}-4}{d\,b^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$

[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*((-a^4+2*a^2*b^2-b^4)/b^4/a*ln(b+a*cos(d*x+c))-1/2/b^2*a/cos(d*x+c)^2-(-a^2+2*b^2)/b^3/cos(d*x+c)+(a^2-2*b^2)/b^4*a*ln(cos(d*x+c))+1/3/b/cos(d*x+c)^3)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.19

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{3\,a^2b^2\cos\left(dx+c\right)+6\left(a^4-2\,a^2b^2+b^4\right)\cos\left(dx+c\right)^3\log\left(a\cos\left(dx+c\right)+b\right)-6\left(a^4-2\,a^2b^2\right)\cos\left(dx-\right.}{6\,ab^4d\cos\left(dx+c\right)^3}$$

[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/6*(3*a^2*b^2*cos(d*x + c) + 6*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^3*log(a*cos(d*x + c) + b) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c)^3*log(-cos(d*x + c)) - 2*a*b^3 - 6*(a^3*b - 2*a*b^3)*cos(d*x + c)^2)/(a*b^4*d*cos(d*x + c)^3)

Sympy [F]

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\tan^5\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c)),x)

[Out] Integral(tan(c + d*x)**5/(a + b*sec(c + d*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{6\,(a^3-2\,ab^2)\log(\cos(dx+c))}{b^4}-\frac{6\,(a^4-2\,a^2b^2+b^4)\log(a\cos(dx+c)+b)}{ab^4}-\frac{3\,ab\cos(dx+c)-6\,(a^2-2\,b^2)\cos(dx+c)^2-2\,b^2}{b^3\cos(dx+c)^3}}{6\,d}$$

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(a^3 - 2*a*b^2)*log(cos(d*x + c))/b^4 - 6*(a^4 - 2*a^2*b^2 + b^4)*lo
g(a*cos(d*x + c) + b)/(a*b^4) - (3*a*b*cos(d*x + c) - 6*(a^2 - 2*b^2)*cos(d
*x + c)^2 - 2*b^2)/(b^3*cos(d*x + c)^3))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(104) = 208.

Time = 1.53 (sec) , antiderivative size = 560, normalized size of antiderivative = 5.19

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{3\,(a^3-2\,ab^2)\log\left(\left|a+b-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{b^4}-\frac{6\,(a^3-2\,ab^2)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^4}-\frac{3\,(a^4-2\,a^2b^2+2}{}}{}$$

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*(a^3 - 2*a*b^2)*log(abs(a + b - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2))/b^4 - 6*(a^3 - 2*a*b^2)*log(abs(-(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 1))/b^4 - 3*(a^4 - 2*a^2*b^2 + 2*b^4)*log(abs(2*b +
 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/(b^4*abs(a)) + (1
1*a^3 - 12*a^2*b - 22*a*b^2 + 20*b^3 + 33*a^3*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 24*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 78*a*b^2*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 48*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 33*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12*a^2*b*(cos(d*x
+ c) - 1)^2/(cos(d*x + c) + 1)^2 - 78*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 + 12*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*a^3*(cos
(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 22*a*b^2*(cos(d*x + c) - 1)^3/(cos(
d*x + c) + 1)^3)/(b^4*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 15.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.10

$$\int \frac{\tan^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d} - \frac{\frac{2\,(3\,a^2-5\,b^2)}{3\,b^3}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,(2\,a^2+a\,b-4\,b^2)}{b^3}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,(a^2+a\,b-b^2)}{b^3}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)} + \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\,(a^2-2\,b^2)}{b^4\,d} - \frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\,(a^2-b^2)^2}{a\,b^4\,d}$$

```
[In] int(tan(c + d*x)^5/(a + b/cos(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - ((2*(3*a^2 - 5*b^2))/(3*b^3) - (2*tan
(c/2 + (d*x)/2)^2*(a*b + 2*a^2 - 4*b^2))/b^3 + (2*tan(c/2 + (d*x)/2)^4*(a*b
 + a^2 - b^2))/b^3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + t
an(c/2 + (d*x)/2)^6 - 1)) + (a*log(tan(c/2 + (d*x)/2)^2 - 1)*(a^2 - 2*b^2))
/(b^4*d) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a
^2 - b^2)^2)/(a*b^4*d)
```

3.289 $\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\log(\cos(c+dx))}{ad} - \frac{(a^2-b^2)\log(a+b\sec(c+dx))}{ab^2d} + \frac{\sec(c+dx)}{bd}$$

```
[Out] ln(cos(d*x+c))/a/d-(a^2-b^2)*ln(a+b*sec(d*x+c))/a/b^2/d+sec(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3970, 908}

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{(a^2-b^2)\log(a+b\sec(c+dx))}{ab^2d} + \frac{\log(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{bd}$$

```
[In] Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) - ((a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a*b^2*d) +
 Sec[c + d*x]/(b*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{b^2 d}\\
&= -\frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{ax}+\frac{a^2-b^2}{a(a+x)}\right)\,dx, x, b\sec(c+dx)\right)}{b^2 d}\\
&= \frac{\log(\cos(c+dx))}{ad} - \frac{(a^2-b^2)\log(a+b\sec(c+dx))}{ab^2 d} + \frac{\sec(c+dx)}{bd}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$
\begin{aligned}
&\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx\\
&= \frac{b^2\log(\cos(c+dx)) + (-a^2+b^2)\log(a+b\sec(c+dx)) + ab\sec(c+dx)}{ab^2 d}
\end{aligned}
$$

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x]),x]
[Out] (b^2*Log[Cos[c + d*x]] + (-a^2 + b^2)*Log[a + b*Sec[c + d*x]] + a*b*Sec[c +
 d*x])/(a*b^2*d)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{a\ln(\cos(dx+c))}{b^2}+\frac{1}{b\cos(dx+c)}+\frac{\left(-a^2+b^2\right)\ln(b+a\cos(dx+c))}{b^2 a}}{d}$
default	$\frac{\frac{a\ln(\cos(dx+c))}{b^2}+\frac{1}{b\cos(dx+c)}+\frac{\left(-a^2+b^2\right)\ln(b+a\cos(dx+c))}{b^2 a}}{d}$
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{2\,\mathrm{e}^{i(dx+c)}}{db\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{a\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2b\,\mathrm{e}^{i(dx+c)}}{a}+1\right)}{b^2 d}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2b\,\mathrm{e}^{i(dx+c)}}{a}+1\right)}{ad}+\frac{a\ln(\mathrm{e}^{2i(}}{b}$

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*a*ln(cos(d*x+c))+1/b/cos(d*x+c)+(-a^2+b^2)/b^2/a*ln(b+a*cos(d*x+
c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a^2\cos(dx+c)\log(-\cos(dx+c)) - (a^2-b^2)\cos(dx+c)\log(a\cos(dx+c)+b) + ab}{ab^2d\cos(dx+c)}$$

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a^2*cos(d*x + c)*log(-cos(d*x + c)) - (a^2 - b^2)*cos(d*x + c)*log(a*cos(d
*x + c) + b) + a*b)/(a*b^2*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{a\log(\cos(dx+c))}{b^2} - \frac{(a^2-b^2)\log(a\cos(dx+c)+b)}{ab^2} + \frac{1}{b\cos(dx+c)}}{d}$$

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(cos(d*x + c))/b^2 - (a^2 - b^2)*log(a*cos(d*x + c) + b)/(a*b^2) + 1/
(b*cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(59) = 118$.

Time = 0.61 (sec) , antiderivative size = 289, normalized size of antiderivative = 4.90

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx =$$

$$-\frac{\frac{a\log\left(\left|a+b-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{b^2}-\frac{2a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^2}-\frac{(a^2-2b^2)\log\left(\frac{\left|2b+\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}\cdots\right|}{\left|2b+\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}\cdots\right|}\right)}{b^2|a|}\cdots}{2d}$$

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(abs(a + b - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2))/b^2 - 2*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^
2 - (a^2 - 2*b^2)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + 2*abs(a)))/(b^2*abs(a)) + 2*(a - 2*b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/(b^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.91 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.95

$$\int \frac{\tan^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}{b^2\,d} - \frac{2}{b\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d} - \frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\,\left(\frac{a}{b^2}-\frac{1}{a}\right)}{d}$$

```
[In] int(tan(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 - 1))/(b^2*d) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 -
1)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - (log(a + b - a*tan(c/2 + (d*x)/
2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a/b^2 - 1/a))/d
```

3.290 $\int \frac{\tan(c+dx)}{a+b\sec(c+dx)} dx$

Optimal result

Integrand size = 19, antiderivative size = 35

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\log(\cos(c+dx))}{ad} - \frac{\log(a+b\sec(c+dx))}{ad}$$

```
[Out] -ln(cos(d*x+c))/a/d-ln(a+b*sec(d*x+c))/a/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3970, 36, 29, 31}

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\log(a+b\sec(c+dx))}{ad} - \frac{\log(\cos(c+dx))}{ad}$$

```
[In] Int[Tan[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - Log[a + b*Sec[c + d*x]]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sec(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sec(c+dx)\right)}{ad}\\
&= -\frac{\log(\cos(c+dx))}{ad} - \frac{\log(a+b\sec(c+dx))}{ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\log(b+a\cos(c+dx))}{ad}$$

```
[In] Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x]),x]
[Out] -(Log[b + a*Cos[c + d*x]]/(a*d))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\sec(dx+c))}{a}+\frac{\ln(\sec(dx+c))}{a}}{d}$	33
default	$\frac{-\frac{\ln(a+b\sec(dx+c))}{a}+\frac{\ln(\sec(dx+c))}{a}}{d}$	33
risch	$\frac{ix}{a}+\frac{2ic}{ad}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2b\,\mathrm{e}^{i(dx+c)}}{a}+1\right)}{ad}$	54

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/a*ln(a+b*sec(d*x+c))+1/a*ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\log\left(a\cos\left(dx+c\right)+b\right)}{ad}$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] -log(a*cos(d*x + c) + b)/(a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(27) = 54$.

Time = 2.92 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.34

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x\tan\left(c\right)}{\sec\left(c\right)} & \text{for}\: a = 0 \wedge b = 0 \wedge d = 0 \\ -\frac{1}{bd\sec\left(c+dx\right)} & \text{for}\: a = 0 \\ \frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2ad} & \text{for}\: b = 0 \\ \frac{x\tan\left(c\right)}{a+b\sec\left(c\right)} & \text{for}\: d = 0 \\ -\frac{\log\left(\frac{a}{b}+\sec\left(c+dx\right)\right)}{ad} + \frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2ad} & \text{otherwise} \end{cases}$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x)
[Out] Piecewise((zoo*x*tan(c)/sec(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-1/(b*d*s
ec(c + d*x)), Eq(a, 0)), (log(tan(c + d*x)**2 + 1)/(2*a*d), Eq(b, 0)), (x*t
an(c)/(a + b*sec(c)), Eq(d, 0)), (-log(a/b + sec(c + d*x))/(a*d) + log(tan(
c + d*x)**2 + 1)/(2*a*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\log\left(a\cos\left(dx+c\right)+b\right)}{ad}$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log(a*cos(d*x + c) + b)/(a*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(35) = 70.

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.26

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\log\left(\frac{\left|2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|a|\right|}{\left|2\,b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|a|\right|}\right)}{d|a|}$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/(d*a
bs(a))
```

Mupad [B] (verification not implemented)

Time = 14.70 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.03

$$\int \frac{\tan(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\mathrm{atan}\left(\frac{a\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,1{}\mathrm{i}+b\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,1{}\mathrm{i}+b\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,1{}\mathrm{i}}\right)\,2{}\mathrm{i}}{a\,d}$$

```
[In] int(tan(c + d*x)/(a + b/cos(c + d*x)),x)
```

```
[Out] (atan((a*sin(c/2 + (d*x)/2)^2)/(a*cos(c/2 + (d*x)/2)^2*1i + b*cos(c/2 + (d*
x)/2)^2*1i + b*sin(c/2 + (d*x)/2)^2*1i))*2i)/(a*d)
```

3.291 $\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\log(\cos(c+dx))}{ad}+\frac{\log(1-\sec(c+dx))}{2(a+b)d} + \frac{\log(1+\sec(c+dx))}{2(a-b)d}-\frac{b^2\log(a+b\sec(c+dx))}{a\left(a^2-b^2\right)d}$$

```
[Out] ln(cos(d*x+c))/a/d+1/2*ln(1-sec(d*x+c))/(a+b)/d+1/2*ln(1+sec(d*x+c))/(a-b)/
d-b^2*ln(a+b*sec(d*x+c))/a/(a^2-b^2)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3970, 908}

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{b^2\log(a+b\sec(c+dx))}{ad\left(a^2-b^2\right)}+\frac{\log(1-\sec(c+dx))}{2d(a+b)} + \frac{\log(\sec(c+dx)+1)}{2d(a-b)}+\frac{\log(\cos(c+dx))}{ad}$$

```
[In] Int[Cot[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + Log[1 - Sec[c + d*x]]/(2*(a + b)*d) + Log[1 + Sec
[c + d*x]]/(2*(a - b)*d) - (b^2*Log[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b^2 \text{Subst}\left(\int \frac{1}{x(a+x)(b^2-x^2)} \, dx, x, b\sec(c+dx)\right)}{d} \\
&= -\frac{b^2 \text{Subst}\left(\int \left(\frac{1}{2b^2(a+b)(b-x)} + \frac{1}{ab^2x} + \frac{1}{a(a-b)(a+b)(a+x)} - \frac{1}{2(a-b)b^2(b+x)}\right) \, dx, x, b\sec(c+dx)\right)}{d} \\
&= \frac{\log(\cos(c+dx))}{ad} + \frac{\log(1-\sec(c+dx))}{2(a+b)d} + \frac{\log(1+\sec(c+dx))}{2(a-b)d} - \frac{b^2\log(a+b\sec(c+dx))}{a\left(a^2-b^2\right)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)} \, dx = \frac{\frac{2\log(\cos(c+dx))}{a} + \frac{\log(1-\sec(c+dx))}{a+b} + \frac{\log(1+\sec(c+dx))}{a-b} - \frac{2b^2\log(a+b\sec(c+dx))}{a^3-ab^2}}{2d}$$

```
[In] Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x]),x]
[Out] ((2*Log[Cos[c + d*x]])/a + Log[1 - Sec[c + d*x]]/(a + b) + Log[1 + Sec[c +
d*x]]/(a - b) - (2*b^2*Log[a + b*Sec[c + d*x]])/(a^3 - a*b^2))/(2*d)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{-\frac{b^2 \ln(b+a\cos(dx+c))}{(a+b)(a-b)a}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
default	$\frac{-\frac{b^2 \ln(b+a\cos(dx+c))}{(a+b)(a-b)a}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
risch	$\frac{ix}{a}-\frac{ix}{a-b}-\frac{ic}{d(a-b)}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}+\frac{2ib^2x}{a(a^2-b^2)}+\frac{2ib^2c}{da(a^2-b^2)}+\frac{\ln(e^{i(dx+c)}+1)}{d(a-b)}+\frac{\ln(e^{i(dx+c)}-1)}{d(a+b)}-\frac{b^2 \mathrm{l}}{}$

[In] int(cot(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(-b^2/(a+b)/(a-b)/a*ln(b+a*cos(d*x+c))+1/(2*a+2*b)*ln(cos(d*x+c)-1)+1/(2*a-2*b)*ln(cos(d*x+c)+1))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2\,b^2\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^2+ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2-ab\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,(a^3-ab^2)d}$$

[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/2*(2*b^2*log(a*cos(d*x + c) + b) - (a^2 + a*b)*log(1/2*cos(d*x + c) + 1/2) - (a^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2))/((a^3 - a*b^2)*d)

Sympy [F]

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cot\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x)

[Out] Integral(cot(c + d*x)/(a + b*sec(c + d*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{2\,b^2\log(a\cos(dx+c)+b)}{a^3-ab^2} - \frac{\log(\cos(dx+c)+1)}{a-b} - \frac{\log(\cos(dx+c)-1)}{a+b}}{2\,d}$$

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*log(a*cos(d*x + c) + b)/(a^3 - a*b^2) - log(cos(d*x + c) + 1)/(
a - b) - log(cos(d*x + c) - 1)/(a + b))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(90) = 180$.

Time = 0.31 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.73

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{a\log\left(\left|-a-b+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^2-b^2} - \frac{(a^2-2\,b^2)\log\left(\frac{\left|-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|a|\right|}{\left|-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|a|\right|}\right)}{(a^2-b^2)|a|} -}{2\,d}$$

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(abs(-a - b + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2))/(a^2 - b^2) - (a^2 - 2*b^2)*log(abs(-2*b - 2*a*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a)
)/abs(-2*b - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 2*abs(a)))/((a^2 - b^2)*abs(a)) - log(abs(-cos(d*
x + c) + 1)/abs(cos(d*x + c) + 1))/(a + b))/d
```

Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \frac{\cot(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,(a+b)} - \frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a\,d} + \frac{b^2\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d\,(a\,b^2-a^3)}$$

```
[In] int(cot(c + d*x)/(a + b/cos(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) +
 (b^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a*b
^2 - a^3))
```

3.292 $\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\log(\cos(c+dx))}{ad} - \frac{(2a+3b)\log(1-\sec(c+dx))}{4(a+b)^2d} - \frac{(2a-3b)\log(1+\sec(c+dx))}{4(a-b)^2d} - \frac{b^4\log(a+b\sec(c+dx))}{a(a^2-b^2)^2d} + \frac{1}{4(a+b)d(1-\sec(c+dx))} + \frac{1}{4(a-b)d(1+\sec(c+dx))}$$

```
[Out] -ln(cos(d*x+c))/a/d-1/4*(2*a+3*b)*ln(1-sec(d*x+c))/(a+b)^2/d-1/4*(2*a-3*b)*
ln(1+sec(d*x+c))/(a-b)^2/d-b^4*ln(a+b*sec(d*x+c))/a/(a^2-b^2)^2/d+1/4/(a+b)
/d/(1-sec(d*x+c))+1/4/(a-b)/d/(1+sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3970, 908}

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{b^4\log(a+b\sec(c+dx))}{ad(a^2-b^2)^2} + \frac{1}{4d(a+b)(1-\sec(c+dx))} + \frac{1}{4d(a-b)(\sec(c+dx)+1)} - \frac{(2a+3b)\log(1-\sec(c+dx))}{4d(a+b)^2} - \frac{(2a-3b)\log(\sec(c+dx)+1)}{4d(a-b)^2} - \frac{\log(\cos(c+dx))}{ad}$$

```
[In] Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a*d)) - ((2*a + 3*b)*Log[1 - Sec[c + d*x]])/(4*(a + b)
^2*d) - ((2*a - 3*b)*Log[1 + Sec[c + d*x]])/(4*(a - b)^2*d) - (b^4*Log[a +
b*Sec[c + d*x]])/(a*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sec[c + d*x])) +
 1/(4*(a - b)*d*(1 + Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{b^4\text{Subst}\left(\int \frac{1}{x(a+x)(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{b^4\text{Subst}\left(\int \left(\frac{1}{4b^3(a+b)(b-x)^2} + \frac{2a+3b}{4b^4(a+b)^2(b-x)} + \frac{1}{ab^4x} - \frac{1}{a(a-b)^2(a+b)^2(a+x)} - \frac{1}{4(a-b)b^3(b+x)^2} + \frac{-2a+3b}{4(a-b)^2b^4(b+x)}\right)\right.}{d}$$

$$= -\frac{\log(\cos(c+dx))}{ad} - \frac{(2a+3b)\log(1-\sec(c+dx))}{4(a+b)^2d} - \frac{(2a-3b)\log(1+\sec(c+dx))}{4(a-b)^2d} - \frac{b^4\log(a+b\sec(c+dx))}{a\left(a^2-b^2\right)^2d} + \frac{1}{4(a+b)d(1-\sec(c+dx))} + \frac{1}{4(a-b)d(1+\sec(c+dx))}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{b^4\left(-\frac{4\log(\cos(c+dx))}{ab^4} - \frac{(2a+3b)\log(1-\sec(c+dx))}{b^4(a+b)^2} - \frac{(2a-3b)\log(1+\sec(c+dx))}{(a-b)^2b^4} - \frac{4\log(a+b\sec(c+dx))}{a(a-b)^2(a+b)^2} - \frac{1}{b^4(a+b)(-1+\sec(c+dx))} + \right.}{4d}$$

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] (b^4*((-4*Log[Cos[c + d*x]])/(a*b^4) - ((2*a + 3*b)*Log[1 - Sec[c + d*x]])/
(b^4*(a + b)^2) - ((2*a - 3*b)*Log[1 + Sec[c + d*x]])/((a - b)^2*b^4) - (4*
Log[a + b*Sec[c + d*x]])/(a*(a - b)^2*(a + b)^2) - 1/(b^4*(a + b)*(-1 + Sec
[c + d*x])) + 1/((a - b)*b^4*(1 + Sec[c + d*x]))))/(4*d)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{-\frac{b^{4}\ln(b+a\cos(dx+c))}{(a+b)^{2}(a-b)^{2}a}-\frac{1}{(4a-4b)(\cos(dx+c)+1)}+\frac{(-2a+3b)\ln(\cos(dx+c)+1)}{4(a-b)^{2}}+\frac{1}{(4a+4b)(\cos(dx+c)-1)}+\frac{(-2a-3b)\ln(\cos(dx+c)-1}{4(a+b)^{2}}}{d}$
default	$\frac{-\frac{b^{4}\ln(b+a\cos(dx+c))}{(a+b)^{2}(a-b)^{2}a}-\frac{1}{(4a-4b)(\cos(dx+c)+1)}+\frac{(-2a+3b)\ln(\cos(dx+c)+1)}{4(a-b)^{2}}+\frac{1}{(4a+4b)(\cos(dx+c)-1)}+\frac{(-2a-3b)\ln(\cos(dx+c)-1}{4(a+b)^{2}}}{d}$
risch	$-\frac{3ibc}{2d(a^{2}-2ab+b^{2})}+\frac{2ib^{4}x}{a(a^{4}-2a^{2}b^{2}+b^{4})}+\frac{iac}{d(a^{2}-2ab+b^{2})}+\frac{iax}{a^{2}+2ab+b^{2}}+\frac{3ibc}{2d(a^{2}+2ab+b^{2})}-\frac{3ibx}{2(a^{2}-2ab+b^{2})}-$

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^4/(a+b)^2/(a-b)^2/a*ln(b+a*cos(d*x+c))-1/(4*a-4*b)/(cos(d*x+c)+1)+1
/4/(a-b)^2*(-2*a+3*b)*ln(cos(d*x+c)+1)+1/(4*a+4*b)/(cos(d*x+c)-1)+1/4/(a+b)
^2*(-2*a-3*b)*ln(cos(d*x+c)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.68

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$=\frac{2a^4-2a^2b^2-2(a^3b-ab^3)\cos(dx+c)-4\left(b^4\cos(dx+c)^2-b^4\right)\log(a\cos(dx+c)+b)+\left(2a^4+a^3b\right.}{}$$

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^4 - 2*a^2*b^2 - 2*(a^3*b - a*b^3)*cos(d*x + c) - 4*(b^4*cos(d*x +
c)^2 - b^4)*log(a*cos(d*x + c) + b) + (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3
- (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c
) + 1/2) + (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 - (2*a^4 - a^3*b - 4*a^2*b^
2 + 3*a*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^5 - 2*a^3*b^
2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c)),x)

[Out] Integral(cot(c + d*x)**3/(a + b*sec(c + d*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{4\,b^4\log(a\cos(dx+c)+b)}{a^5-2\,a^3b^2+ab^4}+\frac{(2\,a-3\,b)\log(\cos(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{(2\,a+3\,b)\log(\cos(dx+c)-1)}{a^2+2\,ab+b^2}+\frac{2\,(b\cos(dx+c)-a)}{(a^2-b^2)\cos(dx+c)^2-a^2+b^2}}{4\,d}$$

[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/4*(4*b^4*log(a*cos(d*x + c) + b)/(a^5 - 2*a^3*b^2 + a*b^4) + (2*a - 3*b)*log(cos(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (2*a + 3*b)*log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^2) + 2*(b*cos(d*x + c) - a)/((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(147) = 294.

Time = 0.35 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.57

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{2\,(2\,a+3\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2\,ab+b^2}-\frac{4\,(a^3-2\,ab^2)\log\left(\left|a+b-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^4-2\,a^2b^2+b^4}-\frac{\left(a+b+\frac{4\,a(\cos(dx+c)-}{\cos(dx+c)+1}\right.}{(a^2+2\,}}{8\,d}$$

[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] -1/8*(2*(2*a + 3*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2) - 4*(a^3 - 2*a*b^2)*log(abs(a + b - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*

```
x + c) - 1)^2/(cos(d*x + c) + 1)^2))/(a^4 - 2*a^2*b^2 + b^4) - (a + b + 4*a
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1))*(cos(d*x + c) + 1)/((a^2 + 2*a*b + b^2)*(cos(d*x + c) - 1)) - 4*(a
^4 - 2*a^2*b^2 + 2*b^4)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 2*abs(a)))/((a^4 - 2*a^2*b^2 + b^4)*abs(a)) - (cos(d*x + c) - 1)/(
(a - b)*(cos(d*x + c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.11

$$\int \frac{\cot^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,d\,\left(4\,a-4\,b\right)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a+3\,b\right)}{d\,\left(2\,a^2+4\,a\,b+2\,b^2\right)} - \frac{a-b}{2\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a+b\right)\,\left(4\,a-4\,b\right)} - \frac{b^4\,\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a\,d\,\left(a^2-b^2\right)^2}$$

```
[In] int(cot(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - tan(c/2 + (d*x)/2)^2/(2*d*(4*a - 4*b)
) - (log(tan(c/2 + (d*x)/2))*(2*a + 3*b))/(d*(4*a*b + 2*a^2 + 2*b^2)) - (a
- b)/(2*d*tan(c/2 + (d*x)/2)^2*(a + b)*(4*a - 4*b)) - (b^4*log(a + b - a*ta
n(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(a*d*(a^2 - b^2)^2)
```

3.293 $\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 234

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\log(\cos(c+dx))}{ad} + \frac{(8a^2+21ab+15b^2)\log(1-\sec(c+dx))}{16(a+b)^3d} \\ &\quad + \frac{(8a^2-21ab+15b^2)\log(1+\sec(c+dx))}{16(a-b)^3d} \\ &\quad - \frac{b^6\log(a+b\sec(c+dx))}{a\left(a^2-b^2\right)^3d} - \frac{1}{16(a+b)d(1-\sec(c+dx))^2} \\ &\quad - \frac{5a+7b}{16(a+b)^2d(1-\sec(c+dx))} - \frac{1}{16(a-b)d(1+\sec(c+dx))^2} \\ &\quad - \frac{5a-7b}{16(a-b)^2d(1+\sec(c+dx))}\end{aligned}$$

```
[Out] ln(cos(d*x+c))/a/d+1/16*(8*a^2+21*a*b+15*b^2)*ln(1-sec(d*x+c))/(a+b)^3/d+1/
16*(8*a^2-21*a*b+15*b^2)*ln(1+sec(d*x+c))/(a-b)^3/d-b^6*ln(a+b*sec(d*x+c))/
a/(a^2-b^2)^3/d-1/16/(a+b)/d/(1-sec(d*x+c))^2+1/16*(-5*a-7*b)/(a+b)^2/d/(1-
sec(d*x+c))-1/16/(a-b)/d/(1+sec(d*x+c))^2+1/16*(-5*a+7*b)/(a-b)^2/d/(1+sec(
d*x+c))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3970, 908}

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(8a^2+21ab+15b^2)\log(1-\sec(c+dx))}{16d(a+b)^3} + \frac{(8a^2-21ab+15b^2)\log(\sec(c+dx)+1)}{16d(a-b)^3} - \frac{b^6\log(a+b\sec(c+dx))}{ad(a^2-b^2)^3} - \frac{5a+7b}{16d(a+b)^2(1-\sec(c+dx))} - \frac{5a-7b}{16d(a-b)^2(\sec(c+dx)+1)} - \frac{1}{16d(a+b)(1-\sec(c+dx))^2} - \frac{1}{16d(a-b)(\sec(c+dx)+1)^2} + \frac{\log(\cos(c+dx))}{ad}$$

```
[In] Int[Cot[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[Cos[c + d*x]]/(a*d) + ((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sec[c + d*x]])
/(16*(a + b)^3*d) + ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sec[c + d*x]])/(16*(
a - b)^3*d) - (b^6*Log[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)^3*d) - 1/(16*(a
+ b)*d*(1 - Sec[c + d*x])^2) - (5*a + 7*b)/(16*(a + b)^2*d*(1 - Sec[c + d*x
])) - 1/(16*(a - b)*d*(1 + Sec[c + d*x])^2) - (5*a - 7*b)/(16*(a - b)^2*d*(
1 + Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b^6\text{Subst}\left(\int \frac{1}{x(a+x)(b^2-x^2)^3}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{b^6\text{Subst}\left(\int\left(\frac{1}{8b^4(a+b)(b-x)^3}+\frac{5a+7b}{16b^5(a+b)^2(b-x)^2}+\frac{8a^2+21ab+15b^2}{16b^6(a+b)^3(b-x)}+\frac{1}{ab^6x}+\frac{1}{a(a-b)^3(a+b)^3(a+x)}+\frac{1}{8b^4(-a+b)(b+x)^3}\right.\right.}{d}$$

$$=\frac{\log(\cos(c+dx))}{ad}+\frac{(8a^2+21ab+15b^2)\log(1-\sec(c+dx))}{16(a+b)^3d}+\frac{(8a^2-21ab+15b^2)\log(1+\sec(c+dx))}{16(a-b)^3d}-\frac{b^6\log(a+b\sec(c+dx))}{a(a^2-b^2)^3d}-\frac{1}{16(a+b)d(1-\sec(c+dx))^2}-\frac{5a+7b}{16(a+b)^2d(1-\sec(c+dx))}-\frac{1}{16(a-b)d(1+\sec(c+dx))^2}-\frac{5a-7b}{16(a-b)^2d(1+\sec(c+dx))}$$

Mathematica [A] (verified)

Time = 4.50 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.94

$$\int\frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{b^6\left(-\frac{16\log(\cos(c+dx))}{ab^6}-\frac{(8a^2+21ab+15b^2)\log(1-\sec(c+dx))}{b^6(a+b)^3}-\frac{(8a^2-21ab+15b^2)\log(1+\sec(c+dx))}{(a-b)^3b^6}+\frac{16\log(a+b\sec(c+dx))}{a(a-b)^3(a+b)^3}+\frac{1}{b^6(\cdots}\right.}{16d}$$

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/16*(b^6*((-16*Log[Cos[c + d*x]])/(a*b^6) - ((8*a^2 + 21*a*b + 15*b^2)*Lo
g[1 - Sec[c + d*x]])/(b^6*(a + b)^3) - ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + S
ec[c + d*x]])/((a - b)^3*b^6) + (16*Log[a + b*Sec[c + d*x]])/(a*(a - b)^3*(
a + b)^3) + 1/(b^6*(a + b)*(-1 + Sec[c + d*x])^2) + (-5*a - 7*b)/(b^6*(a +
b)^2*(-1 + Sec[c + d*x])) + 1/((a - b)*b^6*(1 + Sec[c + d*x])^2) + (5*a - 7
*b)/((a - b)^2*b^6*(1 + Sec[c + d*x]))))/d
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{b^6\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3a}-\frac{1}{2(8a+8b)(\cos(dx+c)-1)^2}-\frac{7a+9b}{16(a+b)^2(\cos(dx+c)-1)}+\frac{\left(8a^2+21ab+15b^2\right)\ln(\cos(dx+c)-1)}{16(a+b)^3}-\frac{1}{2(8a-8b)(\cos(d\cdots}}{d}$
default	$\frac{-\frac{b^6\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3a}-\frac{1}{2(8a+8b)(\cos(dx+c)-1)^2}-\frac{7a+9b}{16(a+b)^2(\cos(dx+c)-1)}+\frac{\left(8a^2+21ab+15b^2\right)\ln(\cos(dx+c)-1)}{16(a+b)^3}-\frac{1}{2(8a-8b)(\cos(d\cdots}}{d}$
risch	$\frac{ix}{a}-\frac{ia^2c}{d(a^3-3a^2b+3a\,b^2-b^3)}+\frac{2ib^6c}{da(a^6-3a^4b^2+3a^2b^4-b^6)}+\frac{21iabc}{8d(a^3-3a^2b+3a\,b^2-b^3)}-\frac{ia^2x}{a^3-3a^2b+3a\,b^2-b^3}+\frac{\cdots}{a(a\cdots}$

```
[In] int(cot(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^6/(a+b)^3/(a-b)^3/a*ln(b+a*cos(d*x+c))-1/2/(8*a+8*b)/(cos(d*x+c)-1)
^2-1/16*(7*a+9*b)/(a+b)^2/(cos(d*x+c)-1)+1/16*(8*a^2+21*a*b+15*b^2)/(a+b)^3
*ln(cos(d*x+c)-1)-1/2/(8*a-8*b)/(cos(d*x+c)+1)^2-1/16*(-7*a+9*b)/(a-b)^2/(c
os(d*x+c)+1)+1/16*(8*a^2-21*a*b+15*b^2)/(a-b)^3*ln(cos(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(218) = 436$.

Time = 0.43 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.46

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{12\,a^6 - 32\,a^4b^2 + 20\,a^2b^4 + 2\,(5\,a^5b - 14\,a^3b^3 + 9\,ab^5)\cos\left(dx+c\right)^3 - 8\,(2\,a^6 - 5\,a^4b^2 + 3\,a^2b^4)\cos\left(dx+c\right.}{}$$

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(12*a^6 - 32*a^4*b^2 + 20*a^2*b^4 + 2*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)
*cos(d*x + c)^3 - 8*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 - 2*(3*a
^5*b - 10*a^3*b^3 + 7*a*b^5)*cos(d*x + c) - 16*(b^6*cos(d*x + c)^4 - 2*b^6*
cos(d*x + c)^2 + b^6)*log(a*cos(d*x + c) + b) + (8*a^6 + 3*a^5*b - 24*a^4*b
^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5 + (8*a^6 + 3*a^5*b - 24*a^4*b^2 - 1
0*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^4 - 2*(8*a^6 + 3*a^5*b - 24
*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^2)*log(1/2*cos(
d*x + c) + 1/2) + (8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 -
 15*a*b^5 + (8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*
b^5)*cos(d*x + c)^4 - 2*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2
*b^4 - 15*a*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^7 - 3*a^
5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^4 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^
4 - a*b^6)*d*cos(d*x + c)^2 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)
```

Sympy [F]

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cot^5\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

```
[In] integrate(cot(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.24

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{16\,b^6\log(a\cos(dx+c)+b)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6}-\frac{(8\,a^2-21\,ab+15\,b^2)\log(\cos(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{(8\,a^2+21\,ab+15\,b^2)\log(\cos(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left((5\,a^2b-9\,b^3)\cos(dx+c)^3\right.}{(a^4-2\,a^2b^2+b^4)}}{16\,d}$$

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*b^6*log(a*cos(d*x + c) + b)/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
 - (8*a^2 - 21*a*b + 15*b^2)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2
 - b^3) - (8*a^2 + 21*a*b + 15*b^2)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b +
3*a*b^2 + b^3) - 2*((5*a^2*b - 9*b^3)*cos(d*x + c)^3 + 6*a^3 - 10*a*b^2 - 4
*(2*a^3 - 3*a*b^2)*cos(d*x + c)^2 - (3*a^2*b - 7*b^3)*cos(d*x + c))/((a^4 -
 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b
^2 + b^4)*cos(d*x + c)^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(218) = 436.

Time = 0.39 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.77

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{4\,(8\,a^2+21\,ab+15\,b^2)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{32\,(a^5-3\,a^3b^2+3\,ab^4)\log\left(\left|a+b-\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{12\,a(c}{\cos}}{}$$

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/64*(4*(8*a^2 + 21*a*b + 15*b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x +
c) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 32*(a^5 - 3*a^3*b^2 + 3*a*b^4)*l
og(abs(a + b - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2))
/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (12*a*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) - 16*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a^2
 - 2*a*b + b^2) - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 28*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16*b^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
```

```
)^2 + 126*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 90*b^2*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*(cos(d*x + c) - 1)^2) - 32*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - 2*b^6
)*log(abs(2*b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 2*abs(a))/abs(2*b + 2*a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*abs(a)))/d
```

Mupad [B] (verification not implemented)

Time = 15.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.24

$$\int \frac{\cot^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(8\,a^2+21\,a\,b+15\,b^2\right)}{d\,\left(8\,a^3+24\,a^2\,b+24\,a\,b^2+8\,b^3\right)} - \frac{{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^4}{4\,d\,\left(16\,a-16\,b\right)} - \frac{\ln\left({\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+1\right)}{a\,d} - \frac{{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2\,\left(\frac{16\,b}{{\left(16\,a-16\,b\right)}^2}-\frac{3}{16\,a-16\,b}\right)}{d} - \frac{\frac{a^2-2\,a\,b+b^2}{4\,\left(a+b\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2\,\left(-3\,a^3+2\,a^2\,b+5\,a\,b^2-4\,b^3\right)}{{\left(a+b\right)}^2}}{d\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^4\,\left(16\,a^2-32\,a\,b+16\,b^2\right)} - \frac{b^6\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2\right)}{a\,d\,{\left(a^2-b^2\right)}^3}$$

```
[In] int(cot(c + d*x)^5/(a + b/cos(c + d*x)),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))*(21*a*b + 8*a^2 + 15*b^2))/(d*(24*a*b^2 + 24*a^2*b
 + 8*a^3 + 8*b^3)) - tan(c/2 + (d*x)/2)^4/(4*d*(16*a - 16*b)) - log(tan(c/2
 + (d*x)/2)^2 + 1)/(a*d) - (tan(c/2 + (d*x)/2)^2*((16*b)/(16*a - 16*b)^2 -
3/(16*a - 16*b)))/d - ((a^2 - 2*a*b + b^2)/(4*(a + b)) + (tan(c/2 + (d*x)/2
)^2*(5*a*b^2 + 2*a^2*b - 3*a^3 - 4*b^3))/(a + b)^2)/(d*tan(c/2 + (d*x)/2)^4
*(16*a^2 - 32*a*b + 16*b^2)) - (b^6*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*
tan(c/2 + (d*x)/2)^2))/(a*d*(a^2 - b^2)^3)
```

3.294 $\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 198

$$\begin{aligned}\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = & -\frac{x}{a}+\frac{(8a^4-20a^2b^2+15b^4)\,\text{arctanh}(\sin(c+dx))}{8b^5d} \\ & -\frac{2(a-b)^{5/2}(a+b)^{5/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^5d} \\ & -\frac{a(a^2-2b^2)\tan(c+dx)}{b^4d}+\frac{(4a^2-7b^2)\sec(c+dx)\tan(c+dx)}{8b^3d} \\ & -\frac{a\tan^3(c+dx)}{3b^2d}+\frac{\sec(c+dx)\tan^3(c+dx)}{4bd}\end{aligned}$$

```
[Out] -x/a+1/8*(8*a^4-20*a^2*b^2+15*b^4)*arctanh(sin(d*x+c))/b^5/d-2*(a-b)^(5/2)*
(a+b)^(5/2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/b^5/d-a*(
a^2-2*b^2)*tan(d*x+c)/b^4/d+1/8*(4*a^2-7*b^2)*sec(d*x+c)*tan(d*x+c)/b^3/d-1
/3*a*tan(d*x+c)^3/b^2/d+1/4*sec(d*x+c)*tan(d*x+c)^3/b/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.37, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

$= \{3983, 2976, 2738, 214, 3855, 3852, 8, 3853\}$

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(a^2-3b^2)\,\text{arctanh}(\sin(c+dx))}{2b^3d} - \frac{a(a^2-3b^2)\tan(c+dx)}{b^4d} + \frac{(a^2-3b^2)\tan(c+dx)\sec(c+dx)}{2b^3d} + \frac{(a^4-3a^2b^2+3b^4)\,\text{arctanh}(\sin(c+dx))}{b^5d} - \frac{2(a-b)^{5/2}(a+b)^{5/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^5d} - \frac{a\tan^3(c+dx)}{3b^2d} - \frac{a\tan(c+dx)}{b^2d} - \frac{x}{a} + \frac{3\text{arctanh}(\sin(c+dx))}{8bd} + \frac{\tan(c+dx)\sec^3(c+dx)}{4bd} + \frac{3\tan(c+dx)\sec(c+dx)}{8bd}$$

```
[In] Int[Tan[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (3*ArcTanh[Sin[c + d*x]])/(8*b*d) + ((a^2 - 3*b^2)*ArcTanh[Sin[c +
 d*x]])/(2*b^3*d) + ((a^4 - 3*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/(b^5*
d) - (2*(a - b)^(5/2)*(a + b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/(a*b^5*d) - (a*Tan[c + d*x])/(b^2*d) - (a*(a^2 - 3*b^2)*Tan[c
 + d*x])/(b^4*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(8*b*d) + ((a^2 - 3*b^2)*S
ec[c + d*x]*Tan[c + d*x])/(2*b^3*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*b*d)
 - (a*Tan[c + d*x]^3)/(3*b^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
```

```
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sin(c+dx)\tan^5(c+dx)}{b+a\cos(c+dx)}\,dx \\
&= \int \left(-\frac{1}{a}-\frac{\left(a^2-b^2\right)^3}{ab^5(b+a\cos(c+dx))}+\frac{\left(a^4-3a^2b^2+3b^4\right)\sec(c+dx)}{b^5} \right. \\
&\quad +\frac{\left(-a^3+3ab^2\right)\sec^2(c+dx)}{b^4}+\frac{\left(a^2-3b^2\right)\sec^3(c+dx)}{b^3}-\frac{a\sec^4(c+dx)}{b^2} \\
&\quad \left. +\frac{\sec^5(c+dx)}{b}\right)dx \\
&= -\frac{x}{a}-\frac{a\int\sec^4(c+dx)\,dx}{b^2}+\frac{\int\sec^5(c+dx)\,dx}{b} \\
&\quad -\frac{\left(a\left(a^2-3b^2\right)\right)\int\sec^2(c+dx)\,dx}{b^4}+\frac{\left(a^2-3b^2\right)\int\sec^3(c+dx)\,dx}{b^3} \\
&\quad -\frac{\left(a^2-b^2\right)^3\int\frac{1}{b+a\cos(c+dx)}\,dx}{ab^5}+\frac{\left(a^4-3a^2b^2+3b^4\right)\int\sec(c+dx)\,dx}{b^5}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{x}{a} + \frac{(a^4 - 3a^2b^2 + 3b^4)\operatorname{arctanh}(\sin(c+dx))}{b^5d} + \frac{(a^2 - 3b^2)\sec(c+dx)\tan(c+dx)}{2b^3d} \\
&\quad + \frac{\sec^3(c+dx)\tan(c+dx)}{4bd} + \frac{3\int \sec^3(c+dx)\,dx}{4b} \\
&\quad + \frac{(a^2-3b^2)\int \sec(c+dx)\,dx}{2b^3} + \frac{a\text{Subst}\left(\int (1+x^2)\, dx, x, -\tan(c+dx)\right)}{b^2d} \\
&\quad + \frac{(a(a^2-3b^2))\,\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{b^4d} \\
&\quad - \frac{\left(2(a^2-b^2)^3\right)\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{ab^5d} \\
&= -\frac{x}{a} + \frac{(a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^3d} + \frac{(a^4 - 3a^2b^2 + 3b^4)\operatorname{arctanh}(\sin(c+dx))}{b^5d} \\
&\quad - \frac{2(a-b)^{5/2}(a+b)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^5d} \\
&\quad - \frac{a\tan(c+dx)}{b^2d} - \frac{a(a^2-3b^2)\tan(c+dx)}{b^4d} \\
&\quad + \frac{3\sec(c+dx)\tan(c+dx)}{8bd} + \frac{(a^2-3b^2)\sec(c+dx)\tan(c+dx)}{2b^3d} \\
&\quad + \frac{\sec^3(c+dx)\tan(c+dx)}{4bd} - \frac{a\tan^3(c+dx)}{3b^2d} + \frac{3\int \sec(c+dx)\,dx}{8b} \\
&= -\frac{x}{a} + \frac{3\operatorname{arctanh}(\sin(c+dx))}{8bd} + \frac{(a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^3d} \\
&\quad + \frac{(a^4 - 3a^2b^2 + 3b^4)\operatorname{arctanh}(\sin(c+dx))}{b^5d} \\
&\quad - \frac{2(a-b)^{5/2}(a+b)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^5d} - \frac{a\tan(c+dx)}{b^2d} \\
&\quad - \frac{a(a^2-3b^2)\tan(c+dx)}{b^4d} + \frac{3\sec(c+dx)\tan(c+dx)}{8bd} \\
&\quad + \frac{(a^2-3b^2)\sec(c+dx)\tan(c+dx)}{2b^3d} + \frac{\sec^3(c+dx)\tan(c+dx)}{4bd} - \frac{a\tan^3(c+dx)}{3b^2d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 907 vs. $2(198) = 396$.

Time = 6.53 (sec) , antiderivative size = 907, normalized size of antiderivative = 4.58

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{(c+dx)(b+a\cos(c+dx))\sec(c+dx)}{ad(a+b\sec(c+dx))} - \frac{2(-a^2+b^2)^3\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))\sec(c+dx)}{ab^5\sqrt{a^2-b^2}d(a+b\sec(c+dx))} + \frac{(-8a^4+20a^2b^2-15b^4)(b+a\cos(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)}{8b^5d(a+b\sec(c+dx))} + \frac{(8a^4-20a^2b^2+15b^4)(b+a\cos(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)}{8b^5d(a+b\sec(c+dx))} + \frac{(b+a\cos(c+dx))\sec(c+dx)}{16bd(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4} + \frac{(12a^2-4ab-27b^2)(b+a\cos(c+dx))\sec(c+dx)}{48b^3d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{a(b+a\cos(c+dx))\sec(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)}{6b^2d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - \frac{(b+a\cos(c+dx))\sec(c+dx)}{16bd(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4} - \frac{a(b+a\cos(c+dx))\sec(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)}{6b^2d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{(-12a^2+4ab+27b^2)(b+a\cos(c+dx))\sec(c+dx)}{48b^3d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{(b+a\cos(c+dx))\sec(c+dx)\left(-3a^3\sin\left(\frac{1}{2}(c+dx)\right)+7ab^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3b^4d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{(b+a\cos(c+dx))\sec(c+dx)\left(-3a^3\sin\left(\frac{1}{2}(c+dx)\right)+7ab^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3b^4d(a+b\sec(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(a*d*(a + b*Sec[c + d*x])))
 - (2*(-a^2 + b^2)^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(
b + a*Cos[c + d*x])*Sec[c + d*x])/(a*b^5*Sqrt[a^2 - b^2]*d*(a + b*Sec[c + d
*x])) + ((-8*a^4 + 20*a^2*b^2 - 15*b^4)*(b + a*Cos[c + d*x])*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x])/(8*b^5*d*(a + b*Sec[c + d*x])) + (
(8*a^4 - 20*a^2*b^2 + 15*b^4)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]*Sec[c + d*x])/(8*b^5*d*(a + b*Sec[c + d*x])) + ((b + a*Cos
[c + d*x])*Sec[c + d*x])/(16*b*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])^4) + ((12*a^2 - 4*a*b - 27*b^2)*(b + a*Cos[c + d*x])*Sec[c
 + d*x])/(48*b^3*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
```

```
])^2) - (a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[(c + d*x)/2])/(6*b^2*d*(a
+ b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) - ((b + a*Cos[c
+ d*x])*Sec[c + d*x])/(16*b*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^4) - (a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[(c + d*x)/2])/(
6*b^2*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + ((-
12*a^2 + 4*a*b + 27*b^2)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(48*b^3*d*(a +
b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + ((b + a*Cos[c +
d*x])*Sec[c + d*x]*(-3*a^3*Sin[(c + d*x)/2] + 7*a*b^2*Sin[(c + d*x)/2]))/(3
*b^4*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + ((b +
a*Cos[c + d*x])*Sec[c + d*x]*(-3*a^3*Sin[(c + d*x)/2] + 7*a*b^2*Sin[(c + d*
x)/2]))/(3*b^4*d*(a + b*Sec[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(181) = 362$.

Time = 1.54 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.08

method	result
derivativedivides	$-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2(a-b)\left(a^5+a^4b-2a^3b^2-2a^2b^3+a\,b^4+b^5\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^5a\sqrt{(a-b)(a+b)}}+\frac{1}{4b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{}{6b^2}$
default	$-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2(a-b)\left(a^5+a^4b-2a^3b^2-2a^2b^3+a\,b^4+b^5\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^5a\sqrt{(a-b)(a+b)}}+\frac{1}{4b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{}{6b^2}$
risch	$-\frac{x}{a}-\frac{i\left(12a^2b\,e^{7i(dx+c)}-27b^3e^{7i(dx+c)}+24a^3e^{6i(dx+c)}-72a\,b^2e^{6i(dx+c)}+12a^2b\,e^{5i(dx+c)}-3b^3e^{5i(dx+c)}+72a^3e^{4i(dx}\right.}{}$

```
[In] int(tan(d*x+c)^6/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a*arctan(tan(1/2*d*x+1/2*c))-2/b^5*(a-b)*(a^5+a^4*b-2*a^3*b^2-2*a^2
*b^3+a*b^4+b^5)/a/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-
b)*(a+b))^(1/2))+1/4/b/(tan(1/2*d*x+1/2*c)-1)^4-1/6*(-2*a-3*b)/b^2/(tan(1/2
*d*x+1/2*c)-1)^3-1/8*(-4*a^2-4*a*b+5*b^2)/b^3/(tan(1/2*d*x+1/2*c)-1)^2+1/8/
b^5*(-8*a^4+20*a^2*b^2-15*b^4)*ln(tan(1/2*d*x+1/2*c)-1)-1/8*(-8*a^3-4*a^2*b
+16*a*b^2+7*b^3)/b^4/(tan(1/2*d*x+1/2*c)-1)-1/4/b/(tan(1/2*d*x+1/2*c)+1)^4-
1/6*(-2*a-3*b)/b^2/(tan(1/2*d*x+1/2*c)+1)^3-1/8*(4*a^2+4*a*b-5*b^2)/b^3/(ta
n(1/2*d*x+1/2*c)+1)^2+1/8*(8*a^4-20*a^2*b^2+15*b^4)/b^5*ln(tan(1/2*d*x+1/2*
c)+1)-1/8*(-8*a^3-4*a^2*b+16*a*b^2+7*b^3)/b^4/(tan(1/2*d*x+1/2*c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 603, normalized size of antiderivative = 3.05

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[-\frac{48\,b^5dx\cos\left(dx+c\right)^4-24\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}\cos\left(dx+c\right)^4\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{}}{a^2\cos(dx+c)^2+2}\right.}{}\right.$$
$$-\frac{48\,b^5dx\cos\left(dx+c\right)^4+48\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\cos\left(dx+c\right)^4-3}{}$$

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/48*(48*b^5*d*x*cos(d*x + c)^4 - 24*(a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b
^2)*cos(d*x + c)^4*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 -
 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^
4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)
*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a^2*b^3*cos(d*x + c) - 6*a*b^
4 + 8*(3*a^4*b - 7*a^2*b^3)*cos(d*x + c)^3 - 3*(4*a^3*b^2 - 9*a*b^4)*cos(d*
x + c)^2)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4), -1/48*(48*b^5*d*x*cos(d*x
 + c)^4 + 48*(a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^4 - 3*(8*
a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + 3*(8*a^
5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a^2
*b^3*cos(d*x + c) - 6*a*b^4 + 8*(3*a^4*b - 7*a^2*b^3)*cos(d*x + c)^3 - 3*(4
*a^3*b^2 - 9*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4)]
```

Sympy [F]

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\tan^6\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

```
[In] integrate(tan(d*x+c)**6/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**6/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(181) = 362$.

Time = 2.11 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.77

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(24*((a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + b^4)*sqrt(-a^2 + b^2)*abs(a
)*abs(-a + b)*abs(b) + (a^5*b + a^4*b^2 - 2*a^3*b^3 - 2*a^2*b^4 + a*b^5 + 2
*b^6)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arc
tan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^6 + sqrt(b^12 + (a*b^5 + b^6)*(a*b^5 - b^
6)))/(a*b^5 - b^6))))/((a*b^4 - b^5)*a^2*b^2 + (a*b^6 - b^7)*abs(a)*abs(b))
 + 24*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 + a*b^6 - 2*b^7 - a^5*abs(a)*abs(b) +
3*a^3*b^2*abs(a)*abs(b) - 3*a*b^4*abs(a)*abs(b) + b^5*abs(a)*abs(b))*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^6 - sqrt
(b^12 + (a*b^5 + b^6)*(a*b^5 - b^6)))/(a*b^5 - b^6))))/(a^2*b^6 - b^6*abs(a
)*abs(b)) - 3*(8*a^4 - 20*a^2*b^2 + 15*b^4)*log(abs(tan(1/2*d*x + 1/2*c) +
1))/b^5 + 3*(8*a^4 - 20*a^2*b^2 + 15*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)
)/b^5 - 2*(24*a^3*tan(1/2*d*x + 1/2*c)^7 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^7
- 48*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 21*b^3*tan(1/2*d*x + 1/2*c)^7 - 72*a^3*
tan(1/2*d*x + 1/2*c)^5 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 176*a*b^2*tan(1/
2*d*x + 1/2*c)^5 + 45*b^3*tan(1/2*d*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2
*c)^3 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 176*a*b^2*tan(1/2*d*x + 1/2*c)^3
+ 45*b^3*tan(1/2*d*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c) + 12*a^2*b*ta
n(1/2*d*x + 1/2*c) + 48*a*b^2*tan(1/2*d*x + 1/2*c) - 21*b^3*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*b^4))/d
```

Mupad [B] (verification not implemented)

Time = 16.74 (sec) , antiderivative size = 9148, normalized size of antiderivative = 46.20

$$\int \frac{\tan^6(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

[In] int(tan(c + d*x)^6/(a + b/cos(c + d*x)),x)

[Out] ((tan(c/2 + (d*x)/2)*(16*a*b^2 + 4*a^2*b - 8*a^3 - 7*b^3))/(4*b^4) - (tan(c/2 + (d*x)/2)^7*(16*a*b^2 - 4*a^2*b - 8*a^3 + 7*b^3))/(4*b^4) - (tan(c/2 + (d*x)/2)^3*(176*a*b^2 + 12*a^2*b - 72*a^3 - 45*b^3))/(12*b^4) + (tan(c/2 + (d*x)/2)^5*(176*a*b^2 - 12*a^2*b - 72*a^3 + 45*b^3))/(12*b^4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (2*atan(((((((((((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 - (tan(c/2 + (d*x)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18)*128i)/(a*b^16))*1i)/a - (128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16)*1i)/a - (128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16)*1i)/a - (128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16)/a - ((((((((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 + (tan(c/2 + (d*x)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18)*128i)/(a*b^16))*1i)/a + (128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16)*1i)/a - (128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16)*1i)/a + (128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^

5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16)/a)/(((((((((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 - (tan(c/2 + (d*x)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18)*128i)/(a*b^16))*1i)/a - (128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16)*1i)/a - (128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16)*1i)/a - (128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16)*1i)/a + (((((((((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 + (tan(c/2 + (d*x)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18)*128i)/(a*b^16))*1i)/a + (128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16)*1i)/a - (128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16)*1i)/a + (128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16)*1i)/a - (256*(64*a^19*b - 2145*a*b^19 - 64*a^20 + 795*b^20 - 3130*a^2*b^18 + 12805*a^3*b^17 + 2569*a^4*b^16 - 33634*a^5*b^15 + 7876*a^6*b^14 + 51074*a^7*b^13 - 23883*a^8*b^12 - 49501*a^9*b^11 + 30942*a^10*b^10 + 31881*a^11*b^9 - 23865*a^12*b^8 - 13776*a^13*b^7 + 11768*a^14*b^6 + 3936*a^15*b^5 - 3712*a^16*b^4 - 704*a^17*b^3 + 704*a^18*b^2))/b^16)))/(a*d) + (atan(((((128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29

678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a
^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 7
04*a^19*b^2))/b^16 + (((128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a
^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 -
7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^
12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 +
416*a^17*b^5 - 192*a^18*b^4))/b^16 + (((128*tan(c/2 + (d*x)/2)*(128*b^23 -
384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 +
755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13
 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15
b^8))/b^16 - (((128(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5
*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^
10*b^14))/b^16 - (128*tan(c/2 + (d*x)/2)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2)
*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19
 - 128*a^7*b^18))/b^21)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*
b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4
 + (15*b^4)/8 - (5*a^2*b^2)/2)*1i)/b^5 + (((128*tan(c/2 + (d*x)/2)*(1414*a*
b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340
*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^1
3 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 2
0913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17
*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16 - (((128*(576*a*b^21 - 192*b^22 +
 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b
^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21
295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15
*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 - (((128*tan(c/2
+ (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*
b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^
9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 -
384*a^14*b^9 + 128*a^15*b^8))/b^16 + (((128*(192*a^2*b^22 - 256*a^3*b^21 -
568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16
+ 416*a^9*b^15 - 192*a^10*b^14))/b^16 + (128*tan(c/2 + (d*x)/2)*(a^4 + (15*
b^4)/8 - (5*a^2*b^2)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a
^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18))/b^21)*(a^4 + (15*b^4)/8 - (5*a^2*b^
2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5
*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2)*1i)/b^5)/((256*(64*a^
19*b - 2145*a*b^19 - 64*a^20 + 795*b^20 - 3130*a^2*b^18 + 12805*a^3*b^17 +
2569*a^4*b^16 - 33634*a^5*b^15 + 7876*a^6*b^14 + 51074*a^7*b^13 - 23883*a^8
*b^12 - 49501*a^9*b^11 + 30942*a^10*b^10 + 31881*a^11*b^9 - 23865*a^12*b^8
 - 13776*a^13*b^7 + 11768*a^14*b^6 + 3936*a^15*b^5 - 3712*a^16*b^4 - 704*a^1
7*b^3 + 704*a^18*b^2))/b^16 + (((128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a
^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 +
 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a
^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b
^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*

a^18*b^3 - 704*a^19*b^2))/b^16 + (((128*(576*a*b^21 - 192*b^22 + 1043*a^2*b
^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310
*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^
11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248
*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 + (((128*tan(c/2 + (d*x)/2)*
(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047
*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 17
92*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^
9 + 128*a^15*b^8))/b^16 - (((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^2
0 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b
^15 - 192*a^10*b^14))/b^16 - (128*tan(c/2 + (d*x)/2)*(a^4 + (15*b^4)/8 - (5
*a^2*b^2)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 3
84*a^6*b^19 - 128*a^7*b^18))/b^21)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)
*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2
))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5 - (((128*tan(c/2 + (d*x)/2)
*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^
18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 2798
3*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^1
2*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 +
3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16 - (((128*(576*a*b^21 - 1
92*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7
376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*
b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 -
3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 - (((128
*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 +
903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15
+ 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^1
3*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16 + (((128*(192*a^2*b^22 - 256*a^
3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*
a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 + (128*tan(c/2 + (d*x)/2)*(a
^4 + (15*b^4)/8 - (5*a^2*b^2)/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^2
1 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18))/b^21)*(a^4 + (15*b^4)/8 -
(5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^
4)/8 - (5*a^2*b^2)/2))/b^5)*(a^4 + (15*b^4)/8 - (5*a^2*b^2)/2))/b^5))*(a^4
+ (15*b^4)/8 - (5*a^2*b^2)/2)*2i)/(b^5*d) + (atan(((((128*tan(c/2 + (d*x)/2
)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b
^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 279
83*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^
12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 +
3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16 + (((128*(576*a*b^21 -
192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 +
7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10
*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 -
3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 + (((a
+ b)^5*(a - b)^5)^(1/2)*((128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 3

22*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16 - (((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 - (128*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18))/(a*b^21))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5)))/(a*b^5))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5))*((a + b)^5*(a - b)^5)^(1/2)*1i)/(a*b^5) + (((128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16 - (((128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 - (((a + b)^5*(a - b)^5)^(1/2)*((128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16 + (((128*(192*a^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 + (128*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18))/(a*b^21))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5)))/(a*b^5))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5))*((a + b)^5*(a - b)^5)^(1/2)*1i)/(a*b^5))/((256*(64*a^19*b - 2145*a*b^19 - 64*a^20 + 795*b^20 - 3130*a^2*b^18 + 12805*a^3*b^17 + 2569*a^4*b^16 - 33634*a^5*b^15 + 7876*a^6*b^14 + 51074*a^7*b^13 - 23883*a^8*b^12 - 49501*a^9*b^11 + 30942*a^10*b^10 + 31881*a^11*b^9 - 23865*a^12*b^8 - 13776*a^13*b^7 + 11768*a^14*b^6 + 3936*a^15*b^5 - 3712*a^16*b^4 - 704*a^17*b^3 + 704*a^18*b^2))/b^16 + (((128*tan(c/2 + (d*x)/2)*(1414*a*b^20 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 + 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 20913*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16 + (((128*(576*a*b^21 - 192*b^22 + 1043*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 + (((a + b)^5*(a - b)^5)^(1/2)*((128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 + 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^12*b^1

```
1 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16 - (((128*(192*a^2*b^
22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176*a^7*
b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 - (128*tan(c/2 +
(d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^2*b^23 - 384*a^3*b^22 + 512*a^4
*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18))/(a*b^21))*((a + b)^5*(
a - b)^5)^(1/2))/(a*b^5)))/(a*b^5))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5))*(
(a + b)^5*(a - b)^5)^(1/2))/(a*b^5) - (((128*tan(c/2 + (d*x)/2)*(1414*a*b^2
0 - 64*a^20*b + 64*a^21 - 514*b^21 + 684*a^2*b^19 - 3084*a^3*b^18 - 4340*a^
4*b^17 + 6000*a^5*b^16 + 15860*a^6*b^15 - 14740*a^7*b^14 - 27983*a^8*b^13 +
 25679*a^9*b^12 + 29678*a^10*b^11 - 28398*a^11*b^10 - 21169*a^12*b^9 + 2091
3*a^13*b^8 + 10520*a^14*b^7 - 10520*a^15*b^6 - 3520*a^16*b^5 + 3520*a^17*b^
4 + 704*a^18*b^3 - 704*a^19*b^2))/b^16 - (((128*(576*a*b^21 - 192*b^22 + 10
43*a^2*b^20 - 2996*a^3*b^19 - 3575*a^4*b^18 + 8886*a^5*b^17 + 7376*a^6*b^16
 - 18310*a^7*b^15 - 7672*a^8*b^14 + 24883*a^9*b^13 + 2308*a^10*b^12 - 21295
*a^11*b^11 + 2736*a^12*b^10 + 11096*a^13*b^9 - 3080*a^14*b^8 - 3256*a^15*b^
7 + 1248*a^16*b^6 + 416*a^17*b^5 - 192*a^18*b^4))/b^16 - (((a + b)^5*(a - b
)^5)^(1/2)*((128*tan(c/2 + (d*x)/2)*(128*b^23 - 384*a*b^22 - 322*a^2*b^21 +
 1222*a^3*b^20 + 903*a^4*b^19 - 3047*a^5*b^18 + 755*a^6*b^17 + 905*a^7*b^16
 + 120*a^8*b^15 + 1000*a^9*b^14 - 1792*a^10*b^13 - 512*a^11*b^12 + 1472*a^1
2*b^11 - 192*a^13*b^10 - 384*a^14*b^9 + 128*a^15*b^8))/b^16 + (((128*(192*a
^2*b^22 - 256*a^3*b^21 - 568*a^4*b^20 + 1016*a^5*b^19 + 280*a^6*b^18 - 1176
*a^7*b^17 + 288*a^8*b^16 + 416*a^9*b^15 - 192*a^10*b^14))/b^16 + (128*tan(c
/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(128*a^2*b^23 - 384*a^3*b^22 + 51
2*a^4*b^21 - 512*a^5*b^20 + 384*a^6*b^19 - 128*a^7*b^18))/(a*b^21))*((a + b
)^5*(a - b)^5)^(1/2))/(a*b^5)))/(a*b^5))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^
5))*((a + b)^5*(a - b)^5)^(1/2))/(a*b^5)))*((a + b)^5*(a - b)^5)^(1/2)*2i)/
(a*b^5*d)
```

3.295 $\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{x}{a}+\frac{(2a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^3d} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^3d} - \frac{a\tan(c+dx)}{b^2d}+\frac{\sec(c+dx)\tan(c+dx)}{2bd}$$

```
[Out] x/a+1/2*(2*a^2-3*b^2)*arctanh(sin(d*x+c))/b^3/d-2*(a-b)^(3/2)*(a+b)^(3/2)*a
rctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/b^3/d-a*tan(d*x+c)/b^2
/d+1/2*sec(d*x+c)*tan(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3983, 2972, 3136, 2738, 214, 3855}

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(2a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^3d} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^3d} - \frac{a\tan(c+dx)}{b^2d}+\frac{x}{a}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

```
[In] Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] x/a + ((2*a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) - (2*(a - b)^(3/2)*
(a + b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*b^3*d
) - (a*Tan[c + d*x])/(b^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
```

```
+ n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\text{integral} = \int \frac{\sin(c+dx)\tan^3(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{\int \frac{(-2a^2+3b^2-ab\cos(c+dx)-2b^2\cos^2(c+dx))\sec(c+dx)}{b+a\cos(c+dx)}\,dx}{2b^2}$$

$$= \frac{x}{a} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{(a^2-b^2)^2\int \frac{1}{b+a\cos(c+dx)}\,dx}{ab^3} - \frac{(-2a^2+3b^2)\int \sec(c+dx)\,dx}{2b^3}$$

$$= \frac{x}{a} + \frac{(2a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{\left(2(a^2-b^2)^2\right)\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{ab^3d}$$

$$= \frac{x}{a} + \frac{(2a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^3d} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ab^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 287 vs. $2(126) = 252$.

Time = 1.91 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.28

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{(b+a\cos(c+dx))\sec(c+dx)\left(\frac{4c}{a} + \frac{4dx}{a} + \frac{8(a^2-b^2)^{3/2}\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^3} - \frac{4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.}{b^3}\right.}{}$$

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((4*c)/a + (4*d*x)/a + (8*(a^2 - b^2)^(3
/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^3) - (4*a^2*
```

```
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/b^3 + (6*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]])/b + (4*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b^3
- (6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b + 1/(b*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2) - 1/(b*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (4*
a*Tan[c + d*x])/b^2))/(4*d*(a + b*Sec[c + d*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(113) = 226$.

Time = 0.88 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{-\frac{2(a-b)\left(a^{3}+a^{2}b-a\,b^{2}-b^{3}\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^{3}a\sqrt{(a-b)(a+b)}}-\frac{1}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{-2a-b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(2a^{2}-3b^{2}\right)\ln\left(\tan\left(\right.\right.}{2b^{3}}}{d}$
default	$\frac{-\frac{2(a-b)\left(a^{3}+a^{2}b-a\,b^{2}-b^{3}\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^{3}a\sqrt{(a-b)(a+b)}}-\frac{1}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{-2a-b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(2a^{2}-3b^{2}\right)\ln\left(\tan\left(\right.\right.}{2b^{3}}}{d}$
risch	$\frac{x}{a}-\frac{i\left(b\,\mathrm{e}^{3i(dx+c)}+2a\,\mathrm{e}^{2i(dx+c)}-b\,\mathrm{e}^{i(dx+c)}+2a\right)}{d\,b^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)a^{2}}{d\,b^{3}}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2db}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)a^{2}}{d\,b^{3}}+$

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^3*(a-b)*(a^3+a^2*b-a*b^2-b^3)/a/((a-b)*(a+b))^(1/2)*arctanh((a-b)
*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/2/b/(tan(1/2*d*x+1/2*c)+1)^2-1/2
*(-2*a-b)/b^2/(tan(1/2*d*x+1/2*c)+1)+1/2*(2*a^2-3*b^2)/b^3*ln(tan(1/2*d*x+1
/2*c)+1)+2/a*arctan(tan(1/2*d*x+1/2*c))+1/2/b/(tan(1/2*d*x+1/2*c)-1)^2-1/2*
(-2*a-b)/b^2/(tan(1/2*d*x+1/2*c)-1)+1/2/b^3*(-2*a^2+3*b^2)*ln(tan(1/2*d*x+1
/2*c)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.52

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{4\,b^{3}dx\cos\left(dx+c\right)^{2}-2\left(a^{2}-b^{2}\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^{2}-2\,b^{2}\right)\cos(dx+c)^{2}+2\,\sqrt{a^{2}-b^{2}}(b\cos(dx+c)+a)\sin}{a^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+b^{2}}\right.}{}\right.$$

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b^3*d*x*cos(d*x + c)^2 - 2*(a^2 - b^2)^(3/2)*cos(d*x + c)^2*log((2*
a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(
d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d
*x + c) + b^2)) + (2*a^3 - 3*a*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) -
(2*a^3 - 3*a*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(2*a^2*b*cos(d*
x + c) - a*b^2)*sin(d*x + c))/(a*b^3*d*cos(d*x + c)^2), 1/4*(4*b^3*d*x*cos(
d*x + c)^2 - 4*(a^2 - b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos
(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^3 - 3*a*b^
2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^3 - 3*a*b^2)*cos(d*x + c)^2*
log(-sin(d*x + c) + 1) - 2*(2*a^2*b*cos(d*x + c) - a*b^2)*sin(d*x + c))/(a*
b^3*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\tan^4{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(113) = 226$.

Time = 0.90 (sec) , antiderivative size = 476, normalized size of antiderivative = 3.78

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{2\left(\left(a^2+ab-b^2\right)\sqrt{-a^2+b^2}|a||-a+b||b|+\left(a^3b+a^2b^2-ab^3-2\,b^4\right)\sqrt{-a^2+b^2}|-a+b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-\frac{b^4+\sqrt{b^8+\left(ab^3+b^4\right)\left(ab^3-b^4\right)}}{ab^3-b^4}}}\right)\right)}{(ab^2-b^3)a^2b^2+(ab^4-b^5)|a||b|}}{}$$

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*((a^2 + a*b - b^2)*sqrt(-a^2 + b^2)*abs(a)*abs(-a + b)*abs(b) + (a^
3*b + a^2*b^2 - a*b^3 - 2*b^4)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*
(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^4 + sqrt(b^8 + (
a*b^3 + b^4)*(a*b^3 - b^4)))/(a*b^3 - b^4))))/((a*b^2 - b^3)*a^2*b^2 + (a*b
^4 - b^5)*abs(a)*abs(b)) + 2*(a^4*b - 2*a^2*b^3 - a*b^4 + 2*b^5 - a^3*abs(a
)*abs(b) + 2*a*b^2*abs(a)*abs(b) - b^3*abs(a)*abs(b))*(pi*floor(1/2*(d*x +
c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b^4 - sqrt(b^8 + (a*b^3 +
 b^4)*(a*b^3 - b^4)))/(a*b^3 - b^4))))/(a^2*b^4 - b^4*abs(a)*abs(b)) - (2*a
^2 - 3*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + (2*a^2 - 3*b^2)*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a*tan(1/2*d*x + 1/2*c)^3 + b*tan(1/
2*d*x + 1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*c))/((tan
(1/2*d*x + 1/2*c)^2 - 1)^2*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 16.06 (sec) , antiderivative size = 6062, normalized size of antiderivative = 48.11

$$\int \frac{\tan^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

```
[In] int(tan(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

```
[Out] (atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^4*3i)/
(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*
sin(c/2 + (d*x)/2)^2)) + (atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*
sin(c/2 + (d*x)/2)^4*3i)/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4
- 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (cos(c/2 + (d*x)/2)*sin(c
/2 + (d*x)/2)^3)/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(
c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (cos(c/2 + (d*x)/2)^3*sin(c/2 + (
d*x)/2))/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (2*atan((4*a^13*sin(c/2 + (d*x)/2) + 4*a^
12*b*sin(c/2 + (d*x)/2) + 12*a^2*b^11*sin(c/2 + (d*x)/2) + 12*a^3*b^10*sin(
```

c/2 + (d*x)/2) + 15*a^4*b^9*sin(c/2 + (d*x)/2) + 15*a^5*b^8*sin(c/2 + (d*x)/2) - 59*a^6*b^7*sin(c/2 + (d*x)/2) - 59*a^7*b^6*sin(c/2 + (d*x)/2) + 57*a^8*b^5*sin(c/2 + (d*x)/2) + 57*a^9*b^4*sin(c/2 + (d*x)/2) - 24*a^10*b^3*sin(c/2 + (d*x)/2) - 24*a^11*b^2*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2)*(12*a*b^11 + 4*a^11*b + 4*a^12 + 12*a^2*b^10 + 15*a^3*b^9 + 15*a^4*b^8 - 59*a^5*b^7 - 59*a^6*b^6 + 57*a^7*b^5 + 57*a^8*b^4 - 24*a^9*b^3 - 24*a^10*b^2)))*cos(c/2 + (d*x)/2)^4)/(a*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (2*atan((4*a^13*sin(c/2 + (d*x)/2) + 4*a^12*b*sin(c/2 + (d*x)/2) + 12*a^2*b^11*sin(c/2 + (d*x)/2) + 12*a^3*b^10*sin(c/2 + (d*x)/2) + 15*a^4*b^9*sin(c/2 + (d*x)/2) + 15*a^5*b^8*sin(c/2 + (d*x)/2) - 59*a^6*b^7*sin(c/2 + (d*x)/2) - 59*a^7*b^6*sin(c/2 + (d*x)/2) + 57*a^8*b^5*sin(c/2 + (d*x)/2) + 57*a^9*b^4*sin(c/2 + (d*x)/2) - 24*a^10*b^3*sin(c/2 + (d*x)/2) - 24*a^11*b^2*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2)*(12*a*b^11 + 4*a^11*b + 4*a^12 + 12*a^2*b^10 + 15*a^3*b^9 + 15*a^4*b^8 - 59*a^5*b^7 - 59*a^6*b^6 + 57*a^7*b^5 + 57*a^8*b^4 - 24*a^9*b^3 - 24*a^10*b^2)))*sin(c/2 + (d*x)/2)^4)/(a*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (a^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^4*2i)/(b^3*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (a^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(c/2 + (d*x)/2)^4*2i)/(b^3*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2*6i)/(b*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (2*a*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^3)/(b^2*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (2*a*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2))/(b^2*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (4*atan((4*a^13*sin(c/2 + (d*x)/2) + 4*a^12*b*sin(c/2 + (d*x)/2) + 12*a^2*b^11*sin(c/2 + (d*x)/2) + 12*a^3*b^10*sin(c/2 + (d*x)/2) + 15*a^4*b^9*sin(c/2 + (d*x)/2) + 15*a^5*b^8*sin(c/2 + (d*x)/2) - 59*a^6*b^7*sin(c/2 + (d*x)/2) - 59*a^7*b^6*sin(c/2 + (d*x)/2) + 57*a^8*b^5*sin(c/2 + (d*x)/2) + 57*a^9*b^4*sin(c/2 + (d*x)/2) - 24*a^10*b^3*sin(c/2 + (d*x)/2) - 24*a^11*b^2*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2)*(12*a*b^11 + 4*a^11*b + 4*a^12 + 12*a^2*b^10 + 15*a^3*b^9 + 15*a^4*b^8 - 59*a^5*b^7 - 59*a^6*b^6 + 57*a^7*b^5 + 57*a^8*b^4 - 24*a^9*b^3 - 24*a^10*b^2)))*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)/(a*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (a^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2*4i)/(b^3*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (atan(((8*a^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 8*a^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*b^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 26*a^2*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/

2) - 6*a^3*b^6*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2)
 + 21*a^4*b^5*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2)
+ 9*a^5*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) +
12*a^6*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 2
0*a^7*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 22
*a^2*b^13*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 14
*a^3*b^12*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 36
*a^4*b^11*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 24
*a^5*b^10*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 47
*a^6*b^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 79*
a^7*b^8*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 49*a
^8*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 65*a^
9*b^6*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 24*a^1
0*b^5*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 24*a^1
1*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 4*a^12
*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 4*a^13*
b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 8*a*b^2*
sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*a^2*b*sin(
c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) - 8*a*b^8*sin(c/2
+ (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 8*a^8*b*sin(c/2 + (d
x)/2)(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2))*1i)/(a*b*cos(c/2 + (d*x)/
2)*(12*a*b^15 - 24*a^2*b^14 - 33*a^3*b^13 + 90*a^4*b^12 + 22*a^5*b^11 - 134
*a^6*b^10 + 17*a^7*b^9 + 100*a^8*b^8 - 31*a^9*b^7 - 38*a^10*b^6 + 16*a^11*b
^5 + 6*a^12*b^4 - 3*a^13*b^3)))*cos(c/2 + (d*x)/2)^4*((a + b)^3*(a - b)^3)^
(1/2)*2i)/(a*b^3*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2
 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) + (atan(((8*a^9*sin(c/2 + (d*x)/2)*(a^
6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 8*a^3*sin(c/2 + (d*x)/2)*(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*b^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^
4 - 3*a^4*b^2)^(3/2) - 26*a^2*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2)^(3/2) - 6*a^3*b^6*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 -
 3*a^4*b^2)^(3/2) + 21*a^4*b^5*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 -
3*a^4*b^2)^(3/2) + 9*a^5*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*
a^4*b^2)^(3/2) + 12*a^6*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a
^4*b^2)^(3/2) - 20*a^7*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2)^(3/2) - 22*a^2*b^13*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2)^(1/2) + 14*a^3*b^12*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2)^(1/2) + 36*a^4*b^11*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2)^(1/2) + 24*a^5*b^10*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2)^(1/2) - 47*a^6*b^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4
*b^2)^(1/2) - 79*a^7*b^8*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*
b^2)^(1/2) + 49*a^8*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b
^2)^(1/2) + 65*a^9*b^6*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2)^(1/2) - 24*a^10*b^5*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2)^(1/2) - 24*a^11*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^

```
2)^(1/2) + 4*a^12*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2
)^(1/2) + 4*a^13*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)
^(1/2) - 8*a*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/
2) + 8*a^2*b*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) -
 8*a*b^8*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 8*a
^8*b*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2))*1i)/(a*b
*cos(c/2 + (d*x)/2)*(12*a*b^15 - 24*a^2*b^14 - 33*a^3*b^13 + 90*a^4*b^12 +
22*a^5*b^11 - 134*a^6*b^10 + 17*a^7*b^9 + 100*a^8*b^8 - 31*a^9*b^7 - 38*a^1
0*b^6 + 16*a^11*b^5 + 6*a^12*b^4 - 3*a^13*b^3)))*sin(c/2 + (d*x)/2)^4*((a +
 b)^3*(a - b)^3)^(1/2)*2i)/(a*b^3*d*(cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)
/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2)) - (atan(((8*a^9*sin(c
/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 8*a^3*sin(c/2 + (
d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*b^3*sin(c/2 + (d*x)/2
)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(5/2) + 8*b^9*sin(c/2 + (d*x)/2)*(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 26*a^2*b^7*sin(c/2 + (d*x)/2)*(a^6
- b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 6*a^3*b^6*sin(c/2 + (d*x)/2)*(a^6 -
b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) + 21*a^4*b^5*sin(c/2 + (d*x)/2)*(a^6 - b
^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) + 9*a^5*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6
 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) + 12*a^6*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6
+ 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 20*a^7*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2)^(3/2) - 22*a^2*b^13*sin(c/2 + (d*x)/2)*(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 14*a^3*b^12*sin(c/2 + (d*x)/2)*(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 36*a^4*b^11*sin(c/2 + (d*x)/2)*(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 24*a^5*b^10*sin(c/2 + (d*x)/2)*(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 47*a^6*b^9*sin(c/2 + (d*x)/2)*(a^6 - b^6 +
3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 79*a^7*b^8*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3
*a^2*b^4 - 3*a^4*b^2)^(1/2) + 49*a^8*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2)^(1/2) + 65*a^9*b^6*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a
^2*b^4 - 3*a^4*b^2)^(1/2) - 24*a^10*b^5*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a
^2*b^4 - 3*a^4*b^2)^(1/2) - 24*a^11*b^4*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a
^2*b^4 - 3*a^4*b^2)^(1/2) + 4*a^12*b^3*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^
2*b^4 - 3*a^4*b^2)^(1/2) + 4*a^13*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2
*b^4 - 3*a^4*b^2)^(1/2) - 8*a*b^2*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2)^(5/2) + 8*a^2*b*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3
*a^4*b^2)^(5/2) - 8*a*b^8*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4
*b^2)^(3/2) - 8*a^8*b*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2
)^(3/2))*1i)/(a*b*cos(c/2 + (d*x)/2)*(12*a*b^15 - 24*a^2*b^14 - 33*a^3*b^13
 + 90*a^4*b^12 + 22*a^5*b^11 - 134*a^6*b^10 + 17*a^7*b^9 + 100*a^8*b^8 - 31
*a^9*b^7 - 38*a^10*b^6 + 16*a^11*b^5 + 6*a^12*b^4 - 3*a^13*b^3)))*cos(c/2 +
 (d*x)/2)^2*sin(c/2 + (d*x)/2)^2*((a + b)^3*(a - b)^3)^(1/2)*4i)/(a*b^3*d*(
cos(c/2 + (d*x)/2)^4 + sin(c/2 + (d*x)/2)^4 - 2*cos(c/2 + (d*x)/2)^2*sin(c/
2 + (d*x)/2)^2))
```

3.296 $\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{x}{a}+\frac{\operatorname{arctanh}(\sin(c+dx))}{bd} - \frac{2\sqrt{a-b}\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}$$

```
[Out] -x/a+arctanh(sin(d*x+c))/b/d-2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a/b/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3979, 4136, 3855, 4004, 3916, 2738, 214}

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2\sqrt{a-b}\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd} - \frac{x}{a}+\frac{\operatorname{arctanh}(\sin(c+dx))}{bd}$$

```
[In] Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + ArcTanh[Sin[c + d*x]]/(b*d) - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(
Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*b*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4136

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rubi steps

$$\text{integral} = \int \frac{-1+\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{\int \sec(c+dx)\,dx}{b} + \frac{\int \frac{-b-a\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$

$$= -\frac{x}{a} + \frac{\operatorname{arctanh}(\sin(c+dx))}{bd} - \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= -\frac{x}{a} + \frac{\operatorname{arctanh}(\sin(c+dx))}{bd} - \frac{\left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b}$$

$$= -\frac{x}{a} + \frac{\operatorname{arctanh}(\sin(c+dx))}{bd} - \frac{\left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$

$$= -\frac{x}{a} + \frac{\operatorname{arctanh}(\sin(c+dx))}{bd} - \frac{2\sqrt{a-b}\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.51

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx =$$
$$-\frac{bc+bdx-2\sqrt{a^2-b^2}\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-a\log\left(\right.}{abd}$$

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x]),x]
[Out] -((b*c + b*d*x - 2*Sqrt[a^2 - b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]] + a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - a*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]])/(a*b*d))
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{2(a-b)(a+b)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{ba\sqrt{(a-b)(a+b)}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{2(a-b)(a+b)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{ba\sqrt{(a-b)(a+b)}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$-\frac{x}{a}-\frac{\sqrt{a^2-b^2}\ln\left(e^{i(dx+c)}+\frac{b+i\sqrt{a^2-b^2}}{a}\right)}{dba}+\frac{\sqrt{a^2-b^2}\ln\left(e^{i(dx+c)}-\frac{i\sqrt{a^2-b^2}-b}{a}\right)}{dba}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{db}-\frac{\ln\left(e^{i(dx-\right.}}{db}$

[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/b*ln(tan(1/2*d*x+1/2*c)+1)-2/b*(a-b)*(a+b)/a/((a-b)*(a+b))^(1/2)*arc
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/b*ln(tan(1/2*d*x+1/2*c
)-1)-2/a*arctan(tan(1/2*d*x+1/2*c)))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.33

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx = \left[-\frac{2\,bdx - a\log\left(\sin\left(dx+c\right)+1\right) + a\log\left(-\sin\left(dx+c\right)+1\right) - \sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)}{a^2\cos(dx+}\right.}{2\,abd}\right.$$
$$-\frac{2\,bdx - a\log\left(\sin\left(dx+c\right)+1\right) + a\log\left(-\sin\left(dx+c\right)+1\right) + 2\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a}{(a^2-b^2)\sin(dx+c)}\right.}{2\,abd}$$

[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] [-1/2*(2*b*d*x - a*log(sin(d*x + c) + 1) + a*log(-sin(d*x + c) + 1) - sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(
a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + b^2)))/(a*b*d), -1/2*(2*b*d*x - a*log(sin(d*x +
 c) + 1) + a*log(-sin(d*x + c) + 1) + 2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))))/(a*b*d)]

Sympy [F]

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\tan^2\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c)),x)

[Out] Integral(tan(c + d*x)**2/(a + b*sec(c + d*x)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(67) = 134$.

Time = 0.49 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.84

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx =$$

$$-\frac{\frac{dx+c}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b} + \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)(a^2-b^2)}{\sqrt{-a^2+b^2}\,ab}}{d}$$

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c) + 1))/b + log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/b + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + ar
ctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*
(a^2 - b^2)/(sqrt(-a^2 + b^2)*a*b))/d
```

Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.59

$$\int \frac{\tan^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d} - \frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d} - \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)(a+b)}\right)\sqrt{a^2-b^2}}{a\,b\,d}$$

```
[In] int(tan(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

```
[Out] (2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) - (2*atan(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) - (2*atanh((sin(c/2 + (d*x)/2)*(a^2 - b
^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a + b)))*(a^2 - b^2)^(1/2))/(a*b*d)
```

3.297 $\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{x}{a}-\frac{2b^3\operatorname{arctanh}\left(\frac{\sqrt{a^2-b^2}\tan\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)}{a\left(a^2-b^2\right)^{3/2}d}-\frac{a\cot(c+dx)}{\left(a^2-b^2\right)d}+\frac{b\csc(c+dx)}{\left(a^2-b^2\right)d}$$

```
[Out] -x/a-2*b^3*arctanh((a^2-b^2)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b))/a/(a^2-b^2)^(3
/2)/d-a*cot(d*x+c)/(a^2-b^2)/d+b*csc(d*x+c)/(a^2-b^2)/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3983, 2981, 2686, 8, 3554, 2814, 2738, 214}

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{a\cot(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}+\frac{b^2x}{a\left(a^2-b^2\right)}-\frac{ax}{a^2-b^2}-\frac{2b^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{3/2}(a+b)^{3/2}}$$

```
[In] Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*x)/(a^2 - b^2)) + (b^2*x)/(a*(a^2 - b^2)) - (2*b^3*ArcTanh[(Sqrt[a - b
]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d) - (a*Co
t[c + d*x])/((a^2 - b^2)*d) + (b*Csc[c + d*x])/((a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
```

```
+ n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos(c+dx)\cot^2(c+dx)}{b+a\cos(c+dx)}\,dx \\
&= \frac{a\int \cot^2(c+dx)\,dx}{a^2-b^2} - \frac{b\int \cot(c+dx)\csc(c+dx)\,dx}{a^2-b^2} + \frac{b^2\int \frac{\cos(c+dx)}{b+a\cos(c+dx)}\,dx}{a^2-b^2} \\
&= \frac{b^2x}{a\left(a^2-b^2\right)} - \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} - \frac{a\int 1\,dx}{a^2-b^2} - \frac{b^3\int \frac{1}{b+a\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)} + \frac{b\text{Subst}\left(\int 1\,dx, x, \csc(c+dx)\right)}{\left(a^2-b^2\right)d} \\
&= -\frac{ax}{a^2-b^2} + \frac{b^2x}{a\left(a^2-b^2\right)} - \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} + \frac{b\csc(c+dx)}{\left(a^2-b^2\right)d} \\
&\quad - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a\left(a^2-b^2\right)d} \\
&= -\frac{ax}{a^2-b^2} + \frac{b^2x}{a\left(a^2-b^2\right)} - \frac{2b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}d} - \frac{a\cot(c+dx)}{\left(a^2-b^2\right)d} + \frac{b\csc(c+dx)}{\left(a^2-b^2\right)d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

$$
\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-2b^3\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\sin(c+dx)+\sqrt{a^2-b^2}(-ab+a^2\cos(\cdots}{2a(a-b)(a+b)\sqrt{a^2-b^2}d}
$$

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/2*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-2*b^3*ArcTanh[((-a + b)*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]]*Sin[c + d*x] + Sqrt[a^2 - b^2]*(-(a*b) + a^2*Cos[
c + d*x] + (a^2 - b^2)*(c + d*x)*Sin[c + d*x])))/(a*(a - b)*(a + b)*Sqrt[a^
2 - b^2]*d)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b^{3}\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)a\sqrt{(a-b)(a+b)}}-\frac{1}{2(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b^{3}\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)a\sqrt{(a-b)(a+b)}}-\frac{1}{2(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
risch	$-\frac{x}{a}-\frac{2i\left(-b\,e^{i(dx+c)}+a\right)}{d(a^{2}-b^{2})\left(e^{2i(dx+c)}-1\right)}-\frac{b^{3}\ln\left(e^{i(dx+c)}+\frac{ia^{2}-ib^{2}+b\sqrt{a^{2}-b^{2}}}{\sqrt{a^{2}-b^{2}}\,a}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)da}+\frac{b^{3}\ln\left(e^{i(dx+c)}+\frac{-ia^{2}+ib^{2}+b\sqrt{a^{2}-b^{2}}}{a\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)da}$

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*tan(1/2*d*x+1/2*c)/(a-b)-2/a*arctan(tan(1/2*d*x+1/2*c))-2/(a-b)/(a
+b)*b^3/a/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b)
)^(1/2))-1/2/(a+b)/tan(1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.42

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = \left[-\frac{\sqrt{a^2-b^2}b^3\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)\sin(dx+c)-2\,a^3b}{2\,(a^5-2\,a^3b^2+ab^4)d\sin(dx+}\right.$$
$$-\frac{\sqrt{-a^2+b^2}b^3\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\sin(dx+c)-a^3b+ab^3+(a^4-2\,a^2b^2+b^4)dx\sin(dx+}{(a^5-2\,a^3b^2+ab^4)d\sin(dx+c)}$$

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 - b^2)*b^3*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)
/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*a^3*b +
2*a*b^3 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*x*sin(d*x + c) + 2*(a^4 - a^2*b^2)*co
s(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*sin(d*x + c)), -(sqrt(-a^2 + b^2)*
b^3*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)
))*sin(d*x + c) - a^3*b + a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*d*x*sin(d*x + c)
+ (a^4 - a^2*b^2)*cos(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cot^2{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c)),x)
[Out] Integral(cot(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(101) = 202$.

Time = 0.38 (sec) , antiderivative size = 582, normalized size of antiderivative = 5.49

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{2\left(a^5-a^4b-2a^3b^2+3a^2b^3+ab^4-2b^5-a^2|-a^3+ab^2|+ab|-a^3+ab^2|+b^2|-a^3+ab^2|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}dx+\cdots\right)}{\sqrt{-\frac{a^2b-b^3+\sqrt{\left(a^3+a^2b-ab^2-b^3\right)\left(a\cdots\right.}}{a^3-a^2b-\cdots}}}\right.\right.}{a^2b|-a^3+ab^2|-b^3|-a^3+ab^2|+\left(a^3-ab^2\right)^2}}{\cdots}$$

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] -1/2*(2*(a^5 - a^4*b - 2*a^3*b^2 + 3*a^2*b^3 + a*b^4 - 2*b^5 - a^2*abs(-a^3
 + a*b^2) + a*b*abs(-a^3 + a*b^2) + b^2*abs(-a^3 + a*b^2))*(pi*floor(1/2*(d
*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^2*b - b^3 + sqrt((
a^3 + a^2*b - a*b^2 - b^3)*(a^3 - a^2*b - a*b^2 + b^3) + (a^2*b - b^3)^2))/
(a^3 - a^2*b - a*b^2 + b^3))))/(a^2*b*abs(-a^3 + a*b^2) - b^3*abs(-a^3 + a*
```

```
b^2) + (a^3 - a*b^2)^2) + 2*((a^2 - a*b - b^2)*sqrt(-a^2 + b^2)*abs(-a^3 +
a*b^2)*abs(-a + b) + (a^5 - a^4*b - 2*a^3*b^2 + 3*a^2*b^3 + a*b^4 - 2*b^5)*
sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(ta
n(1/2*d*x + 1/2*c)/sqrt(-(a^2*b - b^3 - sqrt((a^3 + a^2*b - a*b^2 - b^3)*(a
^3 - a^2*b - a*b^2 + b^3) + (a^2*b - b^3)^2))/(a^3 - a^2*b - a*b^2 + b^3)))
)/((a^3 - a*b^2)^2*(a^2 - 2*a*b + b^2) - (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5
)*abs(-a^3 + a*b^2)) - tan(1/2*d*x + 1/2*c)/(a - b) + 1/((a + b)*tan(1/2*d*
x + 1/2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 16.55 (sec) , antiderivative size = 1002, normalized size of antiderivative = 9.45

$$\int \frac{\cot^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{1\mathrm{i}\cos(c+d\,x)\,a^6-a^5\,b\,1\mathrm{i}-2\mathrm{i}\cos(c+d\,x)\,a^4\,b^2+a^3\,b^3\,2\mathrm{i}+1\mathrm{i}\cos(c+d\,x)\,a^2\,b^4-a\,b^5\,1\mathrm{i}}{1\mathrm{i}\,d\sin(c+d\,x)\,a^7-3\mathrm{i}\,d\sin(c+d\,x)\,a^5\,b^2+3\mathrm{i}\,d\sin(c+d\,x)\,a^3\,b^4-1\mathrm{i}\,d\sin(c+d\,x)\,a\,b^6} + \frac{-a^6\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)2\mathrm{i}+b^6\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)2\mathrm{i}-a^2\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)6\mathrm{i}+a^4\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right.}{}$$

```
[In] int(cot(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

```
[Out] (b^6*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*2i - a^6*atan(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2))*2i - a^2*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*6i + a^4*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*6i + b^3*
atanh((2*b^7*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(3/2) -
 a^13*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 2*b^13
*sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 9*a^2*b^11*
sin(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 3*a^3*b^10*s
in(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 18*a^4*b^9*si
n(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 12*a^5*b^8*sin
(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 21*a^6*b^7*sin(
c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 19*a^7*b^6*sin(c
/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 15*a^8*b^5*sin(c/
2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 15*a^9*b^4*sin(c/2
 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) - 6*a^10*b^3*sin(c/2
+ (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + 6*a^11*b^2*sin(c/2 +
 (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2) + a^12*b*sin(c/2 + (d*x
)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a^16 -
 3*a^2*b^14 + 18*a^4*b^12 - 46*a^6*b^10 + 65*a^8*b^8 - 55*a^10*b^6 + 28*a^1
2*b^4 - 8*a^14*b^2)))*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)*2i)/(a^7*d*
1i + a^3*b^4*d*3i - a^5*b^2*d*3i - a*b^6*d*1i) - (a^6*cos(c + d*x)*1i - a*b
^5*1i - a^5*b*1i + a^3*b^3*2i + a^2*b^4*cos(c + d*x)*1i - a^4*b^2*cos(c + d
```

```
*x)*2i)/(a^7*d*sin(c + d*x)*1i - a*b^6*d*sin(c + d*x)*1i + a^3*b^4*d*sin(c
+ d*x)*3i - a^5*b^2*d*sin(c + d*x)*3i)
```

3.298 $\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{x}{a}-\frac{2b^5\operatorname{arctanh}\left(\frac{\sqrt{a^2-b^2}\tan\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)}{a\left(a^2-b^2\right)^{5/2}d}+\frac{a(a^2-2b^2)\cot(c+dx)}{\left(a^2-b^2\right)^2d}-\frac{a\cot^3(c+dx)}{3\left(a^2-b^2\right)d}-\frac{b(a^2-2b^2)\csc(c+dx)}{\left(a^2-b^2\right)^2d}+\frac{b\csc^3(c+dx)}{3\left(a^2-b^2\right)d}$$

```
[Out] x/a-2*b^5*arctanh((a^2-b^2)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b))/a/(a^2-b^2)^(5/
2)/d+a*(a^2-2*b^2)*cot(d*x+c)/(a^2-b^2)^2/d-1/3*a*cot(d*x+c)^3/(a^2-b^2)/d-
b*(a^2-2*b^2)*csc(d*x+c)/(a^2-b^2)^2/d+1/3*b*csc(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.45, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3983, 2981, 2686, 3554, 8, 2814, 2738, 214}

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{a\cot^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{ab^2\cot(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{a\cot(c+dx)}{d\left(a^2-b^2\right)}+\frac{b\csc^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)}-\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{ax}{a^2-b^2}+\frac{b^4x}{a\left(a^2-b^2\right)^2}+\frac{b^3\csc(c+dx)}{d\left(a^2-b^2\right)^2}-\frac{2b^5\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}$$

```
[In] Int[Cot[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) + (b^4*x)/(a*(a^2 - b^2)^2) + (a*x)/(a^2 - b^2)
- (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b)^(5
/2)*(a + b)^(5/2)*d) - (a*b^2*Cot[c + d*x])/((a^2 - b^2)^2*d) + (a*Cot[c +
d*x])/((a^2 - b^2)*d) - (a*Cot[c + d*x]^3)/(3*(a^2 - b^2)*d) + (b^3*Csc[c +
 d*x])/((a^2 - b^2)^2*d) - (b*Csc[c + d*x])/((a^2 - b^2)*d) + (b*Csc[c + d*
x]^3)/(3*(a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos(c+dx)\cot^4(c+dx)}{b+a\cos(c+dx)}\,dx \\
&= \frac{a\int \cot^4(c+dx)\,dx}{a^2-b^2} - \frac{b\int \cot^3(c+dx)\csc(c+dx)\,dx}{a^2-b^2} + \frac{b^2\int \frac{\cos(c+dx)\cot^2(c+dx)}{b+a\cos(c+dx)}\,dx}{a^2-b^2} \\
&= -\frac{a\cot^3(c+dx)}{3(a^2-b^2)d} + \frac{(ab^2)\int \cot^2(c+dx)\,dx}{(a^2-b^2)^2} - \frac{b^3\int \cot(c+dx)\csc(c+dx)\,dx}{(a^2-b^2)^2} \\
&\quad + \frac{b^4\int \frac{\cos(c+dx)}{b+a\cos(c+dx)}\,dx}{(a^2-b^2)^2} - \frac{a\int \cot^2(c+dx)\,dx}{a^2-b^2} + \frac{b\text{Subst}\left(\int (-1+x^2)\,dx, x, \csc(c+dx)\right)}{(a^2-b^2)d} \\
&= \frac{b^4x}{a(a^2-b^2)^2} - \frac{ab^2\cot(c+dx)}{(a^2-b^2)^2d} + \frac{a\cot(c+dx)}{(a^2-b^2)d} - \frac{a\cot^3(c+dx)}{3(a^2-b^2)d} \\
&\quad - \frac{b\csc(c+dx)}{(a^2-b^2)d} + \frac{b\csc^3(c+dx)}{3(a^2-b^2)d} - \frac{(ab^2)\int 1\,dx}{(a^2-b^2)^2} \\
&\quad - \frac{b^5\int \frac{1}{b+a\cos(c+dx)}\,dx}{a(a^2-b^2)^2} + \frac{a\int 1\,dx}{a^2-b^2} + \frac{b^3\text{Subst}(\int 1\,dx, x, \csc(c+dx))}{(a^2-b^2)^2d} \\
&= -\frac{ab^2x}{(a^2-b^2)^2} + \frac{b^4x}{a(a^2-b^2)^2} + \frac{ax}{a^2-b^2} - \frac{ab^2\cot(c+dx)}{(a^2-b^2)^2d} \\
&\quad + \frac{a\cot(c+dx)}{(a^2-b^2)d} - \frac{a\cot^3(c+dx)}{3(a^2-b^2)d} + \frac{b^3\csc(c+dx)}{(a^2-b^2)^2d} - \frac{b\csc(c+dx)}{(a^2-b^2)d} \\
&\quad + \frac{b\csc^3(c+dx)}{3(a^2-b^2)d} - \frac{(2b^5)\,\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a(a^2-b^2)^2d} \\
&= -\frac{ab^2x}{(a^2-b^2)^2} + \frac{b^4x}{a(a^2-b^2)^2} + \frac{ax}{a^2-b^2} - \frac{2b^5\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} - \frac{ab^2\cot(c+dx)}{(a^2-b^2)^2d} \\
&\quad + \frac{a\cot(c+dx)}{(a^2-b^2)d} - \frac{a\cot^3(c+dx)}{3(a^2-b^2)d} + \frac{b^3\csc(c+dx)}{(a^2-b^2)^2d} - \frac{b\csc(c+dx)}{(a^2-b^2)d} + \frac{b\csc^3(c+dx)}{3(a^2-b^2)d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 416 vs. $2(177) = 354$.

Time = 6.44 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.35

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(c+dx)(b+a\cos(c+dx))\sec(c+dx)}{ad(a+b\sec(c+dx))} + \frac{2b^5\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))\sec(c+dx)}{a\sqrt{a^2-b^2}\,(-a^2+b^2)^2\,d(a+b\sec(c+dx))} + \frac{\left(8a\cos\left(\frac{1}{2}(c+dx)\right)+11b\cos\left(\frac{1}{2}(c+dx)\right)\right)(b+a\cos(c+dx))\csc\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}{12(a+b)^2d(a+b\sec(c+dx))} - \frac{(b+a\cos(c+dx))\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}{24(a+b)d(a+b\sec(c+dx))} + \frac{(b+a\cos(c+dx))\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-8a\sin\left(\frac{1}{2}(c+dx)\right)+11b\sin\left(\frac{1}{2}(c+dx)\right)\right)}{12(-a+b)^2d(a+b\sec(c+dx))} - \frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{24(-a+b)d(a+b\sec(c+dx))}$$

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((c + d*x)*(b + a*Cos[c + d*x])*Sec[c + d*x])/(a*d*(a + b*Sec[c + d*x])) +
(2*b^5*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c +
d*x])*Sec[c + d*x])/(a*Sqrt[a^2 - b^2]*(-a^2 + b^2)^2*d*(a + b*Sec[c + d*x]
)) + ((8*a*Cos[(c + d*x)/2] + 11*b*Cos[(c + d*x)/2])*(b + a*Cos[c + d*x])*C
sc[(c + d*x)/2]*Sec[c + d*x])/(12*(a + b)^2*d*(a + b*Sec[c + d*x])) - ((b +
 a*Cos[c + d*x])*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*Sec[c + d*x])/(24*(a +
 b)*d*(a + b*Sec[c + d*x])) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]*Sec[c
+ d*x]*(-8*a*Sin[(c + d*x)/2] + 11*b*Sin[(c + d*x)/2]))/(12*(-a + b)^2*d*(a
 + b*Sec[c + d*x])) - ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sec[c + d*x]
*Tan[(c + d*x)/2])/(24*(-a + b)*d*(a + b*Sec[c + d*x]))
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8(a-b)^2}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{1}{24(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-5a}{8(a+b)^2\,\mathrm{ta}}}{d}$
default	$\frac{\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8(a-b)^2}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{1}{24(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-5a}{8(a+b)^2\,\mathrm{ta}}}{d}$
risch	$\frac{x}{a}-\frac{2i\left(3a^2b\,e^{5i(dx+c)}-6b^3e^{5i(dx+c)}-6a^3e^{4i(dx+c)}+9a\,b^2e^{4i(dx+c)}-2a^2b\,e^{3i(dx+c)}+8b^3e^{3i(dx+c)}+6a^3e^{2i(dx+c)}-12a\right.}{3\left(a^4-2a^2b^2+b^4\right)\left(e^{2i(dx+c)}-1\right)^3d}$

```
[In] int(cot(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/(a-b)^2*(1/3*tan(1/2*d*x+1/2*c)^3*a-1/3*tan(1/2*d*x+1/2*c)^3*b-5*t
an(1/2*d*x+1/2*c)*a+7*tan(1/2*d*x+1/2*c)*b)+2/a*arctan(tan(1/2*d*x+1/2*c))-
1/24/(a+b)/tan(1/2*d*x+1/2*c)^3-1/8/(a+b)^2*(-5*a-7*b)/tan(1/2*d*x+1/2*c)-2
/(a+b)^2/(a-b)^2*b^5/a/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)
/((a-b)*(a+b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(168) = 336$.

Time = 0.31 (sec) , antiderivative size = 742, normalized size of antiderivative = 4.19

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{4\,a^5b-14\,a^3b^3+10\,ab^5+2\left(4\,a^6-11\,a^4b^2+7\,a^2b^4\right)\cos\left(dx+c\right)^3+3\left(b^5\cos\left(dx+c\right)^2-b^5\right)\sqrt{a^2-b^2}\,\mathrm{lo}}{}\right.$$

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(4*a^5*b - 14*a^3*b^3 + 10*a*b^5 + 2*(4*a^6 - 11*a^4*b^2 + 7*a^2*b^4)*
cos(d*x + c)^3 + 3*(b^5*cos(d*x + c)^2 - b^5)*sqrt(a^2 - b^2)*log((2*a*b*co
s(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x +
c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + b^2))*sin(d*x + c) - 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)^2 - 6
*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c) + 6*((a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*d*x*cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*x)*sin
(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^2 - (a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)*sin(d*x + c)), 1/3*(2*a^5*b - 7*a^3*b^3
 + 5*a*b^5 + (4*a^6 - 11*a^4*b^2 + 7*a^2*b^4)*cos(d*x + c)^3 - 3*(b^5*cos(d
*x + c)^2 - b^5)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^5*b - 3*a^3*b^3 + 2*a*
```

```
b^5)*cos(d*x + c)^2 - 3*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c) + 3*((a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*x*cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*d*x)*sin(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d
*cos(d*x + c)^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cot^4\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

```
[In] integrate(cot(d*x+c)**4/(a+b*sec(d*x+c)),x)
[Out] Integral(cot(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(168) = 336$.

Time = 0.48 (sec) , antiderivative size = 1073, normalized size of antiderivative = 6.06

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] -1/24*(24*((a^4 - a^3*b - 2*a^2*b^2 + 2*a*b^3 + b^4)*sqrt(-a^2 + b^2)*abs(a
^5 - 2*a^3*b^2 + a*b^4)*abs(-a + b) - (a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3
+ 6*a^5*b^4 - 7*a^4*b^5 - 4*a^3*b^6 + 6*a^2*b^7 + a*b^8 - 2*b^9)*sqrt(-a^2
+ b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x
+ 1/2*c)/sqrt(-(a^4*b - 2*a^2*b^3 + b^5 + sqrt((a^5 + a^4*b - 2*a^3*b^2 - 2
```

```
*a^2*b^3 + a*b^4 + b^5)*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)
 + (a^4*b - 2*a^2*b^3 + b^5)^2))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b
^4 - b^5))))/((a^5 - 2*a^3*b^2 + a*b^4)^2*(a^2 - 2*a*b + b^2) + (a^6*b - 2*
a^5*b^2 - a^4*b^3 + 4*a^3*b^4 - a^2*b^5 - 2*a*b^6 + b^7)*abs(a^5 - 2*a^3*b^
2 + a*b^4)) + 24*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 7*a^4*b
^5 - 4*a^3*b^6 + 6*a^2*b^7 + a*b^8 - 2*b^9 + a^4*abs(a^5 - 2*a^3*b^2 + a*b^
4) - a^3*b*abs(a^5 - 2*a^3*b^2 + a*b^4) - 2*a^2*b^2*abs(a^5 - 2*a^3*b^2 + a
*b^4) + 2*a*b^3*abs(a^5 - 2*a^3*b^2 + a*b^4) + b^4*abs(a^5 - 2*a^3*b^2 + a*
b^4))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(
-(a^4*b - 2*a^2*b^3 + b^5 - sqrt((a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b
^4 + b^5)*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5) + (a^4*b - 2*
a^2*b^3 + b^5)^2))/(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5))))/(
a^4*b*abs(a^5 - 2*a^3*b^2 + a*b^4) - 2*a^2*b^3*abs(a^5 - 2*a^3*b^2 + a*b^4)
 + b^5*abs(a^5 - 2*a^3*b^2 + a*b^4) - (a^5 - 2*a^3*b^2 + a*b^4)^2) - (a^2*t
an(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(1/2*d*x + 1/
2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c) + 36*a*b*tan(1/2*d*x + 1/2*c) - 21*b^2
*tan(1/2*d*x + 1/2*c))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (15*a*tan(1/2*d*x
+ 1/2*c)^2 + 21*b*tan(1/2*d*x + 1/2*c)^2 - a - b)/((a^2 + 2*a*b + b^2)*tan(
1/2*d*x + 1/2*c)^3))/d
```

Mupad [B] (verification not implemented)

Time = 23.38 (sec) , antiderivative size = 3859, normalized size of antiderivative = 21.80

$$\int \frac{\cot^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

```
[In] int(cot(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

```
[Out] (a^10*((cos(3*c + 3*d*x)*4i)/3 - sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2))*6i + atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3
*d*x)*2i) + a*((b^9*8i)/3 - b^9*cos(2*c + 2*d*x)*4i) - a^7*((b^3*14i)/3 - b
^3*cos(2*c + 2*d*x)*10i) + a^5*(b^5*10i - b^5*cos(2*c + 2*d*x)*18i) - a^3*(
(b^7*26i)/3 - b^7*cos(2*c + 2*d*x)*14i) + a^9*((b*2i)/3 - b*cos(2*c + 2*d*x
)*2i) + a^8*(b^2*cos(c + d*x)*1i - (b^2*cos(3*c + 3*d*x)*19i)/3 + b^2*sin(c
 + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*30i - b^2*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*10i) - a^2*(b^8*cos(c + d*x
)*1i - (b^8*cos(3*c + 3*d*x)*7i)/3 + b^8*sin(c + d*x)*atan(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2))*30i - b^8*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
)*sin(3*c + 3*d*x)*10i) - a^6*(b^4*cos(c + d*x)*3i - b^4*cos(3*c + 3*d*x)*1
1i + b^4*sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*60i - b^4
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*20i) + a^4*(b
^6*cos(c + d*x)*3i - (b^6*cos(3*c + 3*d*x)*25i)/3 + b^6*sin(c + d*x)*atan(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*60i - b^6*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*20i) + b^10*sin(c + d*x)*atan(sin(c/2 + (
```

d*x)/2)/cos(c/2 + (d*x)/2))*6i - b^10*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*2i + b^5*atanh((2*b^11*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(3/2) - a^21*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 2*b^21*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + a^20*b*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 15*a^2*b^19*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 5*a^3*b^18*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 55*a^4*b^17*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 35*a^5*b^16*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 130*a^6*b^15*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 110*a^7*b^14*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 215*a^8*b^13*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 205*a^9*b^12*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 253*a^10*b^11*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 251*a^11*b^10*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 210*a^12*b^9*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 210*a^13*b^8*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 120*a^14*b^7*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 120*a^15*b^6*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 45*a^16*b^5*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 45*a^17*b^4*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 10*a^18*b^3*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 10*a^19*b^2*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a^26 + 5*a^2*b^24 - 50*a^4*b^22 + 230*a^6*b^20 - 645*a^8*b^18 + 1231*a^10*b^16 - 1688*a^12*b^14 + 1708*a^14*b^12 - 1286*a^16*b^10 + 715*a^18*b^8 - 286*a^20*b^6 + 78*a^22*b^4 - 13*a^24*b^2)))*sin(3*c + 3*d*x)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)*2i - b^5*atanh((2*b^11*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(3/2) - a^21*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 2*b^21*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + a^20*b*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 15*a^2*b^19*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 5*a^3*b^18*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 55*a^4*b^17*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(

1/2) - 35*a^5*b^16*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 130*a^6*b^15*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 110*a^7*b^14*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 215*a^8*b^13*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 205*a^9*b^12*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 253*a^10*b^11*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 251*a^11*b^10*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 210*a^12*b^9*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 210*a^13*b^8*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 120*a^14*b^7*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 120*a^15*b^6*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 45*a^16*b^5*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 45*a^17*b^4*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) - 10*a^18*b^3*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2) + 10*a^19*b^2*sin(c/2 + (d*x)/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a^26 + 5*a^2*b^24 - 50*a^4*b^22 + 230*a^6*b^20 - 645*a^8*b^18 + 1231*a^10*b^16 - 1688*a^12*b^14 + 1708*a^14*b^12 - 1286*a^16*b^10 + 715*a^18*b^8 - 286*a^20*b^6 + 78*a^22*b^4 - 13*a^24*b^2)))*sin(c + d*x)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)*6i)/(a^11*d*sin(3*c + 3*d*x)*1i - a^11*d*sin(c + d*x)*3i + a^3*b^8*d*sin(3*c + 3*d*x)*5i - a^5*b^6*d*sin(3*c + 3*d*x)*10i + a^7*b^4*d*sin(3*c + 3*d*x)*10i - a^9*b^2*d*sin(3*c + 3*d*x)*5i + a*b^10*d*sin(c + d*x)*3i - a*b^10*d*sin(3*c + 3*d*x)*1i - a^3*b^8*d*sin(c + d*x)*15i + a^5*b^6*d*sin(c + d*x)*30i - a^7*b^4*d*sin(c + d*x)*30i + a^9*b^2*d*sin(c + d*x)*15i)

3.299 $\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 255

$$\begin{aligned}\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx = &-\frac{\log(\cos(c+dx))}{a^2d}+\frac{(a^2-b^2)^3(7a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^8d}\\&-\frac{2a(3a^4-8a^2b^2+6b^4)\sec(c+dx)}{b^7d}\\&+\frac{(5a^4-12a^2b^2+6b^4)\sec^2(c+dx)}{2b^6d}\\&-\frac{4a(a^2-2b^2)\sec^3(c+dx)}{3b^5d}+\frac{(3a^2-4b^2)\sec^4(c+dx)}{4b^4d}\\&-\frac{2a\sec^5(c+dx)}{5b^3d}+\frac{\sec^6(c+dx)}{6b^2d}+\frac{(a^2-b^2)^4}{ab^8d(a+b\sec(c+dx))}\end{aligned}$$

```
[Out] -ln(cos(d*x+c))/a^2/d+(a^2-b^2)^3*(7*a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^8/d-
2*a*(3*a^4-8*a^2*b^2+6*b^4)*sec(d*x+c)/b^7/d+1/2*(5*a^4-12*a^2*b^2+6*b^4)*s
ec(d*x+c)^2/b^6/d-4/3*a*(a^2-2*b^2)*sec(d*x+c)^3/b^5/d+1/4*(3*a^2-4*b^2)*se
c(d*x+c)^4/b^4/d-2/5*a*sec(d*x+c)^5/b^3/d+1/6*sec(d*x+c)^6/b^2/d+(a^2-b^2)^
4/a/b^8/d/(a+b*sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used

$= \{3970, 908\}$

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{(a^2-b^2)^4}{ab^8d(a+b\sec(c+dx))} + \frac{(a^2-b^2)^3(7a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^8d} - \frac{4a(a^2-2b^2)\sec^3(c+dx)}{3b^5d} + \frac{(3a^2-4b^2)\sec^4(c+dx)}{4b^4d} - \frac{\log(\cos(c+dx))}{a^2d} - \frac{2a(3a^4-8a^2b^2+6b^4)\sec(c+dx)}{b^7d} + \frac{(5a^4-12a^2b^2+6b^4)\sec^2(c+dx)}{2b^6d} - \frac{2a\sec^5(c+dx)}{5b^3d} + \frac{\sec^6(c+dx)}{6b^2d}$$

```
[In] Int[Tan[c + d*x]^9/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + ((a^2 - b^2)^3*(7*a^2 + b^2)*Log[a + b*Sec[c
 + d*x]])/(a^2*b^8*d) - (2*a*(3*a^4 - 8*a^2*b^2 + 6*b^4)*Sec[c + d*x])/(b^7
*d) + ((5*a^4 - 12*a^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/(2*b^6*d) - (4*a*(a^2 -
 2*b^2)*Sec[c + d*x]^3)/(3*b^5*d) + ((3*a^2 - 4*b^2)*Sec[c + d*x]^4)/(4*b^4
*d) - (2*a*Sec[c + d*x]^5)/(5*b^3*d) + Sec[c + d*x]^6/(6*b^2*d) + (a^2 - b^
2)^4/(a*b^8*d*(a + b*Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^4}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{b^8d}$$

$$= \frac{\text{Subst}\left(\int \left(-2a(3a^4-8a^2b^2+6b^4) + \frac{b^8}{a^2x} + (5a^4-12a^2b^2+6b^4)\,x - 4a(a^2-2b^2)\,x^2 + (3a^2-4b^2\right.\right.}{b^8d}$$

$$= -\frac{\log(\cos(c+dx))}{a^2d} + \frac{(a^2-b^2)^3(7a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^8d} - \frac{2a(3a^4-8a^2b^2+6b^4)\sec(c+dx)}{b^7d} + \frac{(5a^4-12a^2b^2+6b^4)\sec^2(c+dx)}{2b^6d} - \frac{4a(a^2-2b^2)\sec^3(c+dx)}{3b^5d} + \frac{(3a^2-4b^2)\sec^4(c+dx)}{4b^4d} - \frac{2a\sec^5(c+dx)}{5b^3d} + \frac{\sec^6(c+dx)}{6b^2d} + \frac{(a^2-b^2)^4}{ab^8d(a+b\sec(c+dx))}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.90

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-\frac{b^8\log(\cos(c+dx))}{a^2} + \frac{(a^2-b^2)^3(7a^2+b^2)\log(a+b\sec(c+dx))}{a^2} - 2ab(3a^4-8a^2b^2+6b^4)\sec(c+dx) + \frac{1}{2}b^2(5a^4-12a^2b^2+}{}$$

```
[In] Integrate[Tan[c + d*x]^9/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-((b^8*Log[Cos[c + d*x]])/a^2) + ((a^2 - b^2)^3*(7*a^2 + b^2)*Log[a + b*Se
c[c + d*x]])/a^2 - 2*a*b*(3*a^4 - 8*a^2*b^2 + 6*b^4)*Sec[c + d*x] + (b^2*(5
*a^4 - 12*a^2*b^2 + 6*b^4)*Sec[c + d*x]^2)/2 - (4*a*b^3*(a^2 - 2*b^2)*Sec[c
 + d*x]^3)/3 + (b^4*(3*a^2 - 4*b^2)*Sec[c + d*x]^4)/4 - (2*a*b^5*Sec[c + d*
x]^5)/5 + (b^6*Sec[c + d*x]^6)/6 + (a^2 - b^2)^4/(a*(a + b*Sec[c + d*x])))/
(b^8*d)
```

Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{-\frac{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}{a^2b^7(b+a\cos(dx+c))} + \frac{(7a^8-20a^6b^2+18a^4b^4-4a^2b^6-b^8)\ln(b+a\cos(dx+c))}{b^8a^2} - \frac{-3a^2+4b^2}{4b^4\cos(dx+c)^4} - \frac{-5a^4+12a^2b^2-6b^4}{2b^6\cos(dx+c)^2} + \frac{(-}{}}{d}$
default	$\frac{-\frac{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}{a^2b^7(b+a\cos(dx+c))} + \frac{(7a^8-20a^6b^2+18a^4b^4-4a^2b^6-b^8)\ln(b+a\cos(dx+c))}{b^8a^2} - \frac{-3a^2+4b^2}{4b^4\cos(dx+c)^4} - \frac{-5a^4+12a^2b^2-6b^4}{2b^6\cos(dx+c)^2} + \frac{(-}{}}{d}$
risch	Expression too large to display

```
[In] int(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(a^8-4*a^6*b^2+6*a^4*b^4-4*a^2*b^6+b^8)/a^2/b^7/(b+a*cos(d*x+c))+(7*a
^8-20*a^6*b^2+18*a^4*b^4-4*a^2*b^6-b^8)/b^8/a^2*ln(b+a*cos(d*x+c))-1/4*(-3*
a^2+4*b^2)/b^4/cos(d*x+c)^4-1/2*(-5*a^4+12*a^2*b^2-6*b^4)/b^6/cos(d*x+c)^2+
(-7*a^6+20*a^4*b^2-18*a^2*b^4+4*b^6)/b^8*ln(cos(d*x+c))+1/6/b^2/cos(d*x+c)^
```

```
6-2/5/b^3*a/cos(d*x+c)^5-4/3*a*(a^2-2*b^2)/b^5/cos(d*x+c)^3-2*a*(3*a^4-8*a^
2*b^2+6*b^4)/b^7/cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.66

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{14\,a^3b^6\cos(dx+c) - 10\,a^2b^7 + 60\,(7\,a^8b - 20\,a^6b^3 + 18\,a^4b^5 - 4\,a^2b^7 + b^9)\cos(dx+c)^6 + 30\,(7\,a^7b^2 - \cdots}{\cdots}$$

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(14*a^3*b^6*cos(d*x + c) - 10*a^2*b^7 + 60*(7*a^8*b - 20*a^6*b^3 + 18
*a^4*b^5 - 4*a^2*b^7 + b^9)*cos(d*x + c)^6 + 30*(7*a^7*b^2 - 20*a^5*b^4 + 1
8*a^3*b^6)*cos(d*x + c)^5 - 10*(7*a^6*b^3 - 20*a^4*b^5 + 18*a^2*b^7)*cos(d*
x + c)^4 + 5*(7*a^5*b^4 - 20*a^3*b^6)*cos(d*x + c)^3 - 3*(7*a^4*b^5 - 20*a^
2*b^7)*cos(d*x + c)^2 - 60*((7*a^9 - 20*a^7*b^2 + 18*a^5*b^4 - 4*a^3*b^6 -
a*b^8)*cos(d*x + c)^7 + (7*a^8*b - 20*a^6*b^3 + 18*a^4*b^5 - 4*a^2*b^7 - b^
9)*cos(d*x + c)^6)*log(a*cos(d*x + c) + b) + 60*((7*a^9 - 20*a^7*b^2 + 18*a
^5*b^4 - 4*a^3*b^6)*cos(d*x + c)^7 + (7*a^8*b - 20*a^6*b^3 + 18*a^4*b^5 - 4
*a^2*b^7)*cos(d*x + c)^6)*log(-cos(d*x + c)))/(a^3*b^8*d*cos(d*x + c)^7 + a
^2*b^9*d*cos(d*x + c)^6)
```

Sympy [F]

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

```
[In] integrate(tan(d*x+c)**9/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**9/(a + b*sec(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.26

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{14\,a^3b^5\cos(dx+c)-10\,a^2b^6+60\,(7\,a^8-20\,a^6b^2+18\,a^4b^4-4\,a^2b^6+b^8)\cos(dx+c)^6+30\,(7\,a^7b-20\,a^5b^3+18\,a^3b^5)\cos(dx+c)^5-10\,(7\,a^6b^2-20\,a^4b^4}{a^3b^7\cos(dx+c)^7+a^2b^8\cos(dx+c)^6}}{}$$

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*((14*a^3*b^5*cos(d*x + c) - 10*a^2*b^6 + 60*(7*a^8 - 20*a^6*b^2 + 18*
a^4*b^4 - 4*a^2*b^6 + b^8)*cos(d*x + c)^6 + 30*(7*a^7*b - 20*a^5*b^3 + 18*a
^3*b^5)*cos(d*x + c)^5 - 10*(7*a^6*b^2 - 20*a^4*b^4 + 18*a^2*b^6)*cos(d*x +
 c)^4 + 5*(7*a^5*b^3 - 20*a^3*b^5)*cos(d*x + c)^3 - 3*(7*a^4*b^4 - 20*a^2*b
^6)*cos(d*x + c)^2)/(a^3*b^7*cos(d*x + c)^7 + a^2*b^8*cos(d*x + c)^6) + 60*
(7*a^6 - 20*a^4*b^2 + 18*a^2*b^4 - 4*b^6)*log(cos(d*x + c))/b^8 - 60*(7*a^8
 - 20*a^6*b^2 + 18*a^4*b^4 - 4*a^2*b^6 - b^8)*log(a*cos(d*x + c) + b)/(a^2*
b^8))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1696 vs. 2(245) = 490.

Time = 5.91 (sec) , antiderivative size = 1696, normalized size of antiderivative = 6.65

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(d*x+c)^9/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*(7*a^9 - 7*a^8*b - 20*a^7*b^2 + 20*a^6*b^3 + 18*a^5*b^4 - 18*a^4*b
^5 - 4*a^3*b^6 + 4*a^2*b^7 - a*b^8 + b^9)*log(abs(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3*b^8
 - a^2*b^9) + 60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 -
 60*(7*a^6 - 20*a^4*b^2 + 18*a^2*b^4 - 4*b^6)*log(abs(-(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 1))/b^8 - 60*(7*a^9 + 9*a^8*b - 18*a^7*b^2 - 26*a^6*b^3
 + 12*a^5*b^4 + 24*a^4*b^5 + 2*a^3*b^6 - 6*a^2*b^7 - 3*a*b^8 - b^9 + 7*a^9*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 7*a^8*b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - 20*a^7*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a^6*b^3*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18*a^5*b^4*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 18*a^4*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^
6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*a^2*b^7*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - a*b^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b^9*(cos(d*x
```

```
 + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^2*b^8) + (1029*a^6 - 720
*a^5*b - 2940*a^4*b^2 + 1760*a^3*b^3 + 2646*a^2*b^4 - 1168*a*b^5 - 588*b^6
+ 6174*a^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3600*a^5*b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - 18240*a^4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 9120*a^3*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16956*a^2*b^4*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 6288*a*b^5*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 3888*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 15435*a^6*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 7200*a^5*b*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 - 46500*a^4*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 18240*a^3*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 44730*a^2*b^
4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12960*a*b^5*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 - 10740*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 + 20580*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 7200*a^5*b*(cos
(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 62400*a^4*b^2*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 + 17600*a^3*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1
)^3 + 60840*a^2*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 11680*a*b^5
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 15520*b^6*(cos(d*x + c) - 1)^3
/(cos(d*x + c) + 1)^3 + 15435*a^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4
 - 3600*a^5*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 46500*a^4*b^2*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 8160*a^3*b^3*(cos(d*x + c) - 1)^4/
(cos(d*x + c) + 1)^4 + 44730*a^2*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1
)^4 - 4560*a*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 10740*b^6*(cos
(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 6174*a^6*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 - 720*a^5*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 182
40*a^4*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 1440*a^3*b^3*(cos(d*
x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 16956*a^2*b^4*(cos(d*x + c) - 1)^5/(co
s(d*x + c) + 1)^5 - 720*a*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 3
888*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 1029*a^6*(cos(d*x + c)
- 1)^6/(cos(d*x + c) + 1)^6 - 2940*a^4*b^2*(cos(d*x + c) - 1)^6/(cos(d*x +
c) + 1)^6 + 2646*a^2*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 588*b^
6*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/(b^8*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 1)^6))/d
```

Mupad [B] (verification not implemented)

Time = 16.98 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.98

$$\int \frac{\tan^9(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{2\left(-105\,a^7-105\,a^6\,b+265\,a^5\,b^2+265\,a^4\,b^3-191\,a^3\,b^4-191\,a^2\,b^5+15\,a\,b^6+15\,b^7\right)}{15\,a\,b^7}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\left(-42\,a^7-7\,a^6\,b+113\,a^5\,b^2+13\,a^4\,b^3-95\,a^3\,b^4\right.}{a\,b^7}}{}$$
$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\left(7\,a^6-20\,a^4\,b^2+18\,a^2\,b^4-4\,b^6\right)}{b^8\,d}$$
$$+\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\left(a^2-b^2\right)^3\left(7\,a^2+b^2\right)}{a^2\,b^8\,d}$$

```
[In] int(tan(c + d*x)^9/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*(15*a*b^6 - 105*a^6*b - 105*a^7 + 15*b^7 - 191*a^2*b^5 - 191*a^3*b^4 +
265*a^4*b^3 + 265*a^5*b^2))/(15*a*b^7) - (2*tan(c/2 + (d*x)/2)^10*(19*a*b^6
 - 7*a^6*b - 42*a^7 + 6*b^7 - 5*a^2*b^5 - 95*a^3*b^4 + 13*a^4*b^3 + 113*a^5
*b^2))/(a*b^7) - (4*tan(c/2 + (d*x)/2)^6*(7*a*b^6 - 105*a^6*b - 210*a^7 + 3
0*b^7 - 145*a^2*b^5 - 362*a^3*b^4 + 244*a^4*b^3 + 523*a^5*b^2))/(3*a*b^7) +
 (2*tan(c/2 + (d*x)/2)^8*(91*a*b^6 - 105*a^6*b - 315*a^7 + 45*b^7 - 99*a^2*
b^5 - 613*a^3*b^4 + 223*a^4*b^3 + 809*a^5*b^2))/(3*a*b^7) + (2*tan(c/2 + (d
*x)/2)^4*(10*a*b^6 - 350*a^6*b - 525*a^7 + 75*b^7 - 598*a^2*b^5 - 862*a^3*b
^4 + 860*a^4*b^3 + 1290*a^5*b^2))/(5*a*b^7) - (2*tan(c/2 + (d*x)/2)^2*(45*a
*b^6 - 525*a^6*b - 630*a^7 + 90*b^7 - 955*a^2*b^5 - 1067*a^3*b^4 + 1325*a^4
*b^3 + 1555*a^5*b^2))/(15*a*b^7) + (2*tan(c/2 + (d*x)/2)^12*(4*a*b^6 - 7*a^
7 + b^7 - 18*a^3*b^4 + 20*a^5*b^2))/(a*b^7))/(d*(a + b - tan(c/2 + (d*x)/2)
^14*(a - b) - tan(c/2 + (d*x)/2)^2*(7*a + 5*b) + tan(c/2 + (d*x)/2)^12*(7*a
 - 5*b) + tan(c/2 + (d*x)/2)^4*(21*a + 9*b) - tan(c/2 + (d*x)/2)^10*(21*a -
 9*b) - tan(c/2 + (d*x)/2)^6*(35*a + 5*b) + tan(c/2 + (d*x)/2)^8*(35*a - 5*
b))) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (log(tan(c/2 + (d*x)/2)^2 -
1)*(7*a^6 - 4*b^6 + 18*a^2*b^4 - 20*a^4*b^2))/(b^8*d) + (log(a + b - a*tan(
c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(a^2 - b^2)^3*(7*a^2 + b^2))/(a^
2*b^8*d)
```

3.300 $\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\log(\cos(c+dx))}{a^2d} + \frac{(a^2-b^2)^2(5a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^6d} - \frac{2a(2a^2-3b^2)\sec(c+dx)}{b^5d} + \frac{3(a^2-b^2)\sec^2(c+dx)}{2b^4d} - \frac{2a\sec^3(c+dx)}{3b^3d} + \frac{\sec^4(c+dx)}{4b^2d} + \frac{(a^2-b^2)^3}{ab^6d(a+b\sec(c+dx))}$$

```
[Out] ln(cos(d*x+c))/a^2/d+(a^2-b^2)^2*(5*a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^6/d-2
*a*(2*a^2-3*b^2)*sec(d*x+c)/b^5/d+3/2*(a^2-b^2)*sec(d*x+c)^2/b^4/d-2/3*a*se
c(d*x+c)^3/b^3/d+1/4*sec(d*x+c)^4/b^2/d+(a^2-b^2)^3/a/b^6/d/(a+b*sec(d*x+c)
)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3970, 908}

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{(a^2-b^2)^3}{ab^6d(a+b\sec(c+dx))} + \frac{(a^2-b^2)^2(5a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^6d} - \frac{2a(2a^2-3b^2)\sec(c+dx)}{b^5d} + \frac{3(a^2-b^2)\sec^2(c+dx)}{2b^4d} + \frac{\log(\cos(c+dx))}{a^2d} - \frac{2a\sec^3(c+dx)}{3b^3d} + \frac{\sec^4(c+dx)}{4b^2d}$$

```
[In] Int[Tan[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((a^2 - b^2)^2*(5*a^2 + b^2)*Log[a + b*Sec[c +
d*x]])/(a^2*b^6*d) - (2*a*(2*a^2 - 3*b^2)*Sec[c + d*x])/(b^5*d) + (3*(a^2 -
 b^2)*Sec[c + d*x]^2)/(2*b^4*d) - (2*a*Sec[c + d*x]^3)/(3*b^3*d) + Sec[c +
d*x]^4/(4*b^2*d) + (a^2 - b^2)^3/(a*b^6*d*(a + b*Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(b^2-x^2)^3}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{b^6 d}\\
&= -\frac{\text{Subst}\left(\int \left(2(2a^3-3ab^2)+\frac{b^6}{a^2x}-3(a^2-b^2)\,x+2ax^2-x^3+\frac{(a^2-b^2)^3}{a(a+x)^2}-\frac{(a^2-b^2)^2(5a^2+b^2)}{a^2(a+x)}\right)\,dx, x, b\,\text{s}}{b^6 d}\\
&= \frac{\log(\cos(c+dx))}{a^2d}+\frac{(a^2-b^2)^2(5a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^6d}-\frac{2a(2a^2-3b^2)\sec(c+dx)}{b^5d}\\
&\quad+\frac{3(a^2-b^2)\sec^2(c+dx)}{2b^4d}-\frac{2a\sec^3(c+dx)}{3b^3d}+\frac{\sec^4(c+dx)}{4b^2d}+\frac{(a^2-b^2)^3}{ab^6d(a+b\sec(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.91

$$
\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{-\frac{b^6\log(\cos(c+dx))}{a^2}-\frac{(a^2-b^2)^2(5a^2+b^2)\log(a+b\sec(c+dx))}{a^2}+2ab(2a^2-3b^2)\sec(c+dx)-\frac{3}{2}(a-b)b^2(a+b)\sec^2(c}{b^6d}
$$

```
[In] Integrate[Tan[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((-((b^6*Log[Cos[c + d*x]])/a^2) - ((a^2 - b^2)^2*(5*a^2 + b^2)*Log[a + b*
Sec[c + d*x]])/a^2 + 2*a*b*(2*a^2 - 3*b^2)*Sec[c + d*x] - (3*(a - b)*b^2*(a
 + b)*Sec[c + d*x]^2)/2 + (2*a*b^3*Sec[c + d*x]^3)/3 - (b^4*Sec[c + d*x]^4)
/4 - (a^2 - b^2)^3/(a*(a + b*Sec[c + d*x])))/(b^6*d))
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{-\frac{a^6-3a^4b^2+3a^2b^4-b^6}{a^2b^5(b+a\cos(dx+c))}+\frac{\left(5a^6-9a^4b^2+3a^2b^4+b^6\right)\ln(b+a\cos(dx+c))}{b^6a^2}-\frac{-3a^2+3b^2}{2b^4\cos(dx+c)^2}+\frac{\left(-5a^4+9a^2b^2-3b^4\right)\ln(\cos(dx+c))}{b^6}+\frac{}{4b}}{d}$
default	$\frac{-\frac{a^6-3a^4b^2+3a^2b^4-b^6}{a^2b^5(b+a\cos(dx+c))}+\frac{\left(5a^6-9a^4b^2+3a^2b^4+b^6\right)\ln(b+a\cos(dx+c))}{b^6a^2}-\frac{-3a^2+3b^2}{2b^4\cos(dx+c)^2}+\frac{\left(-5a^4+9a^2b^2-3b^4\right)\ln(\cos(dx+c))}{b^6}+\frac{}{4b}}{d}$
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}-\frac{2\left(-12b^6e^{3i(dx+c)}-3b^6e^{i(dx+c)}+15a^6e^{i(dx+c)}+60a^6e^{3i(dx+c)}+60a^6e^{7i(dx+c)}+90a^6e^{5i(dx+c)}-3b^6e^{9}\right.}{}$

```
[In] int(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/a^2/b^5/(b+a*cos(d*x+c))+(5*a^6-9*a^4*b
^2+3*a^2*b^4+b^6)/b^6/a^2*ln(b+a*cos(d*x+c))-1/2*(-3*a^2+3*b^2)/b^4/cos(d*x
+c)^2+(-5*a^4+9*a^2*b^2-3*b^4)/b^6*ln(cos(d*x+c))+1/4/b^2/cos(d*x+c)^4-2/3/
b^3*a/cos(d*x+c)^3-2*a*(2*a^2-3*b^2)/b^5/cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.74

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{5\,a^3b^4\cos(dx+c)-3\,a^2b^5+12\,(5\,a^6b-9\,a^4b^3+3\,a^2b^5-b^7)\cos(dx+c)^4+6\,(5\,a^5b^2-9\,a^3b^4)\cos(dx}{}$$

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(5*a^3*b^4*cos(d*x + c) - 3*a^2*b^5 + 12*(5*a^6*b - 9*a^4*b^3 + 3*a^2
*b^5 - b^7)*cos(d*x + c)^4 + 6*(5*a^5*b^2 - 9*a^3*b^4)*cos(d*x + c)^3 - 2*(
5*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c)^2 - 12*((5*a^7 - 9*a^5*b^2 + 3*a^3*b^4
+ a*b^6)*cos(d*x + c)^5 + (5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x +
 c)^4)*log(a*cos(d*x + c) + b) + 12*((5*a^7 - 9*a^5*b^2 + 3*a^3*b^4)*cos(d*
x + c)^5 + (5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^4)*log(-cos(d*x +
 c)))/(a^3*b^6*d*cos(d*x + c)^5 + a^2*b^7*d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^7{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

```
[In] integrate(tan(d*x+c)**7/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**7/(a + b*sec(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.27

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{5\,a^3b^3\cos(dx+c)-3\,a^2b^4+12\,(5\,a^6-9\,a^4b^2+3\,a^2b^4-b^6)\cos(dx+c)^4+6\,(5\,a^5b-9\,a^3b^3)\cos(dx+c)^3-2\,(5\,a^4b^2-9\,a^2b^4)\cos(dx+c)^2}{a^3b^5\cos(dx+c)^5+a^2b^6\cos(dx+c)^4}+\frac{12\,(5\,a^4-}{}}{12\,d}$$

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*((5*a^3*b^3*cos(d*x + c) - 3*a^2*b^4 + 12*(5*a^6 - 9*a^4*b^2 + 3*a^2*
b^4 - b^6)*cos(d*x + c)^4 + 6*(5*a^5*b - 9*a^3*b^3)*cos(d*x + c)^3 - 2*(5*a
^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^2)/(a^3*b^5*cos(d*x + c)^5 + a^2*b^6*cos(d
*x + c)^4) + 12*(5*a^4 - 9*a^2*b^2 + 3*b^4)*log(cos(d*x + c))/b^6 - 12*(5*a
^6 - 9*a^4*b^2 + 3*a^2*b^4 + b^6)*log(a*cos(d*x + c) + b)/(a^2*b^6))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. 2(173) = 346.

Time = 3.43 (sec) , antiderivative size = 1023, normalized size of antiderivative = 5.72

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(12*(5*a^7 - 5*a^6*b - 9*a^5*b^2 + 9*a^4*b^3 + 3*a^3*b^4 - 3*a^2*b^5 +
 a*b^6 - b^7)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3*b^6 - a^2*b^7) - 12*log(abs(-(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - 12*(5*a^4 - 9*a^2*b^2 + 3*b
^4)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^6 - 12*(5*a^7 +
7*a^6*b - 7*a^5*b^2 - 13*a^4*b^3 - a^3*b^4 + 5*a^2*b^5 + 3*a*b^6 + b^7 + 5*
```

```
a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 5*a^6*b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 9*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^4*b^
3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^3*b^4*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 3*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*b^6*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - b^7*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1))*a^2*b^6) + (125*a^4 - 96*a^3*b - 225*a^2*b^2 + 128*
a*b^3 + 75*b^4 + 500*a^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 288*a^3*b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 972*a^2*b^2*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 416*a*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 348*b^4*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 750*a^4*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 - 288*a^3*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1494
*a^2*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 384*a*b^3*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 + 594*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 + 500*a^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 96*a^3*b*(cos(
d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 972*a^2*b^2*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 96*a*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 34
8*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 125*a^4*(cos(d*x + c) - 1
)^4/(cos(d*x + c) + 1)^4 - 225*a^2*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
 1)^4 + 75*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/(b^6*((cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 1)^4))/d
```

Mupad [B] (verification not implemented)

Time = 15.73 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.82

$$\int \frac{\tan^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(20\,a^5+5\,a^4\,b-31\,a^3\,b^2-4\,a^2\,b^3+8\,a\,b^4+4\,b^5\right)}{a\,b^5}-\frac{2\,\left(15\,a^5+15\,a^4\,b-22\,a^3\,b^2-22\,a^2\,b^3+3\,a\,b^4+3\,b^5\right)}{3\,a\,b^5}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(60\,a^5+4\right.}{}}{d\,\left(\left(b-a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\left(5\,a-3\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\left(2\,b-10\right.\right.} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(5\,a^4-9\,a^2\,b^2+3\,b^4\right)}{b^6\,d} + \frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(a^2-b^2\right)^2\,\left(5\,a^2+b^2\right)}{a^2\,b^6\,d}$$

```
[In] int(tan(c + d*x)^7/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^6*(8*a*b^4 + 5*a^4*b + 20*a^5 + 4*b^5 - 4*a^2*b^3 -
31*a^3*b^2))/(a*b^5) - (2*(3*a*b^4 + 15*a^4*b + 15*a^5 + 3*b^5 - 22*a^2*b^3
 - 22*a^3*b^2))/(3*a*b^5) + (2*tan(c/2 + (d*x)/2)^2*(6*a*b^4 + 45*a^4*b + 6
0*a^5 + 12*b^5 - 66*a^2*b^3 - 83*a^3*b^2))/(3*a*b^5) - (2*tan(c/2 + (d*x)/2
)^4*(6*a*b^4 + 45*a^4*b + 90*a^5 + 18*b^5 - 56*a^2*b^3 - 127*a^3*b^2))/(3*a
```

```
*b^5) + (2*tan(c/2 + (d*x)/2)^8*(a - b)*(4*a*b^3 - 5*a^3*b - 5*a^4 + b^4 +
4*a^2*b^2))/(a*b^5))/(d*(a + b - tan(c/2 + (d*x)/2)^10*(a - b) - tan(c/2 +
(d*x)/2)^2*(5*a + 3*b) + tan(c/2 + (d*x)/2)^4*(10*a + 2*b) + tan(c/2 + (d*x
)/2)^8*(5*a - 3*b) - tan(c/2 + (d*x)/2)^6*(10*a - 2*b))) - log(tan(c/2 + (d
*x)/2)^2 + 1)/(a^2*d) - (log(tan(c/2 + (d*x)/2)^2 - 1)*(5*a^4 + 3*b^4 - 9*a
^2*b^2))/(b^6*d) + (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/
2)^2)*(a^2 - b^2)^2*(5*a^2 + b^2))/(a^2*b^6*d)
```

3.301 $\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\log(\cos(c+dx))}{a^2d} + \frac{(a^2-b^2)\,(3a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^4d} - \frac{2a\sec(c+dx)}{b^3d} + \frac{\sec^2(c+dx)}{2b^2d} + \frac{(a^2-b^2)^2}{ab^4d(a+b\sec(c+dx))}$$

```
[Out] -ln(cos(d*x+c))/a^2/d+(a^2-b^2)*(3*a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^4/d-2*
a*sec(d*x+c)/b^3/d+1/2*sec(d*x+c)^2/b^2/d+(a^2-b^2)^2/a/b^4/d/(a+b*sec(d*x+
c))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3970, 908}

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{(a^2-b^2)^2}{ab^4d(a+b\sec(c+dx))} + \frac{(3a^2+b^2)\,(a^2-b^2)\log(a+b\sec(c+dx))}{a^2b^4d} - \frac{\log(\cos(c+dx))}{a^2d} - \frac{2a\sec(c+dx)}{b^3d} + \frac{\sec^2(c+dx)}{2b^2d}$$

```
[In] Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) + ((a^2 - b^2)*(3*a^2 + b^2)*Log[a + b*Sec[c +
 d*x]])/(a^2*b^4*d) - (2*a*Sec[c + d*x])/(b^3*d) + Sec[c + d*x]^2/(2*b^2*d)
 + (a^2 - b^2)^2/(a*b^4*d*(a + b*Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{b^4 d}\\
&= \frac{\text{Subst}\left(\int \left(-2a+\frac{b^4}{a^2x}+x-\frac{(a^2-b^2)^2}{a(a+x)^2}+\frac{(a^2-b^2)(3a^2+b^2)}{a^2(a+x)}\right)dx, x, b\sec(c+dx)\right)}{b^4 d}\\
&= -\frac{\log(\cos(c+dx))}{a^2 d}+\frac{(a^2-b^2)(3a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^4d}\\
&\quad -\frac{2a\sec(c+dx)}{b^3d}+\frac{\sec^2(c+dx)}{2b^2d}+\frac{(a^2-b^2)^2}{ab^4d(a+b\sec(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$
\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-\frac{b^4\log(\cos(c+dx))}{a^2}+\frac{(a-b)(a+b)(3a^2+b^2)\log(a+b\sec(c+dx))}{a^2}-2ab\sec(c+dx)+\frac{1}{2}b^2\sec^2(c+dx)+\frac{(a^2-b^2)^2}{a(a+b\sec(c+dx))}}{b^4 d}
$$

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-((b^4*Log[Cos[c + d*x]])/a^2) + ((a - b)*(a + b)*(3*a^2 + b^2)*Log[a + b*
Sec[c + d*x]])/a^2 - 2*a*b*Sec[c + d*x] + (b^2*Sec[c + d*x]^2)/2 + (a^2 - b
^2)^2/(a*(a + b*Sec[c + d*x])))/(b^4*d)
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{\left(-3a^2+2b^2\right)\ln(\cos(dx+c))}{b^4}+\frac{1}{2b^2\cos(dx+c)^2}-\frac{2a}{b^3\cos(dx+c)}-\frac{a^4-2a^2b^2+b^4}{a^2b^3(b+a\cos(dx+c))}+\frac{\left(3a^4-2a^2b^2-b^4\right)\ln(b+a\cos(dx+c))}{b^4a^2}}{d}$
default	$\frac{\frac{\left(-3a^2+2b^2\right)\ln(\cos(dx+c))}{b^4}+\frac{1}{2b^2\cos(dx+c)^2}-\frac{2a}{b^3\cos(dx+c)}-\frac{a^4-2a^2b^2+b^4}{a^2b^3(b+a\cos(dx+c))}+\frac{\left(3a^4-2a^2b^2-b^4\right)\ln(b+a\cos(dx+c))}{b^4a^2}}{d}$
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2\left(3a^4\mathrm{e}^{5i(dx+c)}-2a^2b^2\mathrm{e}^{5i(dx+c)}+b^4\mathrm{e}^{5i(dx+c)}+3a^3b\,\mathrm{e}^{4i(dx+c)}+6a^4\mathrm{e}^{3i(dx+c)}-6a^2b^2\mathrm{e}^{3i(dx+c)}+2b^4\mathrm{e}^{3i(dx}\right.}{d\,b^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^2a^2\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}\right.}$

[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*((-3*a^2+2*b^2)/b^4*ln(cos(d*x+c))+1/2/b^2/cos(d*x+c)^2-2/b^3*a/cos(d*x+c)-(a^4-2*a^2*b^2+b^4)/a^2/b^3/(b+a*cos(d*x+c))+(3*a^4-2*a^2*b^2-b^4)/b^4/a^2*ln(b+a*cos(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.81

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$
$$-\frac{3\,a^3b^2\cos\left(dx+c\right)-a^2b^3+2\left(3\,a^4b-2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^2-2\left(\left(3\,a^5-2\,a^3b^2-ab^4\right)\cos\left(dx+c\right)^3\right.}{2}$$

[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] -1/2*(3*a^3*b^2*cos(d*x + c) - a^2*b^3 + 2*(3*a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^2 - 2*((3*a^5 - 2*a^3*b^2 - a*b^4)*cos(d*x + c)^3 + (3*a^4*b - 2*a^2*b^3 - b^5)*cos(d*x + c)^2)*log(a*cos(d*x + c) + b) + 2*((3*a^5 - 2*a^3*b^2)*cos(d*x + c)^3 + (3*a^4*b - 2*a^2*b^3)*cos(d*x + c)^2)*log(-cos(d*x + c)))/(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2)

Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
[Out] Integral(tan(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.23

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{3\,a^3b\cos(dx+c)-a^2b^2+2\,(3\,a^4-2\,a^2b^2+b^4)\cos(dx+c)^2}{a^3b^3\cos(dx+c)^3+a^2b^4\cos(dx+c)^2}+\frac{2\,(3\,a^2-2\,b^2)\log(\cos(dx+c))}{b^4}-\frac{2\,(3\,a^4-2\,a^2b^2-b^4)\log(a\cos(dx+c)+b)}{a^2b^4}}{2\,d}$$

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -1/2*((3*a^3*b*cos(d*x + c) - a^2*b^2 + 2*(3*a^4 - 2*a^2*b^2 + b^4)*cos(d*x
 + c)^2)/(a^3*b^3*cos(d*x + c)^3 + a^2*b^4*cos(d*x + c)^2) + 2*(3*a^2 - 2*b
^2)*log(cos(d*x + c))/b^4 - 2*(3*a^4 - 2*a^2*b^2 - b^4)*log(a*cos(d*x + c)
+ b)/(a^2*b^4))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. 2(119) = 238.

Time = 1.56 (sec) , antiderivative size = 568, normalized size of antiderivative = 4.69

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{2\,(3\,a^5-3\,a^4b-2\,a^3b^2+2\,a^2b^3-ab^4+b^5)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3b^4-a^2b^5}+\frac{2\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{2\,(3\,a^2-2\,b^2)\log\left(\left|-\frac{\cos(\ldots}{\cos(\ldots}\right.\right.}{b^4}}{}$$

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] 1/2*(2*(3*a^5 - 3*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 + b^5)*log(abs(a +
b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1)))/(a^3*b^4 - a^2*b^5) + 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x +
```

```
c) + 1) + 1))/a^2 - 2*(3*a^2 - 2*b^2)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - 1))/b^4 + (9*a^2 - 8*a*b - 6*b^2 + 18*a^2*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*b^2*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 6*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(b^4*((cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 1)^2) - 2*(3*a^5 + 5*a^4*b - 4*a^2*b^3 - 3
*a*b^4 - b^5 + 3*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^4*b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 2*a^2*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*b^4*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((
a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1))*a^2*b^4))/d
```

Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.36

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^2\,d} - \frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,\left(-6\,a^3-3\,a^2\,b+a\,b^2+2\,b^3\right)}{a\,b^3}-\frac{2\,\left(-3\,a^3-3\,a^2\,b+a\,b^2+b^3\right)}{a\,b^3}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,(a-b)\,\left(3\,a^2+3\,a\,b+b^2\right)}{a\,b^3}}{d\,\left((b-a)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+(3\,a-b)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+(-3\,a-b)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a+b\right)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\,\left(3\,a^2-2\,b^2\right)}{b^4\,d} - \frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\,\left(-3\,a^4+2\,a^2\,b^2+b^4\right)}{a^2\,b^4\,d}$$

```
[In] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^2,x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - ((2*tan(c/2 + (d*x)/2)^2*(a*b^2 - 3
*a^2*b - 6*a^3 + 2*b^3))/(a*b^3) - (2*(a*b^2 - 3*a^2*b - 3*a^3 + b^3))/(a*b
^3) + (2*tan(c/2 + (d*x)/2)^4*(a - b)*(3*a*b + 3*a^2 + b^2))/(a*b^3))/(d*(a
 + b - tan(c/2 + (d*x)/2)^2*(3*a + b) - tan(c/2 + (d*x)/2)^6*(a - b) + tan(
c/2 + (d*x)/2)^4*(3*a - b))) - (log(tan(c/2 + (d*x)/2)^2 - 1)*(3*a^2 - 2*b^
2))/(b^4*d) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)
*(b^4 - 3*a^4 + 2*a^2*b^2))/(a^2*b^4*d)
```

3.302 $\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\frac{\log(\cos(c+dx))}{a^2d}+\frac{(a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^2d}+\frac{a^2-b^2}{ab^2d(a+b\sec(c+dx))}$$

```
[Out] ln(cos(d*x+c))/a^2/d+(a^2+b^2)*ln(a+b*sec(d*x+c))/a^2/b^2/d+(a^2-b^2)/a/b^2
/d/(a+b*sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3970, 908}

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\frac{a^2-b^2}{ab^2d(a+b\sec(c+dx))}+\frac{(a^2+b^2)\log(a+b\sec(c+dx))}{a^2b^2d}+\frac{\log(\cos(c+dx))}{a^2d}$$

```
[In] Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((a^2 + b^2)*Log[a + b*Sec[c + d*x]])/(a^2*b^2*
d) + (a^2 - b^2)/(a*b^2*d*(a + b*Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{b^2 d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2 x}+\frac{a^2-b^2}{a(a+x)^2}+\frac{-a^2-b^2}{a^2(a+x)}\right)\,dx, x, b\sec(c+dx)\right)}{b^2 d}\\ &= \frac{\log(\cos(c+dx))}{a^2 d}+\frac{(a^2+b^2)\log(a+b\sec(c+dx))}{a^2 b^2 d}+\frac{a^2-b^2}{ab^2 d(a+b\sec(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{b-\frac{b^3}{a^2}}{b+a\cos(c+dx)}+\log(\cos(c+dx))-\frac{(a^2+b^2)\log(b+a\cos(c+dx))}{a^2}}{b^2 d}$$

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
[Out] -(((b - b^3/a^2)/(b + a*Cos[c + d*x]) + Log[Cos[c + d*x]] - ((a^2 + b^2)*Lo
g[b + a*Cos[c + d*x]])/a^2)/(b^2*d))
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-\frac{a^2-b^2}{a^2b(b+a\cos(dx+c))}+\frac{\left(a^2+b^2\right)\ln(b+a\cos(dx+c))}{b^2a^2}-\frac{\ln(\cos(dx+c))}{b^2}}{d}$
default	$\frac{-\frac{a^2-b^2}{a^2b(b+a\cos(dx+c))}+\frac{\left(a^2+b^2\right)\ln(b+a\cos(dx+c))}{b^2a^2}-\frac{\ln(\cos(dx+c))}{b^2}}{d}$
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2d}-\frac{2(a^2-b^2)e^{i(dx+c)}}{a^2bd(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a)}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+1\right)}{b^2d}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+1\right.}{a^2d}$

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(a^2-b^2)/a^2/b/(b+a*cos(d*x+c))+(a^2+b^2)/b^2/a^2*ln(b+a*cos(d*x+c))
-1/b^2*ln(cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2b-b^3-(a^2b+b^3+(a^3+ab^2)\cos(dx+c))\log(a\cos(dx+c)+b)+(a^3\cos(dx+c)+a^2b)\log(-\mathrm{co}}{a^3b^2d\cos(dx+c)+a^2b^3d}$$

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b - b^3 - (a^2*b + b^3 + (a^3 + a*b^2)*cos(d*x + c))*log(a*cos(d*x +
c) + b) + (a^3*cos(d*x + c) + a^2*b)*log(-cos(d*x + c)))/(a^3*b^2*d*cos(d*x
 + c) + a^2*b^3*d)
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{a^2-b^2}{a^3b\cos(dx+c)+a^2b^2}+\frac{\log(\cos(dx+c))}{b^2}-\frac{(a^2+b^2)\log(a\cos(dx+c)+b)}{a^2b^2}}{d}$$

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] -((a^2 - b^2)/(a^3*b*cos(d*x + c) + a^2*b^2) + log(cos(d*x + c))/b^2 - (a^2
 + b^2)*log(a*cos(d*x + c) + b)/(a^2*b^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(74) = 148$.

Time = 0.65 (sec) , antiderivative size = 313, normalized size of antiderivative = 4.23

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{(a^3-a^2b+ab^2-b^3)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3b^2-a^2b^3}-\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^2}-\frac{a^3+3a^2b+3ab^2\cdots}{\cdots}}{d}$$

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] ((a^3 - a^2*b + a*b^2 - b^3)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3*b^2 - a^2*b^3) -
 log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2 - log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1))/b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3
+ a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^2*b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b^3*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^2*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 14.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.68

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\,\left(\frac{1}{a^2}+\frac{1}{b^2}\right)}{d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}{b^2\,d} - \frac{2\,(a+b)}{a\,b\,d\,\left(\left(b-a\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a+b\right)}$$

```
[In] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(1/a^2 + 1/b^
2))/d - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - log(tan(c/2 + (d*x)/2)^2 -
1)/(b^2*d) - (2*(a + b))/(a*b*d*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))
```

3.303 $\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\log(\cos(c+dx))}{a^2d} - \frac{\log(a+b\sec(c+dx))}{a^2d} + \frac{1}{ad(a+b\sec(c+dx))}$$

```
[Out] -ln(cos(d*x+c))/a^2/d-ln(a+b*sec(d*x+c))/a^2/d+1/a/d/(a+b*sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3970, 46}

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\log(a+b\sec(c+dx))}{a^2d} - \frac{\log(\cos(c+dx))}{a^2d} + \frac{1}{ad(a+b\sec(c+dx))}$$

```
[In] Int[Tan[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - Log[a + b*Sec[c + d*x]]/(a^2*d) + 1/(a*d*(a
+ b*Sec[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx, x, b\sec(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{1}{a(a+x)^2} - \frac{1}{a^2(a+x)}\right)\,dx, x, b\sec(c+dx)\right)}{d} \\ &= -\frac{\log(\cos(c+dx))}{a^2d} - \frac{\log(a+b\sec(c+dx))}{a^2d} + \frac{1}{ad(a+b\sec(c+dx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{b+b\log(b+a\cos(c+dx))+a\cos(c+dx)\log(b+a\cos(c+dx))}{a^2d(b+a\cos(c+dx))}$$

```
[In] Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x])^2,x]
[Out] -((b + b*Log[b + a*Cos[c + d*x]] + a*Cos[c + d*x]*Log[b + a*Cos[c + d*x]])/
(a^2*d*(b + a*Cos[c + d*x])))
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\sec(dx+c))}{a^2}+\frac{1}{a(a+b\sec(dx+c))}+\frac{\ln(\sec(dx+c))}{a^2}}{d}$	49
default	$\frac{-\frac{\ln(a+b\sec(dx+c))}{a^2}+\frac{1}{a(a+b\sec(dx+c))}+\frac{\ln(\sec(dx+c))}{a^2}}{d}$	49
risch	$\frac{ix}{a^2}+\frac{2ic}{a^2d}-\frac{2b\,e^{i(dx+c)}}{a^2d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+1\right)}{a^2d}$	99

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2*ln(a+b*sec(d*x+c))+1/a/(a+b*sec(d*x+c))+1/a^2*ln(sec(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx=-\frac{(a\cos(dx+c)+b)\log(a\cos(dx+c)+b)+b}{a^3d\cos(dx+c)+a^2bd}$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((a*cos(d*x + c) + b)*log(a*cos(d*x + c) + b) + b)/(a^3*d*cos(d*x + c) + a
^2*b*d)
```

Sympy [F]

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$=\begin{cases}\frac{\tilde{\infty}x\tan(c)}{\sec^2(c)}\\ \frac{\log(\tan^2(c+dx)+1)}{2a^2d}\\ -\frac{1}{2b^2d\sec^2(c+dx)}\\ \frac{\int\frac{\tan(c+dx)}{\cos^2(c+dx)\sec^2(c+dx)-2\cos(c+dx)\sec(c+dx)+1}\,dx}{a^2}\\ \frac{x\tan(c)}{(a+b\sec(c))^2}\\ -\frac{2a\log\left(\frac{a}{b}+\sec(c+dx)\right)}{2a^3d+2a^2bd\sec(c+dx)}+\frac{a\log\left(\tan^2(c+dx)+1\right)}{2a^3d+2a^2bd\sec(c+dx)}+\frac{2a}{2a^3d+2a^2bd\sec(c+dx)}-\frac{2b\log\left(\frac{a}{b}+\sec(c+dx)\right)\sec(c+dx)}{2a^3d+2a^2bd\sec(c+dx)}+\frac{b\log\left(\tan^2(c+dx\right.}{2a^3d+2a^2bd}\end{cases}$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x*tan(c)/sec(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(ta
n(c + d*x)**2 + 1)/(2*a**2*d), Eq(b, 0)), (-1/(2*b**2*d*sec(c + d*x)**2), E
q(a, 0)), (Integral(tan(c + d*x)/(cos(c + d*x)**2*sec(c + d*x)**2 - 2*cos(c
 + d*x)*sec(c + d*x) + 1), x)/a**2, Eq(b, -a*cos(c + d*x))), (x*tan(c)/(a +
 b*sec(c))**2, Eq(d, 0)), (-2*a*log(a/b + sec(c + d*x))/(2*a**3*d + 2*a**2*
b*d*sec(c + d*x)) + a*log(tan(c + d*x)**2 + 1)/(2*a**3*d + 2*a**2*b*d*sec(c
 + d*x)) + 2*a/(2*a**3*d + 2*a**2*b*d*sec(c + d*x)) - 2*b*log(a/b + sec(c +
 d*x))*sec(c + d*x)/(2*a**3*d + 2*a**2*b*d*sec(c + d*x)) + b*log(tan(c + d*
x)**2 + 1)*sec(c + d*x)/(2*a**3*d + 2*a**2*b*d*sec(c + d*x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{b}{a^3\cos(dx+c)+a^2b}+\frac{\log(a\cos(dx+c)+b)}{a^2}}{d}$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(b/(a^3*cos(d*x + c) + a^2*b) + log(a*cos(d*x + c) + b)/a^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(54) = 108.

Time = 0.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.41

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{(a-b)\log\left(\left|a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3-a^2b}-\frac{a^2-2ab-b^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{(a^3-a^2b)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}-\frac{\log\left(\left|-\frac{\cos(dx+c}{\cos(dx+c}\right.\right.}{a^2}}{d}$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((a - b)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3 - a^2*b) - (a^2 - 2*a*b - b^2 + a^2
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^3 - a^2*b)*(a +
b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1))) - log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.37 (sec) , antiderivative size = 257, normalized size of antiderivative = 4.76

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{a}{2\left(\frac{a}{2}+b+\frac{a\,\cos(c+d\,x)}{2}\right)}-\frac{a\,\cos(c+d\,x)}{2\left(\frac{a}{2}+b+\frac{a\,\cos(c+d\,x)}{2}\right)}\right)}{a^2\,d} - \frac{b\left(a+a\,\cos\left(c+d\,x\right)-2\,a\,\mathrm{atanh}\left(\frac{a}{2\left(\frac{a}{2}+b+\frac{a\,\cos(c+d\,x)}{2}\right)}-\frac{a\,\cos(c+d\,x)}{2\left(\frac{a}{2}+b+\frac{a\,\cos(c+d\,x)}{2}\right)}\right)+2\,a\,\cos\left(c+d\,x\right)\,\mathrm{atanh}\left(\frac{}{}\right.\right.}{a^2\,d\,\left(a-b\right)\,\left(b+\right.}$$

[In] int(tan(c + d*x)/(a + b/cos(c + d*x))^2,x)

[Out] (2*atanh(a/(2*(a/2 + b + (a*cos(c + d*x))/2)) - (a*cos(c + d*x))/(2*(a/2 + b + (a*cos(c + d*x))/2))))/(a^2*d) - (b*(a + a*cos(c + d*x) - 2*a*atanh(a/(2*(a/2 + b + (a*cos(c + d*x))/2)) - (a*cos(c + d*x))/(2*(a/2 + b + (a*cos(c + d*x))/2))) + 2*a*cos(c + d*x)*atanh(a/(2*(a/2 + b + (a*cos(c + d*x))/2)) - (a*cos(c + d*x))/(2*(a/2 + b + (a*cos(c + d*x))/2))) + (2*atanh(a/(2*(a/2 + b + (a*cos(c + d*x))/2)) - (a*cos(c + d*x))/(2*(a/2 + b + (a*cos(c + d*x))/2)))*(a^3*d - a^3*d*cos(c + d*x)))/(a^2*d)))/(a^2*d*(a - b)*(b + a*cos(c + d*x)))

3.304 $\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 138

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\log(\cos(c+dx))}{a^2d}+\frac{\log(1-\sec(c+dx))}{2(a+b)^2d}+\frac{\log(1+\sec(c+dx))}{2(a-b)^2d}-\frac{b^2(3a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a^2-b^2)^2d}+\frac{b^2}{a(a^2-b^2)d(a+b\sec(c+dx))}$$

```
[Out] ln(cos(d*x+c))/a^2/d+1/2*ln(1-sec(d*x+c))/(a+b)^2/d+1/2*ln(1+sec(d*x+c))/(a
-b)^2/d-b^2*(3*a^2-b^2)*ln(a+b*sec(d*x+c))/a^2/(a^2-b^2)^2/d+b^2/a/(a^2-b^2
)/d/(a+b*sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3970, 908}

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2}{ad(a^2-b^2)(a+b\sec(c+dx))}-\frac{b^2(3a^2-b^2)\log(a+b\sec(c+dx))}{a^2d(a^2-b^2)^2}+\frac{\log(\cos(c+dx))}{a^2d}+\frac{\log(1-\sec(c+dx))}{2d(a+b)^2}+\frac{\log(\sec(c+dx)+1)}{2d(a-b)^2}$$

```
[In] Int[Cot[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + Log[1 - Sec[c + d*x]]/(2*(a + b)^2*d) + Log[1 +
 Sec[c + d*x]]/(2*(a - b)^2*d) - (b^2*(3*a^2 - b^2)*Log[a + b*Sec[c + d*x]]
)/(a^2*(a^2 - b^2)^2*d) + b^2/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{b^2\text{Subst}\left(\int \frac{1}{x(a+x)^2(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d}\\ &= -\frac{b^2\text{Subst}\left(\int \left(\frac{1}{2b^2(a+b)^2(b-x)}+\frac{1}{a^2b^2x}+\frac{1}{a(a-b)(a+b)(a+x)^2}+\frac{3a^2-b^2}{a^2(a-b)^2(a+b)^2(a+x)}-\frac{1}{2(a-b)^2b^2(b+x)}\right)dx, x,\right.}{d}\\ &= \frac{\log(\cos(c+dx))}{a^2d}+\frac{\log(1-\sec(c+dx))}{2(a+b)^2d}+\frac{\log(1+\sec(c+dx))}{2(a-b)^2d}\\ &\quad-\frac{b^2(3a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a^2-b^2)^2d}+\frac{b^2}{a(a^2-b^2)d(a+b\sec(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{b^2\left(-\frac{\log(\cos(c+dx))}{a^2b^2}-\frac{\log(1-\sec(c+dx))}{2b^2(a+b)^2}-\frac{\log(1+\sec(c+dx))}{2(a-b)^2b^2}+\frac{(3a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a-b)^2(a+b)^2}-\frac{1}{a(a^2-b^2)(a+b\sec(c+dx))}\right)}{d}$$

```
[In] Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((b^2*(-(Log[Cos[c + d*x]]/(a^2*b^2)) - Log[1 - Sec[c + d*x]]/(2*b^2*(a +
b)^2) - Log[1 + Sec[c + d*x]]/(2*(a - b)^2*b^2) + ((3*a^2 - b^2)*Log[a + b*
Sec[c + d*x]])/(a^2*(a - b)^2*(a + b)^2) - 1/(a*(a^2 - b^2)*(a + b*Sec[c +
d*x]))))/d)
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)+1)}{2(a-b)^2}+\frac{\ln(\cos(dx+c)-1)}{2(a+b)^2}-\frac{b^3}{a^2(a+b)(a-b)(b+a\cos(dx+c))}-\frac{b^2\left(3a^2-b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2a^2}}{d}$
default	$\frac{\frac{\ln(\cos(dx+c)+1)}{2(a-b)^2}+\frac{\ln(\cos(dx+c)-1)}{2(a+b)^2}-\frac{b^3}{a^2(a+b)(a-b)(b+a\cos(dx+c))}-\frac{b^2\left(3a^2-b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2a^2}}{d}$
risch	$\frac{ix}{a^2}-\frac{ix}{a^2+2ab+b^2}-\frac{ic}{d(a^2+2ab+b^2)}-\frac{ix}{a^2-2ab+b^2}-\frac{ic}{d(a^2-2ab+b^2)}+\frac{6ib^2x}{a^4-2a^2b^2+b^4}+\frac{6ib^2c}{d(a^4-2a^2b^2+b^4)}-\frac{}{a^2}$

[In] int(cot(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2/(a-b)^2*ln(cos(d*x+c)+1)+1/2/(a+b)^2*ln(cos(d*x+c)-1)-1/a^2*b^3/(a+b)/(a-b)/(b+a*cos(d*x+c))-b^2*(3*a^2-b^2)/(a+b)^2/(a-b)^2/a^2*ln(b+a*cos(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.70

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2\,a^2b^3-2\,b^5+2\,(3\,a^2b^3-b^5+(3\,a^3b^2-ab^4)\cos{(dx+c)})\log{(a\cos{(dx+c)}+b)}-(a^4b+2\,a^3b^2+a^2b^3}{2\,((a^7-2\,a}$$

[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] -1/2*(2*a^2*b^3 - 2*b^5 + 2*(3*a^2*b^3 - b^5 + (3*a^3*b^2 - a*b^4)*cos(d*x + c))*log(a*cos(d*x + c) + b) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)

Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))**2,x)

[Out] Integral(cot(c + d*x)/(a + b*sec(c + d*x))**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{2\,b^3}{a^4b-a^2b^3+(a^5-a^3b^2)\cos(dx+c)} + \frac{2\,(3\,a^2b^2-b^4)\log(a\cos(dx+c)+b)}{a^6-2\,a^4b^2+a^2b^4} - \frac{\log(\cos(dx+c)+1)}{a^2-2\,ab+b^2} - \frac{\log(\cos(dx+c)-1)}{a^2+2\,ab+b^2}}{2\,d}$$

[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] -1/2*(2*b^3/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(d*x + c)) + 2*(3*a^2*b^2 - b^4)*log(a*cos(d*x + c) + b)/(a^6 - 2*a^4*b^2 + a^2*b^4) - log(cos(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(134) = 268$.

Time = 0.32 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.20

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{2\,(3\,a^2b^2-b^4)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-2\,a^4b^2+a^2b^4} - \frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2\,ab+b^2} - \frac{2\left(3\,a^2b^2+4\,ab^3+b^4+\frac{3\,a^2b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b^4(\cos}{\cos}\right.}{(a^5+a^4b-a^3b^2-a^2b^3)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos}{\cos}\right.}}{2\,d}$$

[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")

[Out] -1/2*(2*(3*a^2*b^2 - b^4)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 2*a^4*b^2 + a^2*b^4) - log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2) - 2*(3*a^2*b^2 + 4*a*b^3 + b^4 + 3*a^2*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^5 + a^4*b - a^3*b^2 - a^2*b^3)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))) + 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2)/d

Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.16

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d\,(a^2+2\,a\,b+b^2)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^2\,d} - \frac{b^2\,\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\,(3\,a^2-b^2)}{a^2\,d\,(a^2-b^2)^2} - \frac{2\,b^3}{a\,d\,(a+b)\,(a-b)^2\,\left((b-a)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a+b\right)}$$

```
[In] int(cot(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(2*a*b + a^2 + b^2)) - log(tan(c/2 + (d*x)/2)^2
+ 1)/(a^2*d) - (b^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/
2)^2)*(3*a^2 - b^2))/(a^2*d*(a^2 - b^2)^2) - (2*b^3)/(a*d*(a + b)*(a - b)^2
*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))
```

3.305 $\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 197

$$\begin{aligned}\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = &-\frac{\log(\cos(c+dx))}{a^2d}-\frac{(a+2b)\log(1-\sec(c+dx))}{2(a+b)^3d}\\&-\frac{(a-2b)\log(1+\sec(c+dx))}{2(a-b)^3d}\\&-\frac{b^4(5a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a^2-b^2)^3d}\\&+\frac{1}{4(a+b)^2d(1-\sec(c+dx))}+\frac{1}{4(a-b)^2d(1+\sec(c+dx))}\\&+\frac{b^4}{a(a^2-b^2)^2d(a+b\sec(c+dx))}\end{aligned}$$

```
[Out] -ln(cos(d*x+c))/a^2/d-1/2*(a+2*b)*ln(1-sec(d*x+c))/(a+b)^3/d-1/2*(a-2*b)*ln
(1+sec(d*x+c))/(a-b)^3/d-b^4*(5*a^2-b^2)*ln(a+b*sec(d*x+c))/a^2/(a^2-b^2)^3
/d+1/4/(a+b)^2/d/(1-sec(d*x+c))+1/4/(a-b)^2/d/(1+sec(d*x+c))+b^4/a/(a^2-b^2
)^2/d/(a+b*sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used

$= \{3970, 908\}$

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^4}{ad\left(a^2-b^2\right)^2(a+b\sec(c+dx))} - \frac{b^4(5a^2-b^2)\log(a+b\sec(c+dx))}{a^2d\left(a^2-b^2\right)^3} - \frac{\log(\cos(c+dx))}{a^2d} + \frac{1}{4d(a+b)^2(1-\sec(c+dx))} + \frac{1}{4d(a-b)^2(\sec(c+dx)+1)} - \frac{(a+2b)\log(1-\sec(c+dx))}{2d(a+b)^3} - \frac{(a-2b)\log(\sec(c+dx)+1)}{2d(a-b)^3}$$

```
[In] Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Log[Cos[c + d*x]]/(a^2*d)) - ((a + 2*b)*Log[1 - Sec[c + d*x]])/(2*(a + b)
^3*d) - ((a - 2*b)*Log[1 + Sec[c + d*x]])/(2*(a - b)^3*d) - (b^4*(5*a^2 - b
^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)^3*d) + 1/(4*(a + b)^2*d*(1 -
Sec[c + d*x])) + 1/(4*(a - b)^2*d*(1 + Sec[c + d*x])) + b^4/(a*(a^2 - b^2)^
2*d*(a + b*Sec[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{b^4\text{Subst}\left(\int \frac{1}{x(a+x)^2(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{b^4\text{Subst}\left(\int \left(\frac{1}{4b^3(a+b)^2(b-x)^2} + \frac{a+2b}{2b^4(a+b)^3(b-x)} + \frac{1}{a^2b^4x} - \frac{1}{a(a-b)^2(a+b)^2(a+x)^2} + \frac{-5a^2+b^2}{a^2(a-b)^3(a+b)^3(a+x)} - \frac{1}{4(a-b)^2b\cdots}\right.\right.}{d}$$

$$= -\frac{\log(\cos(c+dx))}{a^2d} - \frac{(a+2b)\log(1-\sec(c+dx))}{2(a+b)^3d} - \frac{(a-2b)\log(1+\sec(c+dx))}{2(a-b)^3d} - \frac{b^4(5a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a^2-b^2)^3d} + \frac{1}{4(a+b)^2d(1-\sec(c+dx))} + \frac{1}{4(a-b)^2d(1+\sec(c+dx))} + \frac{b^4}{a(a^2-b^2)^2d(a+b\sec(c+dx))}$$

Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.95

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^4\left(-\frac{4\log(\cos(c+dx))}{a^2b^4} - \frac{2(a+2b)\log(1-\sec(c+dx))}{b^4(a+b)^3} - \frac{2(a-2b)\log(1+\sec(c+dx))}{(a-b)^3b^4} - \frac{4(5a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a-b)^3(a+b)^3} - \frac{1}{b^4(a+b)^2(-1+\mathrm{se}}\right.}{4d}$$

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (b^4*((-4*Log[Cos[c + d*x]])/(a^2*b^4) - (2*(a + 2*b)*Log[1 - Sec[c + d*x]]
)/(b^4*(a + b)^3) - (2*(a - 2*b)*Log[1 + Sec[c + d*x]])/((a - b)^3*b^4) - (
4*(5*a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a - b)^3*(a + b)^3) - 1/(b^4
*(a + b)^2*(-1 + Sec[c + d*x])) + 1/((a - b)^2*b^4*(1 + Sec[c + d*x])) + 4/
(a*(a - b)^2*(a + b)^2*(a + b*Sec[c + d*x]))))/(4*d)
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{1}{4(a+b)^2(\cos(dx+c)-1)} + \frac{(-a-2b)\ln(\cos(dx+c)-1)}{2(a+b)^3} - \frac{1}{4(a-b)^2(\cos(dx+c)+1)} + \frac{(2b-a)\ln(\cos(dx+c)+1)}{2(a-b)^3} - \frac{b^5}{a^2(a+b)^2(a-b)^2(b+a\cos}}{d}$
default	$\frac{\frac{1}{4(a+b)^2(\cos(dx+c)-1)} + \frac{(-a-2b)\ln(\cos(dx+c)-1)}{2(a+b)^3} - \frac{1}{4(a-b)^2(\cos(dx+c)+1)} + \frac{(2b-a)\ln(\cos(dx+c)+1)}{2(a-b)^3} - \frac{b^5}{a^2(a+b)^2(a-b)^2(b+a\cos}}{d}$
risch	$-\frac{2ibx}{a^3-3a^2b+3a\,b^2-b^3} + \frac{10ib^4x}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{2ib^6c}{a^2d(a^6-3a^4b^2+3a^2b^4-b^6)} + \frac{iac}{d(a^3+3a^2b+3a\,b^2+b^3)} + \frac{}{d(a^6-3}$

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/(a+b)^2/(cos(d*x+c)-1)+1/2*(-a-2*b)/(a+b)^3*ln(cos(d*x+c)-1)-1/4/(
a-b)^2/(cos(d*x+c)+1)+1/2*(2*b-a)/(a-b)^3*ln(cos(d*x+c)+1)-b^5/a^2/(a+b)^2/
(a-b)^2/(b+a*cos(d*x+c))-b^4*(5*a^2-b^2)/(a+b)^3/(a-b)^3/a^2*ln(b+a*cos(d*x
+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(187) = 374$.

Time = 0.41 (sec) , antiderivative size = 693, normalized size of antiderivative = 3.52

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{a^6b + a^2b^5 - 2\,b^7 - 2\,(a^6b - a^4b^3 + a^2b^5 - b^7)\cos(dx+c)^2 + (a^7 - 2\,a^5b^2 + a^3b^4)\cos(dx+c) + 2\,\left(5\,a^2b^5 - \right.}{}$$

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^6*b + a^2*b^5 - 2*b^7 - 2*(a^6*b - a^4*b^3 + a^2*b^5 - b^7)*cos(d*x
+ c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*cos(d*x + c) + 2*(5*a^2*b^5 - b^7 - (5
*a^3*b^4 - a*b^6)*cos(d*x + c)^3 - (5*a^2*b^5 - b^7)*cos(d*x + c)^2 + (5*a^
3*b^4 - a*b^6)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (a^6*b + a^5*b^2 - 3
*a^4*b^3 - 5*a^3*b^4 - 2*a^2*b^5 - (a^7 + a^6*b - 3*a^5*b^2 - 5*a^4*b^3 - 2
*a^3*b^4)*cos(d*x + c)^3 - (a^6*b + a^5*b^2 - 3*a^4*b^3 - 5*a^3*b^4 - 2*a^2
*b^5)*cos(d*x + c)^2 + (a^7 + a^6*b - 3*a^5*b^2 - 5*a^4*b^3 - 2*a^3*b^4)*co
s(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^6*b - a^5*b^2 - 3*a^4*b^3 + 5*
a^3*b^4 - 2*a^2*b^5 - (a^7 - a^6*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3*b^4)*cos
(d*x + c)^3 - (a^6*b - a^5*b^2 - 3*a^4*b^3 + 5*a^3*b^4 - 2*a^2*b^5)*cos(d*x
 + c)^2 + (a^7 - a^6*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3*b^4)*cos(d*x + c))*l
og(-1/2*cos(d*x + c) + 1/2))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos
(d*x + c)^3 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*cos(d*x + c)^2 -
(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c) - (a^8*b - 3*a^6*b^3
 + 3*a^4*b^5 - a^2*b^7)*d)
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.54

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{2\,(5\,a^2b^4-b^6)\log(a\cos(dx+c)+b)}{a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6}+\frac{(a-2\,b)\log(\cos(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{(a+2\,b)\log(\cos(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{a^4b+a^2b^3+2\,b^5}{a^6b-2\,a^4b^3+a^2b^5-(a^7-2\,a^5b^2+a^3b^4)\,\mathrm{co}}}{2\,d}$$

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(5*a^2*b^4 - b^6)*log(a*cos(d*x + c) + b)/(a^8 - 3*a^6*b^2 + 3*a^4*
b^4 - a^2*b^6) + (a - 2*b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) + (a + 2*b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (
a^4*b + a^2*b^3 + 2*b^5 - 2*(a^4*b + b^5)*cos(d*x + c)^2 + (a^5 - a^3*b^2)*
cos(d*x + c))/(a^6*b - 2*a^4*b^3 + a^2*b^5 - (a^7 - 2*a^5*b^2 + a^3*b^4)*co
s(d*x + c)^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 + (a^7 - 2*a^5*
b^2 + a^3*b^4)*cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(187) = 374.

Time = 0.38 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.33

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{4\,(a+2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{8\,(5\,a^2b^4-b^6)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6}-\frac{a^5-a^4b-a^3b^2+a^2b^3+\frac{3\,a^5(\cos(dx+c)-1)}{\cos(dx+c)+1}}{}}{}$$

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 +
3*a^2*b + 3*a*b^2 + b^3) + 8*(5*a^2*b^4 - b^6)*log(abs(-a - b - a*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a
^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6) - (a^5 - a^4*b - a^3*b^2 + a^2*b^3 +
3*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^4*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 3*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*
b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a*b^4*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 4*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*a^5*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 4*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 - 2*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 12*a^
```

```
2*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*b^4*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 - 4*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)) - (cos(d*x + c) -
1)/((a^2 - 2*a*b + b^2)*(cos(d*x + c) + 1)) - 8*log(abs(-(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 1))/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.96 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.59

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{a^2-2\,a\,b+b^2}{2\,(a+b)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^5-4\,a^4\,b+6\,a^3\,b^2-4\,a^2\,b^3+a\,b^4-16\,b^5\right)}{2\,a\,{(a+b)}^2\,(a-b)}}{d\,\left(\left(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(-4\,a^3+4\,a^2\,b+4\,a\,b^2-4\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d\,{(a-b)}^2} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a+2\,b\right)}{d\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)} - \frac{b^4\,\ln\left(a+b-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(5\,a^2-b^2\right)}{a^2\,d\,{\left(a^2-b^2\right)}^3}$$

```
[In] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(2*(a + b)) - (tan(c/2 + (d*x)/2)^2*(a*b^4 - 4*a^4*b +
 a^5 - 16*b^5 - 4*a^2*b^3 + 6*a^3*b^2))/(2*a*(a + b)^2*(a - b)))/(d*(tan(c/
2 + (d*x)/2)^2*(4*a*b^2 + 4*a^2*b - 4*a^3 - 4*b^3) + tan(c/2 + (d*x)/2)^4*(
12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))) - tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^
2) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (log(tan(c/2 + (d*x)/2))*(a +
2*b))/(d*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - (b^4*log(a + b - a*tan(c/2 + (d
*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(5*a^2 - b^2))/(a^2*d*(a^2 - b^2)^3)
```

3.306 $\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 278

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{\log(\cos(c+dx))}{a^2d} + \frac{(4a^2+13ab+12b^2)\log(1-\sec(c+dx))}{8(a+b)^4d} \\ &\quad + \frac{(4a^2-13ab+12b^2)\log(1+\sec(c+dx))}{8(a-b)^4d} \\ &\quad - \frac{b^6(7a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a^2-b^2)^4d} \\ &\quad - \frac{1}{16(a+b)^2d(1-\sec(c+dx))^2} \\ &\quad - \frac{5a+9b}{16(a+b)^3d(1-\sec(c+dx))} \\ &\quad - \frac{1}{16(a-b)^2d(1+\sec(c+dx))^2} \\ &\quad - \frac{5a-9b}{16(a-b)^3d(1+\sec(c+dx))} \\ &\quad + \frac{b^6}{a(a^2-b^2)^3d(a+b\sec(c+dx))}\end{aligned}$$

```
[Out] ln(cos(d*x+c))/a^2/d+1/8*(4*a^2+13*a*b+12*b^2)*ln(1-sec(d*x+c))/(a+b)^4/d+1
/8*(4*a^2-13*a*b+12*b^2)*ln(1+sec(d*x+c))/(a-b)^4/d-b^6*(7*a^2-b^2)*ln(a+b*
sec(d*x+c))/a^2/(a^2-b^2)^4/d-1/16/(a+b)^2/d/(1-sec(d*x+c))^2+1/16*(-5*a-9*
b)/(a+b)^3/d/(1-sec(d*x+c))-1/16/(a-b)^2/d/(1+sec(d*x+c))^2+1/16*(-5*a+9*b)
/(a-b)^3/d/(1+sec(d*x+c))+b^6/a/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3970, 908}

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = & \frac{(4a^2+13ab+12b^2)\log(1-\sec(c+dx))}{8d(a+b)^4} \\ & + \frac{(4a^2-13ab+12b^2)\log(\sec(c+dx)+1)}{8d(a-b)^4} \\ & + \frac{b^6}{ad\,(a^2-b^2)^3\,(a+b\sec(c+dx))} \\ & - \frac{b^6(7a^2-b^2)\log(a+b\sec(c+dx))}{a^2d\,(a^2-b^2)^4} + \frac{\log(\cos(c+dx))}{a^2d} \\ & - \frac{5a+9b}{16d(a+b)^3(1-\sec(c+dx))} - \frac{5a-9b}{16d(a-b)^3(\sec(c+dx)+1)} \\ & - \frac{1}{16d(a+b)^2(1-\sec(c+dx))^2} \\ & - \frac{1}{16d(a-b)^2(\sec(c+dx)+1)^2}\end{aligned}$$

```
[In] Int[Cot[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] Log[Cos[c + d*x]]/(a^2*d) + ((4*a^2 + 13*a*b + 12*b^2)*Log[1 - Sec[c + d*x]
])/(8*(a + b)^4*d) + ((4*a^2 - 13*a*b + 12*b^2)*Log[1 + Sec[c + d*x]])/(8*(
a - b)^4*d) - (b^6*(7*a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)^
4*d) - 1/(16*(a + b)^2*d*(1 - Sec[c + d*x])^2) - (5*a + 9*b)/(16*(a + b)^3*
d*(1 - Sec[c + d*x])) - 1/(16*(a - b)^2*d*(1 + Sec[c + d*x])^2) - (5*a - 9*
b)/(16*(a - b)^3*d*(1 + Sec[c + d*x])) + b^6/(a*(a^2 - b^2)^3*d*(a + b*Sec[
c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b^6\text{Subst}\left(\int \frac{1}{x(a+x)^2(b^2-x^2)^3}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{b^6\text{Subst}\left(\int \left(\frac{1}{8b^4(a+b)^2(b-x)^3} + \frac{5a+9b}{16b^5(a+b)^3(b-x)^2} + \frac{4a^2+13ab+12b^2}{8b^6(a+b)^4(b-x)} + \frac{1}{a^2b^6x} + \frac{1}{a(a-b)^3(a+b)^3(a+x)^2} + \frac{7}{a^2(a-b)}\right.\right.}{d}$$

$$= \frac{\log(\cos(c+dx))}{a^2d} + \frac{(4a^2+13ab+12b^2)\log(1-\sec(c+dx))}{8(a+b)^4d} + \frac{(4a^2-13ab+12b^2)\log(1+\sec(c+dx))}{8(a-b)^4d} - \frac{b^6(7a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a^2-b^2)^4d} - \frac{1}{16(a+b)^2d(1-\sec(c+dx))^2} - \frac{5a+9b}{16(a+b)^3d(1-\sec(c+dx))} - \frac{1}{16(a-b)^2d(1+\sec(c+dx))^2} - \frac{5a-9b}{16(a-b)^3d(1+\sec(c+dx))} + \frac{b^6}{a(a^2-b^2)^3d(a+b\sec(c+dx))}$$

Mathematica [A] (verified)

Time = 6.45 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.02

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{b^6\left(-\frac{\log(\cos(c+dx))}{a^2b^6} - \frac{(4a^2+13ab+12b^2)\log(1-\sec(c+dx))}{8b^6(a+b)^4} - \frac{(4a^2-13ab+12b^2)\log(1+\sec(c+dx))}{8(a-b)^4b^6} + \frac{(7a^2-b^2)\log(a+b\sec(c+dx))}{a^2(a-b)^4(a+b)^4}\right.}{}$$

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((b^6*(-(Log[Cos[c + d*x]]/(a^2*b^6)) - ((4*a^2 + 13*a*b + 12*b^2)*Log[1 -
 Sec[c + d*x]])/(8*b^6*(a + b)^4) - ((4*a^2 - 13*a*b + 12*b^2)*Log[1 + Sec[
c + d*x]])/(8*(a - b)^4*b^6) + ((7*a^2 - b^2)*Log[a + b*Sec[c + d*x]])/(a^2
*(a - b)^4*(a + b)^4) + 1/(16*b^4*(a + b)^2*(b - b*Sec[c + d*x])^2) + (5*a
+ 9*b)/(16*b^5*(a + b)^3*(b - b*Sec[c + d*x])) - 1/(a*(a - b)^3*(a + b)^3*(
a + b*Sec[c + d*x])) + 1/(16*(a - b)^2*b^4*(b + b*Sec[c + d*x])^2) + (5*a -
 9*b)/(16*(a - b)^3*b^5*(b + b*Sec[c + d*x]))))/d)
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{-\frac{1}{16(a-b)^2(\cos(dx+c)+1)^2}-\frac{-7a+11b}{16(a-b)^3(\cos(dx+c)+1)}+\frac{(4a^2-13ab+12b^2)\ln(\cos(dx+c)+1)}{8(a-b)^4}-\frac{b^7}{a^2(a+b)^3(a-b)^3(b+a\cos(dx+c))}-\frac{b^6(}{}}{d}$
default	$\frac{-\frac{1}{16(a-b)^2(\cos(dx+c)+1)^2}-\frac{-7a+11b}{16(a-b)^3(\cos(dx+c)+1)}+\frac{(4a^2-13ab+12b^2)\ln(\cos(dx+c)+1)}{8(a-b)^4}-\frac{b^7}{a^2(a+b)^3(a-b)^3(b+a\cos(dx+c))}-\frac{b^6(}{}}{d}$
risch	Expression too large to display

```
[In] int(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/16/(a-b)^2/(cos(d*x+c)+1)^2-1/16*(-7*a+11*b)/(a-b)^3/(cos(d*x+c)+1)
+1/8*(4*a^2-13*a*b+12*b^2)/(a-b)^4*ln(cos(d*x+c)+1)-b^7/a^2/(a+b)^3/(a-b)^3
/(b+a*cos(d*x+c))-b^6*(7*a^2-b^2)/(a+b)^4/(a-b)^4/a^2*ln(b+a*cos(d*x+c))-1/
16/(a+b)^2/(cos(d*x+c)-1)^2-1/16*(7*a+11*b)/(a+b)^3/(cos(d*x+c)-1)+1/8*(4*a
^2+13*a*b+12*b^2)/(a+b)^4*ln(cos(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. 2(262) = 524.

Time = 0.62 (sec) , antiderivative size = 1378, normalized size of antiderivative = 4.96

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*a^8*b - 18*a^6*b^3 + 2*a^4*b^5 + 2*a^2*b^7 + 8*b^9 + 2*(5*a^8*b - 18
*a^6*b^3 + 13*a^4*b^5 - 4*a^2*b^7 + 4*b^9)*cos(d*x + c)^4 - 2*(4*a^9 - 15*a
^7*b^2 + 18*a^5*b^4 - 7*a^3*b^6)*cos(d*x + c)^3 - 2*(7*a^8*b - 24*a^6*b^3 +
 11*a^4*b^5 - 2*a^2*b^7 + 8*b^9)*cos(d*x + c)^2 + 6*(a^9 - 4*a^7*b^2 + 5*a^
5*b^4 - 2*a^3*b^6)*cos(d*x + c) - 8*(7*a^2*b^7 - b^9 + (7*a^3*b^6 - a*b^8)*
cos(d*x + c)^5 + (7*a^2*b^7 - b^9)*cos(d*x + c)^4 - 2*(7*a^3*b^6 - a*b^8)*c
os(d*x + c)^3 - 2*(7*a^2*b^7 - b^9)*cos(d*x + c)^2 + (7*a^3*b^6 - a*b^8)*co
s(d*x + c))*log(a*cos(d*x + c) + b) + (4*a^8*b + 3*a^7*b^2 - 16*a^6*b^3 - 1
4*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 + 12*a^2*b^7 + (4*a^9 + 3*a^8*b - 16*a^
7*b^2 - 14*a^6*b^3 + 24*a^5*b^4 + 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c)^5 +
 (4*a^8*b + 3*a^7*b^2 - 16*a^6*b^3 - 14*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 +
 12*a^2*b^7)*cos(d*x + c)^4 - 2*(4*a^9 + 3*a^8*b - 16*a^7*b^2 - 14*a^6*b^3
+ 24*a^5*b^4 + 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c)^3 - 2*(4*a^8*b + 3*a^7
*b^2 - 16*a^6*b^3 - 14*a^5*b^4 + 24*a^4*b^5 + 35*a^3*b^6 + 12*a^2*b^7)*cos(
d*x + c)^2 + (4*a^9 + 3*a^8*b - 16*a^7*b^2 - 14*a^6*b^3 + 24*a^5*b^4 + 35*a
^4*b^5 + 12*a^3*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (4*a^8*b -
```

```
 3*a^7*b^2 - 16*a^6*b^3 + 14*a^5*b^4 + 24*a^4*b^5 - 35*a^3*b^6 + 12*a^2*b^7
 + (4*a^9 - 3*a^8*b - 16*a^7*b^2 + 14*a^6*b^3 + 24*a^5*b^4 - 35*a^4*b^5 + 1
2*a^3*b^6)*cos(d*x + c)^5 + (4*a^8*b - 3*a^7*b^2 - 16*a^6*b^3 + 14*a^5*b^4
+ 24*a^4*b^5 - 35*a^3*b^6 + 12*a^2*b^7)*cos(d*x + c)^4 - 2*(4*a^9 - 3*a^8*b
 - 16*a^7*b^2 + 14*a^6*b^3 + 24*a^5*b^4 - 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x
+ c)^3 - 2*(4*a^8*b - 3*a^7*b^2 - 16*a^6*b^3 + 14*a^5*b^4 + 24*a^4*b^5 - 35
*a^3*b^6 + 12*a^2*b^7)*cos(d*x + c)^2 + (4*a^9 - 3*a^8*b - 16*a^7*b^2 + 14*
a^6*b^3 + 24*a^5*b^4 - 35*a^4*b^5 + 12*a^3*b^6)*cos(d*x + c))*log(-1/2*cos(
d*x + c) + 1/2))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*co
s(d*x + c)^5 + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos
(d*x + c)^4 - 2*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(
d*x + c)^3 - 2*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos
(d*x + c)^2 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*
x + c) + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d)
```

Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\int \frac{\cot^5{(c+dx)}}{\left(a+b\sec{(c+dx)}\right)^2}\,dx$$

```
[In] integrate(cot(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(262) = 524$.

Time = 0.21 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.01

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx=$$

$$-\frac{\frac{8\,(7\,a^2b^6-b^8)\log(a\cos(dx+c)+b)}{a^{10}-4\,a^8b^2+6\,a^6b^4-4\,a^4b^6+a^2b^8}-\frac{(4\,a^2-13\,ab+12\,b^2)\log(\cos(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{(4\,a^2+13\,ab+12\,b^2)\log(\cos(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{\cdots}{a^8b-3\,a^6b^3+3\,a\cdots}}{\cdots}$$

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(8*(7*a^2*b^6 - b^8)*log(a*cos(d*x + c) + b)/(a^10 - 4*a^8*b^2 + 6*a^6
*b^4 - 4*a^4*b^6 + a^2*b^8) - (4*a^2 - 13*a*b + 12*b^2)*log(cos(d*x + c) +
1)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + 13*a*b + 12*b^2)*
log(cos(d*x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(3*a^
6*b - 6*a^4*b^3 - 5*a^2*b^5 - 4*b^7 + (5*a^6*b - 13*a^4*b^3 - 4*b^7)*cos(d*
x + c)^4 - (4*a^7 - 11*a^5*b^2 + 7*a^3*b^4)*cos(d*x + c)^3 - (7*a^6*b - 17*
```

```
a^4*b^3 - 6*a^2*b^5 - 8*b^7)*cos(d*x + c)^2 + 3*(a^7 - 3*a^5*b^2 + 2*a^3*b^
4)*cos(d*x + c))/(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7 + (a^9 - 3*a^7*b^
2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^5 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 -
a^2*b^7)*cos(d*x + c)^4 - 2*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x
 + c)^3 - 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + (a^9
 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(262) = 524$.

Time = 0.43 (sec) , antiderivative size = 795, normalized size of antiderivative = 2.86

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(8*(4*a^2 + 13*a*b + 12*b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x +
c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 64*(7*a^2*b^6 - b^8)
*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1)))/(a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*
b^8) - (12*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 32*a*b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 20*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a
^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a*b*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a^4 -
 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 32*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 20*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*a^2*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 156*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 144*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^
2/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)^2) + 64*(
7*a^3*b^6 + 5*a^2*b^7 - 3*a*b^8 - b^9 + 7*a^3*b^6*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 7*a^2*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*b^8*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + b^9*(cos(d*x + c) - 1)/(cos(d*x + c) +
1))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*(a + b + a*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))
 - 64*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 15.76 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.69

$$\int \frac{\cot^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{4\,(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-13\,a^4+20\,a^3\,b+18\,a^2\,b^2-44\,a\,b^3+19\,b^4\right)}{4\,(a+b)^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(3\,a^7-10\,a^6\,b+5\,a^5\,b^2+20\,a^4\,b^3-35\,a^3\,b^4+2\right.}{a\,(a+b)^3\,(a-b)}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(16\,a^4-64\,a^3\,b+96\,a^2\,b^2-64\,a\,b^3+16\,b^4\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(16\,a^4-32\,a^3\,b+32\,a\right.\right.}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d\,(a-b)^2}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{16\,a^2+32\,a\,b-48\,b^2}{512\,(a-b)^4}-\frac{7}{32\,(a-b)^2}\right)}{d}$$
$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,a^2+13\,a\,b+12\,b^2\right)}{d\,\left(4\,a^4+16\,a^3\,b+24\,a^2\,b^2+16\,a\,b^3+4\,b^4\right)}$$
$$-\frac{b^6\,\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(7\,a^2-b^2\right)}{a^2\,d\,\left(a^2-b^2\right)^4}$$

```
[In] int(cot(c + d*x)^5/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(4*(a + b)) + (tan(c/2 + (d*x)/2)^2*(20*a^
3*b - 44*a*b^3 - 13*a^4 + 19*b^4 + 18*a^2*b^2))/(4*(a + b)^2) + (tan(c/2 +
(d*x)/2)^4*(3*a^7 - 10*a^6*b - 5*a*b^6 + 32*b^7 + 22*a^2*b^5 - 35*a^3*b^4 +
 20*a^4*b^3 + 5*a^5*b^2))/(a*(a + b)^3*(a - b)))/(d*(tan(c/2 + (d*x)/2)^6*(
16*a^4 - 64*a^3*b - 64*a*b^3 + 16*b^4 + 96*a^2*b^2) - tan(c/2 + (d*x)/2)^4*
(32*a*b^3 - 32*a^3*b + 16*a^4 - 16*b^4))) - tan(c/2 + (d*x)/2)^4/(64*d*(a -
 b)^2) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (tan(c/2 + (d*x)/2)^2*((32
*a*b + 16*a^2 - 48*b^2)/(512*(a - b)^4) - 7/(32*(a - b)^2)))/d + (log(tan(c
/2 + (d*x)/2))*(13*a*b + 4*a^2 + 12*b^2))/(d*(16*a*b^3 + 16*a^3*b + 4*a^4 +
 4*b^4 + 24*a^2*b^2)) - (b^6*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2
 + (d*x)/2)^2)*(7*a^2 - b^2))/(a^2*d*(a^2 - b^2)^4)
```

3.307 $\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 200

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{x}{a^2}-\frac{a(4a^2-5b^2)\operatorname{arctanh}(\sin(c+dx))}{b^5d} + \frac{2(a-b)^{3/2}(a+b)^{3/2}(4a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^5d} + \frac{(a^2-b^2)^2\sin(c+dx)}{ab^4d(b+a\cos(c+dx))}+\frac{(3a^2-2b^2)\tan(c+dx)}{b^4d} - \frac{a\sec(c+dx)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}$$

```
[Out] -x/a^2-a*(4*a^2-5*b^2)*arctanh(sin(d*x+c))/b^5/d+2*(a-b)^(3/2)*(a+b)^(3/2)*
(4*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/b^5/d+(
a^2-b^2)^2*sin(d*x+c)/a/b^4/d/(b+a*cos(d*x+c))+(3*a^2-2*b^2)*tan(d*x+c)/b^4
/d-a*sec(d*x+c)*tan(d*x+c)/b^3/d+1/3*tan(d*x+c)^3/b^2/d
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.42, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules

used = {3983, 2976, 2743, 12, 2738, 214, 3855, 3852, 8, 3853}

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2(a-b)^{3/2}(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d} - \frac{2a(2a^2-3b^2)\,\text{arctanh}(\sin(c+dx))}{b^5d} + \frac{4(a-b)^{3/2}(a+b)^{3/2}\left(2a^2+b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^5d} + \frac{3(a^2-b^2)\tan(c+dx)}{b^4d} + \frac{(a^2-b^2)^2\sin(c+dx)}{ab^4d(a\cos(c+dx)+b)} - \frac{x}{a^2} - \frac{a\text{arctanh}(\sin(c+dx))}{b^3d} - \frac{a\tan(c+dx)\sec(c+dx)}{b^3d} + \frac{\tan^3(c+dx)}{3b^2d} + \frac{\tan(c+dx)}{b^2d}$$

```
[In] Int[Tan[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (a*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*a*(2*a^2 - 3*b^2)*ArcTanh
[Sin[c + d*x]])/(b^5*d) - (2*(a - b)^(3/2)*(a + b)^(3/2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*b^3*d) + (4*(a - b)^(3/2)*(a + b)^(
3/2)*(2*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
2*b^5*d) + ((a^2 - b^2)^2*Sin[c + d*x])/(a*b^4*d*(b + a*Cos[c + d*x])) + Ta
n[c + d*x]/(b^2*d) + (3*(a^2 - b^2)*Tan[c + d*x])/(b^4*d) - (a*Sec[c + d*x]
*Tan[c + d*x])/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\text{integral} = \int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{a^2} + \frac{\left(a^2-b^2\right)^3}{a^2b^4(b+a\cos(c+dx))^2} + \frac{2(2a^6-3a^4b^2+b^6)}{a^2b^5(b+a\cos(c+dx))} + \frac{2(-2a^3+3ab^2)\sec(c+dx)}{b^5} - \frac{3(-a^2+b^2)\sec^2(c+dx)}{b^4} - \frac{2a\sec^3(c+dx)}{b^3} + \frac{\sec^4(c+dx)}{b^2} \right) dx$$

$$= -\frac{x}{a^2} - \frac{(2a)\int \sec^3(c+dx)\,dx}{b^3} + \frac{\int \sec^4(c+dx)\,dx}{b^2} - \frac{(2a(2a^2-3b^2))\int \sec(c+dx)\,dx}{b^5} + \frac{(3(a^2-b^2))\int \sec^2(c+dx)\,dx}{b^4} + \frac{\left(a^2-b^2\right)^3\int \frac{1}{(b+a\cos(c+dx))^2}\,dx}{a^2b^4} + \frac{(2(2a^6-3a^4b^2+b^6))\int \frac{1}{b+a\cos(c+dx)}\,dx}{a^2b^5}$$

$$= -\frac{x}{a^2} - \frac{2a(2a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{b^5d} + \frac{\left(a^2-b^2\right)^2\sin(c+dx)}{ab^4d(b+a\cos(c+dx))} - \frac{a\sec(c+dx)\tan(c+dx)}{b^3d} - \frac{a\int \sec(c+dx)\,dx}{b^3} - \frac{\left(a^2-b^2\right)^2\int \frac{b}{b+a\cos(c+dx)}\,dx}{a^2b^4} - \frac{\text{Subst}\left(\int (1+x^2)\,dx, x, -\tan(c+dx)\right)}{b^2d} - \frac{(3(a^2-b^2))\,\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{b^4d} + \frac{(4(2a^6-3a^4b^2+b^6))\,\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2b^5d}$$

$$= -\frac{x}{a^2} - \frac{a\operatorname{arctanh}(\sin(c+dx))}{b^3d} - \frac{2a(2a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{b^5d} + \frac{4(a-b)^{3/2}(a+b)^{3/2}\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^5d} + \frac{\left(a^2-b^2\right)^2\sin(c+dx)}{ab^4d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{b^2d} + \frac{3(a^2-b^2)\tan(c+dx)}{b^4d} - \frac{a\sec(c+dx)\tan(c+dx)}{b^3d} + \frac{\tan^3(c+dx)}{3b^2d} - \frac{\left(a^2-b^2\right)^2\int \frac{1}{b+a\cos(c+dx)}\,dx}{a^2b^3}$$

$$
\begin{aligned}
&= -\frac{x}{a^2} - \frac{a\text{arctanh}(\sin(c+dx))}{b^3 d} - \frac{2a(2a^2-3b^2)\,\text{arctanh}(\sin(c+dx))}{b^5 d} \\
&\quad + \frac{4(a-b)^{3/2}(a+b)^{3/2}\,(2a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^5d} \\
&\quad + \frac{(a^2-b^2)^2\sin(c+dx)}{ab^4d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{b^2d} + \frac{3(a^2-b^2)\tan(c+dx)}{b^4d} \\
&\quad - \frac{a\sec(c+dx)\tan(c+dx)}{b^3d} + \frac{\tan^3(c+dx)}{3b^2d} \\
&\quad - \frac{\left(2(a^2-b^2)^2\right)\text{Subst}\left(\int\frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2b^3d} \\
&= -\frac{x}{a^2} - \frac{a\text{arctanh}(\sin(c+dx))}{b^3 d} - \frac{2a(2a^2-3b^2)\,\text{arctanh}(\sin(c+dx))}{b^5 d} \\
&\quad - \frac{2(a-b)^{3/2}(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d} \\
&\quad + \frac{4(a-b)^{3/2}(a+b)^{3/2}\,(2a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^5d} \\
&\quad + \frac{(a^2-b^2)^2\sin(c+dx)}{ab^4d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{b^2d} + \frac{3(a^2-b^2)\tan(c+dx)}{b^4d} \\
&\quad - \frac{a\sec(c+dx)\tan(c+dx)}{b^3d} + \frac{\tan^3(c+dx)}{3b^2d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 559 vs. $2(200) = 400$.

Time = 6.18 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.80

$$
\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx
$$

$$
= \frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(-\frac{12(c+dx)(b+a\cos(c+dx))}{a^2} - \frac{24(a^2-b^2)^{3/2}(4a^2+b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\,\mathrm{c}}{a^2b^5}\right.}{}
$$

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((-12*(c + d*x)*(b + a*Cos[c + d*x]))/
a^2 - (24*(a^2 - b^2)^(3/2)*(4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2
])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x]))/(a^2*b^5) + (12*(4*a^3 - 5*a*b^2)
*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/b^5 + (12*a
*(-4*a^2 + 5*b^2)*(b + a*Cos[c + d*x])*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]])/b^5 + ((-6*a + b)*(b + a*Cos[c + d*x]))/(b^3*(Cos[(c + d*x)/2] - Sin[
```

```
(c + d*x)/2])^2) + (2*(b + a*Cos[c + d*x])*Sin[(c + d*x)/2])/(b^2*(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])^3) + (4*(9*a^2 - 7*b^2)*(b + a*Cos[c + d*x])*S
in[(c + d*x)/2])/(b^4*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*(b + a*Co
s[c + d*x])*Sin[(c + d*x)/2])/(b^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
 + ((6*a - b)*(b + a*Cos[c + d*x]))/(b^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2) + (4*(9*a^2 - 7*b^2)*(b + a*Cos[c + d*x])*Sin[(c + d*x)/2])/(b^4*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])) + (12*(a^2 - b^2)^2*Sin[c + d*x])/(a*b^
4)))/(12*d*(a + b*Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{-\frac{1}{3b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{2a+b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a^2+ab-2b^2}{b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(4a^2-5b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^5}-\frac{2\left(\frac{\left(a^5b-2a^3b^3+\right.}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-}\right.}{}}{}$
default	$\frac{-\frac{1}{3b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{2a+b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a^2+ab-2b^2}{b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(4a^2-5b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^5}-\frac{2\left(\frac{\left(a^5b-2a^3b^3+\right.}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-}\right.}{}}{}$
risch	$-\frac{x}{a^2}+\frac{2i\left(6a^4b\,e^{7i(dx+c)}-6a^2b^3e^{7i(dx+c)}+3b^5e^{7i(dx+c)}+12a^5e^{6i(dx+c)}-9a^3b^2e^{6i(dx+c)}+3a\,b^4e^{6i(dx+c)}+30a^4b\,e^{5i(}\right.}{}$

```
[In] int(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/b^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(2*a+b)/b^3/(tan(1/2*d*x+1/2*c)-
1)^2-(3*a^2+a*b-2*b^2)/b^4/(tan(1/2*d*x+1/2*c)-1)+a*(4*a^2-5*b^2)/b^5*ln(ta
n(1/2*d*x+1/2*c)-1)-2/b^5/a^2*((a^5*b-2*a^3*b^3+a*b^5)*tan(1/2*d*x+1/2*c)/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(4*a^6-7*a^4*b^2+2*a^2*b
^4+b^6)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^
(1/2)))-2/a^2*arctan(tan(1/2*d*x+1/2*c))-1/3/b^2/(tan(1/2*d*x+1/2*c)+1)^3-1
/2*(-2*a-b)/b^3/(tan(1/2*d*x+1/2*c)+1)^2-(3*a^2+a*b-2*b^2)/b^4/(tan(1/2*d*x
+1/2*c)+1)-a*(4*a^2-5*b^2)/b^5*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(189) = 378.

Time = 0.56 (sec) , antiderivative size = 843, normalized size of antiderivative = 4.22

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \Bigg[-\frac{6\,ab^5dx\cos{(dx+c)}^4+6\,b^6dx\cos{(dx+c)}^3+3\,\left(\left(4\,a^5-3\,a^3b^2-ab^4\right)\cos{(dx+c)}^4+\left(4\,a^4b-3\,a^2b^3-b\right.\right.}{}$$
$$-\frac{6\,ab^5dx\cos{(dx+c)}^4+6\,b^6dx\cos{(dx+c)}^3-6\,\left(\left(4\,a^5-3\,a^3b^2-ab^4\right)\cos{(dx+c)}^4+\left(4\,a^4b-3\,a^2b^3-b\right.\right.}{}$$

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*a*b^5*d*x*cos(d*x + c)^4 + 6*b^6*d*x*cos(d*x + c)^3 + 3*((4*a^5 -
3*a^3*b^2 - a*b^4)*cos(d*x + c)^4 + (4*a^4*b - 3*a^2*b^3 - b^5)*cos(d*x + c
)^3)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 3*((4*a^6 - 5*a^4*b^2)*cos(d*
x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*
((4*a^6 - 5*a^4*b^2)*cos(d*x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)
*log(-sin(d*x + c) + 1) + 2*(2*a^3*b^3*cos(d*x + c) - a^2*b^4 - (12*a^5*b -
 13*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - (6*a^4*b^2 - 7*a^2*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/(a^3*b^5*d*cos(d*x + c)^4 + a^2*b^6*d*cos(d*x + c)^3), -
1/6*(6*a*b^5*d*x*cos(d*x + c)^4 + 6*b^6*d*x*cos(d*x + c)^3 - 6*((4*a^5 - 3*
a^3*b^2 - a*b^4)*cos(d*x + c)^4 + (4*a^4*b - 3*a^2*b^3 - b^5)*cos(d*x + c)^
3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) + 3*((4*a^6 - 5*a^4*b^2)*cos(d*x + c)^4 + (4*a^5*b - 5*a
^3*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((4*a^6 - 5*a^4*b^2)*cos(
d*x + c)^4 + (4*a^5*b - 5*a^3*b^3)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) +
 2*(2*a^3*b^3*cos(d*x + c) - a^2*b^4 - (12*a^5*b - 13*a^3*b^3 + 3*a*b^5)*co
s(d*x + c)^3 - (6*a^4*b^2 - 7*a^2*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(a^3*b
^5*d*cos(d*x + c)^4 + a^2*b^6*d*cos(d*x + c)^3)]
```

Sympy [F]

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^6{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

```
[In] integrate(tan(d*x+c)**6/(a+b*sec(d*x+c))**2,x)
[Out] Integral(tan(c + d*x)**6/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(189) = 378$.

Time = 2.11 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.06

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$

$$-\frac{\frac{3(dx+c)}{a^2} + \frac{3(4a^3-5ab^2)\log(|\tan(\frac{1}{2}dx+\frac{1}{2}c)+1|)}{b^5} - \frac{3(4a^3-5ab^2)\log(|\tan(\frac{1}{2}dx+\frac{1}{2}c)-1|)}{b^5} + \frac{6(a^4\tan(\frac{1}{2}dx+\frac{1}{2}c)-2a^2b^2\tan(\frac{1}{2}dx\cdots}{(a\tan(\frac{1}{2}dx+\frac{1}{2}c)^2-b\tan(\frac{1}{2}dx\cdots}}{\cdots}$$

```
[In] integrate(tan(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] -1/3*(3*(d*x + c)/a^2 + 3*(4*a^3 - 5*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) +
1))/b^5 - 3*(4*a^3 - 5*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^5 + 6*(a
^4*tan(1/2*d*x + 1/2*c) - 2*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x
+ 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a*
b^4) - 6*(4*a^6 - 7*a^4*b^2 + 2*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^2*b^5) + 2*(9*a^2*tan(1/2*d*x
```

```
 + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*b^2*tan(1/2*d*x + 1/2*c)^5 -
 18*a^2*tan(1/2*d*x + 1/2*c)^3 + 16*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(
1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) - 6*b^2*tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^4))/d
```

Mupad [B] (verification not implemented)

Time = 17.23 (sec) , antiderivative size = 9452, normalized size of antiderivative = 47.26

$$\int \frac{\tan^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] int(tan(c + d*x)^6/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^5*(10*a*b^3 - 6*a^3*b + 36*a^4 + 9*b^4 - 37*a^2*b^2)
)/(3*a*b^4) - (2*tan(c/2 + (d*x)/2)^3*(6*a^3*b - 10*a*b^3 + 36*a^4 + 9*b^4
- 37*a^2*b^2))/(3*a*b^4) + (2*tan(c/2 + (d*x)/2)*(2*a^3*b - 2*a*b^3 + 4*a^4
 + b^4 - 5*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^7*(a - b)*(3*a*b^2 - 2
*a^2*b - 4*a^3 + b^3))/(a*b^4))/(d*(a + b + tan(c/2 + (d*x)/2)^8*(a - b) -
tan(c/2 + (d*x)/2)^2*(4*a + 2*b) - tan(c/2 + (d*x)/2)^6*(4*a - 2*b) + 6*a*t
an(c/2 + (d*x)/2)^4)) - (2*atan(((((((((((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13
*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*
b^15 - 12*a^12*b^14))/(a^3*b^16) - (tan(c/2 + (d*x)/2)*(2*a^6*b^23 - 6*a^7*
b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18)*8192i)/(a^6*b^1
6))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^
21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 -
60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11
 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16))*1i)/a^2 + (8192*(
4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 +
74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 +
 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*
b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16))*1i)/a^2 - (8
192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19
+ 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 58
8*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 +
2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b
^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16))/a^2 - ((((((((8192*(9*a^5*
b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 +
3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) + (tan(c/2 + (d*x)/2
)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11
*b^18)*8192i)/(a^6*b^16))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 -
6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*
b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^1
3*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^
16))*1i)/a^2 + (8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4
```

*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b
^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20
*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a
^3*b^16))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^2
1 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b
^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b
^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*
a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16))/a^2)
/(((((((((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^
8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^1
6) - (tan(c/2 + (d*x)/2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20
 + 6*a^10*b^19 - 2*a^11*b^18)*8192i)/(a^6*b^16))*1i)/a^2 - (8192*tan(c/2 +
(d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19
+ 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 +
 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9
+ 32*a^17*b^8))/(a^4*b^16))*1i)/a^2 + (8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20
 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 1
92*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^
10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*
b^5 + 192*a^18*b^4))/(a^3*b^16))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(a*b^20
 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 1
40*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 13
2*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7
- 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19
*b^2))/(a^4*b^16))*1i)/a^2 + ((((((((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^
6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^1
5 - 12*a^12*b^14))/(a^3*b^16) + (tan(c/2 + (d*x)/2)*(2*a^6*b^23 - 6*a^7*b^2
2 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18)*8192i)/(a^6*b^16))
*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21
- 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*
a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 +
32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16))*1i)/a^2 + (8192*(4*a
*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*
a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 10
40*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7
 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16))*1i)/a^2 + (8192
*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4
*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a
^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 229
7*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4
+ 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16))*1i)/a^2 - (16384*(5*a*b^17 -
256*a^17*b + 256*a^18 - 5*b^18 - 91*a^2*b^16 + 116*a^3*b^15 - 14*a^4*b^14 +
 174*a^5*b^13 + 582*a^6*b^12 - 1036*a^7*b^11 - 1133*a^8*b^10 + 101*a^9*b^9
+ 2245*a^10*b^8 + 2624*a^11*b^7 - 3792*a^12*b^6 - 3264*a^13*b^5 + 3488*a^14
*b^4 + 1536*a^15*b^3 - 1536*a^16*b^2))/(a^3*b^16))))/(a^2*d) - (a*atan(((a*

((8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16) + (a*((8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16) + (a*((8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16) + (a*((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) + (8192*tan(c/2 + (d*x)/2)*(4*a^2 - 5*b^2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18))/(a^3*b^21))*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2)*1i)/b^5 + (a*((8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16) - (a*((8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16) - (a*((8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16) - (a*((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) - (8192*tan(c/2 + (d*x)/2)*(4*a^2 - 5*b^2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18))/(a^3*b^21))*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2)*1i)/b^5)/((16384*(5*a*b^17 - 256*a^17*b + 256*a^18 - 5*b^18 - 91*a^2*b^16 + 116*a^3*b^15 - 14*a^4*b^14 + 174*a^5*b^13 + 582*a^6*b^12 - 1036*a^7*b^11 - 1133*a^8*b^10 + 101*a^9*b^9 + 2245*a^10*b^8 + 2624*a^11*b^7 - 3792*a^12*b^6 - 3264*a^13*b^5 + 3488*a^14*b^4 + 1536*a^15*b^3 - 1536*a^16*b^2))/(a^3*b^16) - (a*((8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16) + (a*((8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 1

92*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16) + (a*((8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16) + (a*((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) + (8192*tan(c/2 + (d*x)/2)*(4*a^2 - 5*b^2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18))/(a^3*b^21))*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5 + (a*((8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16) - (a*((8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16) - (a*((8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16) - (a*((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) - (8192*tan(c/2 + (d*x)/2)*(4*a^2 - 5*b^2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18))/(a^3*b^21))*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5)*(4*a^2 - 5*b^2))/b^5))*(4*a^2 - 5*b^2)*2i)/(b^5*d) - (atan((((4*a^2 + b^2)*((8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16) + ((4*a^2 + b^2)*((8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16) + (((8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16) + ((4*a^2 + b^2)*((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) + (8192*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^6*b^23 - 6*a^7*b^22 +

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 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18))/(a^6*b^21))*((a + b)
^3*(a - b)^3)^(1/2))/(a^2*b^5))*(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2))/
(a^2*b^5))*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b^5))*((a + b)^3*(a - b)^3)^(1
/2)*1i)/(a^2*b^5) + ((4*a^2 + b^2)*((8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*
a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*
b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*
b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*
a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/
(a^4*b^16) - ((4*a^2 + b^2)*((8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3
*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^
14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366
*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192
*a^18*b^4))/(a^3*b^16) - (((8192*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^2
2 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 16
0*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 +
128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16) - ((4
*a^2 + b^2)*((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25
*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*
b^16) - (8192*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2)*
(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b
^18))/(a^6*b^21))*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b^5))*(4*a^2 + b^2)*((a
 + b)^3*(a - b)^3)^(1/2))/(a^2*b^5))*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b^5)
)*((a + b)^3*(a - b)^3)^(1/2)*1i)/(a^2*b^5))/((16384*(5*a*b^17 - 256*a^17*b
 + 256*a^18 - 5*b^18 - 91*a^2*b^16 + 116*a^3*b^15 - 14*a^4*b^14 + 174*a^5*b
^13 + 582*a^6*b^12 - 1036*a^7*b^11 - 1133*a^8*b^10 + 101*a^9*b^9 + 2245*a^1
0*b^8 + 2624*a^11*b^7 - 3792*a^12*b^6 - 3264*a^13*b^5 + 3488*a^14*b^4 + 153
6*a^15*b^3 - 1536*a^16*b^2))/(a^3*b^16) - ((4*a^2 + b^2)*((8192*tan(c/2 + (
d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a^2*b^19 + 4*a^3*b^18 -
40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b^14 + 588*a^8*b^13 - 62
4*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^12*b^9 + 2297*a^13*b^8 +
 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 3680*a^17*b^4 + 1536*a^18*
b^3 - 1536*a^19*b^2))/(a^4*b^16) + ((4*a^2 + b^2)*((8192*(4*a*b^21 - 3*b^22
 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b^17 + 74*a^6*b^16 - 280
*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*b^12 + 1040*a^11*b^11 +
 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696*a^15*b^7 - 528*a^16*b^
6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16) + (((8192*tan(c/2 + (d*x)/2)*(
2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20 - 8*a^6*b^19 + 12*a^7*b
^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 100*a^11*b^14 + 82*a^12*b
^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 - 96*a^16*b^9 + 32*a^17*
b^8))/(a^4*b^16) + ((4*a^2 + b^2)*((8192*(9*a^5*b^21 - 3*a^4*b^22 - 13*a^6*
b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^10*b^16 + 26*a^11*b^15
- 12*a^12*b^14))/(a^3*b^16) + (8192*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*((a +
b)^3*(a - b)^3)^(1/2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a^8*b^21 - 8*a^9*b^20 +
6*a^10*b^19 - 2*a^11*b^18))/(a^6*b^21))*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b
^5))*(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b^5))*((a + b)^3*(a -
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b)^3)^(1/2))/(a^2*b^5))*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b^5) + ((4*a^2 +
b^2)*((8192*tan(c/2 + (d*x)/2)*(a*b^20 - 256*a^20*b + 256*a^21 - b^21 - 4*a
^2*b^19 + 4*a^3*b^18 - 40*a^4*b^17 + 140*a^5*b^16 - 250*a^6*b^15 + 90*a^7*b
^14 + 588*a^8*b^13 - 624*a^9*b^12 + 132*a^10*b^11 + 28*a^11*b^10 - 2361*a^1
2*b^9 + 2297*a^13*b^8 + 4320*a^14*b^7 - 4320*a^15*b^6 - 3680*a^16*b^5 + 368
0*a^17*b^4 + 1536*a^18*b^3 - 1536*a^19*b^2))/(a^4*b^16) - ((4*a^2 + b^2)*((
8192*(4*a*b^21 - 3*b^22 - 8*a^2*b^20 + 16*a^3*b^19 + 20*a^4*b^18 - 26*a^5*b
^17 + 74*a^6*b^16 - 280*a^7*b^15 + 192*a^8*b^14 + 332*a^9*b^13 - 1088*a^10*
b^12 + 1040*a^11*b^11 + 1129*a^12*b^10 - 2366*a^13*b^9 + 20*a^14*b^8 + 1696
*a^15*b^7 - 528*a^16*b^6 - 416*a^17*b^5 + 192*a^18*b^4))/(a^3*b^16) - (((81
92*tan(c/2 + (d*x)/2)*(2*a^2*b^23 - 6*a^3*b^22 + 12*a^4*b^21 - 12*a^5*b^20
- 8*a^6*b^19 + 12*a^7*b^18 - 60*a^8*b^17 + 160*a^9*b^16 - 60*a^10*b^15 - 10
0*a^11*b^14 + 82*a^12*b^13 - 118*a^13*b^12 + 128*a^14*b^11 + 32*a^15*b^10 -
 96*a^16*b^9 + 32*a^17*b^8))/(a^4*b^16) - ((4*a^2 + b^2)*((8192*(9*a^5*b^21
 - 3*a^4*b^22 - 13*a^6*b^20 + 6*a^7*b^19 + 25*a^8*b^18 - 41*a^9*b^17 + 3*a^
10*b^16 + 26*a^11*b^15 - 12*a^12*b^14))/(a^3*b^16) - (8192*tan(c/2 + (d*x)/
2)*(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2)*(2*a^6*b^23 - 6*a^7*b^22 + 8*a
^8*b^21 - 8*a^9*b^20 + 6*a^10*b^19 - 2*a^11*b^18))/(a^6*b^21))*((a + b)^3*(
a - b)^3)^(1/2))/(a^2*b^5))*(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^2
*b^5))*((a + b)^3*(a - b)^3)^(1/2))/(a^2*b^5))*((a + b)^3*(a - b)^3)^(1/2))
/(a^2*b^5)))*(4*a^2 + b^2)*((a + b)^3*(a - b)^3)^(1/2)*2i)/(a^2*b^5*d)
```

3.308 $\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 150

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2} dx = \frac{x}{a^2} - \frac{2a\text{arctanh}(\sin(c+dx))}{b^3d} + \frac{2\sqrt{a-b}\sqrt{a+b}(2a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d} + \frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))} + \frac{\tan(c+dx)}{bd(b+a\cos(c+dx))}$$

```
[Out] x/a^2-2*a*arctanh(sin(d*x+c))/b^3/d+(2*a^2-b^2)*sin(d*x+c)/a/b^2/d/(b+a*cos
(d*x+c))+2*(2*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*
(a-b)^(1/2)*(a+b)^(1/2)/a^2/b^3/d+tan(d*x+c)/b/d/(b+a*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3983, 2969, 3136, 2738, 214, 3855}

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2} dx = \frac{2\sqrt{a-b}\sqrt{a+b}(2a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d} + \frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(a\cos(c+dx)+b)} + \frac{x}{a^2} - \frac{2a\text{arctanh}(\sin(c+dx))}{b^3d} + \frac{\tan(c+dx)}{bd(a\cos(c+dx)+b)}$$

```
[In] Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (2*a*ArcTanh[Sin[c + d*x]])/(b^3*d) + (2*Sqrt[a - b]*Sqrt[a + b]*(2
*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*b^3*d
) + ((2*a^2 - b^2)*Sin[c + d*x])/(a*b^2*d*(b + a*Cos[c + d*x])) + Tan[c + d
*x]/(b*d*(b + a*Cos[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2969

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[(a^2*(n + 1) - b^2*(m + n + 2))*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2*b*d^2*f*(n
+ 1)*(m + 1))), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx \\
&= \frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))}+\frac{\tan(c+dx)}{bd(b+a\cos(c+dx))}+\frac{\int\frac{(-2a^2-ab\cos(c+dx)+b^2\cos^2(c+dx))\sec(c+dx)}{b+a\cos(c+dx)}\,dx}{ab^2} \\
&= \frac{x}{a^2}+\frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))}+\frac{\tan(c+dx)}{bd(b+a\cos(c+dx))} \\
&\quad -\frac{(2a)\int\sec(c+dx)\,dx}{b^3}-\frac{(-2a^4+a^2b^2+b^4)\int\frac{1}{b+a\cos(c+dx)}\,dx}{a^2b^3} \\
&= \frac{x}{a^2}-\frac{2a\text{arctanh}(\sin(c+dx))}{b^3d}+\frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))}+\frac{\tan(c+dx)}{bd(b+a\cos(c+dx))} \\
&\quad -\frac{(2(-2a^4+a^2b^2+b^4))\,\text{Subst}\left(\int\frac{1}{a+b+(-a+b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2b^3d} \\
&= \frac{x}{a^2}-\frac{2a\text{arctanh}(\sin(c+dx))}{b^3d}+\frac{2\sqrt{a-b}\sqrt{a+b}(2a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2b^3d} \\
&\quad +\frac{(2a^2-b^2)\sin(c+dx)}{ab^2d(b+a\cos(c+dx))}+\frac{\tan(c+dx)}{bd(b+a\cos(c+dx))}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 327 vs. $2(150) = 300$.

Time = 1.53 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.18

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(\frac{(c+dx)(b+a\cos(c+dx))}{a^2}+\frac{2(-2a^4+a^2b^2+b^4)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx)}{a^2b^3\sqrt{a^2-b^2}}\right.}{}$$

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(((c + d*x)*(b + a*Cos[c + d*x]))/a^2
+ (2*(-2*a^4 + a^2*b^2 + b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]]*(b + a*Cos[c + d*x]))/(a^2*b^3*Sqrt[a^2 - b^2]) + (2*a*(b + a*Cos[c
 + d*x])*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/b^3 - (2*a*(b + a*Cos[c
+ d*x])*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/b^3 + ((b + a*Cos[c + d*x
])*Sin[(c + d*x)/2])/(b^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + ((b + a*
Cos[c + d*x])*Sin[(c + d*x)/2])/(b^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
 + ((a^2 - b^2)*Sin[c + d*x])/(a*b^2)))/(d*(a + b*Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}-\frac{2\left(\frac{\cdots}{\tan\cdots}\right.}{\cdots}}{d}$
default	$\frac{-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{1}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}-\frac{2\left(\frac{\cdots}{\tan\cdots}\right.}{\cdots}}{d}$
risch	$\frac{x}{a^2}+\frac{2i\left(a^2b\,e^{3i(dx+c)}-b^3e^{3i(dx+c)}+2a^3e^{2i(dx+c)}-a\,b^2e^{2i(dx+c)}+3a^2b\,e^{i(dx+c)}-b^3e^{i(dx+c)}+2a^3-a\,b^2\right)}{d\,a^2b^2\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)\left(e^{2i(dx+c)}+1\right)}+\frac{2a\ln(e^{i(\cdots}}{b^3\cdots}$

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^2/(tan(1/2*d*x+1/2*c)+1)-2*a/b^3*ln(tan(1/2*d*x+1/2*c)+1)+2/a^2*a
rctan(tan(1/2*d*x+1/2*c))-1/b^2/(tan(1/2*d*x+1/2*c)-1)+2*a/b^3*ln(tan(1/2*d
*x+1/2*c)-1)-2/a^2/b^3*((a^3*b-a*b^3)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^4-a^2*b^2-b^4)/((a-b)*(a+b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.89

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[\frac{2\,ab^3dx\cos(dx+c)^2+2\,b^4dx\cos(dx+c)+\left((2\,a^3+ab^2)\cos(dx+c)^2+(2\,a^2b+b^3)\cos(dx+c)\right)\sqrt{a^2\cdots}}{\cdots}\right.$$

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^3*d*x*cos(d*x + c)^2 + 2*b^4*d*x*cos(d*x + c) + ((2*a^3 + a*b^2
)*cos(d*x + c)^2 + (2*a^2*b + b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b
*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x
 + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
+ c) + b^2)) - 2*(a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(sin(d*x + c)
 + 1) + 2*(a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(-sin(d*x + c) + 1)
+ 2*(a^2*b^2 + (2*a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*cos
(d*x + c)^2 + a^2*b^4*d*cos(d*x + c)), (a*b^3*d*x*cos(d*x + c)^2 + b^4*d*x*
cos(d*x + c) + ((2*a^3 + a*b^2)*cos(d*x + c)^2 + (2*a^2*b + b^3)*cos(d*x +
c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
b^2)*sin(d*x + c))) - (a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(sin(d*x
 + c) + 1) + (a^4*cos(d*x + c)^2 + a^3*b*cos(d*x + c))*log(-sin(d*x + c) +
1) + (a^2*b^2 + (2*a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*co
s(d*x + c)^2 + a^2*b^4*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(141) = 282$.

Time = 0.94 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.96

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{\frac{dx+c}{a^2} - \frac{2a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^3} + \frac{2a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^3} - \frac{2\left(2a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - \right.}}{}$$

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^2 - 2*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + 2*a*log(abs(t
an(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a^2*tan(1/2*d*x + 1/2*c)^3 - a*b*tan(1
/2*d*x + 1/2*c)^3 - b^2*tan(1/2*d*x + 1/2*c)^3 - 2*a^2*tan(1/2*d*x + 1/2*c)
 - a*b*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1
/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*a*
b^2) + 2*(2*a^4 - a^2*b^2 - b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
 + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a
^2 + b^2)))/(sqrt(-a^2 + b^2)*a^2*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 16.19 (sec) , antiderivative size = 7044, normalized size of antiderivative = 46.96

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*atan(((((((((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 +
6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) - (tan(c/2 + (d*x)/2)*(2*a
^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10)
*8192i)/(a^6*b^8))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b
^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^
9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8))*1
i)/a^2 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 +
11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^
4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8))*1i)/a^2 + (8192*tan(c/2 + (d*x)/
2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b
^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^1
0*b^3 + 16*a^11*b^2))/(a^4*b^8))/a^2 - (((((((8192*(8*a^5*b^13 - 3*a^4*b^1
4 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b
```

^8) + (tan(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^1
2 + 6*a^10*b^11 - 2*a^11*b^10)*8192i)/(a^6*b^8))*1i)/a^2 + (8192*tan(c/2 +
(d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 +
3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b
^5 + 8*a^13*b^4))/(a^4*b^8))*1i)/a^2 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^1
2 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8
*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8))*1i
)/a^2 - (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a
^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 1
2*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8))/a^2)/(((((((
(((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 1
3*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) - (tan(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^
7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10)*8192i)/(a^6*b
^8))*1i)/a^2 - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b
^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^1
0*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8))*1i)/a^2 - (8192
*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 1
8*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^
3 + 24*a^12*b^2))/(a^3*b^8))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*(16*a^12*b
- a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^
8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^1
1*b^2))/(a^4*b^8))*1i)/a^2 + ((((((((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6
*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) + (ta
n(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^1
0*b^11 - 2*a^11*b^10)*8192i)/(a^6*b^8))*1i)/a^2 + (8192*tan(c/2 + (d*x)/2)*
(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^1
0 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^
13*b^4))/(a^4*b^8))*1i)/a^2 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3
*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14
*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8))*1i)/a^2 - (
8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 -
 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5
 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8))*1i)/a^2 - (16384*(2*a
*b^9 - 16*a^9*b + 16*a^10 - 2*b^10 - 16*a^2*b^8 + 24*a^3*b^7 - 18*a^4*b^6 +
 26*a^5*b^5 + 12*a^6*b^4 - 36*a^7*b^3 + 8*a^8*b^2))/(a^3*b^8))))/(a^2*d) +
((2*tan(c/2 + (d*x)/2)*(a*b + 2*a^2 - b^2))/(a*b^2) - (2*tan(c/2 + (d*x)/2)
^3*(a - b)*(2*a + b))/(a*b^2))/(d*(a + b + tan(c/2 + (d*x)/2)^4*(a - b) - 2
*a*tan(c/2 + (d*x)/2)^2)) + (atan((((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)
^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*((
(2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/
2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^
6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) - (8
192*tan(c/2 + (d*x)/2)*((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*
b))*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^
11*b^10))/(a^4*b^8)) - (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 +

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10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8
- 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8)) - (8192
*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 1
8*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^
3 + 24*a^12*b^2))/(a^3*b^8)) + (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12
 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a
^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/
(a^4*b^8))*1i - ((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((
2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2
))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^
2)^(1/2)/(a^2*b))*((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11
 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) + (8192*tan(c/2 + (d*x)
/2)*((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(2*a^6*b^15 - 6
*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10))/(a^4*b^8)
) + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^
5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*
a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8)) - (8192*(3*b^14 - 5*a*b^13
 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*
b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(
a^3*b^8)) - (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 +
 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6
 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8))*1i)/(((2
*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2)
)/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2
)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(
(8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*
a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) - (8192*tan(c/2 + (d*x)/2)*((2*(a^2 - b^2)
^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b
^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10))/(a^4*b^8)) - (8192*tan(c/2 +
 (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 +
 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*
b^5 + 8*a^13*b^4))/(a^4*b^8)) - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a
^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 +
14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8)) + (8192*t
an(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3
*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a
^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8)) + ((2*(a^2 - b^2)^(1/2))/b^
3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1
/2)/(a^2*b))*(((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*(((2*
(a^2 - b^2)^(1/2))/b^3 + (a^2 - b^2)^(1/2)/(a^2*b))*((8192*(8*a^5*b^13 - 3*
a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))
/(a^3*b^8) + (8192*tan(c/2 + (d*x)/2)*((2*(a^2 - b^2)^(1/2))/b^3 + (a^2 - b
^2)^(1/2)/(a^2*b))*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a
^10*b^11 - 2*a^11*b^10))/(a^4*b^8)) + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15
- 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^
```

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9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^
4*b^8)) - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 +
 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b
^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8)) - (8192*tan(c/2 + (d*x)/2)*(16*
a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21
*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 +
 16*a^11*b^2))/(a^4*b^8)) - (16384*(2*a*b^9 - 16*a^9*b + 16*a^10 - 2*b^10 -
 16*a^2*b^8 + 24*a^3*b^7 - 18*a^4*b^6 + 26*a^5*b^5 + 12*a^6*b^4 - 36*a^7*b^
3 + 8*a^8*b^2))/(a^3*b^8)))*(((a^2 - b^2)^(1/2)*4i)/b^3 + ((a^2 - b^2)^(1/2
)*2i)/(a^2*b)))/d + (a*atan(((a*((2*a*((2*a*((2*a*((8192*(8*a^5*b^13 - 3*a^
4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(
a^3*b^8) - (16384*tan(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13
- 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10))/(a^3*b^11)))/b^3 - (8192*tan(c/2
 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11
 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^1
2*b^5 + 8*a^13*b^4))/(a^4*b^8)))/b^3 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^1
2 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8
*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8)))/b
^3 + (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*
b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a
^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8))*2i)/b^3 - (a*((
2*a*((2*a*((2*a*((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 +
 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) + (16384*tan(c/2 + (d*x)/
2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^1
1*b^10))/(a^3*b^11)))/b^3 + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^
14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9
*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8)))/b
^3 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a
^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 -
52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8)))/b^3 - (8192*tan(c/2 + (d*x)/2)*(16*
a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21
*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 +
 16*a^11*b^2))/(a^4*b^8))*2i)/b^3)/((2*a*((2*a*((2*a*((2*a*((8192*(8*a^5*b^
13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^1
0*b^8))/(a^3*b^8) - (16384*tan(c/2 + (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*
a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11 - 2*a^11*b^10))/(a^3*b^11)))/b^3 - (819
2*tan(c/2 + (d*x)/2)*(2*a^2*b^15 - 6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 +
 a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9 + 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6
 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*b^8)))/b^3 - (8192*(3*b^14 - 5*a*b^13 +
6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^10 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7
 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^10*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3
*b^8)))/b^3 + (8192*tan(c/2 + (d*x)/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13
 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b
^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^10*b^3 + 16*a^11*b^2))/(a^4*b^8)))/b^3 +
 (2*a*((2*a*((2*a*((2*a*((8192*(8*a^5*b^13 - 3*a^4*b^14 - 9*a^6*b^12 + 5*a^
```

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7*b^11 + 6*a^8*b^10 - 13*a^9*b^9 + 6*a^10*b^8))/(a^3*b^8) + (16384*tan(c/2
+ (d*x)/2)*(2*a^6*b^15 - 6*a^7*b^14 + 8*a^8*b^13 - 8*a^9*b^12 + 6*a^10*b^11
 - 2*a^11*b^10))/(a^3*b^11)))/b^3 + (8192*tan(c/2 + (d*x)/2)*(2*a^2*b^15 -
6*a^3*b^14 + 10*a^4*b^13 - 10*a^5*b^12 + a^6*b^11 + 3*a^7*b^10 - 9*a^8*b^9
+ 25*a^9*b^8 - 28*a^10*b^7 + 28*a^11*b^6 - 24*a^12*b^5 + 8*a^13*b^4))/(a^4*
b^8)))/b^3 - (8192*(3*b^14 - 5*a*b^13 + 6*a^2*b^12 - 8*a^3*b^11 - 5*a^4*b^1
0 + 11*a^5*b^9 - 18*a^6*b^8 + 40*a^7*b^7 - 34*a^8*b^6 + 14*a^9*b^5 + 24*a^1
0*b^4 - 52*a^11*b^3 + 24*a^12*b^2))/(a^3*b^8)))/b^3 - (8192*tan(c/2 + (d*x)
/2)*(16*a^12*b - a*b^12 - 16*a^13 + b^13 + 2*a^2*b^11 - 2*a^3*b^10 + 5*a^4*
b^9 - 21*a^5*b^8 + 44*a^6*b^7 - 44*a^7*b^6 + 12*a^8*b^5 + 4*a^9*b^4 - 16*a^
10*b^3 + 16*a^11*b^2))/(a^4*b^8)))/b^3 - (16384*(2*a*b^9 - 16*a^9*b + 16*a^
10 - 2*b^10 - 16*a^2*b^8 + 24*a^3*b^7 - 18*a^4*b^6 + 26*a^5*b^5 + 12*a^6*b^
4 - 36*a^7*b^3 + 8*a^8*b^2))/(a^3*b^8)))*4i)/(b^3*d)
```

3.309 $\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx=-\frac{x}{a^2}+\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d}+\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}$$

```
[Out] -x/a^2+2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/d/(a-b)^
(1/2)/(a+b)^(1/2)+tan(d*x+c)/a/d/(a+b*sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3979, 4146, 12, 3868, 2738, 214}

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{x}{a^2}+\frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}$$

```
[In] Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) + (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*S
qrt[a - b]*Sqrt[a + b]*d) + Tan[c + d*x]/(a*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4146

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{-1+\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx \\
&= \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} - \frac{\int \frac{a^2-b^2}{a+b\sec(c+dx)}\,dx}{a\,(a^2-b^2)} \\
&= \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} - \frac{\int \frac{1}{a+b\sec(c+dx)}\,dx}{a} \\
&= -\frac{x}{a^2} + \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} + \frac{\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2}
\end{aligned}
$$

$$= -\frac{x}{a^2} + \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))} + \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2 d}$$

$$= -\frac{x}{a^2} + \frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d} + \frac{\tan(c+dx)}{ad(a+b\sec(c+dx))}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{c+dx+\frac{2b\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{a\sin(c+dx)}{b+a\cos(c+dx)}}{a^2 d}$$

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((c + d*x + (2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqr
t[a^2 - b^2] - (a*Sin[c + d*x])/(b + a*Cos[c + d*x]))/(a^2*d))
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b-a-b}+\frac{2b\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}}{a^2}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
default	$\frac{\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b-a-b}+\frac{2b\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}}{a^2}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
risch	$-\frac{x}{a^2}+\frac{2i\left(b\,e^{i(dx+c)}+a\right)}{a^2 d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}-\frac{b\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,a^2}+\frac{b\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,d\,a^2}$

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^2*(-a*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b-a-b)+b/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a
+b))^(1/2)))-2/a^2*arctan(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(76) = 152$.

Time = 0.30 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.51

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \left[-\frac{2\,(a^3-ab^2)dx\cos(dx+c)+2\,(a^2b-b^3)dx-(ab\cos(dx+c)+b^2)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)}{}\right.}{2\,((a^5-a^3b^2)d\cos(dx+c)+(a^4b-}\right.$$

$$-\frac{(a^3-ab^2)dx\cos(dx+c)+(a^2b-b^3)dx-(ab\cos(dx+c)+b^2)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c}{(a^2-b^2)\sin(dx+c}\right.}{(a^5-a^3b^2)d\cos(dx+c)+(a^4b-a^2b^3)d}$$

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^3 - a*b^2)*d*x*cos(d*x + c) + 2*(a^2*b - b^3)*d*x - (a*b*cos(d*
x + c) + b^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d
*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b
^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^3 - a*b^2)*sin(
d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c) + (a^4*b - a^2*b^3)*d), -((a^3 -
a*b^2)*d*x*cos(d*x + c) + (a^2*b - b^3)*d*x - (a*b*cos(d*x + c) + b^2)*sqrt
(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c))) - (a^3 - a*b^2)*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c) +
 (a^4*b - a^2*b^3)*d)]
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.69

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}a^2}-\frac{dx+c}{a^2}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)a}}{d}$$

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2
)*a^2) - (d*x + c)/a^2 - 2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a))/d
```

Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 551, normalized size of antiderivative = 6.48

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^2\,d} - \frac{b^2\left(a\,\sin\left(c+d\,x\right)+2\,\mathrm{atanh}\left(\frac{2\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a^2-b^2\right)}^{3/2}-a^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}+2\,b^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}-3\,a^2\,b^3\,\sin\left(\frac{c}{2}+\frac{d}{}\right.}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a\,b^2-a^3\right)\,\left(b\,\left(a^2-b^2\right)+a\,b^2-a^2\right.}\right.\right.}{}$$

```
[In] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

```
[Out] - (2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^2*d) - (b^2*(a*sin(c +
 d*x) + 2*atanh((2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) - a^5*sin(c/2 +
 (d*x)/2)*(a^2 - b^2)^(1/2) + 2*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) -
3*a^2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + a^3*b^2*sin(c/2 + (d*x)/2)
*(a^2 - b^2)^(1/2) + a^4*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/(cos(c/2 +
 (d*x)/2)*(a*b^2 - a^3)*(b*(a^2 - b^2) + a*b^2 - a^2*b - a^3 + b^3)))*(a^2
- b^2)^(1/2)) - a^3*sin(c + d*x) + 2*a*b*cos(c + d*x)*atanh((2*b^3*sin(c/2
+ (d*x)/2)*(a^2 - b^2)^(3/2) - a^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 2
*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 3*a^2*b^3*sin(c/2 + (d*x)/2)*(a
^2 - b^2)^(1/2) + a^3*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + a^4*b*sin(
c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(b*(a^2
 - b^2) + a*b^2 - a^2*b - a^3 + b^3)))*(a^2 - b^2)^(1/2))/(a^2*d*(a^2 - b^2
)*(b + a*cos(c + d*x)))
```

3.310 $\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 227

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2} dx = -\frac{x}{a^2} - \frac{2b^5\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{4b^3(2a^2-b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{b^4\sin(c+dx)}{a\,(a^2-b^2)^2\,d(b+a\cos(c+dx))}$$

```
[Out] -x/a^2-2*b^5*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-b)^
(5/2)/(a+b)^(5/2)/d-4*b^3*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c
)/(a+b)^(1/2))/a^2/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-co
s(d*x+c))+1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))+b^4*sin(d*x+c)/a/(a^2-b^2
)^2/d/(b+a*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{3983, 2976, 2727, 2743, 12, 2738, 214\}$

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2b^5\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}} - \frac{4b^3(2a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}} + \frac{b^4\sin(c+dx)}{ad\left(a^2-b^2\right)^2\left(a\cos(c+dx)+b\right)} - \frac{x}{a^2} - \frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

```
[In] Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2
*(a - b)^(5/2)*(a + b)^(5/2)*d) - (4*b^3*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]
*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[
c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(
1 + Cos[c + d*x])) + (b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*d*(b + a*Cos[c + d
*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx \\
&= \int \left(-\frac{1}{a^2}-\frac{1}{2(a-b)^2(-1-\cos(c+dx))}+\frac{1}{2(a+b)^2(1-\cos(c+dx))}\right.\\
&\quad \left.+\frac{b^4}{a^2\left(a^2-b^2\right)(-b-a\cos(c+dx))^2}+\frac{2\left(2a^2b^3-b^5\right)}{a^2\left(a^2-b^2\right)^2(-b-a\cos(c+dx))}\right)dx \\
&= -\frac{x}{a^2}-\frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2}+\frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} \\
&\quad +\frac{b^4\int \frac{1}{(-b-a\cos(c+dx))^2}\,dx}{a^2\left(a^2-b^2\right)}+\frac{\left(2b^3\left(2a^2-b^2\right)\right)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^2\left(a^2-b^2\right)^2} \\
&= -\frac{x}{a^2}-\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} \\
&\quad +\frac{b^4\sin(c+dx)}{a\left(a^2-b^2\right)^2d(b+a\cos(c+dx))}+\frac{b^4\int \frac{b}{-b-a\cos(c+dx)}\,dx}{a^2\left(a^2-b^2\right)^2} \\
&\quad +\frac{\left(4b^3\left(2a^2-b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2\left(a^2-b^2\right)^2d}
\end{aligned}
$$

$$= -\frac{x}{a^2} - \frac{4b^3(2a^2-b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{b^4\sin(c+dx)}{a\left(a^2-b^2\right)^2d(b+a\cos(c+dx))} + \frac{b^5\int\frac{1}{-b-a\cos(c+dx)}\,dx}{a^2\left(a^2-b^2\right)^2}$$

$$= -\frac{x}{a^2} - \frac{4b^3(2a^2-b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{b^4\sin(c+dx)}{a\left(a^2-b^2\right)^2d(b+a\cos(c+dx))} + \frac{(2b^5)\,\text{Subst}\left(\int\frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2\left(a^2-b^2\right)^2d}$$

$$= -\frac{x}{a^2} - \frac{2b^5\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{4b^3(2a^2-b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{b^4\sin(c+dx)}{a\left(a^2-b^2\right)^2d(b+a\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.92

$$\int\frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(-\frac{2(c+dx)(b+a\cos(c+dx))}{a^2} - \frac{4b^3(-4a^2+b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx)}{a^2(a^2-b^2)^{5/2}}\right.}{2d(a+b\sec(c+dx))^2}$$

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((-2*(c + d*x)*(b + a*Cos[c + d*x]))/a
^2 - (4*b^3*(-4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]]*(b + a*Cos[c + d*x]))/(a^2*(a^2 - b^2)^(5/2)) - ((b + a*Cos[c + d*x])*
Cot[(c + d*x)/2])/(a + b)^2 + (2*b^4*Sin[c + d*x])/(a*(a - b)^2*(a + b)^2)
+ ((b + a*Cos[c + d*x])*Tan[(c + d*x)/2])/(a - b)^2))/(2*d*(a + b*Sec[c + d
*x])^2)
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{(4a^2-b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2a^2}-}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{(4a^2-b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2a^2}-}{d}$
risch	$-\frac{x}{a^2}-\frac{2i\left(-2a^4b\,e^{3i(dx+c)}-b^5e^{3i(dx+c)}+a^5e^{2i(dx+c)}-3a^3b^2e^{2i(dx+c)}-a\,b^4e^{2i(dx+c)}+2a^2b^3e^{i(dx+c)}+b^5e^{i(dx+c)}+a^5+\right.}{\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)(a^2-b^2)^2a^2\left(e^{2i(dx+c)}-1\right)d}$

[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2*tan(1/2*d*x+1/2*c)/(a^2-2*a*b+b^2)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)+2*b^3/(a-b)^2/(a+b)^2/a^2*(-a*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(4*a^2-b^2)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-2/a^2*arctan(tan(1/2*d*x+1/2*c)))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.11

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$=\left[\frac{4\,a^5b^2-2\,a^3b^4-2\,ab^6-(4\,a^2b^4-b^6+(4\,a^3b^3-ab^5)\cos(dx+c))\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx}{a^2\cos(}\right.}{}\right.$$

[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] [1/2*(4*a^5*b^2 - 2*a^3*b^4 - 2*a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 - a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^7 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c) - 2*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4

```
- a^3*b^6)*d*cos(d*x + c) + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*si
n(d*x + c)), (2*a^5*b^2 - a^3*b^4 - a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 -
 a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
+ c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^7 - a*b^6)*cos(d*x
+ c)^2 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c) - ((a^7 - 3*a^5*b^2 + 3
*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*
d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)
 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
[Out] Integral(cot(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.46

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{4\,(4\,a^2b^3-b^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-2\,a^4b^2+a^2b^4)\sqrt{-a^2+b^2}}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^3}{(a^5-2}}{2\,d}$$

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(4*a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
 + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
2)))/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*c)/
(a^2 - 2*a*b + b^2) + (a^4*tan(1/2*d*x + 1/2*c)^2 - 3*a^3*b*tan(1/2*d*x + 1
/2*c)^2 + 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - a*b^3*tan(1/2*d*x + 1/2*c)^2 +
 4*b^4*tan(1/2*d*x + 1/2*c)^2 - a^4 + a^3*b + a^2*b^2 - a*b^3)/((a^5 - 2*a^
3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))) + 2*(d*x + c)/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 18.96 (sec) , antiderivative size = 6093, normalized size of antiderivative = 26.84

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(a^4 - 3*a^3*b - a*b^3
 + 4*b^4 + 3*a^2*b^2))/(a*(a + b)^2))/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6
*a^2*b + 2*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2
*b^3))) - (2*atan(-((tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23
+ 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^
18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2
176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*
a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 4
80*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) - ((32*a^28 - 32*a^27*b + 32*a^6
*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 547
2*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16
*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 +
 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b
^3 - 352*a^26*b^2 - (tan(c/2 + (d*x)/2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^
29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 48
00*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^
17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^
9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a
^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2)*1i)/a^2)*1i)/a^2)/a^2 + (tan(c/2 + (
d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 -
1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*
b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12
+ 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*
a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224
*a^24*b^2) + ((32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b
^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8
```

256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a
^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 -
2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 + (tan(c/2 + (
d*x)/2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*
b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 -
5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688
*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b
^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^2
8*b^2)*1i)/a^2)*1i)/a^2)/a^2)/(64*a^2*b^22 - 192*a^3*b^21 - 640*a^4*b^20 +
1984*a^5*b^19 + 2624*a^6*b^18 - 8192*a^7*b^17 - 6400*a^8*b^16 + 18496*a^9*b
^15 + 11072*a^10*b^14 - 25856*a^11*b^13 - 14464*a^12*b^12 + 23872*a^13*b^11
+ 13760*a^14*b^10 - 15104*a^15*b^9 - 8704*a^16*b^8 + 6592*a^17*b^7 + 3200*
a^18*b^6 - 1856*a^19*b^5 - 512*a^20*b^4 + 256*a^21*b^3 - ((tan(c/2 + (d*x)/
2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*
a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16
- 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 822
4*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*
b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24
*b^2) - ((32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 +
2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a
^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b
^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688
*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 - (tan(c/2 + (d*x)/
2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20
- 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760
*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19
*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 +
5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2
)*1i)/a^2)*1i)/a^2)*1i)/a^2 + ((tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 6
4*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 +
6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^
12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^
10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a
^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) + ((32*a^28 - 32*a^27
*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11
*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13
- 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880
*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 +
480*a^25*b^3 - 352*a^26*b^2 + (tan(c/2 + (d*x)/2)*(128*a^8*b^22 - 64*a^7*b
^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^1
2*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^1
4 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5
760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b
^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2)*1i)/a^2)*1i)/a^2)*1i)/a^2
)))/(a^2*d) + tan(c/2 + (d*x)/2)/(2*d*(a - b)^2) + (b^3*atan(((b^3*(2*a + b

```
)*(tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 6
72*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17
 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31
744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^
18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*
a^23*b^3 - 224*a^24*b^2) + (b^3*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a
- b)*(32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 208
0*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*
b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10
- 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^2
3*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 + (b^3*tan(c/2 + (d*x)/
2)*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(128*a^8*b^22 - 64*a^7*b
^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^1
2*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^1
4 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5
760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b
^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2))/(a^12 - a^2*b^10 + 5*a^4
*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^2*b^10 + 5*a^4*b^8
 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))*((a + b)^5*(a - b)^5)^(1/2)*(2*a
- b)*1i)/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^
2) + (b^3*(2*a + b)*(tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23
+ 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^
18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2
176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*
a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 4
80*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) - (b^3*(2*a + b)*((a + b)^5*(a -
 b)^5)^(1/2)*(2*a - b)*(32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 +
224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13
*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11
 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a
^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 - (b^
3*tan(c/2 + (d*x)/2)*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(128*a
^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^
11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^1
5 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 2
6880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24
*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2))/(a^12
- a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^
2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))*((a + b)^5*(a -
 b)^5)^(1/2)*(2*a - b)*1i)/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a
^8*b^4 - 5*a^10*b^2))/(64*a^2*b^22 - 192*a^3*b^21 - 640*a^4*b^20 + 1984*a^5
*b^19 + 2624*a^6*b^18 - 8192*a^7*b^17 - 6400*a^8*b^16 + 18496*a^9*b^15 + 11
072*a^10*b^14 - 25856*a^11*b^13 - 14464*a^12*b^12 + 23872*a^13*b^11 + 13760
*a^14*b^10 - 15104*a^15*b^9 - 8704*a^16*b^8 + 6592*a^17*b^7 + 3200*a^18*b^6
 - 1856*a^19*b^5 - 512*a^20*b^4 + 256*a^21*b^3 + (b^3*(2*a + b)*(tan(c/2 +
```

```
(d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 -
 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10
*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12
 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568
*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 22
4*a^24*b^2) + (b^3*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(32*a^28
 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 -
 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*
a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*
b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*
a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 + (b^3*tan(c/2 + (d*x)/2)*(2*a + b)*
((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29
*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 4800
*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17
*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9
- 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^2
6*b^4 + 576*a^27*b^3 + 128*a^28*b^2))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6
*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6
 + 10*a^8*b^4 - 5*a^10*b^2))*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b))/(a^12 -
 a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2) - (b^3*(2*a +
 b)*(tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 +
 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^
17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 -
31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*
a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 92
8*a^23*b^3 - 224*a^24*b^2) - (b^3*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*
a - b)*(32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2
080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^1
4*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^1
0 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a
^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 - (b^3*tan(c/2 + (d*x
)/2)*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(128*a^8*b^22 - 64*a^7
*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a
^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b
^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 -
 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25
*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2))/(a^12 - a^2*b^10 + 5*a
^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^2*b^10 + 5*a^4*b
^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))*((a + b)^5*(a - b)^5)^(1/2)*(2*
a - b))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2
)))*(2*a + b)*((a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*2i)/(d*(a^12 - a^2*b^10
 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))
```

3.311 $\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 360

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{x}{a^2} - \frac{2b^7\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{4b^5(3a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))} + \frac{(3a+5b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))^2} - \frac{(3a-5b)\sin(c+dx)}{4(a-b)^3d(1+\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))} + \frac{b^6\sin(c+dx)}{a\left(a^2-b^2\right)^3d(b+a\cos(c+dx))}$$

```
[Out] x/a^2-2*b^7*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-b)^(
7/2)/(a+b)^(7/2)/d-4*b^5*(3*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)
/(a+b)^(1/2))/a^2/(a-b)^(7/2)/(a+b)^(7/2)/d-1/12*sin(d*x+c)/(a+b)^2/d/(1-co
s(d*x+c))^2-1/12*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))+1/4*(3*a+5*b)*sin(d*x+
c)/(a+b)^3/d/(1-cos(d*x+c))+1/12*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))^2-1/4*
(3*a-5*b)*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))+1/12*sin(d*x+c)/(a-b)^2/d/(1+
cos(d*x+c))+b^6*sin(d*x+c)/a/(a^2-b^2)^3/d/(b+a*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3983, 2976, 2729, 2727, 2743, 12, 2738, 214}

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2b^7\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}} - \frac{4b^5(3a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{7/2}(a+b)^{7/2}} + \frac{b^6\sin(c+dx)}{ad\,(a^2-b^2)^3\,(a\cos(c+dx)+b)} + \frac{x}{a^2} + \frac{(3a+5b)\sin(c+dx)}{4d(a+b)^3(1-\cos(c+dx))} - \frac{\sin(c+dx)}{12d(a+b)^2(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12d(a-b)^2(\cos(c+dx)+1)} - \frac{(3a-5b)\sin(c+dx)}{4d(a-b)^3(\cos(c+dx)+1)} - \frac{\sin(c+dx)}{12d(a+b)^2(1-\cos(c+dx))^2} + \frac{\sin(c+dx)}{12d(a-b)^2(\cos(c+dx)+1)^2}$$

```
[In] Int[Cot[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] x/a^2 - (2*b^7*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a
 - b)^(7/2)*(a + b)^(7/2)*d) - (4*b^5*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(7/2)*(a + b)^(7/2)*d) - Sin[c +
 d*x]/(12*(a + b)^2*d*(1 - Cos[c + d*x])^2) - Sin[c + d*x]/(12*(a + b)^2*d*
(1 - Cos[c + d*x])) + ((3*a + 5*b)*Sin[c + d*x])/(4*(a + b)^3*d*(1 - Cos[c
+ d*x])) + Sin[c + d*x]/(12*(a - b)^2*d*(1 + Cos[c + d*x])^2) - ((3*a - 5*b
)*Sin[c + d*x])/(4*(a - b)^3*d*(1 + Cos[c + d*x])) + Sin[c + d*x]/(12*(a -
b)^2*d*(1 + Cos[c + d*x])) + (b^6*Sin[c + d*x])/(a*(a^2 - b^2)^3*d*(b + a*C
os[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\text{integral} = \int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{1}{a^2} + \frac{1}{4(a-b)^2(-1-\cos(c+dx))^2} + \frac{3a-5b}{4(a-b)^3(-1-\cos(c+dx))} + \frac{1}{4(a+b)^2(1-\cos(c+dx))^2} + \frac{-3a-5b}{4(a+b)^3(1-\cos(c+dx))} + \frac{b^6}{a^2\left(a^2-b^2\right)^2(-b-a\cos(c+dx))^2} + \frac{2b^5(3a^2-b^2)}{a^2\left(a^2-b^2\right)^3(-b-a\cos(c+dx))}\right) dx$$

$$= \frac{x}{a^2} + \frac{(3a-5b)\int \frac{1}{-1-\cos(c+dx)}\,dx}{4(a-b)^3} + \frac{\int \frac{1}{(-1-\cos(c+dx))^2}\,dx}{4(a-b)^2} + \frac{\int \frac{1}{(1-\cos(c+dx))^2}\,dx}{4(a+b)^2} - \frac{(3a+5b)\int \frac{1}{1-\cos(c+dx)}\,dx}{4(a+b)^3} + \frac{b^6\int \frac{1}{(-b-a\cos(c+dx))^2}\,dx}{a^2\left(a^2-b^2\right)^2} + \frac{(2b^5(3a^2-b^2))\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^2\left(a^2-b^2\right)^3}$$

$$= \frac{x}{a^2} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} + \frac{(3a+5b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))^2} - \frac{(3a-5b)\sin(c+dx)}{4(a-b)^3d(1+\cos(c+dx))} + \frac{b^6\sin(c+dx)}{a\left(a^2-b^2\right)^3d(b+a\cos(c+dx))} - \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{12(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{12(a+b)^2} + \frac{b^6\int \frac{b}{-b-a\cos(c+dx)}\,dx}{a^2\left(a^2-b^2\right)^3} + \frac{(4b^5(3a^2-b^2))\,\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2\left(a^2-b^2\right)^3d}$$

$$= \frac{x}{a^2} - \frac{4b^5(3a^2-b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))} + \frac{(3a+5b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))^2} - \frac{(3a-5b)\sin(c+dx)}{4(a-b)^3d(1+\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))} + \frac{b^6\sin(c+dx)}{a\left(a^2-b^2\right)^3d(b+a\cos(c+dx))} + \frac{b^7\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^2\left(a^2-b^2\right)^3}$$

$$= \frac{x}{a^2} - \frac{4b^5(3a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))} + \frac{(3a+5b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))^2} - \frac{(3a-5b)\sin(c+dx)}{4(a-b)^3d(1+\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))} + \frac{b^6\sin(c+dx)}{a(a^2-b^2)^3d(b+a\cos(c+dx))} + \frac{(2b^7)\operatorname{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2(a^2-b^2)^3d}$$

$$= \frac{x}{a^2} - \frac{2b^7\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{4b^5(3a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))} + \frac{(3a+5b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))^2} - \frac{(3a-5b)\sin(c+dx)}{4(a-b)^3d(1+\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx))} + \frac{b^6\sin(c+dx)}{a(a^2-b^2)^3d(b+a\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.84

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(\frac{24(c+dx)(b+a\cos(c+dx))}{a^2} - \frac{48b^5(-6a^2+b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))}{a^2(a^2-b^2)^{7/2}}\right.}{}$$

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((24*(c + d*x)*(b + a*Cos[c + d*x]))/a
^2 - (48*b^5*(-6*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]]*(b + a*Cos[c + d*x]))/(a^2*(a^2 - b^2)^(7/2)) + (4*(4*a + 7*b)*(b + a
*Cos[c + d*x])*Cot[(c + d*x)/2])/(a + b)^3 - ((b + a*Cos[c + d*x])*Cot[(c +
 d*x)/2]*Csc[(c + d*x)/2]^2)/(a + b)^2 + (24*b^6*Sin[c + d*x])/(a*(a - b)^3
```

```
*(a + b)^3) + (4*(-4*a + 7*b)*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2])/(a - b
)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a - b)^2)
)/(24*d*(a + b*Sec[c + d*x])^2)
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^2-2ab+b^2\right)(a-b)}-\frac{1}{24(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-5a-9b}{8(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2\arctan(\text{ta}}{a}}{d}$
default	$\frac{\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{3}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^2-2ab+b^2\right)(a-b)}-\frac{1}{24(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-5a-9b}{8(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2\arctan(\text{ta}}{a}}{d}$
risch	$\frac{x}{a^2}+\frac{2i\left(-3a\,b^6-7b^4a^3+17a^5b^2e^{4i(dx+c)}-53a^3b^4e^{4i(dx+c)}-9a\,b^6e^{4i(dx+c)}-14a^6b\,e^{3i(dx+c)}+26a^4b^3e^{3i(dx+c)}+24a^2b\right.}{}$

```
[In] int(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/(a^2-2*a*b+b^2)/(a-b)*(1/3*tan(1/2*d*x+1/2*c)^3*a-1/3*tan(1/2*d*x+
1/2*c)^3*b-5*tan(1/2*d*x+1/2*c)*a+9*tan(1/2*d*x+1/2*c)*b)-1/24/(a+b)^2/tan(
1/2*d*x+1/2*c)^3-1/8/(a+b)^3*(-5*a-9*b)/tan(1/2*d*x+1/2*c)+2/a^2*arctan(tan
(1/2*d*x+1/2*c))+2*b^5/(a+b)^3/(a-b)^3/a^2*(-a*b*tan(1/2*d*x+1/2*c)/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(6*a^2-b^2)/((a-b)*(a+b))^(1/2
)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(324) = 648$.

Time = 0.36 (sec) , antiderivative size = 1481, normalized size of antiderivative = 4.11

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(8*a^7*b^2 - 40*a^5*b^4 + 26*a^3*b^6 + 6*a*b^8 + 2*(4*a^9 - 13*a^7*b^2
 + 2*a^5*b^4 + 4*a^3*b^6 + 3*a*b^8)*cos(d*x + c)^4 - 2*(2*a^8*b - 11*a^6*b^
3 + 16*a^4*b^5 - 7*a^2*b^7)*cos(d*x + c)^3 - 3*(6*a^2*b^6 - b^8 - (6*a^3*b^
```

```
5 - a*b^7)*cos(d*x + c)^3 - (6*a^2*b^6 - b^8)*cos(d*x + c)^2 + (6*a^3*b^5 -
 a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^
2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2
*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) -
 6*(a^9 - 2*a^7*b^2 - 7*a^5*b^4 + 6*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^2 + 2*(
a^8*b - 8*a^6*b^3 + 13*a^4*b^5 - 6*a^2*b^7)*cos(d*x + c) + 6*((a^9 - 4*a^7*
b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c)^3 + (a^8*b - 4*a^6*b^
3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*x*cos(d*x + c)^2 - (a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c) - (a^8*b - 4*a^6*b^3 + 6*a^4*
b^5 - 4*a^2*b^7 + b^9)*d*x)*sin(d*x + c))/(((a^11 - 4*a^9*b^2 + 6*a^7*b^4 -
 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 -
4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 - (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a
^5*b^6 + a^3*b^8)*d*cos(d*x + c) - (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*
b^7 + a^2*b^9)*d)*sin(d*x + c)), 1/3*(4*a^7*b^2 - 20*a^5*b^4 + 13*a^3*b^6 +
 3*a*b^8 + (4*a^9 - 13*a^7*b^2 + 2*a^5*b^4 + 4*a^3*b^6 + 3*a*b^8)*cos(d*x +
 c)^4 - (2*a^8*b - 11*a^6*b^3 + 16*a^4*b^5 - 7*a^2*b^7)*cos(d*x + c)^3 + 3*
(6*a^2*b^6 - b^8 - (6*a^3*b^5 - a*b^7)*cos(d*x + c)^3 - (6*a^2*b^6 - b^8)*c
os(d*x + c)^2 + (6*a^3*b^5 - a*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-
sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x +
 c) - 3*(a^9 - 2*a^7*b^2 - 7*a^5*b^4 + 6*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^2
+ (a^8*b - 8*a^6*b^3 + 13*a^4*b^5 - 6*a^2*b^7)*cos(d*x + c) + 3*((a^9 - 4*a
^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c)^3 + (a^8*b - 4*a^6
*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*x*cos(d*x + c)^2 - (a^9 - 4*a^7*b^2 +
 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*x*cos(d*x + c) - (a^8*b - 4*a^6*b^3 + 6*a
^4*b^5 - 4*a^2*b^7 + b^9)*d*x)*sin(d*x + c))/(((a^11 - 4*a^9*b^2 + 6*a^7*b^
4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + (a^10*b - 4*a^8*b^3 + 6*a^6*b^5
 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 - (a^11 - 4*a^9*b^2 + 6*a^7*b^4 -
4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c) - (a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a
^4*b^7 + a^2*b^9)*d)*sin(d*x + c))]
```

Sympy [F]

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\int \frac{\cot^4{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

```
[In] integrate(cot(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
[Out] Integral(cot(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.35

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{48\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^7-3\,a^5b^2+3\,a^3b^4-ab^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)} - \frac{48\,(6\,a^2b^5-b^7)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\right.}{}\right.\right.}{(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6)\sqrt{-a^2+b^2}}}{}$$

```
[In] integrate(cot(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(48*b^6*tan(1/2*d*x + 1/2*c)/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*(
a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 48*(6*a^2*b
^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^8 - 3*a^6
*b^2 + 3*a^4*b^4 - a^2*b^6)*sqrt(-a^2 + b^2)) - (a^4*tan(1/2*d*x + 1/2*c)^3
 - 4*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*
b^3*tan(1/2*d*x + 1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*
x + 1/2*c) + 72*a^3*b*tan(1/2*d*x + 1/2*c) - 126*a^2*b^2*tan(1/2*d*x + 1/2*
c) + 96*a*b^3*tan(1/2*d*x + 1/2*c) - 27*b^4*tan(1/2*d*x + 1/2*c))/(a^6 - 6*
a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6) - 24*(d*x + c
)/a^2 - (15*a*tan(1/2*d*x + 1/2*c)^2 + 27*b*tan(1/2*d*x + 1/2*c)^2 - a - b)
/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)^3))/d
```

Mupad [B] (verification not implemented)

Time = 19.43 (sec) , antiderivative size = 8348, normalized size of antiderivative = 23.19

$$\int \frac{\cot^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

[In] int(cot(c + d*x)^4/(a + b/cos(c + d*x))^2,x)

[Out] tan(c/2 + (d*x)/2)^3/(24*d*(a - b)^2) + ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(3*(a + b)) + (2*tan(c/2 + (d*x)/2)^2*(11*a^3*b - 31*a*b^3 - 8*a^4 + 13*b^4 + 15*a^2*b^2))/(3*(a + b)^2) - (tan(c/2 + (d*x)/2)^4*(11*a^5*b - 9*a*b^5 - 5*a^6 + 16*b^6 + 31*a^2*b^4 - 34*a^3*b^3 + 6*a^4*b^2))/(a*(a + b)^3))/(d*(tan(c/2 + (d*x)/2)^5*(8*a^4 - 32*a^3*b - 32*a*b^3 + 8*b^4 + 48*a^2*b^2) - tan(c/2 + (d*x)/2)^3*(16*a*b^3 - 16*a^3*b + 8*a^4 - 8*b^4))) + (tan(c/2 + (d*x)/2)*((16*a*b + 8*a^2 - 24*b^2)/(64*(a - b)^4) - 3/(4*(a - b)^2)))/d + (2*atan(((tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^3*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 15360*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a^12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a^16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a^20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*a^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^29*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 384*a^34*b^2) - ((32*a^38 - 32*a^37*b - 32*a^6*b^32 - 32*a^7*b^31 + 640*a^8*b^30 - 4992*a^10*b^28 + 2624*a^11*b^27 + 21504*a^12*b^26 - 19872*a^13*b^25 - 57920*a^14*b^24 + 77472*a^15*b^23 + 100992*a^16*b^22 - 195008*a^17*b^21 - 107008*a^18*b^20 + 344960*a^19*b^19 + 39424*a^20*b^18 - 446688*a^21*b^17 + 76032*a^22*b^16 + 431904*a^23*b^15 - 161920*a^24*b^14 - 313984*a^25*b^13 + 167552*a^26*b^12 + 171072*a^27*b^11 - 113664*a^28*b^10 - 68960*a^29*b^9 + 53568*a^30*b^8 + 20064*a^31*b^7 - 17536*a^32*b^6 - 4032*a^33*b^5 + 3840*a^34*b^4 + 512*a^35*b^3 - 512*a^36*b^2 + (tan(c/2 + (d*x)/2)*(64*a^39*b - 64*a^7*b^33 + 128*a^8*b^32 + 896*a^9*b^31 - 1920*a^10*b^30 - 5760*a^11*b^29 + 13440*a^12*b^28 + 22400*a^13*b^27 - 58240*a^14*b^26 - 58240*a^15*b^25 + 174720*a^16*b^24 + 104832*a^17*b^23 - 384384*a^18*b^22 - 128128*a^19*b^21 + 640640*a^20*b^20 + 91520*a^21*b^19 - 823680*a^22*b^18 + 823680*a^24*b^16 - 91520*a^25*b^15 - 640640*a^26*b^14 + 128128*a^27*b^13 + 384384*a^28*b^12 - 104832*a^29*b^11 - 174720*a^30*b^10 + 58240*a^31*b^9 + 58240*a^32*b^8 - 22400*a^33*b^7 - 13440*a^34*b^6 + 5760*a^35*b^5 + 1920*a^36*b^4 - 896*a^37*b^3 - 128*a^38*b^2)*1i)/a^2)*1i)/a^2)/a^2 + (tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^3*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 15360*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a^12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a^16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a^20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*a^24*b^12 +

243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^29*b^7 - 448
0*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 384*a^34*b^2)
- ((32*a^37*b - 32*a^38 + 32*a^6*b^32 + 32*a^7*b^31 - 640*a^8*b^30 + 4992*a
^10*b^28 - 2624*a^11*b^27 - 21504*a^12*b^26 + 19872*a^13*b^25 + 57920*a^14*
b^24 - 77472*a^15*b^23 - 100992*a^16*b^22 + 195008*a^17*b^21 + 107008*a^18*
b^20 - 344960*a^19*b^19 - 39424*a^20*b^18 + 446688*a^21*b^17 - 76032*a^22*b
^16 - 431904*a^23*b^15 + 161920*a^24*b^14 + 313984*a^25*b^13 - 167552*a^26*
b^12 - 171072*a^27*b^11 + 113664*a^28*b^10 + 68960*a^29*b^9 - 53568*a^30*b^
8 - 20064*a^31*b^7 + 17536*a^32*b^6 + 4032*a^33*b^5 - 3840*a^34*b^4 - 512*a
^35*b^3 + 512*a^36*b^2 + (tan(c/2 + (d*x)/2)*(64*a^39*b - 64*a^7*b^33 + 128
*a^8*b^32 + 896*a^9*b^31 - 1920*a^10*b^30 - 5760*a^11*b^29 + 13440*a^12*b^2
8 + 22400*a^13*b^27 - 58240*a^14*b^26 - 58240*a^15*b^25 + 174720*a^16*b^24
+ 104832*a^17*b^23 - 384384*a^18*b^22 - 128128*a^19*b^21 + 640640*a^20*b^20
 + 91520*a^21*b^19 - 823680*a^22*b^18 + 823680*a^24*b^16 - 91520*a^25*b^15
- 640640*a^26*b^14 + 128128*a^27*b^13 + 384384*a^28*b^12 - 104832*a^29*b^11
 - 174720*a^30*b^10 + 58240*a^31*b^9 + 58240*a^32*b^8 - 22400*a^33*b^7 - 13
440*a^34*b^6 + 5760*a^35*b^5 + 1920*a^36*b^4 - 896*a^37*b^3 - 128*a^38*b^2)
*1i)/a^2)*1i)/a^2)/a^2)/(((tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^3
*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 1536
0*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a^
12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a^
16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a^
20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*a
^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^2
9*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 384
*a^34*b^2) - ((32*a^38 - 32*a^37*b - 32*a^6*b^32 - 32*a^7*b^31 + 640*a^8*b^
30 - 4992*a^10*b^28 + 2624*a^11*b^27 + 21504*a^12*b^26 - 19872*a^13*b^25 -
57920*a^14*b^24 + 77472*a^15*b^23 + 100992*a^16*b^22 - 195008*a^17*b^21 - 1
07008*a^18*b^20 + 344960*a^19*b^19 + 39424*a^20*b^18 - 446688*a^21*b^17 + 7
6032*a^22*b^16 + 431904*a^23*b^15 - 161920*a^24*b^14 - 313984*a^25*b^13 + 1
67552*a^26*b^12 + 171072*a^27*b^11 - 113664*a^28*b^10 - 68960*a^29*b^9 + 53
568*a^30*b^8 + 20064*a^31*b^7 - 17536*a^32*b^6 - 4032*a^33*b^5 + 3840*a^34*
b^4 + 512*a^35*b^3 - 512*a^36*b^2 + (tan(c/2 + (d*x)/2)*(64*a^39*b - 64*a^7
*b^33 + 128*a^8*b^32 + 896*a^9*b^31 - 1920*a^10*b^30 - 5760*a^11*b^29 + 134
40*a^12*b^28 + 22400*a^13*b^27 - 58240*a^14*b^26 - 58240*a^15*b^25 + 174720
*a^16*b^24 + 104832*a^17*b^23 - 384384*a^18*b^22 - 128128*a^19*b^21 + 64064
0*a^20*b^20 + 91520*a^21*b^19 - 823680*a^22*b^18 + 823680*a^24*b^16 - 91520
*a^25*b^15 - 640640*a^26*b^14 + 128128*a^27*b^13 + 384384*a^28*b^12 - 10483
2*a^29*b^11 - 174720*a^30*b^10 + 58240*a^31*b^9 + 58240*a^32*b^8 - 22400*a^
33*b^7 - 13440*a^34*b^6 + 5760*a^35*b^5 + 1920*a^36*b^4 - 896*a^37*b^3 - 12
8*a^38*b^2)*1i)/a^2)*1i)/a^2)*1i)/a^2 - ((tan(c/2 + (d*x)/2)*(32*a^36 - 96*
a^35*b + 64*a^3*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*
a^7*b^29 - 15360*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b
^25 - 194240*a^12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b
^21 - 580608*a^16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b

^17 - 522720*a^20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*
b^13 - 138880*a^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*
b^9 + 40320*a^29*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408
*a^33*b^3 - 384*a^34*b^2) - ((32*a^37*b - 32*a^38 + 32*a^6*b^32 + 32*a^7*b^
31 - 640*a^8*b^30 + 4992*a^10*b^28 - 2624*a^11*b^27 - 21504*a^12*b^26 + 198
72*a^13*b^25 + 57920*a^14*b^24 - 77472*a^15*b^23 - 100992*a^16*b^22 + 19500
8*a^17*b^21 + 107008*a^18*b^20 - 344960*a^19*b^19 - 39424*a^20*b^18 + 44668
8*a^21*b^17 - 76032*a^22*b^16 - 431904*a^23*b^15 + 161920*a^24*b^14 + 31398
4*a^25*b^13 - 167552*a^26*b^12 - 171072*a^27*b^11 + 113664*a^28*b^10 + 6896
0*a^29*b^9 - 53568*a^30*b^8 - 20064*a^31*b^7 + 17536*a^32*b^6 + 4032*a^33*b
^5 - 3840*a^34*b^4 - 512*a^35*b^3 + 512*a^36*b^2 + (tan(c/2 + (d*x)/2)*(64*
a^39*b - 64*a^7*b^33 + 128*a^8*b^32 + 896*a^9*b^31 - 1920*a^10*b^30 - 5760*
a^11*b^29 + 13440*a^12*b^28 + 22400*a^13*b^27 - 58240*a^14*b^26 - 58240*a^1
5*b^25 + 174720*a^16*b^24 + 104832*a^17*b^23 - 384384*a^18*b^22 - 128128*a^
19*b^21 + 640640*a^20*b^20 + 91520*a^21*b^19 - 823680*a^22*b^18 + 823680*a^
24*b^16 - 91520*a^25*b^15 - 640640*a^26*b^14 + 128128*a^27*b^13 + 384384*a^
28*b^12 - 104832*a^29*b^11 - 174720*a^30*b^10 + 58240*a^31*b^9 + 58240*a^32
*b^8 - 22400*a^33*b^7 - 13440*a^34*b^6 + 5760*a^35*b^5 + 1920*a^36*b^4 - 89
6*a^37*b^3 - 128*a^38*b^2)*1i)/a^2)*1i)/a^2)*1i)/a^2 + 64*a^2*b^32 - 256*a^
3*b^31 - 960*a^4*b^30 + 3840*a^5*b^29 + 6144*a^6*b^28 - 23168*a^7*b^27 - 25
088*a^8*b^26 + 78784*a^9*b^25 + 76800*a^10*b^24 - 173760*a^11*b^23 - 183168
*a^12*b^22 + 269952*a^13*b^21 + 334080*a^14*b^20 - 314880*a^15*b^19 - 45388
8*a^16*b^18 + 291456*a^17*b^17 + 449856*a^18*b^16 - 221568*a^19*b^15 - 3184
00*a^20*b^14 + 136960*a^21*b^13 + 155904*a^22*b^12 - 64896*a^23*b^11 - 4992
0*a^24*b^10 + 21440*a^25*b^9 + 9344*a^26*b^8 - 4288*a^27*b^7 - 768*a^28*b^6
 + 384*a^29*b^5)))/(a^2*d) + (b^5*atan(((b^5*(tan(c/2 + (d*x)/2)*(32*a^36 -
 96*a^35*b + 64*a^3*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7
680*a^7*b^29 - 15360*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^
11*b^25 - 194240*a^12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^
15*b^21 - 580608*a^16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^
19*b^17 - 522720*a^20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a
^23*b^13 - 138880*a^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a
^27*b^9 + 40320*a^29*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 +
1408*a^33*b^3 - 384*a^34*b^2) - (b^5*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1
/2)*(32*a^38 - 32*a^37*b - 32*a^6*b^32 - 32*a^7*b^31 + 640*a^8*b^30 - 4992*
a^10*b^28 + 2624*a^11*b^27 + 21504*a^12*b^26 - 19872*a^13*b^25 - 57920*a^14
*b^24 + 77472*a^15*b^23 + 100992*a^16*b^22 - 195008*a^17*b^21 - 107008*a^18
*b^20 + 344960*a^19*b^19 + 39424*a^20*b^18 - 446688*a^21*b^17 + 76032*a^22*
b^16 + 431904*a^23*b^15 - 161920*a^24*b^14 - 313984*a^25*b^13 + 167552*a^26
*b^12 + 171072*a^27*b^11 - 113664*a^28*b^10 - 68960*a^29*b^9 + 53568*a^30*b
^8 + 20064*a^31*b^7 - 17536*a^32*b^6 - 4032*a^33*b^5 + 3840*a^34*b^4 + 512*
a^35*b^3 - 512*a^36*b^2 + (b^5*tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*((a + b)^7*
(a - b)^7)^(1/2)*(64*a^39*b - 64*a^7*b^33 + 128*a^8*b^32 + 896*a^9*b^31 - 1
920*a^10*b^30 - 5760*a^11*b^29 + 13440*a^12*b^28 + 22400*a^13*b^27 - 58240*
a^14*b^26 - 58240*a^15*b^25 + 174720*a^16*b^24 + 104832*a^17*b^23 - 384384*

a^18*b^22 - 128128*a^19*b^21 + 640640*a^20*b^20 + 91520*a^21*b^19 - 823680*
a^22*b^18 + 823680*a^24*b^16 - 91520*a^25*b^15 - 640640*a^26*b^14 + 128128*
a^27*b^13 + 384384*a^28*b^12 - 104832*a^29*b^11 - 174720*a^30*b^10 + 58240*
a^31*b^9 + 58240*a^32*b^8 - 22400*a^33*b^7 - 13440*a^34*b^6 + 5760*a^35*b^5
 + 1920*a^36*b^4 - 896*a^37*b^3 - 128*a^38*b^2))/(a^16 - a^2*b^14 + 7*a^4*b
^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2)))/
(a^16 - a^2*b^14 + 7*a^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21
*a^12*b^4 - 7*a^14*b^2))*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*1i)/(a^1
6 - a^2*b^14 + 7*a^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^1
2*b^4 - 7*a^14*b^2) + (b^5*(tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^
3*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 153
60*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a
^12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a
^16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a
^20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*
a^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^
29*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 38
4*a^34*b^2) - (b^5*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*(32*a^37*b - 3
2*a^38 + 32*a^6*b^32 + 32*a^7*b^31 - 640*a^8*b^30 + 4992*a^10*b^28 - 2624*a
^11*b^27 - 21504*a^12*b^26 + 19872*a^13*b^25 + 57920*a^14*b^24 - 77472*a^15
*b^23 - 100992*a^16*b^22 + 195008*a^17*b^21 + 107008*a^18*b^20 - 344960*a^1
9*b^19 - 39424*a^20*b^18 + 446688*a^21*b^17 - 76032*a^22*b^16 - 431904*a^23
*b^15 + 161920*a^24*b^14 + 313984*a^25*b^13 - 167552*a^26*b^12 - 171072*a^2
7*b^11 + 113664*a^28*b^10 + 68960*a^29*b^9 - 53568*a^30*b^8 - 20064*a^31*b^
7 + 17536*a^32*b^6 + 4032*a^33*b^5 - 3840*a^34*b^4 - 512*a^35*b^3 + 512*a^3
6*b^2 + (b^5*tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*(
64*a^39*b - 64*a^7*b^33 + 128*a^8*b^32 + 896*a^9*b^31 - 1920*a^10*b^30 - 57
60*a^11*b^29 + 13440*a^12*b^28 + 22400*a^13*b^27 - 58240*a^14*b^26 - 58240*
a^15*b^25 + 174720*a^16*b^24 + 104832*a^17*b^23 - 384384*a^18*b^22 - 128128
*a^19*b^21 + 640640*a^20*b^20 + 91520*a^21*b^19 - 823680*a^22*b^18 + 823680
*a^24*b^16 - 91520*a^25*b^15 - 640640*a^26*b^14 + 128128*a^27*b^13 + 384384
*a^28*b^12 - 104832*a^29*b^11 - 174720*a^30*b^10 + 58240*a^31*b^9 + 58240*a
^32*b^8 - 22400*a^33*b^7 - 13440*a^34*b^6 + 5760*a^35*b^5 + 1920*a^36*b^4 -
 896*a^37*b^3 - 128*a^38*b^2))/(a^16 - a^2*b^14 + 7*a^4*b^12 - 21*a^6*b^10
+ 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2)))/(a^16 - a^2*b^14 +
 7*a^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14
b^2))(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*1i)/(a^16 - a^2*b^14 + 7*a
^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2
))/(64*a^2*b^32 - 256*a^3*b^31 - 960*a^4*b^30 + 3840*a^5*b^29 + 6144*a^6*b^
28 - 23168*a^7*b^27 - 25088*a^8*b^26 + 78784*a^9*b^25 + 76800*a^10*b^24 - 1
73760*a^11*b^23 - 183168*a^12*b^22 + 269952*a^13*b^21 + 334080*a^14*b^20 -
314880*a^15*b^19 - 453888*a^16*b^18 + 291456*a^17*b^17 + 449856*a^18*b^16 -
 221568*a^19*b^15 - 318400*a^20*b^14 + 136960*a^21*b^13 + 155904*a^22*b^12
- 64896*a^23*b^11 - 49920*a^24*b^10 + 21440*a^25*b^9 + 9344*a^26*b^8 - 4288
*a^27*b^7 - 768*a^28*b^6 + 384*a^29*b^5 + (b^5*(tan(c/2 + (d*x)/2)*(32*a^36

- 96*a^35*b + 64*a^3*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 +
7680*a^7*b^29 - 15360*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*
a^11*b^25 - 194240*a^12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*
a^15*b^21 - 580608*a^16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*
a^19*b^17 - 522720*a^20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632
*a^23*b^13 - 138880*a^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480
*a^27*b^9 + 40320*a^29*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4
+ 1408*a^33*b^3 - 384*a^34*b^2) - (b^5*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^
(1/2)*(32*a^38 - 32*a^37*b - 32*a^6*b^32 - 32*a^7*b^31 + 640*a^8*b^30 - 499
2*a^10*b^28 + 2624*a^11*b^27 + 21504*a^12*b^26 - 19872*a^13*b^25 - 57920*a^
14*b^24 + 77472*a^15*b^23 + 100992*a^16*b^22 - 195008*a^17*b^21 - 107008*a^
18*b^20 + 344960*a^19*b^19 + 39424*a^20*b^18 - 446688*a^21*b^17 + 76032*a^2
2*b^16 + 431904*a^23*b^15 - 161920*a^24*b^14 - 313984*a^25*b^13 + 167552*a^
26*b^12 + 171072*a^27*b^11 - 113664*a^28*b^10 - 68960*a^29*b^9 + 53568*a^30
*b^8 + 20064*a^31*b^7 - 17536*a^32*b^6 - 4032*a^33*b^5 + 3840*a^34*b^4 + 51
2*a^35*b^3 - 512*a^36*b^2 + (b^5*tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*((a + b)^
7*(a - b)^7)^(1/2)*(64*a^39*b - 64*a^7*b^33 + 128*a^8*b^32 + 896*a^9*b^31 -
1920*a^10*b^30 - 5760*a^11*b^29 + 13440*a^12*b^28 + 22400*a^13*b^27 - 5824
0*a^14*b^26 - 58240*a^15*b^25 + 174720*a^16*b^24 + 104832*a^17*b^23 - 38438
4*a^18*b^22 - 128128*a^19*b^21 + 640640*a^20*b^20 + 91520*a^21*b^19 - 82368
0*a^22*b^18 + 823680*a^24*b^16 - 91520*a^25*b^15 - 640640*a^26*b^14 + 12812
8*a^27*b^13 + 384384*a^28*b^12 - 104832*a^29*b^11 - 174720*a^30*b^10 + 5824
0*a^31*b^9 + 58240*a^32*b^8 - 22400*a^33*b^7 - 13440*a^34*b^6 + 5760*a^35*b
^5 + 1920*a^36*b^4 - 896*a^37*b^3 - 128*a^38*b^2))/(a^16 - a^2*b^14 + 7*a^4
*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2))
)/(a^16 - a^2*b^14 + 7*a^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 +
21*a^12*b^4 - 7*a^14*b^2))*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2))/(a^16
- a^2*b^14 + 7*a^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12
*b^4 - 7*a^14*b^2) - (b^5*(tan(c/2 + (d*x)/2)*(32*a^36 - 96*a^35*b + 64*a^3
*b^33 - 128*a^4*b^32 - 1056*a^5*b^31 + 2080*a^6*b^30 + 7680*a^7*b^29 - 1536
0*a^8*b^28 - 31360*a^9*b^27 + 67200*a^10*b^26 + 77760*a^11*b^25 - 194240*a^
12*b^24 - 114240*a^13*b^23 + 393792*a^14*b^22 + 68096*a^15*b^21 - 580608*a^
16*b^20 + 96000*a^17*b^19 + 636160*a^18*b^18 - 300960*a^19*b^17 - 522720*a^
20*b^16 + 412640*a^21*b^15 + 319520*a^22*b^14 - 373632*a^23*b^13 - 138880*a
^24*b^12 + 243456*a^25*b^11 + 36096*a^26*b^10 - 116480*a^27*b^9 + 40320*a^2
9*b^7 - 4480*a^30*b^6 - 9600*a^31*b^5 + 1920*a^32*b^4 + 1408*a^33*b^3 - 384
*a^34*b^2) - (b^5*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*(32*a^37*b - 32
*a^38 + 32*a^6*b^32 + 32*a^7*b^31 - 640*a^8*b^30 + 4992*a^10*b^28 - 2624*a^
11*b^27 - 21504*a^12*b^26 + 19872*a^13*b^25 + 57920*a^14*b^24 - 77472*a^15*
b^23 - 100992*a^16*b^22 + 195008*a^17*b^21 + 107008*a^18*b^20 - 344960*a^19
*b^19 - 39424*a^20*b^18 + 446688*a^21*b^17 - 76032*a^22*b^16 - 431904*a^23*
b^15 + 161920*a^24*b^14 + 313984*a^25*b^13 - 167552*a^26*b^12 - 171072*a^27
*b^11 + 113664*a^28*b^10 + 68960*a^29*b^9 - 53568*a^30*b^8 - 20064*a^31*b^7
+ 17536*a^32*b^6 + 4032*a^33*b^5 - 3840*a^34*b^4 - 512*a^35*b^3 + 512*a^36
*b^2 + (b^5*tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*(6

```
4*a^39*b - 64*a^7*b^33 + 128*a^8*b^32 + 896*a^9*b^31 - 1920*a^10*b^30 - 576
0*a^11*b^29 + 13440*a^12*b^28 + 22400*a^13*b^27 - 58240*a^14*b^26 - 58240*a
^15*b^25 + 174720*a^16*b^24 + 104832*a^17*b^23 - 384384*a^18*b^22 - 128128*
a^19*b^21 + 640640*a^20*b^20 + 91520*a^21*b^19 - 823680*a^22*b^18 + 823680*
a^24*b^16 - 91520*a^25*b^15 - 640640*a^26*b^14 + 128128*a^27*b^13 + 384384*
a^28*b^12 - 104832*a^29*b^11 - 174720*a^30*b^10 + 58240*a^31*b^9 + 58240*a^
32*b^8 - 22400*a^33*b^7 - 13440*a^34*b^6 + 5760*a^35*b^5 + 1920*a^36*b^4 -
896*a^37*b^3 - 128*a^38*b^2))/(a^16 - a^2*b^14 + 7*a^4*b^12 - 21*a^6*b^10 +
 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2)))/(a^16 - a^2*b^14 +
7*a^4*b^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*
b^2))*(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2))/(a^16 - a^2*b^14 + 7*a^4*b
^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2)))*
(6*a^2 - b^2)*((a + b)^7*(a - b)^7)^(1/2)*2i)/(d*(a^16 - a^2*b^14 + 7*a^4*b
^12 - 21*a^6*b^10 + 35*a^8*b^8 - 35*a^10*b^6 + 21*a^12*b^4 - 7*a^14*b^2))
```

3.312 $\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 761

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \frac{ae^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d}$$
$$-\frac{(a^2-b^2)\,e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}$$
$$-\frac{ae^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d}+\frac{(a^2-b^2)\,e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}$$
$$-\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d}$$
$$+\frac{(a^2-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d}$$
$$+\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d}$$
$$-\frac{(a^2-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d}$$
$$+\frac{2\sqrt{2}\sqrt{a-b}\sqrt{a+b}e^2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{abd\sqrt{\sin(c+dx)}}$$
$$-\frac{2\sqrt{2}\sqrt{a-b}\sqrt{a+b}e^2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{abd\sqrt{\sin(c+dx)}}$$
$$-\frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{bd\sqrt{\sin(2c+2dx)}}+\frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd}$$

```
[Out] 1/2*a*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/d*2^(1/2)-
1/2*(a^2-b^2)*e^(5/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/b^2/
d*2^(1/2)-1/2*a*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/
d*2^(1/2)+1/2*(a^2-b^2)*e^(5/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/
2))/a/b^2/d*2^(1/2)-1/4*a*e^(5/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e
^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)+1/4*(a^2-b^2)*e^(5/2)*ln(e^(1/2)-2^(1/2)*(
e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/b^2/d*2^(1/2)+1/4*a*e^(5/2)*ln(e^
(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)-1/4*(a
^2-b^2)*e^(5/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
/a/b^2/d*2^(1/2)+2*e^2*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),-(a
-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(a-b)^(1/2)*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(
e*tan(d*x+c))^(1/2)/a/b/d/sin(d*x+c)^(1/2)-2*e^2*EllipticPi(sin(d*x+c)^(1/2
```

```
)/(1+cos(d*x+c))^(1/2),(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(a-b)^(1/2)*(a+b)
^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/a/b/d/sin(d*x+c)^(1/2)+2*e^2*c
os(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4
*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/b/d/sin(2*d*x+2*c)^(1/2)+2*e*cos(d*x
+c)*(e*tan(d*x+c))^(3/2)/b/d
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3976, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719, 3975, 2812, 2809, 2985, 2984, 504, 1227, 551}

$$\begin{aligned}
\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx &= -\frac{e^{5/2}(a^2-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d} \\
&+ \frac{e^{5/2}(a^2-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ab^2d} \\
&+ \frac{e^{5/2}(a^2-b^2)\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ab^2d} \\
&- \frac{e^{5/2}(a^2-b^2)\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ab^2d} \\
&+ \frac{2\sqrt{2}e^2\sqrt{a-b}\sqrt{a+b}\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\,\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{abd\sqrt{\sin(c+dx)}} \\
&- \frac{2\sqrt{2}e^2\sqrt{a-b}\sqrt{a+b}\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\,\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{abd\sqrt{\sin(c+dx)}} \\
&+ \frac{ae^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} - \frac{ae^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}b^2d} \\
&- \frac{ae^{5/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}b^2d} \\
&+ \frac{ae^{5/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}b^2d} \\
&- \frac{2e^2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}}{bd\sqrt{\sin(2c+2dx)}} + \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd}
\end{aligned}$$

```
[In] Int[(e*Tan[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*b^2
*d) - ((a^2 - b^2)*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e
```

```
]])/(Sqrt[2]*a*b^2*d) - (a*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*b^2*d) - (a*e^(5/2)*Log[Sqrt[e] + Sq
rt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) + ((a
^2 - b^2)*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*a*b^2*d) + (a*e^(5/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d
*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) - ((a^2 - b^2)*e^(5/
2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*S
qrt[2]*a*b^2*d) + (2*Sqrt[2]*Sqrt[a - b]*Sqrt[a + b]*e^2*Sqrt[Cos[c + d*x]]
*EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 +
Cos[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*b*d*Sqrt[Sin[c + d*x]]) - (2*S
qrt[2]*Sqrt[a - b]*Sqrt[a + b]*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[a - b
]/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*Sqrt[
e*Tan[c + d*x]])/(a*b*d*Sqrt[Sin[c + d*x]]) - (2*e^2*Cos[c + d*x]*EllipticE
[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/(b*d*Sqrt[Sin[2*c + 2*d*x]]) + (2
*e*Cos[c + d*x]*(e*Tan[c + d*x])^(3/2))/(b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2809

```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]]), Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x]
)), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2812

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
```

```
0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3975

```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[1/a, Int[Sqrt[e*Cot[c + d*x]], x], x] - Dist[b/a, In
t[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3976

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Dist[-e^2/b^2, Int[(e*Cot[c + d*x])^(m - 2)*(a - b*Csc[c
 + d*x]), x], x] + Dist[e^2*((a^2 - b^2)/b^2), Int[(e*Cot[c + d*x])^(m - 2)
/(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2
, 0] && IGtQ[m - 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e^2 \int (a-b\sec(c+dx))\sqrt{e\tan(c+dx)}\,dx}{b^2} + \frac{\left((a^2-b^2)\,e^2\right)\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx}{b^2} \\
&= -\frac{(ae^2)\int \sqrt{e\tan(c+dx)}\,dx}{b^2} + \frac{e^2\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{b} \\
&\quad + \frac{\left((a^2-b^2)\,e^2\right)\int \sqrt{e\tan(c+dx)}\,dx}{ab^2} - \frac{\left((a^2-b^2)\,e^2\right)\int \frac{\sqrt{e\tan(c+dx)}}{b+a\cos(c+dx)}\,dx}{ab}
\end{aligned}$$

$$=\frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd}-\frac{(2e^2)\int\cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{b}$$
$$-\frac{(ae^3)\operatorname{Subst}\left(\int\frac{\sqrt{x}}{e^2+x^2}\,dx,x,e\tan(c+dx)\right)}{b^2d}$$
$$+\frac{((a^2-b^2)\,e^3)\operatorname{Subst}\left(\int\frac{\sqrt{x}}{e^2+x^2}\,dx,x,e\tan(c+dx)\right)}{ab^2d}$$
$$-\frac{\left((a^2-b^2)\,e^2\sqrt{e\cot(c+dx)}\sqrt{e\tan(c+dx)}\right)\int\frac{1}{(b+a\cos(c+dx))\sqrt{e\cot(c+dx)}}\,dx}{ab}$$

$$=\frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd}-\frac{(2ae^3)\operatorname{Subst}\left(\int\frac{x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{b^2d}$$
$$+\frac{(2(a^2-b^2)\,e^3)\operatorname{Subst}\left(\int\frac{x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ab^2d}$$
$$-\frac{\left((a^2-b^2)\,e^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int\frac{\sqrt{\sin(c+dx)}}{\sqrt{-\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{ab\sqrt{\sin(c+dx)}}$$
$$-\frac{\left(2e^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{b\sqrt{\sin(c+dx)}}$$

$$=\frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd}+\frac{(ae^3)\operatorname{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{b^2d}$$
$$-\frac{(ae^3)\operatorname{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{b^2d}$$
$$-\frac{((a^2-b^2)\,e^3)\operatorname{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ab^2d}$$
$$+\frac{((a^2-b^2)\,e^3)\operatorname{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ab^2d}$$
$$-\frac{\left((a^2-b^2)\,e^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{ab\sqrt{\sin(c+dx)}}$$
$$-\frac{\left(2e^2\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int\sqrt{\sin(2c+2dx)}\,dx}{b\sqrt{\sin(2c+2dx)}}$$

$$
\begin{aligned}
&= -\frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{bd\sqrt{\sin(2c+2dx)}} \\
&\quad + \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd} \\
&\quad - \frac{(ae^{5/2})\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d} \\
&\quad - \frac{(ae^{5/2})\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d} \\
&\quad + \frac{((a^2-b^2)\,e^{5/2})\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d} \\
&\quad + \frac{((a^2-b^2)\,e^{5/2})\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d} \\
&\quad - \frac{(ae^3)\,\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2b^2d} \\
&\quad - \frac{(ae^3)\,\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2b^2d} \\
&\quad + \frac{((a^2-b^2)\,e^3)\,\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ab^2d} \\
&\quad + \frac{((a^2-b^2)\,e^3)\,\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ab^2d} \\
&\quad - \frac{\left(4\sqrt{2}(a^2-b^2)\,e^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}(a+b+(-a+b)x^4)}\,dx, x, \frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)}{abd\sqrt{\sin(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
= &-\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d} \\
&+\frac{(a^2-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d} \\
&+\frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d} \\
&-\frac{(a^2-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d} \\
&-\frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{bd\sqrt{\sin(2c+2dx)}} \\
&+\frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd} \\
&-\frac{\left(ae^{5/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} \\
&+\frac{\left(ae^{5/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} \\
&+\frac{\left((a^2-b^2)\,e^{5/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d} \\
&-\frac{\left((a^2-b^2)\,e^{5/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d} \\
&-\frac{\left(2\sqrt{2}(a^2-b^2)\,e^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\text{Subst}\Bigg(\int\frac{1}{\left(\sqrt{a+b}-\sqrt{a-b}x^2\right)\sqrt{1-x^4}}\,dx,x,\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}}{a\sqrt{a-b}bd\sqrt{\sin(c+dx)}} \\
&+\frac{\left(2\sqrt{2}(a^2-b^2)\,e^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\text{Subst}\Bigg(\int\frac{1}{\left(\sqrt{a+b}+\sqrt{a-b}x^2\right)\sqrt{1-x^4}}\,dx,x,\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}}{a\sqrt{a-b}bd\sqrt{\sin(c+dx)}}
\end{aligned}
$$

$$= \frac{ae^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} - \frac{(a^2-b^2)\,e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}$$

$$- \frac{ae^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} + \frac{(a^2-b^2)\,e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}$$

$$- \frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d}$$

$$+ \frac{(a^2-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d}$$

$$+ \frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d}$$

$$- \frac{(a^2-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d}$$

$$- \frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|\,2\right)\sqrt{e\tan(c+dx)}}{bd\sqrt{\sin(2c+2dx)}}$$

$$+ \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd}$$

$$- \frac{\left(2\sqrt{2}(a^2-b^2)\,e^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a+b}-\sqrt{a-b}x^2\right)}\,dx,x,\frac{\sqrt{\sin(c+}}{\sqrt{1+\cos(}}\right.}{a\sqrt{a-b}bd\sqrt{\sin(c+dx)}}$$

$$+ \frac{\left(2\sqrt{2}(a^2-b^2)\,e^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a+b}+\sqrt{a-b}x^2\right)}\,dx,x,\frac{\sqrt{\sin(c+}}{\sqrt{1+\cos(}}\right.}{a\sqrt{a-b}bd\sqrt{\sin(c+dx)}}$$

$$
\begin{aligned}
&= \frac{ae^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} - \frac{(a^2-b^2)\,e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d} \\
&- \frac{ae^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} + \frac{(a^2-b^2)\,e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d} \\
&- \frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d} \\
&+ \frac{(a^2-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d} \\
&+ \frac{ae^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d} \\
&- \frac{(a^2-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d} \\
&+ \frac{2\sqrt{2}\sqrt{a-b}\sqrt{a+b}e^2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+d}}{abd\sqrt{\sin(c+dx)}} \\
&- \frac{2\sqrt{2}\sqrt{a-b}\sqrt{a+b}e^2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx}}{abd\sqrt{\sin(c+dx)}} \\
&- \frac{2e^2\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{bd\sqrt{\sin(2c+2dx)}} \\
&+ \frac{2e\cos(c+dx)(e\tan(c+dx))^{3/2}}{bd}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 52.97 (sec) , antiderivative size = 1846, normalized size of antiderivative = 2.43

$$
\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \frac{2(b+a\cos(c+dx))\cot(c+dx)(e\tan(c+dx))^{5/2}}{bd(a+b\sec(c+dx))}
$$

$$
- \frac{(b+a\cos(c+dx))\sec(c+dx)(e\tan(c+dx))^{5/2}\left(\frac{4a\sec^2(c+dx)\left(\frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\tan(c+}}{\sqrt[4]{-a^2+}}\right.}{}\right.}{}\right.}{}
$$

```
[In] Integrate[(e*Tan[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Cot[c + d*x]*(e*Tan[c + d*x])^(5/2))/(b*d*(a + b*Se
c[c + d*x])) - ((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Tan[c + d*x])^(5/2)*((
4*a*Sec[c + d*x]^2*((-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a
^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^2
+ b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1/4)*S
qrt[Tan[c + d*x]] + b*Tan[c + d*x]] - Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[b
]*(-a^2 + b^2)^(1/4)*Sqrt[Tan[c + d*x]] + b*Tan[c + d*x]])/(4*Sqrt[2]*Sqrt[
b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^
2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))/(3*a^2 - 3*b^2))*(a + b*
Sqrt[1 + Tan[c + d*x]^2]))/((b + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)^(3/2)
) - (b*Sec[c + d*x]*(6*Sqrt[2]*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]] - 6*Sqrt[2]*a^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 6*Sqrt[2]*b
^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(
3/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] + (
6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c +
 d*x]])/(a^2 - b^2)^(1/4)] - 3*Sqrt[2]*a^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]] + 3*Sqrt[2]*b^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan
[c + d*x]] + 3*Sqrt[2]*a^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]] - 3*Sqrt[2]*b^2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + (3
+ 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] - (3 + 3*I)*Sqrt[b]*(a
^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqr
t[Tan[c + d*x]] + I*b*Tan[c + d*x]] + 8*a*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan
[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))*(a + b*S
qrt[1 + Tan[c + d*x]^2]))/(4*(a^3 - a*b^2)*(b + a*Cos[c + d*x])*(1 + Tan[c
+ d*x]^2)) + (Cos[2*(c + d*x)]*Sec[c + d*x]^2*(-84*Sqrt[2]*b*ArcTan[1 - Sqr
t[2]*Sqrt[Tan[c + d*x]]] + 84*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x
]]] + ((42 + 42*I)*(-a^2 + 2*b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c +
d*x]])/(a^2 - b^2)^(1/4)])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + ((42 + 42*I)*(a^2
- 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)]
)/(Sqrt[b]*(a^2 - b^2)^(1/4)) + 42*Sqrt[2]*b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]] - 42*Sqrt[2]*b*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]] + ((21 + 21*I)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]])/(Sqrt[b]*(a^2
 - b^2)^(1/4)) + ((21 + 21*I)*(-a^2 + 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*
Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]])/(Sqrt[b]*
(a^2 - b^2)^(1/4)) + (112*a^3*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (
b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))/(a^2 - b^2) - (168*a*b
^2*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)]*Tan[c + d*x]^(3/2))/(a^2 - b^2) - (24*a*b^2*AppellF1[7/4, 1/2, 1, 11/
4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(7/2))/(
a^2 - b^2) - (168*a*Tan[c + d*x]^(3/2))/Sqrt[1 + Tan[c + d*x]^2])*(a + b*Sq
rt[1 + Tan[c + d*x]^2]))/(84*a*(b + a*Cos[c + d*x])*(-1 + Tan[c + d*x]^2)*S
```

```
qrt[1 + Tan[c + d*x]^2]))))/(b*d*(a + b*Sec[c + d*x])*Tan[c + d*x]^(5/2))
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2966 vs. 2(646) = 1292.

Time = 3.69 (sec) , antiderivative size = 2967, normalized size of antiderivative = 3.90

method	result	size
default	Expression too large to display	2967

```
[In] int((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)/b/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/a*(a-b)*e^2*(e*ta
n(d*x+c))^(1/2)*(-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)
^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(
1/2),1/2+1/2*I,1/2*2^(1/2))*b^2*cos(d*x+c)-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*
(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((c
sc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2*cos(d*x+c)+I*(csc(
d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(
d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1
/2))*b^2-I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*
(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/
2+1/2*I,1/2*2^(1/2))*b^2+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*
x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+
c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^2*cos(d*x+c)-2
*2^(1/2)*a*b+I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1
/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2
),1/2-1/2*I,1/2*2^(1/2))*b^2*cos(d*x+c)+(a^2-b^2)^(1/2)*(csc(d*x+c)-cot(d*x
+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),
1/2*2^(1/2))*a+(a^2-b^2)^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-
csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-co
t(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b-(a^2-b^2)^
(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(
d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/
(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a-(a^2-b^2)^(1/2)*(csc(d*x+c)-cot(d
*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2
)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/
2)),1/2*2^(1/2))*b-4*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)
+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)
^(1/2),1/2*2^(1/2))*a*b+2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d
*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+
c)+1)^(1/2),1/2*2^(1/2))*a*b-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-cs
c(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(
```

```
d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a^2*cos(d*x+c)
+(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c
)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+(
(a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^2*cos(d*x+c)-(csc(d*x+c)-cot(d*x+c)+1)^(
1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elliptic
Pi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^
(1/2))*a^2*cos(d*x+c)-I*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x
+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c
)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^2*cos(d*x+c)+(a^2-b^2)^(1/2)*(csc(d*x+c
)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c
))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b)
)^(1/2)),1/2*2^(1/2))*a*cos(d*x+c)+(a^2-b^2)^(1/2)*(csc(d*x+c)-cot(d*x+c)+1
)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ellip
ticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2
^(1/2))*b*cos(d*x+c)-(a^2-b^2)^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d
*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*co
s(d*x+c)-(a^2-b^2)^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*
x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+
c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b*cos(d*x+c)-4*(
csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-
csc(d*x+c))^(1/2)*EllipticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*a*
b*cos(d*x+c)+2*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1
/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2)
,1/2*2^(1/2))*a*b*cos(d*x+c)+2*2^(1/2)*a*b*cos(d*x+c)+(csc(d*x+c)-cot(d*x+c
)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/
2*2^(1/2))*b^2-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1
/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2
),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a^2+(csc(d*x+c)-cot(d*x+c)
+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Ell
ipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1
/2*2^(1/2))*b^2-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(
1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/
2),1/2+1/2*I,1/2*2^(1/2))*b^2-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(cot(d*x+c)-c
sc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot
(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^2-(csc(d*x+c)-cot(d*x+c)+1)^(1/2)
*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((
csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))
*a^2)*sin(d*x+c)/(cos(d*x+c)^2-1)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\tan{(c+dx)})^{\frac{5}{2}}}{a+b\sec{(c+dx)}}\,dx$$

```
[In] integrate((e*tan(d*x+c))**(5/2)/(a+b*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**(5/2)/(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\tan{(dx+c)})^{\frac{5}{2}}}{b\sec{(dx+c)}+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\tan{(dx+c)})^{\frac{5}{2}}}{b\sec{(dx+c)}+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,\left(e\tan(c+d\,x)\right)^{5/2}}{b+a\,\cos(c+d\,x)}\,dx$$

```
[In] int((e*tan(c + d*x))^(5/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(5/2))/(b + a*cos(c + d*x)), x)
```

3.313 $\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 740

$$
\begin{aligned}
\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx &= \frac{ae^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} \\
&- \frac{(a^2-b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d} \\
&- \frac{ae^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} + \frac{(a^2-b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d} \\
&+ \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d} \\
&- \frac{(a^2-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d} \\
&- \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d} \\
&+ \frac{(a^2-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d} \\
&- \frac{2\sqrt{2}\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&+ \frac{2\sqrt{2}\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&+ \frac{e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{bd\sqrt{e\tan(c+dx)}}
\end{aligned}
$$

```
[Out] 1/2*a*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/d*2^(1/2)-
1/2*(a^2-b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/b^2/
d*2^(1/2)-1/2*a*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/b^2/
d*2^(1/2)+1/2*(a^2-b^2)*e^(3/2)*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/
2))/a/b^2/d*2^(1/2)+1/4*a*e^(3/2)*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e
^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)-1/4*(a^2-b^2)*e^(3/2)*ln(e^(1/2)-2^(1/2)*(
e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/b^2/d*2^(1/2)-1/4*a*e^(3/2)*ln(e^
(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/b^2/d*2^(1/2)+1/4*(a
^2-b^2)*e^(3/2)*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))
/a/b^2/d*2^(1/2)-2*e^2*EllipticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c))^(1/2),
b/(a-(a^2-b^2)^(1/2)),I)*2^(1/2)*(a^2-b^2)^(1/2)*sin(d*x+c)^(1/2)/a/b/d/(-c
os(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)+2*e^2*EllipticPi((-cos(d*x+c))^(1/2)/
```

```
(1+sin(d*x+c))^(1/2),b/(a+(a^2-b^2)^(1/2)),I)*2^(1/2)*(a^2-b^2)^(1/2)*sin(d
*x+c)^(1/2)/a/b/d/(-cos(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)-e^2*(sin(c+1/4*P
i+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2))*sec(
d*x+c)*sin(2*d*x+2*c)^(1/2)/b/d/(e*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.760, Rules used = {3976, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720, 3977, 2812, 2808, 2986, 1227, 551}

$$\begin{aligned}
&\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \\
&-\frac{2\sqrt{2}e^2\sqrt{a^2-b^2}\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&+\frac{2\sqrt{2}e^2\sqrt{a^2-b^2}\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&-\frac{e^{3/2}(a^2-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}+\frac{e^{3/2}(a^2-b^2)\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ab^2d} \\
&-\frac{e^{3/2}(a^2-b^2)\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ab^2d} \\
&+\frac{e^{3/2}(a^2-b^2)\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ab^2d} \\
&+\frac{ae^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d}-\frac{ae^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}b^2d} \\
&+\frac{ae^{3/2}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}b^2d} \\
&-\frac{ae^{3/2}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}b^2d} \\
&+\frac{e^2\sqrt{\sin(2c+2dx)}\sec(c+dx)\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)}{bd\sqrt{e\tan(c+dx)}}
\end{aligned}$$

```
[In] Int[(e*Tan[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*b^2
*d) - ((a^2 - b^2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e
]])/(Sqrt[2]*a*b^2*d) - (a*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
```

```
)/Sqrt[e]])/(Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*b^2*d) + (a*e^(3/2)*Log[Sqrt[e] + Sq
rt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) - ((a
^2 - b^2)*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c
 + d*x]]])/(2*Sqrt[2]*a*b^2*d) - (a*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d
*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*b^2*d) + ((a^2 - b^2)*e^(3/
2)*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*S
qrt[2]*a*b^2*d) - (2*Sqrt[2]*Sqrt[a^2 - b^2]*e^2*EllipticPi[b/(a - Sqrt[a^2
 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[
c + d*x]])/(a*b*d*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt[2]*Sq
rt[a^2 - b^2]*e^2*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c +
d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*b*d*Sqrt[-Cos[c +
 d*x]]*Sqrt[e*Tan[c + d*x]]) + (e^2*EllipticF[c - Pi/4 + d*x, 2]*Sec[c + d*
x]*Sqrt[Sin[2*c + 2*d*x]])/(b*d*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2808

```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[g*Tan[e + f*x]]/Sqrt[Sin[e
+ f*x]]), Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x])),
 x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2812

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3976

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Dist[-e^2/b^2, Int[(e*Cot[c + d*x])^(m - 2)*(a - b*Csc[c
 + d*x]), x], x] + Dist[e^2*((a^2 - b^2)/b^2), Int[(e*Cot[c + d*x])^(m - 2)
/(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2
, 0] && IGtQ[m - 1/2, 0]
```

Rule 3977

```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[1/a, Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Dist[b
/a, Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e^2 \int \frac{a-b\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{b^2}+\frac{\left(\left(a^2-b^2\right)e^2\right)\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{b^2}\\
&= -\frac{\left(ae^2\right)\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{b^2}+\frac{e^2\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{b}\\
&\quad+\frac{\left(\left(a^2-b^2\right)e^2\right)\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{ab^2}-\frac{\left(\left(a^2-b^2\right)e^2\right)\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{ab}\\
&= -\frac{\left(ae^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx,x,e\tan(c+dx)\right)}{b^2d}\\
&\quad+\frac{\left(\left(a^2-b^2\right)e^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(e^2+x^2\right)}\,dx,x,e\tan(c+dx)\right)}{ab^2d}\\
&\quad-\frac{\left(\left(a^2-b^2\right)e^2\right)\int \frac{\sqrt{e\cot(c+dx)}}{b+a\cos(c+dx)}\,dx}{ab\sqrt{e\cot(c+dx)}\sqrt{e\tan(c+dx)}}+\frac{\left(e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}\\
&= -\frac{\left(2ae^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{b^2d}\\
&\quad+\frac{\left(2\left(a^2-b^2\right)e^3\right)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ab^2d}\\
&\quad-\frac{\left(\left(a^2-b^2\right)e^2\sqrt{\sin(c+dx)}\right)\int \frac{\sqrt{-\cos(c+dx)}}{(b+a\cos(c+dx))\sqrt{\sin(c+dx)}}\,dx}{ab\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}\\
&\quad+\frac{\left(e^2\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{b\sqrt{e\tan(c+dx)}}
\end{aligned}$$

$$= \frac{e^2 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{bd\sqrt{e\tan(c+dx)}} - \frac{(ae^2)\operatorname{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{b^2d} - \frac{(ae^2)\operatorname{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{b^2d} + \frac{((a^2-b^2)\,e^2)\operatorname{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ab^2d} + \frac{((a^2-b^2)\,e^2)\operatorname{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ab^2d} - \frac{\left(2\sqrt{2}(a^2-b^2)\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)e^2\sqrt{\sin(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{\left(-a+\sqrt{a^2-b^2}+bx^2\right)\sqrt{1-x^4}}\,dx, x, \frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right)}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} - \frac{\left(2\sqrt{2}(a^2-b^2)\left(1+\frac{a}{\sqrt{a^2-b^2}}\right)e^2\sqrt{\sin(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{\left(-a-\sqrt{a^2-b^2}+bx^2\right)\sqrt{1-x^4}}\,dx, x, \frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right)}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$
\begin{aligned}
&= \frac{e^2 \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{bd\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{\left(ae^{3/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d} \\
&\quad + \frac{\left(ae^{3/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d} \\
&\quad - \frac{\left(\left(a^2-b^2\right)e^{3/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d} \\
&\quad - \frac{\left(\left(a^2-b^2\right)e^{3/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d} \\
&\quad - \frac{\left(ae^2\right)\operatorname{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2b^2d} \\
&\quad - \frac{\left(ae^2\right)\operatorname{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2b^2d} \\
&\quad + \frac{\left(\left(a^2-b^2\right)e^2\right)\operatorname{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2ab^2d} \\
&\quad + \frac{\left(\left(a^2-b^2\right)e^2\right)\operatorname{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2ab^2d} \\
&\quad - \frac{\left(2\sqrt{2}\left(a^2-b^2\right)\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)e^2\sqrt{\sin(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(-a+\sqrt{a^2-b^2}+bx^2\right)}\,dx,x,\frac{\sqrt{-\mathrm{co}}}{\sqrt{1+\mathrm{si}}}\right.}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&\quad - \frac{\left(2\sqrt{2}\left(a^2-b^2\right)\left(1+\frac{a}{\sqrt{a^2-b^2}}\right)e^2\sqrt{\sin(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(-a-\sqrt{a^2-b^2}+bx^2\right)}\,dx,x,\frac{\sqrt{-\mathrm{co}}}{\sqrt{1+\mathrm{si}}}\right.}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d} \\
&- \frac{(a^2-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d} \\
&- \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d} \\
&+ \frac{(a^2-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d} \\
&- \frac{2\sqrt{2}\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&+ \frac{2\sqrt{2}\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&+ \frac{e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{bd\sqrt{e\tan(c+dx)}} \\
&- \frac{\left(ae^{3/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} \\
&+ \frac{\left(ae^{3/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} \\
&+ \frac{\left((a^2-b^2)\,e^{3/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d} \\
&- \frac{\left((a^2-b^2)\,e^{3/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}
\end{aligned}
$$

$$= \frac{ae^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} - \frac{(a^2-b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}$$
$$- \frac{ae^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}b^2d} + \frac{(a^2-b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ab^2d}$$
$$+ \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d}$$
$$- \frac{(a^2-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d}$$
$$- \frac{ae^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}b^2d}$$
$$+ \frac{(a^2-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ab^2d}$$
$$- \frac{2\sqrt{2}\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$
$$+ \frac{2\sqrt{2}\sqrt{a^2-b^2}e^2\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{abd\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$
$$+ \frac{e^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{bd\sqrt{e\tan(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 21.45 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.74

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx =$$
$$-\frac{\cos(c+dx)\left(a+b\sqrt{\sec^2(c+dx)}\right)(e\tan(c+dx))^{3/2}\left(-5(a^2-b^2)\left(2\sqrt{2}\sqrt{b}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx}\right.\right.\right.}{}$$

```
[In] Integrate[(e*Tan[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/20*(Cos[c + d*x]*(a + b*Sqrt[Sec[c + d*x]^2])*(e*Tan[c + d*x])^(3/2)*(-5
*(a^2 - b^2)*(2*Sqrt[2]*Sqrt[b]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*
Sqrt[2]*Sqrt[b]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - (2 - 2*I)*(a^2 - b
^2)^(1/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)
] + (2 - 2*I)*(a^2 - b^2)^(1/4)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*
```

```
x]])/(a^2 - b^2)^(1/4)] + Sqrt[2]*Sqrt[b]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]] - Sqrt[2]*Sqrt[b]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]] - (1 - I)*(a^2 - b^2)^(1/4)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[
b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] + (1 - I)*(a^2
- b^2)^(1/4)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[T
an[c + d*x]] + I*b*Tan[c + d*x]]) + 8*a*b^(3/2)*AppellF1[5/4, 1/2, 1, 9/4,
-Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(5/2)))/(a*
Sqrt[b]*(a^2 - b^2)*d*(b + a*Cos[c + d*x])*Tan[c + d*x]^(3/2))
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1650 vs. $2(636) = 1272$.

Time = 3.36 (sec) , antiderivative size = 1651, normalized size of antiderivative = 2.23

method	result	size
default	Expression too large to display	1651

```
[In] int((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/2/d*2^(1/2)/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/(a^2-b^2)^(1/2)/a
*(2*I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^
2-b^2)^(1/2)*a*b-2*I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1
/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b-EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),
1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^2-EllipticPi((csc(d*x+c)-cot(d*x+c
)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2-EllipticPi((csc(d*x+c
)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^2-2*Elliptic
Pi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1
/2))*(a^2-b^2)^(1/2)*a^2+2*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)
/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^2-2*EllipticPi((c
sc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*
a^2*b-2*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))
^(1/2)),1/2*2^(1/2))*a*b^2-2*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a
-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2+2*EllipticP
i((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(
1/2))*(a^2-b^2)^(1/2)*b^2+2*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-
b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a^2*b+2*EllipticPi((csc(d*x+c)-c
ot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*b^2-I*E
llipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^
(3/2)+I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(
a^2-b^2)^(3/2)-EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(
1/2))*(a^2-b^2)^(1/2)*a^2+I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+
1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^2+2*EllipticPi((csc(d*x+c)-cot(d*x+c)+
1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b+2*EllipticPi((csc(d*x+c
)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b-I*Elliptic
```

```
Pi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a
^2-I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2
-b^2)^(1/2)*b^2+I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*
2^(1/2))*(a^2-b^2)^(1/2)*a^2+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2
-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1
/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)+2*EllipticPi((csc(d*x+c)-cot(d*x
+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a^3+2*EllipticPi(
(csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2)
)*b^3-2*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b
))^(1/2)),1/2*2^(1/2))*a^3-2*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a
-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^3)*(cot(d*x+c)-csc(d*x+c))^(1
/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*((1
-cos(d*x+c))^2*csc(d*x+c)^2-1)^2*(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-co
t(d*x+c)+csc(d*x+c)))^(3/2)/((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d
*x+c))^(1/2)/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^
(1/2)/(1-cos(d*x+c))*sin(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{(e\tan{(c+dx)})^{\frac{3}{2}}}{a+b\sec{(c+dx)}}\,dx$$

```
[In] integrate((e*tan(d*x+c))**(3/2)/(a+b*sec(d*x+c)),x)
[Out] Integral((e*tan(c + d*x))**(3/2)/(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{\left(e\tan\left(dx+c\right)\right)^{\frac{3}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{\left(e\tan\left(dx+c\right)\right)^{\frac{3}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\tan(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(e\tan(c+d\,x)\right)^{3/2}}{b+a\,\cos\left(c+d\,x\right)}\,dx$$

```
[In] int((e*tan(c + d*x))^(3/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(3/2))/(b + a*cos(c + d*x)), x)
```

3.314 $\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 415

$$\begin{aligned}
\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx &= -\frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}+\frac{\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}\\
&+\frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\
&-\frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\
&+\frac{2\sqrt{2}b\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{a\sqrt{a-b}\sqrt{a+b}d\sqrt{\sin(c+dx)}}\\
&-\frac{2\sqrt{2}b\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{a\sqrt{a-b}\sqrt{a+b}d\sqrt{\sin(c+dx)}}
\end{aligned}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d*2^(1/2)+1/4*ln(
e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a/d*2^(1/2
)-1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))*e^(1/2)/a
/d*2^(1/2)+2*b*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),-(a-b)^(1/2
)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/a/d/(a-b)^(1
/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2)-2*b*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x
+c))^(1/2),(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c
))^(1/2)/a/d/(a-b)^(1/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3975, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2812, 2809, 2985, 2984, 504, 1227, 551}

$$\begin{aligned}
&\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx \\
&= \frac{2\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\,\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{ad\sqrt{a-b}\sqrt{a+b}\sqrt{\sin(c+dx)}} \\
&\quad - \frac{2\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\,\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)}{ad\sqrt{a-b}\sqrt{a+b}\sqrt{\sin(c+dx)}} \\
&\quad - \frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad} \\
&\quad + \frac{\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad} \\
&\quad - \frac{\sqrt{e}\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad}
\end{aligned}$$

```
[In] Int[Sqrt[e*Tan[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*d
)) + (Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*
a*d) + (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c +
 d*x]]])/(2*Sqrt[2]*a*d) - (Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sq
rt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*d) + (2*Sqrt[2]*b*Sqrt[Cos[c + d*
x]]*EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1
 + Cos[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*Sqrt[a - b]*Sqrt[a + b]*d*S
qrt[Sin[c + d*x]]) - (2*Sqrt[2]*b*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[a - b]
/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*Sqrt[e
*Tan[c + d*x]])/(a*Sqrt[a - b]*Sqrt[a + b]*d*Sqrt[Sin[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2809

```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]]), Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x]
)), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2812

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3975

```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[1/a, Int[Sqrt[e*Cot[c + d*x]], x], x] - Dist[b/a, In
t[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sqrt{e\tan(c+dx)}\,dx}{a} - \frac{b\int \frac{\sqrt{e\tan(c+dx)}}{b+a\cos(c+dx)}\,dx}{a}\\
&= \frac{e\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{ad}\\
&\quad - \frac{\left(b\sqrt{e\cot(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\cot(c+dx)}}\,dx}{a}\\
&= \frac{(2e)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad}\\
&\quad - \frac{\left(b\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{\sqrt{\sin(c+dx)}}{\sqrt{-\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{a\sqrt{\sin(c+dx)}}\\
&= -\frac{e\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad} + \frac{e\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{ad}\\
&\quad - \frac{\left(b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{a\sqrt{\sin(c+dx)}}\\
&= \frac{\sqrt{e}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\
&\quad + \frac{\sqrt{e}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}\\
&\quad + \frac{e\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad}\\
&\quad + \frac{e\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2ad}\\
&\quad - \frac{\left(4\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}(a+b+(-a+b)x^4)}\,dx, x, \frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)}{ad\sqrt{\sin(c+dx)}}
\end{aligned}$$

$$= \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} + \frac{\sqrt{e}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{\sqrt{e}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} - \frac{\left(2\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a+b}-\sqrt{a-b}x^2\right)\sqrt{1-x^4}}\,dx, x, \frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)}{a\sqrt{a-b}d\sqrt{\sin(c+dx)}} + \frac{\left(2\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a+b}+\sqrt{a-b}x^2\right)\sqrt{1-x^4}}\,dx, x, \frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)}{a\sqrt{a-b}d\sqrt{\sin(c+dx)}}$$

$$= -\frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad} - \frac{\left(2\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a+b}-\sqrt{a-b}x^2\right)}\,dx, x, \frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)}{a\sqrt{a-b}d\sqrt{\sin(c+dx)}} + \frac{\left(2\sqrt{2}b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a+b}+\sqrt{a-b}x^2\right)}\,dx, x, \frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)}{a\sqrt{a-b}d\sqrt{\sin(c+dx)}}$$

$$= -\frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad} + \frac{\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad}$$
$$+ \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}$$
$$- \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad}$$
$$+ \frac{2\sqrt{2}b\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{a\sqrt{a-b}\sqrt{a+b}d\sqrt{\sin(c+dx)}}$$
$$- \frac{2\sqrt{2}b\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{a\sqrt{a-b}\sqrt{a+b}d\sqrt{\sin(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.98 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx =$$
$$-\frac{\cos(c+dx)\left(a+b\sqrt{\sec^2(c+dx)}\right)\sqrt{e\tan(c+dx)}\left(6\sqrt{2}(a^2-b^2)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-6\sqrt{}\right.}{}$$

```
[In] Integrate[Sqrt[e*Tan[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/12*(Cos[c + d*x]*(a + b*Sqrt[Sec[c + d*x]^2])*Sqrt[e*Tan[c + d*x]]*(6*Sq
rt[2]*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 6*Sqrt[2]*a^2*Ar
cTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 6*Sqrt[2]*b^2*ArcTan[1 + Sqrt[2]*Sqr
t[Tan[c + d*x]]] - (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - ((1 + I)*
Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] + (6 + 6*I)*Sqrt[b]*(a^2 - b
^2)^(3/4)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)
] - 3*Sqrt[2]*a^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sq
rt[2]*b^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*a^
2*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 3*Sqrt[2]*b^2*Log[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + (3 + 3*I)*Sqrt[b]*(a^2 - b^2
)^(3/4)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c
+ d*x]] + I*b*Tan[c + d*x]] - (3 + 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[
a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan
[c + d*x]] + 8*a*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c +
 d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2)))/(a*(a^2 - b^2)*d*(b + a*Cos[c +
d*x])*Sqrt[Tan[c + d*x]])
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(328) = 656$.

Time = 2.54 (sec) , antiderivative size = 820, normalized size of antiderivative = 1.98

method	result	size
default	Expression too large to display	820

```
[In] int((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)*b/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/a*(I*EllipticPi((
csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*a-I*EllipticPi((csc(d
*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*b-I*EllipticPi((csc(d*x+c)
-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*a+I*EllipticPi((csc(d*x+c)-cot(
d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*b-EllipticPi((csc(d*x+c)-cot(d*x+c)+
1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*a+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2
),1/2-1/2*I,1/2*2^(1/2))*b-EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1
/2*I,1/2*2^(1/2))*a+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/
2*2^(1/2))*b-(a^2-b^2)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-
b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))+(a^2-b^2)^(1/2)*EllipticPi((csc(d
*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))+El
lipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/
2*2^(1/2))*a-b*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)
*(a+b))^(1/2)),1/2*2^(1/2))+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-
b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a-b*EllipticPi((csc(d*x+c)-cot(d
*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2)))*(cot(d*x+c)-
csc(d*x+c))^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c
)+1)^(1/2)*(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(d*x+c)))^
(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(
d*x+c)-csc(d*x+c))^(1/2)/((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*
csc(d*x+c))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{e\tan\left(c+dx\right)}}{a+b\sec\left(c+dx\right)}\,dx$$

```
[In] integrate((e*tan(d*x+c))**(1/2)/(a+b*sec(d*x+c)),x)
[Out] Integral(sqrt(e*tan(c + d*x))/(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{e\tan\left(dx+c\right)}}{b\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(e*tan(d*x + c))/(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{e\tan\left(dx+c\right)}}{b\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(e*tan(d*x + c))/(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\sqrt{e\tan\left(c+d\,x\right)}}{b+a\,\cos\left(c+d\,x\right)}\,dx$$

```
[In] int((e*tan(c + d*x))^(1/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*tan(c + d*x))^(1/2))/(b + a*cos(c + d*x)), x)
```

3.315 $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 422

$$\begin{aligned}
&\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx \\
&= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} \\
&\quad-\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} \\
&\quad+\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} \\
&\quad-\frac{2\sqrt{2}b\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\sqrt{a^2-b^2}d\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&\quad+\frac{2\sqrt{2}b\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\sqrt{a^2-b^2}d\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}
\end{aligned}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)+1/2
*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/d*2^(1/2)/e^(1/2)-1/4*ln(
e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/2)/e^(1/2
)+1/4*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/d*2^(1/
2)/e^(1/2)-2*b*EllipticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c))^(1/2),b/(a-(a^
2-b^2)^(1/2)),I)*2^(1/2)*sin(d*x+c)^(1/2)/a/d/(a^2-b^2)^(1/2)/(-cos(d*x+c))
^(1/2)/(e*tan(d*x+c))^(1/2)+2*b*EllipticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c
))^(1/2),b/(a+(a^2-b^2)^(1/2)),I)*2^(1/2)*sin(d*x+c)^(1/2)/a/d/(a^2-b^2)^(1
/2)/(-cos(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3977, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2812, 2808, 2986, 1227, 551}

$$\begin{aligned}
&\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx\\
&=-\frac{2\sqrt{2}b\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{ad\sqrt{a^2-b^2}\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}\\
&\quad+\frac{2\sqrt{2}b\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)}{ad\sqrt{a^2-b^2}\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}\\
&\quad-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}ad\sqrt{e}}\\
&\quad-\frac{\log\left(\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad\sqrt{e}}\\
&\quad+\frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}+\sqrt{e}\right)}{2\sqrt{2}ad\sqrt{e}}
\end{aligned}$$

```
[In] Int[1/((a + b*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e]))
 + ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]]/(Sqrt[2]*a*d*Sqrt[e])
 - Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]]/(2*Sq
rt[2]*a*d*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Ta
n[c + d*x]]]/(2*Sqrt[2]*a*d*Sqrt[e]) - (2*Sqrt[2]*b*EllipticPi[b/(a - Sqrt[
a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[S
in[c + d*x]])/(a*Sqrt[a^2 - b^2]*d*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]
) + (2*Sqrt[2]*b*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d
*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*Sqrt[a^2 - b^2]*d*
Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2808

```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[g*Tan[e + f*x]]/Sqrt[Sin[e
+ f*x]]), Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x])),
 x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2812

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3977

```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[1/a, Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Dist[b
/a, Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a} - \frac{b\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{a}$$

$$=\frac{e\text{Subst}\left(\int\frac{1}{\sqrt{x}(e^2+x^2)}\,dx,x,e\tan(c+dx)\right)}{ad}-\frac{b\int\frac{\sqrt{e\cot(c+dx)}}{b+a\cos(c+dx)}\,dx}{a\sqrt{e\cot(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$=\frac{(2e)\text{Subst}\left(\int\frac{1}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ad}-\frac{\left(b\sqrt{\sin(c+dx)}\right)\int\frac{\sqrt{-\cos(c+dx)}}{(b+a\cos(c+dx))\sqrt{\sin(c+dx)}}\,dx}{a\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$=\frac{\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ad}+\frac{\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{ad}$$
$$-\frac{\left(2\sqrt{2}b\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\left(-a+\sqrt{a^2-b^2}+bx^2\right)\sqrt{1-x^4}}\,dx,x,\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right)}{ad\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$
$$-\frac{\left(2\sqrt{2}b\left(1+\frac{a}{\sqrt{a^2-b^2}}\right)\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\left(-a-\sqrt{a^2-b^2}+bx^2\right)\sqrt{1-x^4}}\,dx,x,\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right)}{ad\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$=\frac{\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2ad}$$
$$+\frac{\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2ad}$$
$$-\frac{\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}}$$
$$-\frac{\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}}$$
$$-\frac{\left(2\sqrt{2}b\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(-a+\sqrt{a^2-b^2}+bx^2\right)}\,dx,x,\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right)}{ad\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$
$$-\frac{\left(2\sqrt{2}b\left(1+\frac{a}{\sqrt{a^2-b^2}}\right)\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(-a-\sqrt{a^2-b^2}+bx^2\right)}\,dx,x,\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right)}{ad\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$= -\frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} - \frac{2\sqrt{2}b\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\sqrt{a^2-b^2}d\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} + \frac{2\sqrt{2}b\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\sqrt{a^2-b^2}d\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} + \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} - \frac{\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}ad\sqrt{e}} - \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} + \frac{\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}ad\sqrt{e}} - \frac{2\sqrt{2}b\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\sqrt{a^2-b^2}d\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} + \frac{2\sqrt{2}b\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\sqrt{a^2-b^2}d\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 38.30 (sec) , antiderivative size = 1475, normalized size of antiderivative = 3.50

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx$$

$$= \frac{(b+a\cos(c+dx))\sec(c+dx)\sqrt{\tan(c+dx)}\left(\frac{2\sec^3(c+dx)\left(a+b\sqrt{1+\tan^2(c+dx)}\right)\left(-\frac{\left(\frac{1}{8}-\frac{i}{8}\right)a\left(2\arctan\left(1-\frac{(1+i)\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)\right)}{}\right)}{}\right)}{}$$

```
[In] Integrate[1/((a + b*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*Sqrt[Tan[c + d*x]]*((2*Sec[c + d*x]^3*(a
 + b*Sqrt[1 + Tan[c + d*x]^2])*(((-1/8 + I/8)*a*(2*ArcTan[1 - ((1 + I)*Sqrt
[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*
Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] - Log[Sqrt[a^2
 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c
+ d*x]]))/(Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(-a^2 + b^2)*AppellF1[1/4, -1/
2, 1, 5/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Tan[c +
d*x]]*Sqrt[1 + Tan[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4,
 -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(2*b^2*AppellF1[5/4
, -1/2, 2, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (a^2 -
 b^2)*AppellF1[5/4, 1/2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
 - b^2)])*Tan[c + d*x]^2)*(a^2 - b^2*(1 + Tan[c + d*x]^2)))))/((b + a*Cos[c
 + d*x])*(1 + Tan[c + d*x]^2)^2) + (Cos[2*(c + d*x)]*Sec[c + d*x]^3*(a + b*
Sqrt[1 + Tan[c + d*x]^2])*((-20*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]]])/a + (20*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/a + ((10 - 10*
I)*(a^2 - 2*b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2
)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - ((10 - 10*I)*(a^2 - 2*b^2)*ArcTan
[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a
^2 - b^2)^(3/4)) - (10*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]])/a + (10*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])
/a + ((5 - 5*I)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 -
b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]])/(a*Sqrt[b]*(a^2 - b^2)^(
3/4)) - ((5 - 5*I)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]])/(a*Sqrt[b]*(a^2 - b^2
)^(3/4)) - (8*b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*
```

```
x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(5/2))/(-a^2 + b^2) - (200*b*(-a^2 + b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)
]*Sqrt[Tan[c + d*x]])/(Sqrt[1 + Tan[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4
, 1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(2*b^
2*AppellF1[5/4, 1/2, 2, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b
^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)*(-a^2 + b^2*(1 + Tan[c + d*x]^2))))
)/(20*(b + a*Cos[c + d*x])*(1 - Tan[c + d*x]^2)*(1 + Tan[c + d*x]^2))))/(2*
d*(a + b*Sec[c + d*x])*Sqrt[e*Tan[c + d*x]])
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1918 vs. $2(343) = 686$.

Time = 3.58 (sec) , antiderivative size = 1919, normalized size of antiderivative = 4.55

method	result	size
default	Expression too large to display	1919

```
[In] int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*2^(1/2)/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/(a^2-b^2)^(1/2)/(
a-b)/a*(3*I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2
))*(a^2-b^2)^(1/2)*a^2*b-3*I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2
-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2-3*I*EllipticPi((csc(d*x+c)-cot(d*
x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2*b+3*I*EllipticPi((
csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2
-3*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b
^2)^(1/2)*a*b^2+3*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*
2^(1/2))*(a^2-b^2)^(1/2)*a^2*b-3*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2)
,1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a*b^2+2*(a^2-b^2)^(1/2)*EllipticPi(
(csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2)
)*a*b^2+2*(a^2-b^2)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)
/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*b^2-I*EllipticPi((csc(d*x+c)-cot
(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*b-I*EllipticPi((csc
(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^3-I*El
lipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(
3/2)*a-I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*
(a^2-b^2)^(1/2)*b^3+I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,
1/2*2^(1/2))*(a^2-b^2)^(3/2)*a+I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2)
,1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^3+I*EllipticPi((csc(d*x+c)-cot(d*
x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*b-2*EllipticF((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*(a^2-b^2)^(3/2)*a+2*EllipticF((csc(d*x
+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^3+EllipticPi((csc(d*
x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*a-EllipticP
```

```
i((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*b-
EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(a^2-b^2)
^(1/2)*a^3+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*(a^2-b^2)^(1/2)*b^3+EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,
1/2*2^(1/2))*(a^2-b^2)^(3/2)*a-EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1
/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(3/2)*b-EllipticPi((csc(d*x+c)-cot(d*x+c)+1
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^3+EllipticPi((csc(d*x+c)-c
ot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^3-2*(a^2-b^2)^(
1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1
/2)),1/2*2^(1/2))*b^3-2*(a^2-b^2)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1
)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^3-2*EllipticPi((cs
c(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a
^2*b^2+4*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b)
)^(1/2)),1/2*2^(1/2))*a*b^3+2*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(
a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a^2*b^2-4*EllipticPi((csc(d*x+
c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*b^3
+I*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*(a^2-b
^2)^(1/2)*a^3-4*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*(a^2
-b^2)^(1/2)*a^2*b+2*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*
(a^2-b^2)^(1/2)*a*b^2+3*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*
I,1/2*2^(1/2))*(a^2-b^2)^(1/2)*a^2*b+2*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)
^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^4-2*EllipticPi((csc
(d*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^
4)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c)+1)^(1/2)*(csc(d*x+c
)-cot(d*x+c)+1)^(1/2)/(e*tan(d*x+c))^(1/2)*(1+sec(d*x+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\tan{(c+dx)}}\,(a+b\sec{(c+dx)})}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*tan(c + d*x))*(a + b*sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)\sqrt{e\tan{(dx+c)}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)\sqrt{e\tan{(dx+c)}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(e*tan(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx = \int \frac{\cos{(c+d\,x)}}{\sqrt{e\tan{(c+d\,x)}}\,\left(b+a\,\cos{(c+d\,x)}\right)}\,dx$$

```
[In] int(1/((e*tan(c + d*x))^(1/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*tan(c + d*x))^(1/2)*(b + a*cos(c + d*x))), x)
```

3.316 $\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 863

$$
\begin{aligned}
\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx &= \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{3/2}} \\
&- \frac{b^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\,(a^2-b^2)\,de^{3/2}} - \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{3/2}} \\
&+ \frac{b^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\,(a^2-b^2)\,de^{3/2}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,(a^2-b^2)\,de^{3/2}} \\
&+ \frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\,(a^2-b^2)\,de^{3/2}} \\
&+ \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,(a^2-b^2)\,de^{3/2}} \\
&- \frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\,(a^2-b^2)\,de^{3/2}} - \frac{2(a-b\sec(c+dx))}{(a^2-b^2)\,de\sqrt{e\tan(c+dx)}} \\
&+ \frac{2\sqrt{2}b^3\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(c+dx)}} \\
&- \frac{2\sqrt{2}b^3\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right),-1\right)\sqrt{e\tan(c+dx)}}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(c+dx)}} \\
&+ \frac{2b\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{(a^2-b^2)\,de^2\sqrt{\sin(2c+2dx)}} - \frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{(a^2-b^2)\,de^3}
\end{aligned}
$$

```
[Out] 1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b^2)/d/e^(3/2)*2^
(1/2)-1/2*b^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/(a^2-b^2)/d/
e^(3/2)*2^(1/2)-1/2*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b
^2)/d/e^(3/2)*2^(1/2)+1/2*b^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2)
)/a/(a^2-b^2)/d/e^(3/2)*2^(1/2)-1/4*a*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/
2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/d/e^(3/2)*2^(1/2)+1/4*b^2*ln(e^(1/2)-2^(1/
2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/(a^2-b^2)/d/e^(3/2)*2^(1/2)+1
/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/
d/e^(3/2)*2^(1/2)-1/4*b^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*t
an(d*x+c))/a/(a^2-b^2)/d/e^(3/2)*2^(1/2)-2*(a-b*sec(d*x+c))/(a^2-b^2)/d/e/(
e*tan(d*x+c))^(1/2)+2*b^3*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),
-(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/a
/(a-b)^(3/2)/(a+b)^(3/2)/d/e^2/sin(d*x+c)^(1/2)-2*b^3*EllipticPi(sin(d*x+c)
^(1/2)/(1+cos(d*x+c))^(1/2),(a-b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(
1/2)*(e*tan(d*x+c))^(1/2)/a/(a-b)^(3/2)/(a+b)^(3/2)/d/e^2/sin(d*x+c)^(1/2)-
2*b*cos(d*x+c)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(
c+1/4*Pi+d*x),2^(1/2))*(e*tan(d*x+c))^(1/2)/(a^2-b^2)/d/e^2/sin(2*d*x+2*c)^
(1/2)-2*b*cos(d*x+c)*(e*tan(d*x+c))^(3/2)/(a^2-b^2)/d/e^3
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3978, 3967, 3969, 3557, 335, 303, 1176, 631, 210, 1179, 642, 2693, 2695, 2652, 2719, 3975, 2812, 2809, 2985, 2984, 504, 1227, 551}

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \frac{2\sqrt{2}\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right.}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(c+dx)}}$$

$$-\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}},\arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right),-1\right)\sqrt{e\tan(c+dx)}b^3}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(c+dx)}}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)b^2}{\sqrt{2}a\left(a^2-b^2\right)de^{3/2}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)b^2}{\sqrt{2}a\left(a^2-b^2\right)de^{3/2}}$$

$$+\frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{e}-\sqrt{2}\sqrt{e\tan(c+dx)}\right)b^2}{2\sqrt{2}a\left(a^2-b^2\right)de^{3/2}}$$

$$-\frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)b^2}{2\sqrt{2}a\left(a^2-b^2\right)de^{3/2}}-\frac{2\cos(c+dx)(e\tan(c+dx))^{3/2}b}{\left(a^2-b^2\right)de^3}$$

$$+\frac{2\cos(c+dx)E\left(c+dx-\frac{\pi}{4}\middle|2\right)\sqrt{e\tan(c+dx)}b}{\left(a^2-b^2\right)de^2\sqrt{\sin(2c+2dx)}}+\frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2-b^2\right)de^{3/2}}$$

$$-\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\left(a^2-b^2\right)de^{3/2}}-\frac{a\log\left(\sqrt{e}\tan(c+dx)+\sqrt{e}-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right)de^{3/2}}$$

$$+\frac{a\log\left(\sqrt{e}\tan(c+dx)+\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right)de^{3/2}}-\frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\tan(c+dx)}}$$

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)
*d*e^(3/2)) - (b^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*a*(a^2 - b^2)*d*e^(3/2)) - (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)*d*e^(3/2)) + (b^2*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) - (a*Log[Sqrt
[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2
 - b^2)*d*e^(3/2)) + (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt
[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) + (a*Log[Sqrt[e] + S
qrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2 - b^2)
*d*e^(3/2)) - (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[
c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(3/2)) - (2*(a - b*Sec[c + d*x]))/
((a^2 - b^2)*d*e*Sqrt[e*Tan[c + d*x]]) + (2*Sqrt[2]*b^3*Sqrt[Cos[c + d*x]]*
EllipticPi[-(Sqrt[a - b]/Sqrt[a + b]), ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + C
os[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d*e
^2*Sqrt[Sin[c + d*x]]) - (2*Sqrt[2]*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[Sqrt[
a - b]/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*
Sqrt[e*Tan[c + d*x]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*d*e^2*Sqrt[Sin[c + d*x
```

```
]]) + (2*b*Cos[c + d*x]*EllipticE[c - Pi/4 + d*x, 2]*Sqrt[e*Tan[c + d*x]])/
((a^2 - b^2)*d*e^2*Sqrt[Sin[2*c + 2*d*x]]) - (2*b*Cos[c + d*x]*(e*Tan[c + d
*x])^(3/2))/((a^2 - b^2)*d*e^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2809

```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]]), Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x]
)), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2812

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
```

```
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3975

```
Int[Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[1/a, Int[Sqrt[e*Cot[c + d*x]], x], x] - Dist[b/a, In
t[Sqrt[e*Cot[c + d*x]]/(b + a*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3978

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Dist[1/(a^2 - b^2), Int[(e*Cot[c + d*x])^m*(a - b*Csc[c
+ d*x]), x], x] + Dist[b^2/(e^2*(a^2 - b^2)), Int[(e*Cot[c + d*x])^(m + 2)/
(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2,
 0] && ILtQ[m + 1/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{a-b\sec(c+dx)}{(e\tan(c+dx))^{3/2}}\,dx}{a^2-b^2}+\frac{b^2\int \frac{\sqrt{e\tan(c+dx)}}{a+b\sec(c+dx)}\,dx}{(a^2-b^2)\,e^2}\\
&= -\frac{2(a-b\sec(c+dx))}{(a^2-b^2)\,de\sqrt{e\tan(c+dx)}}+\frac{2\int \left(-\frac{a}{2}-\frac{1}{2}b\sec(c+dx)\right)\sqrt{e\tan(c+dx)}\,dx}{(a^2-b^2)\,e^2}\\
&\quad+\frac{b^2\int \sqrt{e\tan(c+dx)}\,dx}{a\,(a^2-b^2)\,e^2}-\frac{b^3\int \frac{\sqrt{e\tan(c+dx)}}{b+a\cos(c+dx)}\,dx}{a\,(a^2-b^2)\,e^2}\\
&= -\frac{2(a-b\sec(c+dx))}{(a^2-b^2)\,de\sqrt{e\tan(c+dx)}}-\frac{a\int \sqrt{e\tan(c+dx)}\,dx}{(a^2-b^2)\,e^2}\\
&\quad-\frac{b\int \sec(c+dx)\sqrt{e\tan(c+dx)}\,dx}{(a^2-b^2)\,e^2}+\frac{b^2\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx,x,e\tan(c+dx)\right)}{a\,(a^2-b^2)\,de}\\
&\quad-\frac{\left(b^3\sqrt{e\cot(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\cot(c+dx)}}\,dx}{a\,(a^2-b^2)\,e^2}
\end{aligned}
$$

$$= -\frac{2(a - b\sec(c+dx))}{(a^2-b^2)\,de\sqrt{e\tan(c+dx)}} - \frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{(a^2-b^2)\,de^3}$$
$$+ \frac{(2b)\int \cos(c+dx)\sqrt{e\tan(c+dx)}\,dx}{(a^2-b^2)\,e^2} - \frac{a\text{Subst}\left(\int \frac{\sqrt{x}}{e^2+x^2}\,dx, x, e\tan(c+dx)\right)}{(a^2-b^2)\,de}$$
$$+ \frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a\,(a^2-b^2)\,de}$$
$$- \frac{\left(b^3\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{\sqrt{\sin(c+dx)}}{\sqrt{-\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{a\,(a^2-b^2)\,e^2\sqrt{\sin(c+dx)}}$$

$$= -\frac{2(a - b\sec(c+dx))}{(a^2-b^2)\,de\sqrt{e\tan(c+dx)}} - \frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{(a^2-b^2)\,de^3}$$
$$- \frac{(2a)\text{Subst}\left(\int \frac{x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{(a^2-b^2)\,de}$$
$$- \frac{b^2\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a\,(a^2-b^2)\,de}$$
$$+ \frac{b^2\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a\,(a^2-b^2)\,de}$$
$$+ \frac{\left(2b\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}\,dx}{(a^2-b^2)\,e^2\sqrt{\sin(c+dx)}}$$
$$- \frac{\left(b^3\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{a\,(a^2-b^2)\,e^2\sqrt{\sin(c+dx)}}$$

$$= -\frac{2(a-b\sec(c+dx))}{(a^2-b^2)\,de\sqrt{e\tan(c+dx)}} - \frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{(a^2-b^2)\,de^3}$$
$$+ \frac{b^2\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\,(a^2-b^2)\,de^{3/2}}$$
$$+ \frac{b^2\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\,(a^2-b^2)\,de^{3/2}}$$
$$+ \frac{a\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{(a^2-b^2)\,de}$$
$$- \frac{a\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{(a^2-b^2)\,de}$$
$$+ \frac{b^2\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2a\,(a^2-b^2)\,de}$$
$$+ \frac{b^2\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2a\,(a^2-b^2)\,de}$$
$$- \frac{\left(4\sqrt{2}b^3\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}(a+b+(-a+b)x^4)}\,dx, x, \frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)}{a\,(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}}$$
$$+ \frac{\left(2b\cos(c+dx)\sqrt{e\tan(c+dx)}\right)\int \sqrt{\sin(2c+2dx)}\,dx}{(a^2-b^2)\,e^2\sqrt{\sin(2c+2dx)}}$$

$$
= \frac{b^2 \log \left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\left(a^2-b^2\right)de^{3/2}}
- \frac{b^2 \log \left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\left(a^2-b^2\right)de^{3/2}}
- \frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\tan(c+dx)}} + \frac{2b\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{\left(a^2-b^2\right)de^2\sqrt{\sin(2c+2dx)}}
- \frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{\left(a^2-b^2\right)de^3}
- \frac{a\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right)de^{3/2}}
- \frac{a\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right)de^{3/2}}
+ \frac{b^2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{3/2}}
- \frac{b^2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{3/2}}
- \frac{a\text{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\left(a^2-b^2\right)de}
- \frac{a\text{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{2\left(a^2-b^2\right)de}
- \frac{\left(2\sqrt{2}b^3\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a+b}-\sqrt{a-b}x^2\right)\sqrt{1-x^4}}\,dx, x, \frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)}{a\sqrt{a-b}\left(a^2-b^2\right)de^2\sqrt{\sin(c+dx)}}
+ \frac{\left(2\sqrt{2}b^3\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a+b}+\sqrt{a-b}x^2\right)\sqrt{1-x^4}}\,dx, x, \frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)}{a\sqrt{a-b}\left(a^2-b^2\right)de^2\sqrt{\sin(c+dx)}}
$$

$$= -\frac{b^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{3/2}}+\frac{b^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{3/2}}$$
$$-\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right)de^{3/2}}$$
$$+\frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\left(a^2-b^2\right)de^{3/2}}$$
$$+\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right)de^{3/2}}$$
$$-\frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\left(a^2-b^2\right)de^{3/2}}$$
$$-\frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\tan(c+dx)}}+\frac{2b\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{\left(a^2-b^2\right)de^2\sqrt{\sin(2c+2dx)}}$$
$$-\frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{\left(a^2-b^2\right)de^3}-\frac{a\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2-b^2\right)de^{3/2}}$$
$$+\frac{a\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2-b^2\right)de^{3/2}}$$
$$-\frac{\left(2\sqrt{2}b^3\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a+b}-\sqrt{a-b}x^2\right)}\,dx,x,\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)}{a\sqrt{a-b}\left(a^2-b^2\right)de^2\sqrt{\sin(c+dx)}}$$
$$+\frac{\left(2\sqrt{2}b^3\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a+b}+\sqrt{a-b}x^2\right)}\,dx,x,\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)}{a\sqrt{a-b}\left(a^2-b^2\right)de^2\sqrt{\sin(c+dx)}}$$

$$
\begin{aligned}
&= \frac{a \arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2-b^2\right) de^{3/2}} - \frac{b^2 \arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right) de^{3/2}} \\
&- \frac{a \arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2-b^2\right) de^{3/2}} + \frac{b^2 \arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right) de^{3/2}} \\
&- \frac{a \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right) de^{3/2}} \\
&+ \frac{b^2 \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\left(a^2-b^2\right) de^{3/2}} \\
&+ \frac{a \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right) de^{3/2}} \\
&- \frac{b^2 \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\left(a^2-b^2\right) de^{3/2}} \\
&- \frac{2(a-b\sec(c+dx))}{\left(a^2-b^2\right) de\sqrt{e\tan(c+dx)}} \\
&+ \frac{2\sqrt{2}b^3\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(-\frac{\sqrt{a-b}}{\sqrt{a+b}}, \arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right), -1\right)\sqrt{e\tan(c+dx)}}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(c+dx)}} \\
&- \frac{2\sqrt{2}b^3\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}}, \arcsin\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right), -1\right)\sqrt{e\tan(c+dx)}}{a(a-b)^{3/2}(a+b)^{3/2}de^2\sqrt{\sin(c+dx)}} \\
&+ \frac{2b\cos(c+dx)E\left(c-\frac{\pi}{4}+dx\middle|2\right)\sqrt{e\tan(c+dx)}}{\left(a^2-b^2\right) de^2\sqrt{\sin(2c+2dx)}} \\
&- \frac{2b\cos(c+dx)(e\tan(c+dx))^{3/2}}{\left(a^2-b^2\right) de^3}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 23.90 (sec) , antiderivative size = 1571, normalized size of antiderivative = 1.82

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \frac{(b+a\cos(c+dx))\sec(c+dx)\left(-\frac{2(b-a\cos(c+dx))\csc(c+dx)}{-a^2+b^2}+\frac{2b\sin(}{-a^2}\right.}{d(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}$$

$$+\frac{(b+a\cos(c+dx))\sec(c+dx)\tan^{\frac{3}{2}}(c+dx)\left(-\frac{(-a^2+3b^2)\sec(c+dx)\left(6\sqrt{2}(a^2-b^2)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-6\sqrt{2}a^2\arctan\right.}{}\right.}{}$$

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-2*(b - a*Cos[c + d*x])*Csc[c + d*x])/
(-a^2 + b^2) + (2*b*Sin[c + d*x])/(-a^2 + b^2))*Tan[c + d*x]^2)/(d*(a + b*S
ec[c + d*x])*(e*Tan[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*T
an[c + d*x]^(3/2)*(-1/12*((-a^2 + 3*b^2)*Sec[c + d*x]*(6*Sqrt[2]*(a^2 - b^2
)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 6*Sqrt[2]*a^2*ArcTan[1 + Sqrt[2]
*Sqrt[Tan[c + d*x]]] + 6*Sqrt[2]*b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
 - (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan
[c + d*x]])/(a^2 - b^2)^(1/4)] + (6 + 6*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan
[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] - 3*Sqrt[2]*a^
2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*b^2*Log[1
- Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 3*Sqrt[2]*a^2*Log[1 + Sqrt[2
]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 3*Sqrt[2]*b^2*Log[1 + Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]] + (3 + 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt
[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Ta
n[c + d*x]] - (3 + 3*I)*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + (1
+ I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]] + 8*a
*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)]*Tan[c + d*x]^(3/2))*(a + b*Sqrt[1 + Tan[c + d*x]^2]))/((a^3 - a*b^2)*
(b + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)) + (b*Cos[2*(c + d*x)]*Sec[c + d*
x]^2*(-84*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 84*Sqrt[2]*b*A
rcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + ((42 + 42*I)*(-a^2 + 2*b^2)*ArcTan[
1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(Sqrt[b]*(a^2
- b^2)^(1/4)) + ((42 + 42*I)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt
[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + 42*Sqrt[2
]*b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 42*Sqrt[2]*b*Log[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + ((21 + 21*I)*(a^2 - 2*b^2)*
```

```
Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]]
+ I*b*Tan[c + d*x]])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + ((21 + 21*I)*(-a^2 + 2*b
^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*
x]] + I*b*Tan[c + d*x]])/(Sqrt[b]*(a^2 - b^2)^(1/4)) + (112*a^3*AppellF1[3/
4, 1/2, 1, 7/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c +
d*x]^(3/2))/(a^2 - b^2) - (168*a*b^2*AppellF1[3/4, 1/2, 1, 7/4, -Tan[c + d*
x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan[c + d*x]^(3/2))/(a^2 - b^2) - (
24*a*b^2*AppellF1[7/4, 1/2, 1, 11/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/
(a^2 - b^2)]*Tan[c + d*x]^(7/2))/(a^2 - b^2) - (168*a*Tan[c + d*x]^(3/2))/S
qrt[1 + Tan[c + d*x]^2])*(a + b*Sqrt[1 + Tan[c + d*x]^2]))/(84*a*(b + a*Cos
[c + d*x])*(-1 + Tan[c + d*x]^2)*Sqrt[1 + Tan[c + d*x]^2])))/((a - b)*(a +
b)*d*(a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4015 vs. 2(746) = 1492.

Time = 3.27 (sec) , antiderivative size = 4016, normalized size of antiderivative = 4.65

method	result	size
default	Expression too large to display	4016

```
[In] int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*2^(1/2)*b/((a^2-b^2)^(1/2)-a+b)/((a^2-b^2)^(1/2)+a-b)/(a-b)/a/(a+b)*(1
-cos(d*x+c))*(-((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c)
)^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)
*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1
/2-1/2*I,1/2*2^(1/2))*a^3-2*((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d
*x+c))^(1/2)*a*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2+4*((1-cos(d*x+c))^3*csc(d*
x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)*a^2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+((1
-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c
)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d
*x+c))^(1/2)*(a^2-b^2)^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),(a-
b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^2-((1-cos(d*x+c))*((1-cos(d*x+c
))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2
*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*(a^2-b^2)^(1/
2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1
/2)),1/2*2^(1/2))*b^2-((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc
(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c)
)^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^
(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*b^2+((1-cos(d*x+c))*((
1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)
^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*El
lipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*b^2-((1-c
```

os(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-
cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x
+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*b^3+2*((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)
*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d
*x+c)-csc(d*x+c))^(1/2)*EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/
2))*a*b^2+((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/
2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot
(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/
2*I,1/2*2^(1/2))*a^2*b+((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*cs
c(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c
))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)
^(1/2),1/2-1/2*I,1/2*2^(1/2))*a*b^2+((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d
*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c
)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-
cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2*b+I*((1-cos(d*x+c))*((1-cos(
d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)
*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elliptic
Pi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^2*b+I*((1-cos(d
x+c))((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(
d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))
^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*
b^2-I*((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(
csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x
+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,
1/2*2^(1/2))*a^2*b-I*((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(
d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))
^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(
1/2),1/2-1/2*I,1/2*2^(1/2))*a*b^2-((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x
+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+
2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-co
t(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*a^3-4*((1-cos(d*x+c))^3*csc(d*x+c)
^3+cot(d*x+c)-csc(d*x+c))^(1/2)*a^2*b+2*((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(
d*x+c)-csc(d*x+c))^(1/2)*a*b^2-((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)
^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*c
ot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d
*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*b^3+((1-cos(d*x+c))*((1-cos(d*x+c))^2
*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc
(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d
*x+c)-cot(d*x+c)+1)^(1/2),(a-b)/(a-b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^3+
((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*
x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-cs
c(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a
-b)*(a+b))^(1/2)),1/2*2^(1/2))*b^3-((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*
x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)

```
+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-c
ot(d*x+c)+1)^(1/2),-(a-b)/(-a+b+((a-b)*(a+b))^(1/2)),1/2*2^(1/2))*a*b^2-I*(
(1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x
+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc
(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+1/2*I,1/2*2^(
1/2))*a^3-I*((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(
1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(c
ot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2+
1/2*I,1/2*2^(1/2))*b^3+I*((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*
csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x
+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c)-cot(d*x+c)+
1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*a^3+I*((1-cos(d*x+c))*((1-cos(d*x+c))^2*csc
(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)*(2-2*csc(d*x
+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*EllipticPi((csc(d*x+c
)-cot(d*x+c)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*b^3+4*((1-cos(d*x+c))*((1-cos(
d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(1/2)
*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elliptic
E((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2*b-4*((1-cos(d*x+c))*((1-
cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+1)^(
1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*Elli
pticE((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*a*b^2-2*((1-cos(d*x+c))*
((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*csc(d*x+c))^(1/2)*(csc(d*x+c)-cot(d*x+c)+
1)^(1/2)*(2-2*csc(d*x+c)+2*cot(d*x+c))^(1/2)*(cot(d*x+c)-csc(d*x+c))^(1/2)*
EllipticF((csc(d*x+c)-cot(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2*b-2*((1-cos(d*x+
c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)*a^3*(1-cos(d*x+c))^2*csc(d*
x+c)^2+2*((1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)*a^3)/(
(1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))^(1/2)/((1-cos(d*x+c))^
2*csc(d*x+c)^2-1)^2/(-e/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-cot(d*x+c)+csc(
d*x+c)))^(3/2)*csc(d*x+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\tan{(c+dx)})^{\frac{3}{2}}\,(a+b\sec{(c+dx)})}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(3/2),x)
[Out] Integral(1/((e*tan(c + d*x))**(3/2)*(a + b*sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)\,(e\tan{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)\,(e\tan{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos{(c+d\,x)}}{(e\tan{(c+d\,x)})^{3/2}\,\left(b+a\,\cos{(c+d\,x)}\right)}\,dx$$

```
[In] int(1/((e*tan(c + d*x))^(3/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*tan(c + d*x))^(3/2)*(b + a*cos(c + d*x))), x)
```

3.317 $\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 836

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{5/2}} - \frac{b^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\,(a^2-b^2)\,de^{5/2}} - \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{5/2}} + \frac{b^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\,(a^2-b^2)\,de^{5/2}} + \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,(a^2-b^2)\,de^{5/2}} - \frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\,(a^2-b^2)\,de^{5/2}} - \frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,(a^2-b^2)\,de^{5/2}} + \frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\,(a^2-b^2)\,de^{5/2}} - \frac{2(a-b\sec(c+dx))}{3\,(a^2-b^2)\,de(e\tan(c+dx))^{3/2}} - \frac{2\sqrt{2}b^3\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\,(a^2-b^2)^{3/2}\,de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} + \frac{2\sqrt{2}b^3\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\,(a^2-b^2)^{3/2}\,de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} + \frac{b\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3\,(a^2-b^2)\,de^2\sqrt{e\tan(c+dx)}}$$

```
[Out] 1/2*a*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b^2)/d/e^(5/2)*2^
(1/2)-1/2*b^2*arctan(1-2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/a/(a^2-b^2)/d/
e^(5/2)*2^(1/2)-1/2*a*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2))/(a^2-b
^2)/d/e^(5/2)*2^(1/2)+1/2*b^2*arctan(1+2^(1/2)*(e*tan(d*x+c))^(1/2)/e^(1/2)
)/a/(a^2-b^2)/d/e^(5/2)*2^(1/2)+1/4*a*ln(e^(1/2)-2^(1/2)*(e*tan(d*x+c))^(1/
2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/d/e^(5/2)*2^(1/2)-1/4*b^2*ln(e^(1/2)-2^(1/
2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/a/(a^2-b^2)/d/e^(5/2)*2^(1/2)-1
/4*a*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*tan(d*x+c))/(a^2-b^2)/
d/e^(5/2)*2^(1/2)+1/4*b^2*ln(e^(1/2)+2^(1/2)*(e*tan(d*x+c))^(1/2)+e^(1/2)*t
```

```
an(d*x+c))/a/(a^2-b^2)/d/e^(5/2)*2^(1/2)-2*b^3*EllipticPi((-cos(d*x+c))^(1/
2)/(1+sin(d*x+c))^(1/2),b/(a-(a^2-b^2)^(1/2)),I)*2^(1/2)*sin(d*x+c)^(1/2)/a
/(a^2-b^2)^(3/2)/d/e^2/(-cos(d*x+c))^(1/2)/(e*tan(d*x+c))^(1/2)+2*b^3*Ellip
ticPi((-cos(d*x+c))^(1/2)/(1+sin(d*x+c))^(1/2),b/(a+(a^2-b^2)^(1/2)),I)*2^(
1/2)*sin(d*x+c)^(1/2)/a/(a^2-b^2)^(3/2)/d/e^2/(-cos(d*x+c))^(1/2)/(e*tan(d*
x+c))^(1/2)-1/3*b*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(c
os(c+1/4*Pi+d*x),2^(1/2))*sec(d*x+c)*sin(2*d*x+2*c)^(1/2)/(a^2-b^2)/d/e^2/(
e*tan(d*x+c))^(1/2)-2/3*(a-b*sec(d*x+c))/(a^2-b^2)/d/e/(e*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 836, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3978, 3967, 3969, 3557, 335, 217, 1179, 642, 1176, 631, 210, 2694, 2653, 2720, 3977, 2812, 2808, 2986, 1227, 551}

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{2}\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)\sqrt{\sin(c+dx)}b^3}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} + \frac{2\sqrt{2}\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{\sin(c+dx)+1}}\right),-1\right)\sqrt{\sin(c+dx)}b^3}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)b^2}{\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)b^2}{\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} - \frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{e}-\sqrt{2}\sqrt{e\tan(c+dx)}\right)b^2}{2\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} + \frac{\log\left(\sqrt{e}\tan(c+dx)+\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)b^2}{2\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} + \frac{\text{EllipticF}\left(c+dx-\frac{\pi}{4},2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}b}{3\left(a^2-b^2\right)de^2\sqrt{e\tan(c+dx)}} + \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2-b^2\right)de^{5/2}} - \frac{a\arctan\left(\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\left(a^2-b^2\right)de^{5/2}} + \frac{a\log\left(\sqrt{e}\tan(c+dx)+\sqrt{e}-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right)de^{5/2}} - \frac{a\log\left(\sqrt{e}\tan(c+dx)+\sqrt{e}+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right)de^{5/2}} - \frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}}$$

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (a*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)
*d*e^(5/2)) - (b^2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Tan[c + d*x]])/Sqrt[e]])/(Sqr
t[2]*a*(a^2 - b^2)*d*e^(5/2)) - (a*ArcTan[1 + (Sqrt[2]*Sqrt[e*Tan[c + d*x]]
)/Sqrt[e]])/(Sqrt[2]*(a^2 - b^2)*d*e^(5/2)) + (b^2*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Tan[c + d*x]])/Sqrt[e]])/(Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) + (a*Log[Sqrt
[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2
 - b^2)*d*e^(5/2)) - (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] - Sqrt[2]*Sqrt
[e*Tan[c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) - (a*Log[Sqrt[e] + S
qrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[c + d*x]]])/(2*Sqrt[2]*(a^2 - b^2)
*d*e^(5/2)) + (b^2*Log[Sqrt[e] + Sqrt[e]*Tan[c + d*x] + Sqrt[2]*Sqrt[e*Tan[
```

```
c + d*x]]])/(2*Sqrt[2]*a*(a^2 - b^2)*d*e^(5/2)) - (2*(a - b*Sec[c + d*x]))/
(3*(a^2 - b^2)*d*e*(e*Tan[c + d*x])^(3/2)) - (2*Sqrt[2]*b^3*EllipticPi[b/(a
 - Sqrt[a^2 - b^2]), ArcSin[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1
]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2)^(3/2)*d*e^2*Sqrt[-Cos[c + d*x]]*Sqrt[e
*Tan[c + d*x]]) + (2*Sqrt[2]*b^3*EllipticPi[b/(a + Sqrt[a^2 - b^2]), ArcSin
[Sqrt[-Cos[c + d*x]]/Sqrt[1 + Sin[c + d*x]]], -1]*Sqrt[Sin[c + d*x]])/(a*(a
^2 - b^2)^(3/2)*d*e^2*Sqrt[-Cos[c + d*x]]*Sqrt[e*Tan[c + d*x]]) + (b*Ellipt
icF[c - Pi/4 + d*x, 2]*Sec[c + d*x]*Sqrt[Sin[2*c + 2*d*x]])/(3*(a^2 - b^2)*
d*e^2*Sqrt[e*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2808

```
Int[Sqrt[(g_.)*tan[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[g*Tan[e + f*x]]/Sqrt[Sin[e
```

```
+ f*x]]), Int[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x])),
 x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2812

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rule 3977

```
Int[1/(Sqrt[cot[(c_.) + (d_.)*(x_)]*(e_.)]*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[1/a, Int[1/Sqrt[e*Cot[c + d*x]], x], x] - Dist[b
/a, Int[1/(Sqrt[e*Cot[c + d*x]]*(b + a*Sin[c + d*x])), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3978

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)/(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Dist[1/(a^2 - b^2), Int[(e*Cot[c + d*x])^m*(a - b*Csc[c
+ d*x]), x], x] + Dist[b^2/(e^2*(a^2 - b^2)), Int[(e*Cot[c + d*x])^(m + 2)/
(a + b*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2,
 0] && ILtQ[m + 1/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{a-b\sec(c+dx)}{(e\tan(c+dx))^{5/2}}\,dx}{a^2-b^2}+\frac{b^2\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{(a^2-b^2)\,e^2}\\
&= -\frac{2(a-b\sec(c+dx))}{3\,(a^2-b^2)\,de(e\tan(c+dx))^{3/2}}+\frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3\,(a^2-b^2)\,e^2}\\
&\quad+\frac{b^2\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{a\,(a^2-b^2)\,e^2}-\frac{b^3\int \frac{1}{(b+a\cos(c+dx))\sqrt{e\tan(c+dx)}}\,dx}{a\,(a^2-b^2)\,e^2}\\
&= -\frac{2(a-b\sec(c+dx))}{3\,(a^2-b^2)\,de(e\tan(c+dx))^{3/2}}-\frac{a\int \frac{1}{\sqrt{e\tan(c+dx)}}\,dx}{(a^2-b^2)\,e^2}\\
&\quad+\frac{b\int \frac{\sec(c+dx)}{\sqrt{e\tan(c+dx)}}\,dx}{3\,(a^2-b^2)\,e^2}+\frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx,x,e\tan(c+dx)\right)}{a\,(a^2-b^2)\,de}\\
&\quad-\frac{b^3\int \frac{\sqrt{e\cot(c+dx)}}{b+a\cos(c+dx)}\,dx}{a\,(a^2-b^2)\,e^2\sqrt{e\cot(c+dx)}\sqrt{e\tan(c+dx)}}\\
&= -\frac{2(a-b\sec(c+dx))}{3\,(a^2-b^2)\,de(e\tan(c+dx))^{3/2}}-\frac{a\text{Subst}\left(\int \frac{1}{\sqrt{x}(e^2+x^2)}\,dx,x,e\tan(c+dx)\right)}{(a^2-b^2)\,de}\\
&\quad+\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{a\,(a^2-b^2)\,de}\\
&\quad-\frac{\left(b^3\sqrt{\sin(c+dx)}\right)\int \frac{\sqrt{-\cos(c+dx)}}{(b+a\cos(c+dx))\sqrt{\sin(c+dx)}}\,dx}{a\,(a^2-b^2)\,e^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}\\
&\quad+\frac{\left(b\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{\sin(c+dx)}}\,dx}{3\,(a^2-b^2)\,e^2\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}}
\end{aligned}
$$

$$= -\frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}} + \frac{b^2\text{Subst}\left(\int \frac{e-x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a\left(a^2-b^2\right)de^2}$$

$$+ \frac{b^2\text{Subst}\left(\int \frac{e+x^2}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{a\left(a^2-b^2\right)de^2}$$

$$- \frac{(2a)\text{Subst}\left(\int \frac{1}{e^2+x^4}\,dx, x, \sqrt{e\tan(c+dx)}\right)}{\left(a^2-b^2\right)de}$$

$$- \frac{\left(2\sqrt{2}b^3\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\left(-a+\sqrt{a^2-b^2}+bx^2\right)\sqrt{1-x^4}}\,dx, x, \frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right)}{a\left(a^2-b^2\right)de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$- \frac{\left(2\sqrt{2}b^3\left(1+\frac{a}{\sqrt{a^2-b^2}}\right)\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\left(-a-\sqrt{a^2-b^2}+bx^2\right)\sqrt{1-x^4}}\,dx, x, \frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right)}{a\left(a^2-b^2\right)de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$+ \frac{\left(b\sec(c+dx)\sqrt{\sin(2c+2dx)}\right)\int \frac{1}{\sqrt{\sin(2c+2dx)}}\,dx}{3\left(a^2-b^2\right)e^2\sqrt{e\tan(c+dx)}}$$

$$= -\frac{2(a-b\sec(c+dx))}{3(a^2-b^2)de(e\tan(c+dx))^{3/2}} + \frac{b\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3(a^2-b^2)de^2\sqrt{e\tan(c+dx)}} - \frac{b^2\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a(a^2-b^2)de^{5/2}} - \frac{b^2\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a(a^2-b^2)de^{5/2}} - \frac{a\text{Subst}\left(\int\frac{e-x^2}{e^2+x^4}dx,x,\sqrt{e\tan(c+dx)}\right)}{(a^2-b^2)de^2} - \frac{a\text{Subst}\left(\int\frac{e+x^2}{e^2+x^4}dx,x,\sqrt{e\tan(c+dx)}\right)}{(a^2-b^2)de^2} + \frac{b^2\text{Subst}\left(\int\frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}dx,x,\sqrt{e\tan(c+dx)}\right)}{2a(a^2-b^2)de^2} + \frac{b^2\text{Subst}\left(\int\frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}dx,x,\sqrt{e\tan(c+dx)}\right)}{2a(a^2-b^2)de^2} - \frac{\left(2\sqrt{2}b^3\left(1-\frac{a}{\sqrt{a^2-b^2}}\right)\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(-a+\sqrt{a^2-b^2}+bx^2\right)}dx,x,\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right)}{a(a^2-b^2)de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} - \frac{\left(2\sqrt{2}b^3\left(1+\frac{a}{\sqrt{a^2-b^2}}\right)\sqrt{\sin(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(-a-\sqrt{a^2-b^2}+bx^2\right)}dx,x,\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right)}{a(a^2-b^2)de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}}$$

$$
\begin{aligned}
&= -\frac{b^2 \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} \\
&\quad + \frac{b^2 \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} \\
&\quad - \frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}} \\
&\quad - \frac{2\sqrt{2}b^3 \operatorname{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{2\sqrt{2}b^3 \operatorname{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{b \operatorname{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3\left(a^2-b^2\right)de^2\sqrt{e\tan(c+dx)}} \\
&\quad + \frac{a\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}+2x}{-e-\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right)de^{5/2}} \\
&\quad + \frac{a\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{e}-2x}{-e+\sqrt{2}\sqrt{e}x-x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right)de^{5/2}} \\
&\quad + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} \\
&\quad - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} \\
&\quad - \frac{a\operatorname{Subst}\left(\int \frac{1}{e-\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\left(a^2-b^2\right)de^2} \\
&\quad - \frac{a\operatorname{Subst}\left(\int \frac{1}{e+\sqrt{2}\sqrt{e}x+x^2}\,dx,x,\sqrt{e\tan(c+dx)}\right)}{2\left(a^2-b^2\right)de^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{5/2}}+\frac{b^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} \\
&+\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right)de^{5/2}} \\
&-\frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} \\
&-\frac{a\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\left(a^2-b^2\right)de^{5/2}} \\
&+\frac{b^2\log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\left(a^2-b^2\right)de^{5/2}} \\
&-\frac{2(a-b\sec(c+dx))}{3\left(a^2-b^2\right)de(e\tan(c+dx))^{3/2}} \\
&-\frac{2\sqrt{2}b^3\,\text{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&+\frac{2\sqrt{2}b^3\,\text{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}},\arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right),-1\right)\sqrt{\sin(c+dx)}}{a\left(a^2-b^2\right)^{3/2}de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&+\frac{b\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3\left(a^2-b^2\right)de^2\sqrt{e\tan(c+dx)}} \\
&-\frac{a\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2-b^2\right)de^{5/2}} \\
&+\frac{a\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2-b^2\right)de^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a \arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{5/2}} - \frac{b^2 \arctan\left(1-\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\,(a^2-b^2)\,de^{5/2}} \\
&- \frac{a \arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2-b^2)\,de^{5/2}} + \frac{b^2 \arctan\left(1+\frac{\sqrt{2}\sqrt{e\tan(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}a\,(a^2-b^2)\,de^{5/2}} \\
&+ \frac{a \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,(a^2-b^2)\,de^{5/2}} \\
&- \frac{b^2 \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)-\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\,(a^2-b^2)\,de^{5/2}} \\
&- \frac{a \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}\,(a^2-b^2)\,de^{5/2}} \\
&+ \frac{b^2 \log\left(\sqrt{e}+\sqrt{e}\tan(c+dx)+\sqrt{2}\sqrt{e\tan(c+dx)}\right)}{2\sqrt{2}a\,(a^2-b^2)\,de^{5/2}} \\
&- \frac{2(a-b\sec(c+dx))}{3\,(a^2-b^2)\,de(e\tan(c+dx))^{3/2}} \\
&- \frac{2\sqrt{2}b^3\,\mathrm{EllipticPi}\left(\frac{b}{a-\sqrt{a^2-b^2}}, \arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right), -1\right)\sqrt{\sin(c+dx)}}{a\,(a^2-b^2)^{3/2}\,de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&+ \frac{2\sqrt{2}b^3\,\mathrm{EllipticPi}\left(\frac{b}{a+\sqrt{a^2-b^2}}, \arcsin\left(\frac{\sqrt{-\cos(c+dx)}}{\sqrt{1+\sin(c+dx)}}\right), -1\right)\sqrt{\sin(c+dx)}}{a\,(a^2-b^2)^{3/2}\,de^2\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}} \\
&+ \frac{b\,\mathrm{EllipticF}\left(c-\frac{\pi}{4}+dx, 2\right)\sec(c+dx)\sqrt{\sin(2c+2dx)}}{3\,(a^2-b^2)\,de^2\sqrt{e\tan(c+dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 33.62 (sec) , antiderivative size = 2169, normalized size of antiderivative = 2.59

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*((2*a)/(3*(a^2 - b^2)) - (2*(-a + b*Cos[c + d*x])*Csc
[c + d*x]^2)/(3*(-a^2 + b^2)))*Sec[c + d*x]*Tan[c + d*x]^3)/(d*(a + b*Sec[c
 + d*x])*(e*Tan[c + d*x])^(5/2)) - ((b + a*Cos[c + d*x])*Sec[c + d*x]*Tan[c
 + d*x]^(5/2)*((2*(3*a^2 - 5*b^2)*Sec[c + d*x]^3*(a + b*Sqrt[1 + Tan[c + d*
x]^2])*(((-1/8 + I/8)*a*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/
(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2
```

```
 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[c + d*x]] + I*b*Tan[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + I*b*Tan[c + d*x]]))/(Sqrt[b]*(a^2
- b^2)^(3/4)) + (5*b*(-a^2 + b^2)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[c + d*x]
^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Tan[c + d*x]]*Sqrt[1 + Tan[c + d
*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*T
an[c + d*x]^2)/(a^2 - b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, -Tan[c +
 d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2,
 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2
)*(a^2 - b^2*(1 + Tan[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 + Tan[c + d*
x]^2)^2) + (8*a*b*Sec[c + d*x]^2*(a + b*Sqrt[1 + Tan[c + d*x]^2])*((Sqrt[b]
*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^2 + b^2)^(1/4)] +
2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Tan[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Tan[c + d*x]] +
 b*Tan[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[b]*(-a^2 + b^2)^(1/4
)*Sqrt[Tan[c + d*x]] + b*Tan[c + d*x]]))/(4*Sqrt[2]*(-a^2 + b^2)^(3/4)) + (
5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)]*Sqrt[Tan[c + d*x]])/(Sqrt[1 + Tan[c + d*x]^2]*(-5*(a^2 -
 b^2)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
 - b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, -Tan[c + d
*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)*(-a^2 + b^2*(1 +
Tan[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 + Tan[c + d*x]^2)^(3/2)) + ((3
*a^2 - 3*b^2)*Cos[2*(c + d*x)]*Sec[c + d*x]^3*(a + b*Sqrt[1 + Tan[c + d*x]^
2])*((-20*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]])/a + (20*Sqrt[2]*A
rcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]])/a + ((10 - 10*I)*(a^2 - 2*b^2)*ArcTa
n[1 - ((1 + I)*Sqrt[b]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(
a^2 - b^2)^(3/4)) - ((10 - 10*I)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*
Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (10
*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/a + (10*Sqrt[2
]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])/a + ((5 - 5*I)*(a^2 -
 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c
+ d*x]] + I*b*Tan[c + d*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - ((5 - 5*I)*(a^
2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Tan
[c + d*x]] + I*b*Tan[c + d*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (8*b*Appell
F1[5/4, 1/2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Tan
[c + d*x]^(5/2))/(-a^2 + b^2) - (200*b*(-a^2 + b^2)*AppellF1[1/4, 1/2, 1, 5
/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Tan[c + d*x]])/
(Sqrt[1 + Tan[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[c
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(2*b^2*AppellF1[5/4, 1/2, 2
, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*Ap
pellF1[5/4, 3/2, 1, 9/4, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
)*Tan[c + d*x]^2)*(-a^2 + b^2*(1 + Tan[c + d*x]^2)))))/(20*(b + a*Cos[c + d
*x])*(1 - Tan[c + d*x]^2)*(1 + Tan[c + d*x]^2))))/(6*(a - b)*(a + b)*d*(a +
 b*Sec[c + d*x])*(e*Tan[c + d*x])^(5/2))
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7580 vs. $2(725) = 1450$.

Time = 4.79 (sec) , antiderivative size = 7581, normalized size of antiderivative = 9.07

method	result	size
default	Expression too large to display	7581

```
[In] int(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(e\tan(c+dx))^{\frac{5}{2}}\,(a+b\sec(c+dx))}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))**(5/2),x)
[Out] Integral(1/((e*tan(c + d*x))**(5/2)*(a + b*sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\,(e\tan(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*tan(d*x + c))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))(e\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)}{(e\tan(c+d\,x))^{5/2}\,(b+a\,\cos(c+d\,x))}\,dx$$

```
[In] int(1/((e*tan(c + d*x))^(5/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*tan(c + d*x))^(5/2)*(b + a*cos(c + d*x))), x)
```

3.318 $\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 169

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx = -\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a(a^2-2b^2)\,(a+b\sec(c+dx))^{3/2}}{3b^4d} + \frac{2(3a^2-2b^2)\,(a+b\sec(c+dx))^{5/2}}{5b^4d} - \frac{6a(a+b\sec(c+dx))^{7/2}}{7b^4d} + \frac{2(a+b\sec(c+dx))^{9/2}}{9b^4d}$$

```
[Out] -2/3*a*(a^2-2*b^2)*(a+b*sec(d*x+c))^(3/2)/b^4/d+2/5*(3*a^2-2*b^2)*(a+b*sec(
d*x+c))^(5/2)/b^4/d-6/7*a*(a+b*sec(d*x+c))^(7/2)/b^4/d+2/9*(a+b*sec(d*x+c))
^(9/2)/b^4/d-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+b*sec
(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3970, 912, 1275, 213}

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx = \frac{2(3a^2-2b^2)\,(a+b\sec(c+dx))^{5/2}}{5b^4d} - \frac{2a(a^2-2b^2)\,(a+b\sec(c+dx))^{3/2}}{3b^4d} - \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2(a+b\sec(c+dx))^{9/2}}{9b^4d} - \frac{6a(a+b\sec(c+dx))^{7/2}}{7b^4d} + \frac{2\sqrt{a+b\sec(c+dx)}}{d}$$

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + b*Se
c[c + d*x]])/d - (2*a*(a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(3/2))/(3*b^4*d) +
 (2*(3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(5/2))/(5*b^4*d) - (6*a*(a + b*Sec
[c + d*x])^(7/2))/(7*b^4*d) + (2*(a + b*Sec[c + d*x])^(9/2))/(9*b^4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{a+x}(b^2-x^2)^2}{x}\,dx, x, b\sec(c+dx)\right)}{b^4 d}$$

$$= \frac{2\text{Subst}\left(\int \frac{x^2(-a^2+b^2+2ax^2-x^4)^2}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4 d}$$

$$= \frac{2\text{Subst}\left(\int \left(b^4 - a(a^2-2b^2)\,x^2 + (3a^2-2b^2)\,x^4 - 3ax^6 + x^8 + \frac{ab^4}{-a+x^2}\right)\,dx, x, \sqrt{a+b\sec(c+dx)}\right.}{b^4 d}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a(a^2-2b^2)\,(a+b\sec(c+dx))^{3/2}}{3b^4 d} + \frac{2(3a^2-2b^2)\,(a+b\sec(c+dx))^{5/2}}{5b^4 d} - \frac{6a(a+b\sec(c+dx))^{7/2}}{7b^4 d} + \frac{2(a+b\sec(c+dx))^{9/2}}{9b^4 d} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= -\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a(a^2-2b^2)\,(a+b\sec(c+dx))^{3/2}}{3b^4 d} + \frac{2(3a^2-2b^2)\,(a+b\sec(c+dx))^{5/2}}{5b^4 d} - \frac{6a(a+b\sec(c+dx))^{7/2}}{7b^4 d} + \frac{2(a+b\sec(c+dx))^{9/2}}{9b^4 d}$$

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.90

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx$$
$$= \frac{-2\sqrt{a}b^4\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right) + 2b^4\sqrt{a+b\sec(c+dx)} - \frac{2}{3}a(a^2-2b^2)\,(a+b\sec(c+dx))^{3/2} + \frac{2}{5}(3a^2 -}{b^4 d}$$

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^5,x]
```

```
[Out] (-2*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]] + 2*b^4*Sqrt[a +
b*Sec[c + d*x]] - (2*a*(a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(3/2))/3 + (2*(3*
a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(5/2))/5 - (6*a*(a + b*Sec[c + d*x])^(7/2
))/7 + (2*(a + b*Sec[c + d*x])^(9/2))/9)/(b^4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(145) = 290$.

Time = 16.53 (sec) , antiderivative size = 883, normalized size of antiderivative = 5.22

method	result	size
default	Expression too large to display	883

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315/d/b^4*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*(315*cos(d*x+c)*ln(4*cos(d*x+c)*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b
+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*a^(1/2)*b^4+32*((b+a
*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^4*cos(d*x+c)-168*((b+a*co
s(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2*cos(d*x+c)-630*cos(d*x
+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^4+32*((b+a*cos(d
*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^4-16*((b+a*cos(d*x+c))*cos(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2)*a^3*b-168*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)*a^2*b^2+84*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1
/2)*a*b^3-630*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^4-16*(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3*b*sec(d*x+c)+12*((b
+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2*sec(d*x+c)+84*((b
+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^3*sec(d*x+c)+252*((b+
a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^4*sec(d*x+c)+12*((b+a*co
s(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2*sec(d*x+c)^2-10*((b+a*
cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^3*sec(d*x+c)^2+252*((b+a
*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^4*sec(d*x+c)^2-10*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^3*sec(d*x+c)^3-70*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^4*sec(d*x+c)^3-70*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^4*sec(d*x+c)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.51

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx$$

$$= \left[\frac{315\sqrt{a}b^4\cos(dx+c)^4\log\left(-8a^2\cos(dx+c)^2 - 8ab\cos(dx+c) - b^2 + 4\left(2a\cos(dx+c)^2 + b\cos(dx\right.\right.}{}\right.$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/630*(315*sqrt(a)*b^4*cos(d*x + c)^4*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*co
s(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))) + 4*(5*a*b^3*cos(d*x + c) - (16*a^4 - 84*a
^2*b^2 - 315*b^4)*cos(d*x + c)^4 + 35*b^4 + 2*(4*a^3*b - 21*a*b^3)*cos(d*x
+ c)^3 - 6*(a^2*b^2 + 21*b^4)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos
(d*x + c)))/(b^4*d*cos(d*x + c)^4), 1/315*(315*sqrt(-a)*b^4*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) +
 b))*cos(d*x + c)^4 + 2*(5*a*b^3*cos(d*x + c) - (16*a^4 - 84*a^2*b^2 - 315*
b^4)*cos(d*x + c)^4 + 35*b^4 + 2*(4*a^3*b - 21*a*b^3)*cos(d*x + c)^3 - 6*(a
^2*b^2 + 21*b^4)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(
b^4*d*cos(d*x + c)^4)]
```

Sympy [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx = \int \sqrt{a+b\sec\left(c+dx\right)}\tan^5\left(c+dx\right)dx$$

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**5,x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.13

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx = \frac{315\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)+630\sqrt{a+\frac{b}{\cos(dx+c)}}+\frac{70\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{9}{2}}}{b^4}-\frac{270\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{7}{2}}a}{b^4}+\frac{378\left(a+\frac{b}{\cos(dx+c)}\right.}{b^4}}{315\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/315*(315*sqrt(a)*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos
(d*x + c)) + sqrt(a))) + 630*sqrt(a + b/cos(d*x + c)) + 70*(a + b/cos(d*x +
 c))^(9/2)/b^4 - 270*(a + b/cos(d*x + c))^(7/2)*a/b^4 + 378*(a + b/cos(d*x
+ c))^(5/2)*a^2/b^4 - 210*(a + b/cos(d*x + c))^(3/2)*a^3/b^4 - 252*(a + b/c
os(d*x + c))^(5/2)/b^2 + 420*(a + b/cos(d*x + c))^(3/2)*a/b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 966 vs. $2(145) = 290$.

Time = 1.76 (sec) , antiderivative size = 966, normalized size of antiderivative = 5.72

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 2/315*(315*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1
/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
 a + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(315*(sqrt(a - b)*tan(1/2*d*x
 + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*
a*tan(1/2*d*x + 1/2*c)^2 + a + b))^8*a - 3150*(sqrt(a - b)*tan(1/2*d*x + 1/
2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan
(1/2*d*x + 1/2*c)^2 + a + b))^7*sqrt(a - b)*a + 210*(sqrt(a - b)*tan(1/2*d*
x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2
*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^6*(39*a^2 - 5*a*b - 32*b^2) - 630*(sqrt
(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*
d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^5*(9*a^2 + 15*a*b - 1
6*b^2)*sqrt(a - b) - 252*(25*a^3 - 37*a^2*b + 80*a*b^2 - 72*b^3)*(sqrt(a -
b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x +
 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^4 + 945*a^5 + 3864*a^4*b +
 2562*a^3*b^2 + 2448*a^2*b^3 - 1083*a*b^4 + 224*b^5 + 42*(255*a^3 + 2*a^2*b
 + 655*a*b^2 - 288*b^3)*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/
2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
a + b))^3*sqrt(a - b) - 18*(175*a^4 - 483*a^3*b + 1113*a^2*b^2 - 773*a*b^3
+ 448*b^4)*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c
)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2 - 1
8*(105*a^4 + 637*a^3*b + 203*a^2*b^2 + 447*a*b^3 - 112*b^4)*(sqrt(a - b)*ta
n(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*
c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))*sqrt(a - b))/(sqrt(a - b)*tan(1
/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^
4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) - sqrt(a - b))^9)*sgn(cos(d*x + c))
/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(c+dx)}\tan^5(c+dx)\,dx = \int \tan(c+d\,x)^5\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

```
[In] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^(1/2), x)
```

3.319 $\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx = \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d} + \frac{2(a+b\sec(c+dx))^{5/2}}{5b^2d}$$

```
[Out] -2/3*a*(a+b*sec(d*x+c))^(3/2)/b^2/d+2/5*(a+b*sec(d*x+c))^(5/2)/b^2/d+2*arct
anh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3970, 912, 1275, 213}

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx = \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2(a+b\sec(c+dx))^{5/2}}{5b^2d} - \frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d} - \frac{2\sqrt{a+b\sec(c+dx)}}{d}$$

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Sec
[c + d*x]])/d - (2*a*(a + b*Sec[c + d*x])^(3/2))/(3*b^2*d) + (2*(a + b*Sec[
c + d*x])^(5/2))/(5*b^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}\,(b^2-x^2)}{x}\,dx, x, b\sec(c+dx)\right)}{b^2d} \\
&= -\frac{2\text{Subst}\left(\int \frac{x^2(-a^2+b^2+2ax^2-x^4)}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2d} \\
&= -\frac{2\text{Subst}\left(\int \left(b^2+ax^2-x^4+\frac{ab^2}{-a+x^2}\right)\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2d} \\
&= -\frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d} \\
&\quad + \frac{2(a+b\sec(c+dx))^{5/2}}{5b^2d} - \frac{(2a)\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}
\end{aligned}
$$

$$= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\sec(c+dx)}}{d} - \frac{2a(a+b\sec(c+dx))^{3/2}}{3b^2d} + \frac{2(a+b\sec(c+dx))^{5/2}}{5b^2d}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx = \frac{2\left(15\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right) + \frac{\sqrt{a+b\sec(c+dx)}(-2a^2-15b^2+ab\sec(c+dx)+3b^2\sec^2(c+dx))}{b^2}\right)}{15d}$$

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^3,x]
```

```
[Out] (2*(15*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]] + (Sqrt[a + b*Sec[
c + d*x]]*(-2*a^2 - 15*b^2 + a*b*Sec[c + d*x] + 3*b^2*Sec[c + d*x]^2))/b^2)
)/(15*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(84) = 168$.

Time = 10.67 (sec) , antiderivative size = 465, normalized size of antiderivative = 4.65

method	result
default	$\frac{\sqrt{a+b\sec(dx+c)}\left(15\sqrt{a}\cos(dx+c)\ln\left(4\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\sqrt{a}+4a\cos(dx+c)+4\sqrt{a}\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}+2\right.\right.}{}$

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/d/b^2*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/((b+a*cos(d*x+c))*cos(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2)*(15*a^(1/2)*cos(d*x+c)*ln(4*cos(d*x+c)*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2
)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^2-4*((b+a*cos
(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*cos(d*x+c)-30*cos(d*x+c)*((
b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^2-4*((b+a*cos(d*x+c))*
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(
d*x+c)+1)^2)^(1/2)*a*b-30*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1
/2)*b^2+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b*sec(d*x+
c)+6*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^2*sec(d*x+c)+6*
((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*b^2*sec(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.11

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx$$

$$= \left[\frac{15\sqrt{a}b^2\cos(dx+c)^2\log\left(-8a^2\cos(dx+c)^2-8ab\cos(dx+c)-b^2-4\left(2a\cos(dx+c)^2+b\cos(dx\right.\right.}{30b^2d\cos(dx}\right.$$

$$-\frac{15\sqrt{-a}b^2\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)}{2a\cos(dx+c)+b}\right)\cos(dx+c)^2-2\left(ab\cos(dx+c)-(2a^2+15b^2)\cos(dx\right.}{15b^2d\cos(dx+c)^2}$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a)*b^2*cos(d*x + c)^2*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(
d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*co
s(d*x + c) + b)/cos(d*x + c))) + 4*(a*b*cos(d*x + c) - (2*a^2 + 15*b^2)*cos
(d*x + c)^2 + 3*b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(b^2*d*cos(d*
x + c)^2), -1/15*(15*sqrt(-a)*b^2*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) +
b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))*cos(d*x + c)^2 - 2*(a
*b*cos(d*x + c) - (2*a^2 + 15*b^2)*cos(d*x + c)^2 + 3*b^2)*sqrt((a*cos(d*x
+ c) + b)/cos(d*x + c)))/(b^2*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx = \int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx$$
$$= -\frac{15\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)+30\sqrt{a+\frac{b}{\cos(dx+c)}}-\frac{6\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}{b^2}+\frac{10\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a}{b^2}}{15\,d}$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a)*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(
d*x + c)) + sqrt(a))) + 30*sqrt(a + b/cos(d*x + c)) - 6*(a + b/cos(d*x + c)
)^(5/2)/b^2 + 10*(a + b/cos(d*x + c))^(3/2)*a/b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(84) = 168$.

Time = 0.70 (sec) , antiderivative size = 539, normalized size of antiderivative = 5.39

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx =$$
$$-\frac{2\left(\frac{15\,a\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\left(15\left(\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{}\right.}{}$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/
2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
a + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(15*(sqrt(a - b)*tan(1/2*d*x +
 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*
tan(1/2*d*x + 1/2*c)^2 + a + b))^4*a - 30*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)
^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2
*d*x + 1/2*c)^2 + a + b))^3*(a + 2*b)*sqrt(a - b) + 20*(sqrt(a - b)*tan(1/2
*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4
- 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2*(4*a*b - 3*b^2) - 15*a^3 - 10*a^2*
b - 35*a*b^2 + 12*b^3 + 10*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan
(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2
 + a + b))*(3*a^2 - a*b + 6*b^2)*sqrt(a - b))/(sqrt(a - b)*tan(1/2*d*x + 1/
2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan
(1/2*d*x + 1/2*c)^2 + a + b) - sqrt(a - b))^5)*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(c+dx)}\tan^3(c+dx)\,dx=\int \tan(c+d\,x)^3\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^(1/2), x)
```

3.320 $\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx = -\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{a+b\sec(c+dx)}}{d}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+b*sec(d*x+c))^(1/
2)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3970, 52, 65, 213}

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx = \frac{2\sqrt{a+b\sec(c+dx)}}{d}-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}$$

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + b*Se
c[c + d*x]])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= \frac{2\sqrt{a+b\sec(c+dx)}}{d} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= \frac{2\sqrt{a+b\sec(c+dx)}}{d} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}\\
&= -\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+b\sec(c+dx)}}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx = \frac{-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)+2\sqrt{a+b\sec(c+dx)}}{d}$$

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x],x]
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]] + 2*Sqrt[a + b*Sec[c
+ d*x]])/d
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\sec(dx+c)}-2\sqrt{a}\ \text{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	42
default	$\frac{2\sqrt{a+b\sec(dx+c)}-2\sqrt{a}\ \text{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d}$	42

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(2*(a+b*sec(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1
/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.76

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx$$
$$= \left[\frac{\sqrt{a}\log\left(-8\,a^2\cos\left(dx+c\right)^2-8\,ab\cos\left(dx+c\right)-b^2+4\left(2\,a\cos\left(dx+c\right)^2+b\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)}{\cos(dx+c}}\right.}{2\,d}\right.$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="fricas")
[Out] [1/2*(sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a
*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x
```

```
 + c))) + 4*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/d, (sqrt(-a)*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x
+ c) + b)) + 2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/d]
```

Sympy [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx = \int \sqrt{a+b\sec\left(c+dx\right)}\tan\left(c+dx\right)dx$$

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx = \frac{\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{\cos(dx+c)}}}{d}$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="maxima")
[Out] (sqrt(a)*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c))
 + sqrt(a))) + 2*sqrt(a + b/cos(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(43) = 86$.

Time = 0.40 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.63

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx = \frac{2\left(\frac{a\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{}{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\,\mathrm{t}}}\right.}{d}$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c),x, algorithm="giac")
[Out] 2*(a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x +
 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)
+ sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*b/(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2
 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d
*x + 1/2*c)^2 + a + b) - sqrt(a - b)))*sgn(cos(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 13.72 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \sqrt{a+b\sec(c+dx)}\tan(c+dx)\,dx=\frac{2\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{d}-\frac{2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{d}$$

```
[In] int(tan(c + d*x)*(a + b/cos(c + d*x))^(1/2),x)
[Out] (2*(a + b/cos(c + d*x))^(1/2))/d - (2*a^(1/2)*atanh((a + b/cos(c + d*x))^(1
/2)/a^(1/2)))/d
```

3.321 $\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d} - \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] 2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-arctanh((a+b*sec(d*x+c)
)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d-arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2))*(a+b)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3970, 912, 1301, 212, 213}

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d} - \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[In] Int[Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d - (Sqrt[a - b]*ArcT
anh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/d - (Sqrt[a + b]*ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{b^2\text{Subst}\left(\int \frac{\sqrt{a+x}}{x(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d}\\ &= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}\end{aligned}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \left(-\frac{a}{b^2(a-x^2)} + \frac{a+b}{2b^2(a+b-x^2)} + \frac{-a+b}{2b^2(-a+b+x^2)}\right) dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= \frac{(2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d} - \frac{\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = -\frac{-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right) + \sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right) + \sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
[Out] -((-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]] + Sqrt[a - b]*ArcTa
nh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]] + Sqrt[a + b]*ArcTanh[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]])/d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(88) = 176$.

Time = 2.24 (sec) , antiderivative size = 543, normalized size of antiderivative = 5.12

method	result
default	$-\frac{\sqrt{a+b\sec(dx+c)}\left(\sqrt{a+b}\,\ln\left(-\frac{2\left(2\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\sqrt{a+b}+2a\cos(dx+c)+\cos(dx+c)b+2\sqrt{a+b}\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\right.}{\cos(dx+c)-1}\right.\right.}{}$

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d/(a-b)^(1/2)*(a+b*sec(d*x+c))^(1/2)*((a+b)^(1/2)*ln(-2*(2*cos(d*x+c)*
((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*
x+c)+cos(d*x+c)*b+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)
^2)^(1/2)+b)/(cos(d*x+c)-1))*(a-b)^(1/2)-2*a^(1/2)*ln(4*cos(d*x+c)*((b+a*co
s(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/
2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*(a-b)^(1/2)-a*
ln(1/(a-b)^(1/2)*(2*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)
^2)^(1/2)*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
*(a-b)^(1/2)-2*a*cos(d*x+c)+cos(d*x+c)*b-b)/(cos(d*x+c)+1))+ln(1/(a-b)^(1/2
)*(2*(a-b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d
*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*
a*cos(d*x+c)+cos(d*x+c)*b-b)/(cos(d*x+c)+1))*b)*cos(d*x+c)/(cos(d*x+c)+1)/(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(88) = 176$.

Time = 0.60 (sec) , antiderivative size = 2132, normalized size of antiderivative = 20.11

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2
*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d
*x + c))) + sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 -
4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 +
2*cos(d*x + c) + 1)) + sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)
^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*
x + c)^2 - 2*cos(d*x + c) + 1)))/d, 1/4*(2*sqrt(-a - b)*arctan(2*sqrt(-a -
b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x
+ c) + b)) + 2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2
 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b
)/cos(d*x + c))) + sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 +
 b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*co
s(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)))/d, -1/4*(2*sqrt(-a + b)*arctan(-2*sqrt(-a + b)
*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x +
c) + b)) - 2*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 -
 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/
```

```
cos(d*x + c))) - sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b
^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)
^2 - 2*cos(d*x + c) + 1)))/d, -1/2*(sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) +
 b)) - sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - sqrt(a)*log(-8*a^2*cos(
d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x +
 c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))))/d, -1/4*(4*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a
*cos(d*x + c) + b)) - sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^
2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a
*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) - sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*co
s(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a +
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c
))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/d, -1/4*(4*sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c
) + b)) - 2*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - sqrt(a - b)*log(-(
(8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 +
b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*
a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/d, -1/4*
(4*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d
*x + c)/(2*a*cos(d*x + c) + b)) + 2*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) +
 b)) - sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2
*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) +
 b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos
(d*x + c) + 1)))/d, -1/2*(2*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + sqrt(-a + b)*arc
tan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((
2*a - b)*cos(d*x + c) + b)) - sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*co
s(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)))/d
]
```

Sympy [F]

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{a+b\sec{(c+dx)}}\cot{(c+dx)}\,dx$$

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x), x)
```

Maxima [F]

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\cot\left(dx+c\right)\,dx$$

[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(88) = 176$.

Time = 0.63 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.76

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = -\frac{\left(\frac{4\,a\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2\,(a+b)\arctan\left(-\frac{\sqrt{a-b}\tan\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] -1/2*(4*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(a + b)*arctan(-(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))/sqrt(-a - b))/sqrt(-a - b) + sqrt(a - b)*log(abs(-(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))*(a - b) + sqrt(a - b)*a)))*sgn(cos(d*x + c))/d

Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \cot(c+d\,x)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

[In] int(cot(c + d*x)*(a + b/cos(c + d*x))^(1/2),x)

[Out] int(cot(c + d*x)*(a + b/cos(c + d*x))^(1/2), x)

3.322 $\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 215

$$\begin{aligned}\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = &-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d}\\ &+\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d}\\ &-\frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4\sqrt{a-b}\,d}\\ &+\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}\\ &+\frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4\sqrt{a+b}\,d}\\ &-\frac{\cot^2(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}\end{aligned}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+a*arctanh((a+b*sec(d*x
+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)-3/4*b*arctanh((a+b*sec(d*x+c))^(1/2)/
(a-b)^(1/2))/d/(a-b)^(1/2)+a*arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/d/
(a+b)^(1/2)+3/4*b*arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
-1/2*cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3970, 912, 1329, 1192, 12, 1107, 212, 1184, 213}

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = -\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{3b\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}} + \frac{a\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}} + \frac{a\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}} + \frac{3b\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4d\sqrt{a+b}} - \frac{\cot^2(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

```
[In] Int[Cot[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/d + (a*ArcTanh[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a - b]])/(Sqrt[a - b]*d) - (3*b*ArcTanh[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a - b]])/(4*Sqrt[a - b]*d) + (a*ArcTanh[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]])/(Sqrt[a + b]*d) + (3*b*ArcTanh[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]])/(4*Sqrt[a + b]*d) - (Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d
*x]])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1329

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Dist[d*e*(f^2/(c*d^2
 - b*d*e + a*e^2)), Int[(f*x)^(m - 2)*((a + b*x^2 + c*x^4)^(p + 1)/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
p, -1] && GtQ[m, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^4\text{Subst}\left(\int \frac{\sqrt{a+x}}{x(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d} \\
&= \frac{(2b^4)\,\text{Subst}\left(\int \frac{x^2}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&= \frac{(2b^2)\,\text{Subst}\left(\int \frac{-a^2+b^2+ax^2}{(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&\quad + \frac{(2ab^2)\,\text{Subst}\left(\int \frac{1}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&= -\frac{b^2\sqrt{a+b\sec(c+dx)}}{2d\left(a^2-b^2-2a(a+b\sec(c+dx))+(a+b\sec(c+dx))^2\right)} \\
&\quad + \frac{(2ab^2)\,\text{Subst}\left(\int \left(-\frac{1}{b^2(a-x^2)}+\frac{1}{2b^2(a+b-x^2)}-\frac{1}{2b^2(-a+b+x^2)}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{6b^2(a^2-b^2)}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{4\left(a^2-b^2\right)d} \\
&= -\frac{b^2\sqrt{a+b\sec(c+dx)}}{2d\left(a^2-b^2-2a(a+b\sec(c+dx))+(a+b\sec(c+dx))^2\right)} \\
&\quad + \frac{a\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&\quad - \frac{a\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&\quad - \frac{(2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&\quad + \frac{(3b^2)\,\text{Subst}\left(\int \frac{1}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{2d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} \\
&\quad + \frac{a\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} + \frac{a\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d} \\
&\quad - \frac{b^2\sqrt{a+b\sec(c+dx)}}{2d\left(a^2-b^2-2a(a+b\sec(c+dx))+(a+b\sec(c+dx))^2\right)} \\
&\quad - \frac{(3b)\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{4d} \\
&\quad + \frac{(3b)\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{4d} \\
&= -\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{d} \\
&\quad + \frac{a\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} - \frac{3b\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4\sqrt{a-b}d} \\
&\quad + \frac{a\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d} + \frac{3b\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4\sqrt{a+b}d} \\
&\quad - \frac{b^2\sqrt{a+b\sec(c+dx)}}{2d\left(a^2-b^2-2a(a+b\sec(c+dx))+(a+b\sec(c+dx))^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.99

$$
\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx
$$

$$
= \frac{-\frac{b\arctan\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - 8\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right) + \frac{4\sqrt{-(a-b)^2}\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{-a+b}} + \frac{4a\text{arctanh}\left(\frac{\sqrt{a+b}}{\ }\right.}{\sqrt{a+b}}}{4d}
$$

```
[In] Integrate[Cot[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-((b*ArcTan[Sqrt[a + b*Sec[c + d*x]]/Sqrt[-a + b]])/Sqrt[-a + b]) - 8*Sqrt
[a]*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]] + (4*Sqrt[-(a - b)^2]*ArcTanh
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/Sqrt[-a + b] + (4*a*ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]])/Sqrt[a + b] + (3*b*ArcTanh[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]])/Sqrt[a + b] - 2*Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d*
x]])/(4*d)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2071 vs. $2(177) = 354$.

Time = 2.20 (sec) , antiderivative size = 2072, normalized size of antiderivative = 9.64

method	result	size
default	Expression too large to display	2072

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/(a-b)^(3/2)/(a+b)*(-((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4
*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*(a-b)^(3/2)*a*
(1-cos(d*x+c))^4*csc(d*x+c)^4+((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c
))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*(a-b)^(3/2
)*b*(1-cos(d*x+c))^4*csc(d*x+c)^4+8*a^(3/2)*ln(2*(-2*a*(1-cos(d*x+c))^2*csc
(d*x+c)^2+b*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*a^(1/2)*((1-cos(d*x+c))^4*a*csc
(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2
+a+b)^(1/2)+2*a+b)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))*(1-cos(d*x+c))^2*(a-b
)^(3/2)*csc(d*x+c)^2+8*a^(1/2)*ln(2*(-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+b*(
1-cos(d*x+c))^2*csc(d*x+c)^2+2*a^(1/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-
cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)+2
*a+b)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))*(1-cos(d*x+c))^2*(a-b)^(3/2)*b*csc
(d*x+c)^2-4*(a+b)^(1/2)*ln(2*(-a*(1-cos(d*x+c))^2*csc(d*x+c)^2+(a+b)^(1/2)*
((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos
(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)+a+b)/(1-cos(d*x+c))^2*sin(d*x+c)^2)*(1-c
os(d*x+c))^2*(a-b)^(3/2)*a*csc(d*x+c)^2-3*(a+b)^(1/2)*ln(2*(-a*(1-cos(d*x+c
))^2*csc(d*x+c)^2+(a+b)^(1/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c
))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)+a+b)/(1-co
s(d*x+c))^2*sin(d*x+c)^2)*(1-cos(d*x+c))^2*(a-b)^(3/2)*b*csc(d*x+c)^2+((1-c
os(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+
c))^2*csc(d*x+c)^2+a+b)^(1/2)*(a-b)^(3/2)*a*(1-cos(d*x+c))^2*csc(d*x+c)^2-(
(1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(
d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*(a-b)^(3/2)*b*(1-cos(d*x+c))^2*csc(d*x+c)
^2+((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-
cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(3/2)*(a-b)^(3/2)+4*a^3*ln((a*(1-cos(d*x+c)
)^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2+((1-cos(d*x+c))^4*a*csc(d*
x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+
b)^(1/2)*(a-b)^(1/2)-a)/(a-b)^(1/2))*(1-cos(d*x+c))^2*csc(d*x+c)^2-3*a^2*ln
((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2+((1-cos(d
*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^
2*csc(d*x+c)^2+a+b)^(1/2)*(a-b)^(1/2)-a)/(a-b)^(1/2))*(1-cos(d*x+c))^2*b*cs
c(d*x+c)^2-4*ln((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x
+c)^2+((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*
(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*(a-b)^(1/2)-a)/(a-b)^(1/2))*a*b^2*
(1-cos(d*x+c))^2*csc(d*x+c)^2+3*ln((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-co
```

```
s(d*x+c))^2*csc(d*x+c)^2+((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*
b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*(a-b)^(1/2)-a)/
(a-b)^(1/2))*b^3*(1-cos(d*x+c))^2*csc(d*x+c)^2)*((1-cos(d*x+c))^2*csc(d*x+c
)^2-1)*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-
b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)/(1-cos(d*x+c))^2*sin(d*x+c)^2/(
(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)*((1-c
os(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(177) = 354$.

Time = 1.81 (sec) , antiderivative size = 3523, normalized size of antiderivative = 16.39

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2
 + 8*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(a)*log(-8*a^2*cos(d*x +
c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*s
qrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - ((4*a^2 + a*b - 3*b^2)*co
s(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)
*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(
a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x
+ c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((4*a^2 - a*b - 3*b^2)*cos(d
*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*co
s(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a +
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c
))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^2 -
(a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
*cos(d*x + c)^2 - 2*((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3
*b^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) + 8*((a^2 - b^2)*cos(d*x +
 c)^2 - a^2 + b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) -
 b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))) - ((4*a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 - a*b
+ 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*(
(2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/16*(8*
(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + 2*((4*
a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(-a + b)*arcta
n(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*
a - b)*cos(d*x + c) + b)) + 8*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt
```

(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) + ((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/8*(4*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + ((4*a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - ((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) + 4*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + 16*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - ((4*a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + 16*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - ((4*a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d), 1/16*(8*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)^2 + 16*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + 2*((4*a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 - a*b + 3*b^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) + ((4*a^2 - a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x +

```
c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2
)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^2 - b^2)*d*cos(
d*x + c)^2 - (a^2 - b^2)*d), 1/8*(4*(a^2 - b^2)*sqrt((a*cos(d*x + c) + b)/c
os(d*x + c))*cos(d*x + c)^2 + 8*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sq
rt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x +
c)/(2*a*cos(d*x + c) + b)) + ((4*a^2 + a*b - 3*b^2)*cos(d*x + c)^2 - 4*a^2
- a*b + 3*b^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b
)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - ((4*a^2 - a*b
- 3*b^2)*cos(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sqrt(-a - b)*arctan(2*sqrt(-
a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(
d*x + c) + b)))/((a^2 - b^2)*d*cos(d*x + c)^2 - (a^2 - b^2)*d)]
```

Sympy [F]

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec{(c+dx)}}\cot^3{(c+dx)}\,dx$$

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x)**3, x)
```

Maxima [F]

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}+a}\cot{(dx+c)}^3\;dx$$

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(177) = 354$.

Time = 0.84 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.39

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$= \frac{\left(\frac{16\,a\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2\,(4\,a+3\,b)\arctan\left(-\frac{\sqrt{a-b}\,\mathrm{ta}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(16*a*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*
d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a
+ b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(4*a + 3*b)*arctan(-(sqrt(a - b)
*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1
/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))/sqrt(-a - b))/sqrt(-a - b) +
 (4*a - 3*b)*log(abs((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d
*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a +
 b))*(a - b) - sqrt(a - b)*a))/sqrt(a - b) + sqrt(a*tan(1/2*d*x + 1/2*c)^4
- b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) - 2*((sqrt
(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*
d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))*a - (a + b)*sqrt(a -
b))/((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 -
b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2 - a - b))
*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \cot(c+d\,x)^3\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^(1/2), x)
```

3.323 $\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 344

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = -\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^2d} - \frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3bd} + \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} + \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}$$

```
[Out] -2/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/b^2/d-2/3*(a+2*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x
+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d
*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/3*(a+b*sec(d*x+c))^
(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3979, 4142, 4143, 4006, 3869, 3917, 4089}

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = -\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d} - \frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bd} + \frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d} + \frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) - (2*Sqrt[a + b]*(a + 2*b
)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/(3*b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a,
 ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*S
qrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4142

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^
m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[
a*A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m,
0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \int \sqrt{a+b\sec(c+dx)}\left(-1+\sec^2(c+dx)\right)\,dx$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2}{3}\int\frac{-\frac{3a}{2}-b\sec(c+dx)+\frac{1}{2}a\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{-\frac{3a}{2} + \left(-\frac{a}{2}-b\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + \frac{1}{3}a\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^2d} + \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} - a\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + \frac{1}{3}(-a-2b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^2d} - \frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+}{a-b}}}{3bd} + \frac{2\sqrt{a+b}\cot(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} + \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 11.57 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.09

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = -\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\sec(c+dx)}\left(2a(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{} + \frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{3b}+\frac{2}{3}\tan(c+dx)\right)}{d}$$

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^2*Sqrt[a + b*Sec[c + d*x]]*(2*a*(a + b)*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 4*(2*a - b)*b*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 12
```

```
*a*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)] + a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2]))/(3*b*d*(b + a*Cos[c + d*x])) + (Sqrt[a + b*Sec[c + d*x]]*((2*a*Sin[
c + d*x])/(3*b) + (2*Tan[c + d*x])/3))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1379 vs. 2(309) = 618.

Time = 11.95 (sec) , antiderivative size = 1380, normalized size of antiderivative = 4.01

method	result	size
default	Expression too large to display	1380

```
[In] int((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d/b*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(-4*Elliptic
F(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)^2+2*Elli
pticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^2*cos(d*x+c)^2+El
lipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*cos(d*x+c)^2+
EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)^
2+6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b*cos
(d*x+c)^2-8*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b
*cos(d*x+c)+4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b
^2*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*a^2*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*a*b*cos(d*x+c)+12*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b
))^(1/2))*a*b*cos(d*x+c)-4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*a*b+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*b^2+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)
-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2+(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c)
,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b+6*(cos(d*x+c)/(
```

```
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b+cos(d*x+c)*sin(d*x+c
)*a^2+cos(d*x+c)*sin(d*x+c)*a*b+2*a*b*sin(d*x+c)+b^2*sin(d*x+c)+b^2*tan(d*x
+c))
```

Fricas [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = \int \sqrt{b\sec(dx+c)+a}\tan(dx+c)^2\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

Sympy [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = \int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*tan(d*x+c)**2,x)
[Out] Integral(sqrt(a + b*sec(c + d*x))*tan(c + d*x)**2, x)
```

Maxima [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = \int \sqrt{b\sec(dx+c)+a}\tan(dx+c)^2\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

Giac [F]

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = \int \sqrt{b\sec(dx+c)+a}\tan(dx+c)^2\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*tan(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(c+dx)}\tan^2(c+dx)\,dx = \int \tan(c+d\,x)^2\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

```
[In] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^(1/2), x)
```

3.324 $\int \sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 125

$$\int \sqrt{a+b\sec(c+dx)}\,dx =$$

$$-\frac{2\cot(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}(a+b\sec(c+ \cdots}{\sqrt{a+b}d}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sec(d*x+c))^(1/2),a/(a+b),((a-b)/
(a+b))^(1/2))*(a+b*sec(d*x+c))*(-b*(1-sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)*(
b*(1+sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3865}

$$\int \sqrt{a+b\sec(c+dx)}\,dx =$$

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\cdots\right.}{d\sqrt{a+b}}$$

```
[In] Int[Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c
+ d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x
]))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x])
)/(Sqrt[a + b]*d)
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

integral =

$$-\frac{2\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}(a+b\sec(c+}{\sqrt{a+b}d}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.21

$$\int\sqrt{a+b\sec(c+dx)}\,dx$$
$$=\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left((-a+b)\operatorname{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)+2a\right.}{d(b+a\cos(c+dx))}$$

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((-a + b)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/(d*(b + a*Cos[c + d*x]))
```

Maple [A] (verified)

Time = 6.57 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.46

method	result
default	$\frac{2(\cos(dx+c)+1)\left(\operatorname{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)a-\operatorname{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)b-2a\operatorname{EllipticPi}\left(\cot(dx+c)-\right.\right.}{d(b+a\cos(dx+c))}$

```
[In] int((a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(cos(d*x+c)+1)*(EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a-
EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi(cot(d
*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
```

```
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(b+
a*cos(d*x+c))
```

Fricas [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec\left(c+dx\right)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((a + b/cos(c + d*x))^(1/2),x)
[Out] int((a + b/cos(c + d*x))^(1/2), x)
```

3.325 $\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 246

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} - \frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d} + \frac{2\cot(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}(a+b\sec(c+dx))}{\sqrt{a+b}\,d}$$

```
[Out] cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2
)/d+2*cot(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sec(d*x+c))^(1/2),a/(a+b),((a-
b)/(a+b))^(1/2))*(a+b*sec(d*x+c))*(-b*(1-sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2
)*(b*(1+sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)/d/(a+b)^(1/2)-cot(d*x+c)*(a+b*s
ec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used

$= \{3981, 3865, 3960, 3917\}$

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$= \frac{\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}$$
$$+\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\right.}{d\sqrt{a+b}}$$
$$-\frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

```
[In] Int[Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/d - (Cot[c + d*x]*Sqrt[a + b*Sec[c + d*x]])/d +
(2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c +
 d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x]
))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x]))
/(Sqrt[a + b]*d)
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3981

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rubi steps

$$\text{integral} = \int \left(-\sqrt{a+b\sec(c+dx)}+\csc^2(c+dx)\sqrt{a+b\sec(c+dx)}\right) dx$$

$$= -\int \sqrt{a+b\sec(c+dx)}\,dx + \int \csc^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$= -\frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d} + \frac{2\cot(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}(a+b\,\text{s}}{\sqrt{a+b}d} + \frac{1}{2}b\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} - \frac{\cot(c+dx)\sqrt{a+b\sec(c+dx)}}{d} + \frac{2\cot(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}(a+b\,\text{s}}{\sqrt{a+b}d}$$

Mathematica [A] (verified)

Time = 3.61 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.63

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\sqrt{a+b\sec(c+dx)}\left(-\cot(c+dx)-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\left((-2a+b)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)+4a\,\text{EllipticPi}\left(-1,\arcsin(\text{t}\right.\right.}{b+a\cos(c+dx)}\right.}{d}$$

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(-Cot[c + d*x] - (2*Cos[(c + d*x)/2]^2*((-2*a + b
)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*a*EllipticPi[-1,
 ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sqrt[(1 + Sec[c + d*x])^(-1)]*
Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))])/(b + a*Cos[c + d*x
]))))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(226) = 452$.

Time = 5.81 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.22

method	result
default	$\frac{\sqrt{a+b\sec(dx+c)}\left(4\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,a\cos(dx+c)-2\,\mathrm{EllipticF}\left(\cot\right.\right.}{}$

[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))*(4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)+EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a+EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b-a*cos(d*x+c)*cot(d*x+c)-cot(d*x+c)*b)

Fricas [F]

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cot(dx+c)^2\,dx$$

[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^2, x)

Sympy [F]

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\cot^2(c+dx)\,dx$$

[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a + b*sec(c + d*x))*cot(c + d*x)**2, x)

Maxima [F]

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\cot\left(dx+c\right)^2\,dx$$

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

Giac [F]

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\cot\left(dx+c\right)^2\,dx$$

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cot(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \cot(c+d\,x)^2\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^(1/2), x)
```

3.326 $\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2a(a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d} + \frac{2(3a^2-2b^2)(a+b\sec(c+dx))^{3/2}}{3b^4d} - \frac{6a(a+b\sec(c+dx))^{5/2}}{5b^4d} + \frac{2(a+b\sec(c+dx))^{7/2}}{7b^4d}$$

```
[Out] 2/3*(3*a^2-2*b^2)*(a+b*sec(d*x+c))^(3/2)/b^4/d-6/5*a*(a+b*sec(d*x+c))^(5/2)
/b^4/d+2/7*(a+b*sec(d*x+c))^(7/2)/b^4/d-2*arctanh((a+b*sec(d*x+c))^(1/2)/a^
(1/2))/d/a^(1/2)-2*a*(a^2-2*b^2)*(a+b*sec(d*x+c))^(1/2)/b^4/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3970, 912, 1167, 213}

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2(3a^2-2b^2)(a+b\sec(c+dx))^{3/2}}{3b^4d} - \frac{2a(a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{2(a+b\sec(c+dx))^{7/2}}{7b^4d} - \frac{6a(a+b\sec(c+dx))^{5/2}}{5b^4d}$$

```
[In] Int[Tan[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*(a^2 - 2*
b^2)*Sqrt[a + b*Sec[c + d*x]])/(b^4*d) + (2*(3*a^2 - 2*b^2)*(a + b*Sec[c +
d*x])^(3/2))/(3*b^4*d) - (6*a*(a + b*Sec[c + d*x])^(5/2))/(5*b^4*d) + (2*(a
 + b*Sec[c + d*x])^(7/2))/(7*b^4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{b^4 d}\\
&= \frac{2\text{Subst}\left(\int \frac{\left(-a^2+b^2+2ax^2-x^4\right)^2}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4 d}\\
&= \frac{2\text{Subst}\left(\int \left(-a^3+2ab^2+\left(3a^2-2b^2\right)x^2-3ax^4+x^6+\frac{b^4}{-a+x^2}\right)\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4 d}
\end{aligned}$$

$$= -\frac{2a(a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d} + \frac{2(3a^2-2b^2)(a+b\sec(c+dx))^{3/2}}{3b^4d} - \frac{6a(a+b\sec(c+dx))^{5/2}}{5b^4d} + \frac{2(a+b\sec(c+dx))^{7/2}}{7b^4d} + \frac{2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2a(a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d} + \frac{2(3a^2-2b^2)(a+b\sec(c+dx))^{3/2}}{3b^4d} - \frac{6a(a+b\sec(c+dx))^{5/2}}{5b^4d} + \frac{2(a+b\sec(c+dx))^{7/2}}{7b^4d}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{-\frac{2b^4\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} - 2a(a^2-2b^2)\sqrt{a+b\sec(c+dx)} + \frac{2}{3}(3a^2-2b^2)(a+b\sec(c+dx))^{3/2} - \frac{6}{5}a(}{b^4d}$$

```
[In] Integrate[Tan[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((-2*b^4*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] - 2*a*(a^2 - 2*
b^2)*Sqrt[a + b*Sec[c + d*x]] + (2*(3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(3/
2))/3 - (6*a*(a + b*Sec[c + d*x])^(5/2))/5 + (2*(a + b*Sec[c + d*x])^(7/2))
/7)/(b^4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(128) = 256.

Time = 16.65 (sec) , antiderivative size = 646, normalized size of antiderivative = 4.36

method	result
default	$-\frac{\sqrt{a+b\sec(dx+c)}\left(105\cos(dx+c)\ln\left(4\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\sqrt{a}+4a\cos(dx+c)+4\sqrt{a}\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\right.\right.}{}$

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105/d/a/b^4*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/((b+a*cos(d*x+c))*cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(105*cos(d*x+c)*ln(4*cos(d*x+c)*((b+a*cos(d*
x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*a^(1/2)*b^4+96*((b
+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^4*cos(d*x+c)-280*((b+a*
cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2*cos(d*x+c)+96*((b+a*
cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^4-48*((b+a*cos(d*x+c))*cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3*b-280*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)*a^2*b^2+140*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)
^2)^(1/2)*a*b^3-48*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3
*b*sec(d*x+c)+36*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b
^2*sec(d*x+c)+140*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^
3*sec(d*x+c)+36*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^
2*sec(d*x+c)^2-30*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^
3*sec(d*x+c)^2-30*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^
3*sec(d*x+c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.57

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \left[\frac{105\,\sqrt{a}b^4\cos\left(dx+c\right)^3\log\left(-8\,a^2\cos\left(dx+c\right)^2-8\,ab\cos\left(dx+c\right)-b^2+4\left(2\,a\cos\left(dx+c\right)^2+b\cos\left(dx\right.\right.\right.}{}\right.$$

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*sqrt(a)*b^4*cos(d*x + c)^3*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*co
s(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))) - 4*(18*a^2*b^2*cos(d*x + c) - 15*a*b^3 +
4*(12*a^4 - 35*a^2*b^2)*cos(d*x + c)^3 - 2*(12*a^3*b - 35*a*b^3)*cos(d*x +
c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a*b^4*d*cos(d*x + c)^3), 1/
105*(105*sqrt(-a)*b^4*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))*cos(d*x + c)^3 - 2*(18*a^2*b^2*co
s(d*x + c) - 15*a*b^3 + 4*(12*a^4 - 35*a^2*b^2)*cos(d*x + c)^3 - 2*(12*a^3*
b - 35*a*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a*b
^4*d*cos(d*x + c)^3)]
```

Sympy [F]

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan^5{(c+dx)}}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**5/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\frac{105\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}} + \frac{30\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{7}{2}}}{b^4} - \frac{126\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}a}{b^4} + \frac{210\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a^2}{b^4} - \frac{210\sqrt{a+\frac{b}{\cos(dx+c)}}a^3}{b^4} - \frac{14}{}}{105\,d}$$

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 1/105*(105*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c
)) + sqrt(a)))/sqrt(a) + 30*(a + b/cos(d*x + c))^(7/2)/b^4 - 126*(a + b/cos
(d*x + c))^(5/2)*a/b^4 + 210*(a + b/cos(d*x + c))^(3/2)*a^2/b^4 - 210*sqrt(
a + b/cos(d*x + c))*a^3/b^4 - 140*(a + b/cos(d*x + c))^(3/2)/b^2 + 420*sqrt
(a + b/cos(d*x + c))*a/b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(128) = 256$.

Time = 1.81 (sec) , antiderivative size = 699, normalized size of antiderivative = 4.72

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\left(\frac{105\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2\left(105\left(\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{}\right.}{}$$

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2
*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a
 + b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(105*(sqrt(a - b)*tan(1/2*d*x +
 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*
tan(1/2*d*x + 1/2*c)^2 + a + b))^6 - 840*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^
2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*
d*x + 1/2*c)^2 + a + b))^5*sqrt(a - b) + 35*(sqrt(a - b)*tan(1/2*d*x + 1/2*
c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1
/2*d*x + 1/2*c)^2 + a + b))^4*(27*a - 23*b) + 280*(sqrt(a - b)*tan(1/2*d*x
+ 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a
*tan(1/2*d*x + 1/2*c)^2 + a + b))^3*(3*a + 4*b)*sqrt(a - b) - 21*(sqrt(a -
b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x +
 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))^2*(65*a^2 - 2*a*b - 15*b^2
) + 315*a^3 + 707*a^2*b - 7*a*b^2 - 55*b^3 - 56*(sqrt(a - b)*tan(1/2*d*x +
1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*t
an(1/2*d*x + 1/2*c)^2 + a + b))*(19*a*b + 5*b^2)*sqrt(a - b))/(sqrt(a - b)*
tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/
2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) - sqrt(a - b))^7)/(d*sgn(cos(d
*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\tan(c+d\,x)^5}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(1/2), x)
```

3.327 $\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2d}+\frac{2(a+b\sec(c+dx))^{3/2}}{3b^2d}$$

```
[Out] 2/3*(a+b*sec(d*x+c))^(3/2)/b^2/d+2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/
d/a^(1/2)-2*a*(a+b*sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3970, 912, 1167, 213}

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+\frac{2(a+b\sec(c+dx))^{3/2}}{3b^2d}-\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2d}$$

```
[In] Int[Tan[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*Sqrt[a + b
*Sec[c + d*x]])/(b^2*d) + (2*(a + b*Sec[c + d*x])^(3/2))/(3*b^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{b^2d} \\
&= -\frac{2\text{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2d} \\
&= -\frac{2\text{Subst}\left(\int \left(a-x^2+\frac{b^2}{-a+x^2}\right)\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2d} \\
&= -\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2d}+\frac{2(a+b\sec(c+dx))^{3/2}}{3b^2d} \\
&\quad -\frac{2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&= \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{2a\sqrt{a+b\sec(c+dx)}}{b^2d}+\frac{2(a+b\sec(c+dx))^{3/2}}{3b^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\left(\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{(-2a+b\sec(c+dx))\sqrt{a+b\sec(c+dx)}}{b^2}\right)}{3d}$$

```
[In] Integrate[Tan[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*((3*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + ((-2*a + b*Sec[
c + d*x])*Sqrt[a + b*Sec[c + d*x]])/b^2))/(3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(67) = 134.

Time = 11.29 (sec) , antiderivative size = 312, normalized size of antiderivative = 3.95

method	result
default	$-\frac{\sqrt{a+b\sec(dx+c)}\left(-3\sqrt{a}\,\cos(dx+c)\ln\left(4\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\,\sqrt{a}+4a\cos(dx+c)+4\sqrt{a}\,\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+}{(\cos(dx+c)+1)^2}}\right.\right.}{3da\,b^2(}$

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d/a/b^2*(a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*(-3*a^(1/2)*cos(d*x+c)*ln(4*cos(d*x+c)*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1
/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^2+4*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*cos(d*x+c)+4*((b+a*cos(d*
x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2-2*((b+a*cos(d*x+c))*cos(d*x+c)
/(cos(d*x+c)+1)^2)^(1/2)*a*b-2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^
2)^(1/2)*a*b*sec(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.46

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \Bigg[\frac{3\,\sqrt{a}b^2\cos\left(dx+c\right)\log\Big(-8\,a^2\cos\left(dx+c\right)^2-8\,ab\cos\left(dx+c\right)-b^2-4\,\big(2\,a\cos\left(dx+c\right)^2+b\cos\left(dx+c\right)}{6\,ab^2d\cos\left(dx+c\right)}$$

$$-\frac{3\,\sqrt{-a}b^2\arctan\left(\frac{2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+b}\right)\cos\left(dx+c\right)+2\,\left(2\,a^2\cos\left(dx+c\right)-ab\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}}{3\,ab^2d\cos\left(dx+c\right)}\Bigg]$$

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a)*b^2*cos(d*x + c)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x
+ c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*
x + c) + b)/cos(d*x + c))) - 4*(2*a^2*cos(d*x + c) - a*b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)))/(a*b^2*d*cos(d*x + c)), -1/3*(3*sqrt(-a)*b^2*arctan
(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d
*x + c) + b))*cos(d*x + c) + 2*(2*a^2*cos(d*x + c) - a*b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c)))/(a*b^2*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan^3\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{\frac{3\,\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}{b^2} + \frac{6\sqrt{a+\frac{b}{\cos(dx+c)}}\,a}{b^2}}{3\,d}$$

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c))
+ sqrt(a)))/sqrt(a) - 2*(a + b/cos(d*x + c))^(3/2)/b^2 + 6*sqrt(a + b/cos(d
*x + c))*a/b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(67) = 134$.

Time = 0.82 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.35

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\left(\frac{3\,\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2\left(3\left(\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{\left(\sqrt{a-b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}\right.}{3\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*
x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a +
b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(3*(sqrt(a - b)*tan(1/2*d*x + 1/2*
c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1
/2*d*x + 1/2*c)^2 + a + b))^2 - 3*a - b)/(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^
2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*
d*x + 1/2*c)^2 + a + b) - sqrt(a - b))^3)/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^3}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(1/2), x)
```

3.328 $\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3970, 65, 213}

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[In] Int[Tan[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}\\
&= -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$
\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}
$$

```
[In] Integrate[Tan[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26
default	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(25) = 50$.

Time = 0.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 4.68

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \left[\frac{\log\left(-8\,a^2\cos\left(dx+c\right)^2 - 8\,ab\cos\left(dx+c\right) - b^2 + 4\left(2\,a\cos\left(dx+c\right)^2 + b\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\right.}{2\,\sqrt{a}d}\right.$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x
+ c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(
sqrt(a)*d), sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
c))*cos(d*x + c)/(2*a*cos(d*x + c) + b))/(a*d)]
```

Sympy [F]

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) + sqrt(a
)))/(sqrt(a)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(25) = 50$.

Time = 0.49 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.29

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}d\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/
2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) + s
qrt(a - b))/sqrt(-a))/(sqrt(-a)*d*sgn(cos(d*x + c)))
```

Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[In] int(tan(c + d*x)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] -(2*atanh((a + b/cos(c + d*x))^(1/2)/a^(1/2)))/(a^(1/2)*d)
```

3.329 $\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

```
[Out] 2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-arctanh((a+b*sec(d*x+c)
)^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)-arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2))/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3970, 912, 1184, 212, 213}

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$$

```
[In] Int[Cot[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - ArcTanh[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d} \\
&= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&= -\frac{(2b^2)\,\text{Subst}\left(\int \left(-\frac{1}{b^2(a-x^2)}+\frac{1}{2b^2(a+b-x^2)}-\frac{1}{2b^2(-a+b+x^2)}\right)\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2} dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2} dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d} \\
&= \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

$$
\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}} dx = -\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{d}
$$

```
[In] Integrate[Cot[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] -(((-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a - b]]/Sqrt[a - b] + ArcTanh[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]]/Sqrt[a + b])/d)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(88) = 176$.

Time = 2.29 (sec) , antiderivative size = 658, normalized size of antiderivative = 6.21

method	result
default	$-\frac{\sqrt{a+b\sec(dx+c)}\left(-2a^{\frac{3}{2}}(a-b)^{\frac{3}{2}}\ln\left(4\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\sqrt{a}+4a\cos(dx+c)+4\sqrt{a}\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\right.\right.}{}$

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d/(a-b)^(3/2)/(a+b)/a*(a+b*sec(d*x+c))^(1/2)*(-2*a^(3/2)*(a-b)^(3/2)*l
n(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)
+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1
```

```
/2)+2*b)+(a+b)^(1/2)*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+cos(d*x+c)*b+2*(a
+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+b)/(cos(d*x+
c)-1))*a-2*a^(1/2)*(a-b)^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)
/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))
*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b-a^3*ln(1/(a-b)^(1/2)*(2*(a-b)^(1
/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+2*((b+a
*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+
cos(d*x+c)*b-b)/(cos(d*x+c)+1))+ln(1/(a-b)^(1/2)*(2*(a-b)^(1/2)*((b+a*cos(d
*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+2*((b+a*cos(d*x+c))*co
s(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+cos(d*x+c)*b-b)
/(cos(d*x+c)+1))*a*b^2)*cos(d*x+c)/(cos(d*x+c)+1)/((b+a*cos(d*x+c))*cos(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(88) = 176$.

Time = 1.42 (sec) , antiderivative size = 2420, normalized size of antiderivative = 22.83

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2 - b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c)
- b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))) + (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)
*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(
a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x
+ c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (a^2 - a*b)*sqrt(a + b)*log(
-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2
+ b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(
4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^3
- a*b^2)*d), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - (a^2 + a*b)
*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b
)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)) - (a^2 - a*b)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c
)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt(
(a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d
*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^3 - a*b^2)*d), -1/4*(2*(a^2 + a*b)*sq
rt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*c
os(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - 2*(a^2 - b^2)*sqrt(a)*log(-8*a^
```

```
2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos
(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - (a^2 - a*b)*s
qrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*
cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(
d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c
) + 1)))/((a^3 - a*b^2)*d), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)
) + 2*(a^2 + a*b)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c)
+ b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^2 - a*b)
*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b
)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x +
 c) + 1)))/((a^3 - a*b^2)*d), 1/4*(2*(a^2 - a*b)*sqrt(-a - b)*arctan(2*sqrt
(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*co
s(d*x + c) + b)) + 2*(a^2 - b^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*
cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) + (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 -
 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*
x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*
b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^3 - a*b^2)*d
), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(a^2 - a*b)*sqrt(-a
 - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x
 + c)/((2*a + b)*cos(d*x + c) + b)) - (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2
- 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d
*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3
*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^3 - a*b^2)*
d), -1/2*((a^2 + a*b)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^2 -
a*b)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - (a^2 - b^2)*sqrt(a)*log(-
8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b
*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))))/((a^3 - a*
b^2)*d), -1/2*(2*(a^2 - b^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + (a^2 + a*b)*sqr
t(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*co
s(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^2 - a*b)*sqrt(-a - b)*arctan(
2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a +
 b)*cos(d*x + c) + b)))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot{(c+dx)}}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot{(dx+c)}}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(cot(c + d*x)/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cot(c + d*x)/(a + b/cos(c + d*x))^(1/2), x)
```

3.330 $\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 260

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} - \frac{b\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d}+\frac{b\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{3/2}d} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}+\frac{\sqrt{a+b\sec(c+dx)}}{4(a+b)d(1-\sec(c+dx))} + \frac{\sqrt{a+b\sec(c+dx)}}{4(a-b)d(1+\sec(c+dx))}$$

```
[Out] -1/4*b*arctanh((a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d+1/4*b*arct
anh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/d-2*arctanh((a+b*sec(d*
x+c))^(1/2)/a^(1/2))/d/a^(1/2)+arctanh((a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2))/
d/(a-b)^(1/2)+arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)+1/4
*(a+b*sec(d*x+c))^(1/2)/(a+b)/d/(1-sec(d*x+c))+1/4*(a+b*sec(d*x+c))^(1/2)/(
a-b)/d/(1+sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3970, 912, 1252, 212, 205, 213}

$$\begin{aligned}\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = &-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}\\&+\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}+\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}\\&+\frac{b\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4d(a+b)^{3/2}}+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a+b)(1-\sec(c+dx))}\\&+\frac{\sqrt{a+b\sec(c+dx)}}{4d(a-b)(\sec(c+dx)+1)}\end{aligned}$$

```
[In] Int[Cot[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + ArcTanh[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - (b*ArcTanh[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(3/2)*d) + (b*ArcTanh[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]])/(4*(a + b)^(3/2)*d) + ArcTanh[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]]/(Sqrt[a + b]*d) + Sqrt[a + b*Sec[c + d*x]]/(4*(a + b)*d*(1 -
 Sec[c + d*x])) + Sqrt[a + b*Sec[c + d*x]]/(4*(a - b)*d*(1 + Sec[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1252

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^4\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= \frac{(2b^4)\,\text{Subst}\left(\int \frac{1}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}\\
&= \frac{(2b^4)\,\text{Subst}\left(\int \left(-\frac{1}{b^4(a-x^2)}+\frac{1}{4b^3(a+b-x^2)^2}+\frac{1}{2b^4(a+b-x^2)}-\frac{1}{4b^3(-a+b+x^2)^2}-\frac{1}{2b^4(-a+b+x^2)}\right)dx, x, \sqrt{a+
}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}\\
&\quad-\frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}\\
&\quad-\frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{1}{(a+b-x^2)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{2d}\\
&\quad-\frac{b\text{Subst}\left(\int \frac{1}{(-a+b+x^2)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{2d}
\end{aligned}$$

$$= -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d} + \frac{\sqrt{a+b\sec(c+dx)}}{4(a+b)d(1-\sec(c+dx))} + \frac{\sqrt{a+b\sec(c+dx)}}{4(a-b)d(1+\sec(c+dx))} + \frac{b\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{4(a-b)d} + \frac{b\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{4(a+b)d}$$

$$= -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d} + \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{3/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d} + \frac{\sqrt{a+b\sec(c+dx)}}{4(a+b)d(1-\sec(c+dx))} + \frac{\sqrt{a+b\sec(c+dx)}}{4(a-b)d(1+\sec(c+dx))}$$

Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.08

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{-\frac{b^2\arctan\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{-a+b}}\right)}{(-a+b)^{3/2}} - \frac{8b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{4b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \cdots}{4bd}$$

```
[In] Integrate[Cot[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-((b^2*ArcTan[Sqrt[a + b*Sec[c + d*x]]/Sqrt[-a + b]])/(-a + b)^(3/2)) - (8
*b*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/Sqrt[a] + (4*b*ArcTanh[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a - b]])/Sqrt[a - b] + (b^2*ArcTanh[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]])/(a + b)^(3/2) - (4*a*ArcTanh[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]])/Sqrt[a + b] + 4*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]] - (b*Sqrt[a + b*Sec[c + d*x]])/((a + b)*(-1 + Sec[c + d*x])
) + (b*Sqrt[a + b*Sec[c + d*x]])/((a - b)*(1 + Sec[c + d*x])))/(4*b*d)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3106 vs. $2(218) = 436$.

Time = 2.01 (sec) , antiderivative size = 3107, normalized size of antiderivative = 11.95

method	result	size
default	Expression too large to display	3107

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/(a+b)^2/(a-b)^(5/2)/a*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+
c))^2*csc(d*x+c)^2-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*
x+c))^2*csc(d*x+c)^2-1)*(8*ln(2*(-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+b*(1-co
s(d*x+c))^2*csc(d*x+c)^2+2*a^(1/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(
d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)+2*a+b
)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))*a^(7/2)*(a-b)^(3/2)*(1-cos(d*x+c))^2*c
sc(d*x+c)^2+8*ln(2*(-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+b*(1-cos(d*x+c))^2*c
sc(d*x+c)^2+2*a^(1/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*c
sc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)+2*a+b)/((1-cos(d*x
+c))^2*csc(d*x+c)^2+1))*a^(5/2)*(a-b)^(3/2)*b*(1-cos(d*x+c))^2*csc(d*x+c)^2
-8*ln(2*(-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+b*(1-cos(d*x+c))^2*csc(d*x+c)^2
+2*a^(1/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4
-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)+2*a+b)/((1-cos(d*x+c))^2*csc(
d*x+c)^2+1))*a^(3/2)*(a-b)^(3/2)*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2-(a-b)^(3
/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1
-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*a^3*(1-cos(d*x+c))^4*csc(d*x+c)^4+2*
(a-b)^(3/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^
4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*a^2*b*(1-cos(d*x+c))^4*csc(d
*x+c)^4-(a-b)^(3/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc
(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*a*b^2*(1-cos(d*x+c))
^4*csc(d*x+c)^4-4*ln(2*(-a*(1-cos(d*x+c))^2*csc(d*x+c)^2+(a+b)^(1/2)*((1-co
s(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c
))^2*csc(d*x+c)^2+a+b)^(1/2)+a+b)/(1-cos(d*x+c))^2*sin(d*x+c)^2)*(a+b)^(1/2
)*(a-b)^(3/2)*a^3*(1-cos(d*x+c))^2*csc(d*x+c)^2-ln(2*(-a*(1-cos(d*x+c))^2*c
sc(d*x+c)^2+(a+b)^(1/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b
*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)+a+b)/(1-cos(d*x+
c))^2*sin(d*x+c)^2)*(a+b)^(1/2)*(a-b)^(3/2)*a^2*b*(1-cos(d*x+c))^2*csc(d*x+
c)^2+5*ln(2*(-a*(1-cos(d*x+c))^2*csc(d*x+c)^2+(a+b)^(1/2)*((1-cos(d*x+c))^4
*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*
x+c)^2+a+b)^(1/2)+a+b)/(1-cos(d*x+c))^2*sin(d*x+c)^2)*(a+b)^(1/2)*(a-b)^(3/
2)*a*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2-8*ln(2*(-2*a*(1-cos(d*x+c))^2*csc(d*
x+c)^2+b*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*a^(1/2)*((1-cos(d*x+c))^4*a*csc(d*
x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+
b)^(1/2)+2*a+b)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))*a^(1/2)*(a-b)^(3/2)*b^3*
(1-cos(d*x+c))^2*csc(d*x+c)^2+(a-b)^(3/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-
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```
(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2
)*a^3*(1-cos(d*x+c))^2*csc(d*x+c)^2-4*(a-b)^(3/2)*((1-cos(d*x+c))^4*a*csc(d
*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a
+b)^(1/2)*a^2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2-(a-b)^(3/2)*((1-cos(d*x+c))^4
*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*
x+c)^2+a+b)^(1/2)*a*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2+4*ln((a*(1-cos(d*x+c)
)^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2+((1-cos(d*x+c))^4*a*csc(d*
x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+
b)^(1/2)*(a-b)^(1/2)-a)/(a-b)^(1/2))*a^5*(1-cos(d*x+c))^2*csc(d*x+c)^2-ln((
a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2+((1-cos(d*x
+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*
csc(d*x+c)^2+a+b)^(1/2)*(a-b)^(1/2)-a)/(a-b)^(1/2))*a^4*(1-cos(d*x+c))^2*b*
csc(d*x+c)^2-9*ln((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d
*x+c)^2+((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*
a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*(a-b)^(1/2)-a)/(a-b)^(1/2))*a^3*
b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2+ln((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-
cos(d*x+c))^2*csc(d*x+c)^2+((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^
4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*(a-b)^(1/2)-a
)/(a-b)^(1/2))*a^2*b^3*(1-cos(d*x+c))^2*csc(d*x+c)^2+5*ln((a*(1-cos(d*x+c))
^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2+((1-cos(d*x+c))^4*a*csc(d*x
+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b
)^(1/2)*(a-b)^(1/2)-a)/(a-b)^(1/2))*a*b^4*(1-cos(d*x+c))^2*csc(d*x+c)^2+(a-
b)^(3/2)*((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2
*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(3/2)*a^2-(a-b)^(3/2)*((1-cos(d*x+c))
^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(
d*x+c)^2+a+b)^(3/2)*a*b)/((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))
^2*csc(d*x+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)/(1-cos(d*x+c)
)^2*sin(d*x+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(216) = 432$.

Time = 25.89 (sec) , antiderivative size = 4336, normalized size of antiderivative = 16.68

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*
sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d
*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
) + (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 -
 5*a*b^3)*cos(d*x + c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x +
```

```
c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqr
t((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3
 - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(
-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2
+ b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(
4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a
^4 - a^2*b^2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + b)/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a
^5 - 2*a^3*b^2 + a*b^4)*d), -1/16*(16*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2
*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + (4*a^4 + 3*a^3*b
 - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x +
c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*
a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) +
b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b
- 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b
^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sq
rt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d
*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^2)*cos(d*
x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b
^4)*d), -1/16*(2*(4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b
- 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c
) + b)) + 8*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2
)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos
(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c
))) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b - 6*a^2*b^2
 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x
 + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*s
qrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(c
os(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^2)*cos(d*x + c)^2 -
(a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/((a^
5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/
16*(16*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqr
t(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c
)/(2*a*cos(d*x + c) + b)) + 2*(4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a
^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a + b)*arctan(-2*
sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b
)*cos(d*x + c) + b)) + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*
a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(a + b)*log(-((8*a^2 + 8*a
*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x +
c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)
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```
*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^2)*
cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2
 + a*b^4)*d), 1/16*(2*(4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a
^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a - b)*arctan(2*sqrt(-a -
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x
 + c) + b)) - 8*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x +
c)^2)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a
*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c))) - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2
*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos
(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a -
b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c)
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 8*((a^4 - a^2*b^2)*cos(d*x + c)^
2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/
((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d),
 -1/16*(16*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)
*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x
 + c)/(2*a*cos(d*x + c) + b)) - 2*(4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 -
(4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a - b)*arctan
(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a
+ b)*cos(d*x + c) + b)) + (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 +
 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(a - b)*log(-((8*a^2 -
8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x
 + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b
^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 8*((a^4 - a^2*b^
2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*
b^2 + a*b^4)*d), -1/8*((4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*
a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a + b)*arctan(-2*sqrt(-a
 + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d
*x + c) + b)) - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b -
 6*a^2*b^2 + 5*a*b^3)*cos(d*x + c)^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sq
rt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c)
+ b)) + 4*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*
sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d
*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
) - 4*((a^4 - a^2*b^2)*cos(d*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt(
(a*cos(d*x + c) + b)/cos(d*x + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x +
c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d), -1/8*(8*(a^4 - 2*a^2*b^2 + b^4 - (a^4
- 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d
*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + (4*a^4 +
3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cos
(d*x + c)^2)*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/
cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (4*a^4 - 3*a^3*b
```

```
 - 6*a^2*b^2 + 5*a*b^3 - (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cos(d*x +
c)^2)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - 4*((a^4 - a^2*b^2)*cos(d
*x + c)^2 - (a^3*b - a*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x
 + c)))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*
b^4)*d)]
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot^3{(c+dx)}}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot{(dx+c)}^3}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(216) = 432$.

Time = 0.92 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.05

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\frac{16\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2(4a+5b)\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\right.}{}\right.}{}}{}$$

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] 1/8*(16*arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*
x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a +
```

```
b) + sqrt(a - b))/sqrt(-a))/sqrt(-a) - 2*(4*a + 5*b)*arctan(-(sqrt(a - b)*t
an(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2
*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))/sqrt(-a - b))/((a + b)*sqrt(-a
 - b)) + (4*a - 5*b)*log(abs((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*t
an(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)
^2 + a + b))*(a - b) - sqrt(a - b)*a))/(a - b)^(3/2) + sqrt(a*tan(1/2*d*x +
 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)/
(a - b) - 2*((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2
*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))*a -
 (a + b)*sqrt(a - b))/(((sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/
2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 +
a + b))^2 - a - b)*(a + b)))/(d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^3}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(1/2), x)
```

3.331 $\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 404

$$
\begin{aligned}
\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = &-\frac{2\sqrt{a+b}\cot(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} \\
&-\frac{2(a-b)\sqrt{a+b}(8a^2-21b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{-\frac{b(-1+\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+\cdots}{a-b}}}{15b^4d} \\
&+\frac{2\sqrt{a+b}(-8a^2+2ab+21b^2)\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{-\frac{b(-1+\sec(c+dx))}{a+b}}\sqrt{\frac{b(\cdots}{}}}{15b^3d} \\
&-\frac{8a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d}+\frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd}
\end{aligned}
$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a/d-2/15*(a-b)*(8*a^2-21*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d
*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(-b*(-1+sec(d*x+c
))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d+2/15*(-8*a^2+2*a*b+21
*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))
^(1/2))*(a+b)^(1/2)*(-b*(-1+sec(d*x+c))/(a+b))^(1/2)*(b*(1+sec(d*x+c))/(-a+
b))^(1/2)/b^3/d-8/15*a*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/5*sec(d*x+
c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.51, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3980, 3869, 3922, 3917, 4089, 3945, 4167, 4090}

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx =$$
$$-\frac{2(a-b)\sqrt{a+b}(8a^2+9b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^4d}$$
$$-\frac{2\sqrt{a+b}(8a^2-2ab+9b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\right.}{15b^3d}$$
$$+\frac{4(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}$$
$$+\frac{4\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}$$
$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$
$$-\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{15b^2d}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

```
[In] Int[Tan[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - b)*Sqrt[a + b]*(8*a^
2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(15*b^4*d) + (4*Sqrt[a + b]*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) -
(2*Sqrt[a + b]*(8*a^2 - 2*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) - (2*Sqrt[a
 + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(a*d) - (8*a*Sqrt[a + b*Sec[c + d*x]]*Tan[c
 + d*x])/(15*b^2*d) + (2*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]
)/(5*b*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3922

```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[Csc[e + f*
x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

Rule 3945

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(2
*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3980

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{\sqrt{a+b\sec(c+dx)}} - \frac{2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}} + \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\right) dx$$

$$= -\left(2\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx\right) + \int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a}, \arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right), \frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} + \frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd} + 2\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx - 2\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx + \frac{\int \frac{\sec(c+dx)\left(2a+3b\sec(c+dx)-4a\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{5b}$$

$$= \frac{4(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d}$$

$$+\frac{4\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx}{a-b}}}{ad}$$

$$-\frac{8a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d}+\frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd}$$

$$+\frac{2\int\frac{\sec(c+dx)\left(ab+\frac{1}{2}(8a^2+9b^2)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{15b^2}$$

$$= \frac{4(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d}$$

$$+\frac{4\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx}{a-b}}}{ad}$$

$$-\frac{8a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d}+\frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd}$$

$$+\frac{1}{15}\left(9+\frac{8a^2}{b^2}\right)\int\frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx-\frac{(8a^2-2ab+9b^2)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{15b^2}$$

$$= \frac{4(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d}$$

$$-\frac{2(a-b)\sqrt{a+b}(8a^2+9b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sec}}{a}}}{15b^4d}$$

$$+\frac{4\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

$$-\frac{2\sqrt{a+b}(8a^2-2ab+9b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{15b^3d}$$

$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx}{a-b}}}{ad}$$

$$-\frac{8a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d}+\frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd}$$

Mathematica [A] (verified)

Time = 15.80 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.10

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{\sec(c+dx)}\left(\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2\left(8a^3+8a^2b-21ab^2-21b^3\right)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx)}}\right.}{}\right.}{}$$

```
[In] Integrate[Tan[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(8*a^3 +
8*a^2*b - 21*a*b^2 - 21*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1
+ Sec[c + d*x]))] + 4*b*(4*a^2 + a*b - 18*b^2)*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c +
d*x])/((a + b)*(1 + Sec[c + d*x]))] + 60*b^3*EllipticPi[-1, ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c
 + d*x])/((a + b)*(1 + Sec[c + d*x]))] + ((8*a^2 - 21*b^2)*(b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/2))/Sqrt
[Sec[(c + d*x)/2]^2] + (b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*((8*a^2 - 21
*b^2)*Sin[c + d*x] + b*(-4*a + 3*b*Sec[c + d*x])*Tan[c + d*x])))/(15*b^3*d*
Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2242 vs. $2(365) = 730$.

Time = 15.87 (sec) , antiderivative size = 2243, normalized size of antiderivative = 5.55

method	result	size
default	Expression too large to display	2243

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d/b^3*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(-a*b^2*s
in(d*x+c)+4*a^2*b*sin(d*x+c)+8*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*a^2*b-21*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*a*b^2+8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2
))*a^3*cos(d*x+c)^2-21*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))
```

```
^(1/2))*b^3*cos(d*x+c)^2-8*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1
/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*a^2*b-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*a*b^2+72*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b
^3*cos(d*x+c)+16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2)
)*a^3*cos(d*x+c)+36*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*b^3*cos(d*x+c)^2-42*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b
))^(1/2))*b^3*cos(d*x+c)-21*b^3*sin(d*x+c)+3*tan(d*x+c)*b^3-30*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)^2-60*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)
+3*b^3*tan(d*x+c)*sec(d*x+c)-a*b^2*tan(d*x+c)+8*a^3*cos(d*x+c)*sin(d*x+c)-1
6*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x
+c)-4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2*cos
(d*x+c)-8*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b
*cos(d*x+c)^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*a*b^2*cos(d*x+c)^2+8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))*a^2*b*cos(d*x+c)^2-21*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))*a*b^2*cos(d*x+c)^2+16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*b*cos(d*x+c)-42*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*
x+c),((a-b)/(a+b))^(1/2))*a*b^2*cos(d*x+c)-30*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-
csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3+36*EllipticF(cot(d*x+c)-csc(d*x+c),(
(a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*b^3-21*a*b^2*cos(d*x+c)*sin(d*x+c)+8*EllipticE(c
ot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3-21*EllipticE(cot(d*x+c
)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^3-4*a^2*b*cos(d*x+c)*sin(d*x+c))
```

Fricas [F]

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**4/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^4}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(1/2), x)
```

3.332 $\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 310

$$
\begin{aligned}
\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = & -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d} \\
& -\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd} \\
& +\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad}
\end{aligned}
$$

```
[Out] -2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-
b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a
-b))^(1/2)/b^2/d-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(
d*x+c))/(a-b))^(1/2)/b/d+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+
b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^
(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3979, 4144, 4006, 3869, 3917, 4089}

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d} - \frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd} + \frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

```
[In] Int[Tan[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4144

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*
x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{-1+\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx \\
&= \int \frac{-1-\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx \\
&= -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d} \\
&\quad - \int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx - \int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx
\end{aligned}$$

$$= -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d}$$
$$-\frac{2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$
$$+\frac{2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx}{a-b}}}{ad}$$

Mathematica [A] (warning: unable to verify)

Time = 12.55 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.23

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{-\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}\left(2(a+b)\sqrt{\frac{b+a\cos(c+d}{(a+b)(1+\cos(c+}}\right.}{}$$

```
[In] Integrate[Tan[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-(Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]*Sqrt[Sec[c + d*
x]]*Sqrt[1 + Sec[c + d*x]]*(2*(a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 4
*b*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*b*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)] + a*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c
 + d*x))/2] - a*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] + 2*
b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])) + 2*(b + a*Cos[c
 + d*x])*Tan[c + d*x])/(b*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. 2(283) = 566.

Time = 10.92 (sec) , antiderivative size = 1069, normalized size of antiderivative = 3.45

method	result	size
default	Expression too large to display	1069

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 2/d/b*(2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b*cos(d*x+c)^2+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)^2+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)^2+4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)-4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b+a*sin(d*x+c)*cos(d*x+c)+sin(d*x+c)*b)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)

Fricas [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)

Sympy [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(tan(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^2}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(1/2), x)
```

3.333 $\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3869}

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

```
[In] Int[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left(\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)-2\,\text{Ellipti}\right.}{d\sqrt{a+b\sec(c+dx)}}$$

```
[In] Integrate[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A] (verified)

Time = 5.44 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

method	result
default	$\frac{2(\cos(dx+c)+1)\left(\text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)-2\,\text{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}}{d(b+a\cos(dx+c))}$

```
[In] int(1/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(cos(d*x+c)+1)*(EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))-2*
EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos
```

```
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(a+b*sec(d
*x+c))^(1/2)/(b+a*cos(d*x+c))
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(1/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(1/(a + b/cos(c + d*x))^(1/2), x)
```

3.334 $\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 361

$$
\begin{aligned}
&\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx \\
&= \frac{\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{\sqrt{a+b}d} \\
&\quad - \frac{\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{\sqrt{a+b}d} \\
&\quad + \frac{2\sqrt{a+b}\cot(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} \\
&\quad - \frac{\cot(c+dx)}{d\sqrt{a+b\sec(c+dx)}} + \frac{b^2\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}
\end{aligned}
$$

```
[Out] cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d/(a+b)^(1
/2)-cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d/(a+b
)^(1/2)+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,
((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(
d*x+c))/(a-b))^(1/2)/a/d-cot(d*x+c)/d/(a+b*sec(d*x+c))^(1/2)+b^2*tan(d*x+c)
/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3981, 3869, 3960, 3918, 21, 3914, 3917, 4089}

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{b^2\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}} - \frac{\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d\sqrt{a+b}} + \frac{\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d\sqrt{a+b}} + \frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad} - \frac{\cot(c+dx)}{d\sqrt{a+b\sec(c+dx)}}$$

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(Sqrt[a + b]*d) - (Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) + (2*Sqrt[a
 + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(a*d) - Cot[c + d*x]/(d*Sqrt[a + b*Sec[c +
d*x]]) + (b^2*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3914

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3981

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{1}{\sqrt{a+b\sec(c+dx)}}+\frac{\csc^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\right)dx$$

$$= -\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx+\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} - \frac{\cot(c+dx)}{d\sqrt{a+b\sec(c+dx)}}-\frac{1}{2}b\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} - \frac{\cot(c+dx)}{d\sqrt{a+b\sec(c+dx)}}+\frac{b^2\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{b\int \frac{\sec(c+dx)\left(-\frac{a}{2}-\frac{1}{2}b\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} - \frac{\cot(c+dx)}{d\sqrt{a+b\sec(c+dx)}}+\frac{b^2\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{b\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx}{2\,(a^2-b^2)}$$

$$= \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} - \frac{\cot(c+dx)}{d\sqrt{a+b\sec(c+dx)}}+\frac{b^2\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{b\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{2(a+b)} - \frac{b^2\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{2\,(a^2-b^2)}$$

$$
= \frac{\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{\sqrt{a+b}d}
- \frac{\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{\sqrt{a+b}d}
+ \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad}
- \frac{\cot(c+dx)}{d\sqrt{a+b\sec(c+dx)}} + \frac{b^2\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}
$$

Mathematica [A] (verified)

Time = 13.15 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.01

$$
\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx
= \frac{\left(-8b(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\cot\left(\frac{1}{2}(c+dx)\right)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-8(2a^2-ab\right.}{}
$$

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((-8*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*Cot[(c + d*x)/2]*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] - 8*(2*a^2 - a*b - 3*b^2)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*C
ot[(c + d*x)/2]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*(a
 - b)*Csc[c + d*x]^2*(2*Cos[c + d*x]*(b + a*Cos[c + d*x]) + 32*(a + b)*Cos[
(c + d*x)/2]^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)]*Sin[(c + d*x)/2]))*Tan[c + d*x])/(8*(-a^2 + b^2)*d*Sqrt[a
+ b*Sec[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. 2(330) = 660.

Time = 7.10 (sec) , antiderivative size = 1374, normalized size of antiderivative = 3.81

method	result	size
default	Expression too large to display	1374

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d/(a-b)/(a+b)*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc
(d*x+c)^2-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^2*c
sc(d*x+c)^2-1)*(-4*(-(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x
+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*Ellip
ticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*(-cot(d*x+c)+csc(d*x+c)
)+2*(-(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+
c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)
-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b*(-cot(d*x+c)+csc(d*x+c))+6*(-(1-cos(d*
x+c))^2*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d
*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a
-b)/(a+b))^(1/2))*b^2*(-cot(d*x+c)+csc(d*x+c))-2*(-(1-cos(d*x+c))^2*csc(d*x
+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*
x+c)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2
))*a*b*(-cot(d*x+c)+csc(d*x+c))-2*(-(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*
(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+
b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(-cot(d*
x+c)+csc(d*x+c))+8*(-(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x
+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*Ellip
ticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(-cot(d*x+c)+csc(d*
x+c))-8*(-(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(
d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticPi(cot(d
*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*(-cot(d*x+c)+csc(d*x+c))-a^2*(
1-cos(d*x+c))^4*csc(d*x+c)^4+2*a*(1-cos(d*x+c))^4*b*csc(d*x+c)^4-(1-cos(d*x
+c))^4*b^2*csc(d*x+c)^4+2*a^2*(1-cos(d*x+c))^2*csc(d*x+c)^2-2*a*(1-cos(d*x+
c))^2*b*csc(d*x+c)^2-a^2+b^2)/((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c
))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)/(1-cos(d*x
+c))*sin(d*x+c)/((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*
x+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)
```

Fricas [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\cot\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cot(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cot(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^2}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(1/2), x)
```

3.335 $\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d}-\frac{2a(a+b\sec(c+dx))^{3/2}}{b^4d}+\frac{2(a+b\sec(c+dx))^{5/2}}{5b^4d}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-2*a*(a+b*sec(d*x+c))^(
3/2)/b^4/d+2/5*(a+b*sec(d*x+c))^(5/2)/b^4/d+2*(a^2-b^2)^2/a/b^4/d/(a+b*sec(
d*x+c))^(1/2)+2*(3*a^2-2*b^2)*(a+b*sec(d*x+c))^(1/2)/b^4/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3970, 912, 1275, 212}

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(3a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d}+\frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\sec(c+dx)}}+\frac{2(a+b\sec(c+dx))^{5/2}}{5b^4d}-\frac{2a(a+b\sec(c+dx))^{3/2}}{b^4d}$$

```
[In] Int[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2)
^2)/(a*b^4*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^2 - 2*b^2)*Sqrt[a + b*Sec[
c + d*x]])/(b^4*d) - (2*a*(a + b*Sec[c + d*x])^(3/2))/(b^4*d) + (2*(a + b*S
ec[c + d*x])^(5/2))/(5*b^4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)^{3/2}}\,dx, x, b\sec(c+dx)\right)}{b^4d}\\ &= \frac{2\text{Subst}\left(\int \frac{(-a^2+b^2+2ax^2-x^4)^2}{x^2(-a+x^2)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4d}\\ &= \frac{2\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)-\frac{(a^2-b^2)^2}{ax^2}-3ax^2+x^4-\frac{b^4}{a(a-x^2)}\right)\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^4d}\end{aligned}$$

$$
\begin{aligned}
&= \frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d} \\
&\quad -\frac{2a(a+b\sec(c+dx))^{3/2}}{b^4d}+\frac{2(a+b\sec(c+dx))^{5/2}}{5b^4d} \\
&\quad -\frac{2\mathrm{Subst}\left(\int\frac{1}{a-x^2}\,dx,x,\sqrt{a+b\sec(c+dx)}\right)}{ad} \\
&= -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\sec(c+dx)}} \\
&\quad +\frac{2(3a^2-2b^2)\sqrt{a+b\sec(c+dx)}}{b^4d} \\
&\quad -\frac{2a(a+b\sec(c+dx))^{3/2}}{b^4d}+\frac{2(a+b\sec(c+dx))^{5/2}}{5b^4d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$
\int\frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\frac{2\left(5b^4\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\sec(c+dx)}{a}\right)+a(4a(4a^2-5b^2)+2b(4\right.}{5ab^4d\sqrt{a+b\sec(c+}}
$$

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(5*b^4*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sec[c + d*x])/a] + a*(4*a*
(4*a^2 - 5*b^2) + 2*b*(4*a^2 - 5*b^2)*Sec[c + d*x] - 2*a*b^2*Sec[c + d*x]^2
 + b^3*Sec[c + d*x]^3)))/(5*a*b^4*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1233 vs. 2(132) = 264.

Time = 15.60 (sec) , antiderivative size = 1234, normalized size of antiderivative = 8.34

method	result	size
default	Expression too large to display	1234

```
[In] int(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/d/a^2/b^4*(a+b*sec(d*x+c))^(1/2)/((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)/(b+a*cos(d*x+c))^2/(cos(d*x+c)+1)*(5*cos(d*x+c)^3*ln(4*cos(
d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos
(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)
```

```
*a^(5/2)*b^4+10*cos(d*x+c)^2*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(
cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*c
os(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*a^(3/2)*b^5-32*((b+a*cos(d*x+c))*cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^6*cos(d*x+c)^3+40*cos(d*x+c)^3*a^4*b^2*((
b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-10*cos(d*x+c)^3*a^2*b^4*
((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+5*cos(d*x+c)*ln(4*cos(
d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos
(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)
*a^(1/2)*b^6-32*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*a^6-48*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^5*b*co
s(d*x+c)^2+40*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*a^4*b^2+60*cos(d*x+c)^2*a^3*b^3*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)-10*cos(d*x+c)^2*a^2*b^4*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)-10*cos(d*x+c)^2*a*b^5*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)-48*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^
2)^(1/2)*a^5*b-12*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^4*
b^2*cos(d*x+c)+60*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*a^3*b^3+20*cos(d*x+c)*a^2*b^4*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)-10*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*a*b^5-12*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^4*b^
2+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3*b^3+20*((b+a*c
os(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^4+2*((b+a*cos(d*x+c))*c
os(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3*b^3*sec(d*x+c)-2*((b+a*cos(d*x+c))*co
s(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^4*sec(d*x+c)-2*((b+a*cos(d*x+c))*cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^4*sec(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.16

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left[\frac{5\left(ab^4\cos\left(dx+c\right)^3+b^5\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(-8\,a^2\cos\left(dx+c\right)^2-8\,ab\cos\right.}{}\right.$$

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/10*(5*(a*b^4*cos(d*x + c)^3 + b^5*cos(d*x + c)^2)*sqrt(a)*log(-8*a^2*cos
(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4*(2*a*cos(d*x + c)^2 + b*cos(d*x
+ c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - 4*(2*a^3*b^2*cos(d
*x + c) - a^2*b^3 - (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3 - 2*(4*a
^4*b - 5*a^2*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/
(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2), 1/5*(5*(a*b^4*cos(d*
```

```
x + c)^3 + b^5*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(2*a^3*b^2*
cos(d*x + c) - a^2*b^3 - (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3 - 2
*(4*a^4*b - 5*a^2*b^3)*cos(d*x + c)^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
c)))/(a^3*b^4*d*cos(d*x + c)^3 + a^2*b^5*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)**5/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(tan(c + d*x)**5/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.31

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\frac{5\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{10}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,a} + \frac{2\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}{b^4} - \frac{10\left(a+\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}a}{b^4} + \frac{30}{}}{5\,d}$$

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] 1/5*(5*log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) +
 sqrt(a)))/a^(3/2) + 10/(sqrt(a + b/cos(d*x + c))*a) + 2*(a + b/cos(d*x + c
))^(5/2)/b^4 - 10*(a + b/cos(d*x + c))^(3/2)*a/b^4 + 30*sqrt(a + b/cos(d*x
+ c))*a^2/b^4 + 10*a^3/(sqrt(a + b/cos(d*x + c))*b^4) - 20*sqrt(a + b/cos(d
*x + c))/b^2 - 20*a/(sqrt(a + b/cos(d*x + c))*b^2))/d
```

Giac [F]

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(dx+c)^5}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^5/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^5}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^5/(a + b/cos(c + d*x))^(3/2), x)
```

3.336 $\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{a+b\sec(c+dx)}}{b^2d}$$

```
[Out] 2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*(a^2-b^2)/a/b^2/d/(a+
b*sec(d*x+c))^(1/2)+2*(a+b*sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3970, 912, 1275, 212}

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{a+b\sec(c+dx)}}{b^2d}$$

```
[In] Int[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2))
/(a*b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[c + d*x]])/(b^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)^{3/2}}\,dx, x, b\sec(c+dx)\right)}{b^2 d}\\
&= -\frac{2\text{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{x^2(-a+x^2)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2 d}\\
&= -\frac{2\text{Subst}\left(\int \left(-1+\frac{a^2-b^2}{ax^2}-\frac{b^2}{a(a-x^2)}\right)dx, x, \sqrt{a+b\sec(c+dx)}\right)}{b^2 d}\\
&= \frac{2(a^2-b^2)}{ab^2 d\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{a+b\sec(c+dx)}}{b^2 d}+\frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{ad}\\
&= \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a^2-b^2)}{ab^2 d\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{a+b\sec(c+dx)}}{b^2 d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\left(-b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\sec(c+dx)}{a}\right)+a(2a+b\sec(c+dx))\right)}{ab^2d\sqrt{a+b\sec(c+dx)}}$$

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
[Out] (2*(-(b^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sec[c + d*x])/a]) + a*(2*a
 + b*Sec[c + d*x])))/(a*b^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. 2(78) = 156.

Time = 9.56 (sec) , antiderivative size = 731, normalized size of antiderivative = 8.31

method	result
default	$\frac{\left(\cos(dx+c)^3 a^{\frac{5}{2}}\ln\left(4\cos(dx+c)\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}\,\sqrt{a}+4a\cos(dx+c)+4\sqrt{a}\,\sqrt{\frac{(b+a\cos(dx+c))\cos(dx+c)}{(\cos(dx+c)+1)^2}}+2b\right)b^2+2\cos(dx+c\right.}{}$

```
[In] int(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/d/a^2/b^2*(cos(d*x+c)^3*a^(5/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^2+2*cos(d*x+c)^2*a^(3/2)*ln(4
*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*
a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
+2*b)*b^3+2*sin(d*x+c)^2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c
)+1)^2)^(1/2)*a^2*b^2+4*cos(d*x+c)^3*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)*a^4+2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a
^2*b^2*sin(d*x+c)^2+4*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)
+1)^2)^(1/2)*a^4+6*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)
^2)^(1/2)*a^3*b-2*cos(d*x+c)^2*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^
2)^(1/2)*a*b^3+cos(d*x+c)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*a^(1/2)*b^4+6*cos(d*x+c)*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3*b-2*cos(d*x+c)*((b+a*cos(d*x+c))
*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^3)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(
d*x+c))^2/((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)/(cos(d*x+c)+
1)
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.60

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left[\frac{(ab^2\cos(dx+c)+b^3)\sqrt{a}\log\left(-8\,a^2\cos(dx+c)^2-8\,ab\cos(dx+c)-b^2-\ldots\right.}{2\ldots}\right.$$

$$-\frac{(ab^2\cos(dx+c)+b^3)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)}{2\,a\cos(dx+c)+b}\right)-2\,(a^2b+(2\,a^3-ab^2)\cos(dx+c))\sqrt{\frac{a\cos(\ldots}{\cos(\ldots}}\ldots}{a^3b^2d\cos(dx+c)+a^2b^3d}$$

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b^2*cos(d*x + c) + b^3)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*
cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) + 4*(a^2*b + (2*a^3 - a*b^2)*cos(d*x + c
))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(a^3*b^2*d*cos(d*x + c) + a^2*b
^3*d), -((a*b^2*cos(d*x + c) + b^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 2*(a^2*b
 + (2*a^3 - a*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)))/(
a^3*b^2*d*cos(d*x + c) + a^2*b^3*d)]
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,a} - \frac{2\sqrt{a+\frac{b}{\cos(dx+c)}}}{b^2} - \frac{2a}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,b^2}}{d}$$

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -(log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) + sqrt
(a)))/a^(3/2) + 2/(sqrt(a + b/cos(d*x + c))*a) - 2*sqrt(a + b/cos(d*x + c))
/b^2 - 2*a/(sqrt(a + b/cos(d*x + c))*b^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(78) = 156.

Time = 1.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.93

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\left(\frac{\frac{\left(2a^3\mathrm{sgn}(\cos(dx+c))-a^2b\mathrm{sgn}(\cos(dx+c))-ab^2\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2b^2}-\frac{2a^3\mathrm{sgn}(\cos(dx+c))+a^2b\mathrm{sgn}(\cos(dx+c))-ab^2\mathrm{sgn}(\cos(dx+c))}{a^2b^2}}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}}+\frac{\arctan\cdots}{\cdots}\right)}{d}$$

```
[In] integrate(tan(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2*(((2*a^3*sgn(cos(d*x + c)) - a^2*b*sgn(cos(d*x + c)) - a*b^2*sgn(cos(d*x
 + c)))*tan(1/2*d*x + 1/2*c)^2/(a^2*b^2) - (2*a^3*sgn(cos(d*x + c)) + a^2*b
*sgn(cos(d*x + c)) - a*b^2*sgn(cos(d*x + c)))/(a^2*b^2))/sqrt(a*tan(1/2*d*x
 + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b
) + arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x +
1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) +
 sqrt(a - b))/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^3}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^3/(a + b/cos(c + d*x))^(3/2), x)
```

3.337 $\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2}{ad\sqrt{a+b\sec(c+dx)}}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2/a/d/(a+b*sec(d*x+c))
^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3970, 53, 65, 213}

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2}{ad\sqrt{a+b\sec(c+dx)}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}$$

```
[In] Int[Tan[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + 2/(a*d*Sqrt[a
+ b*Sec[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= \frac{2}{ad\sqrt{a+b\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sec(c+dx)\right)}{ad}\\
&= \frac{2}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{ad}\\
&= -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2}{ad\sqrt{a+b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\sec(c+dx)}{a}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

```
[In] Integrate[Tan[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sec[c + d*x])/a])/(a*d*Sqrt[a + b
*Sec[c + d*x]])
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+b\sec(dx+c)}}}{d}$	45
default	$\frac{-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{a+b\sec(dx+c)}}}{d}$	45

```
[In] int(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/d*(-2/a^(3/2)*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))+2/a/(a+b*sec(d*x+c)
)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(46) = 92.

Time = 0.42 (sec) , antiderivative size = 260, normalized size of antiderivative = 4.81

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\cos(dx+c)+(a\cos(dx+c)+b)\sqrt{a}\log\left(-8\,a^2\cos(dx+\right.}{2\,(a^3d\,\mathrm{co}}\right.$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] [1/2*(4*a*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + (a*cos(d*x
 + c) + b)*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 + 4
```

```
*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))))/(a^3*d*cos(d*x + c) + a^2*b*d), ((a*cos(d*x + c) + b)*sqrt(-a
)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2
*a*cos(d*x + c) + b)) + 2*a*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x
 + c))/(a^3*d*cos(d*x + c) + a^2*b*d)]
```

Sympy [F]

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\frac{\log\left(\frac{\sqrt{a+\frac{b}{\cos(dx+c)}}-\sqrt{a}}{\sqrt{a+\frac{b}{\cos(dx+c)}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{\cos(dx+c)}}\,a}}{d}$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] (log((sqrt(a + b/cos(d*x + c)) - sqrt(a))/(sqrt(a + b/cos(d*x + c)) + sqrt(
a)))/a^(3/2) + 2/(sqrt(a + b/cos(d*x + c))*a))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(46) = 92.

Time = 0.78 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.96

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\left(\frac{\arctan\left(-\frac{\sqrt{a-b}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}+\sqrt{a-b}}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a\,\mathrm{sgn}(\cos(dx+c))}\right.}{}$$

```
[In] integrate(tan(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 2*(arctan(-1/2*(sqrt(a - b)*tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1
/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b) +
sqrt(a - b))/sqrt(-a))/(sqrt(-a)*a*sgn(cos(d*x + c))) + 2*b/(((sqrt(a - b)*
tan(1/2*d*x + 1/2*c)^2 - sqrt(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/
2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b))*sqrt(a - b) - a - b)*sqrt(a -
 b)*a*sgn(cos(d*x + c))))/d
```

Mupad [B] (verification not implemented)

Time = 15.59 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\tan(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2}{a\,d\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}} - \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{a}}\right)}{a^{3/2}\,d}$$

```
[In] int(tan(c + d*x)/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] 2/(a*d*(a + b/cos(c + d*x))^(1/2)) - (2*atanh((a + b/cos(c + d*x))^(1/2)/a^
(1/2)))/(a^(3/2)*d)
```

3.338 $\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 142

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{2b^2}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-arctanh((a+b*sec(d*x+c)
)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d-arctanh((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2))/(a+b)^(3/2)/d+2*b^2/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3970, 912, 1301, 212}

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2b^2}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

```
[In] Int[Cot[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sec[c
```

```
+ d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b^2)/(a*(a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b^2\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{x^2(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}\\
&= -\frac{(2b^2)\,\text{Subst}\left(\int \left(\frac{1}{a(a^2-b^2)x^2}-\frac{1}{ab^2(a-x^2)}+\frac{1}{2(a-b)b^2(a-b-x^2)}+\frac{1}{2b^2(a+b)(a+b-x^2)}\right)dx, x, \sqrt{a+b\sec(c+
\end{aligned}
$$

d

$$
\begin{aligned}
&= \frac{2b^2}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{ad} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{(a-b)d} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{(a+b)d} \\
&= \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d} \\
&\quad - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{2b^2}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.67 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.49

$$
\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx =
$$

$$
-\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\sec(c+dx)}{a-b}\right)}{(a-b)\sqrt{a+b\sec(c+dx)}} + \frac{a\,\text{Hypergeometric2F1}\left(-\right.}{(a+b)\sqrt{a+b}}}{bd}
$$

```
[In] Integrate[Cot[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] -((-(ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]]/Sqrt[a - b]) + ArcTanh[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]/Sqrt[a + b] - (a*Hypergeometric2F1[-1/
2, 1, 1/2, (a + b*Sec[c + d*x])/(a - b)])/((a - b)*Sqrt[a + b*Sec[c + d*x]]
) + (a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sec[c + d*x])/(a + b)])/((a +
 b)*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b
*Sec[c + d*x])/a])/(a*Sqrt[a + b*Sec[c + d*x]]))/(b*d))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2691 vs. $2(122) = 244$.

Time = 2.22 (sec) , antiderivative size = 2692, normalized size of antiderivative = 18.96

method	result	size
default	Expression too large to display	2692

```
[In] int(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d/(a-b)^(5/2)/a^2/(a+b)^2*(-2*(a-b)^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(d*
x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*cos(d*x+c)*a^(9/2)
-2*(a-b)^(3/2)*a^(7/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b*cos(d*x+c)-2*(a-b)^(3/2)*a^(7/2)*ln(4*co
s(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*c
os(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*
b)*b+2*(a-b)^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)
+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)+2*b)*cos(d*x+c)*a^(5/2)*b^2-2*(a-b)^(3/2)*ln(4*cos(
d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos
(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)
*a^(5/2)*b^2+2*(a-b)^(3/2)*a^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x
+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^3*cos(d*x+c)+(a+b)^(1/2)*(a-
b)^(3/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+cos(d*x+c)*b+2*(a+b)^(1/2)*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+b)/(cos(d*x+c)-1))*cos(d*x+c)*a^4-(a
+b)^(1/2)*(a-b)^(3/2)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)*(a+b)^(1/2)+2*a*cos(d*x+c)+cos(d*x+c)*b+2*(a+b)^(1/2)*(
(b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+b)/(cos(d*x+c)-1))*cos(
d*x+c)*a^3*b+2*(a-b)^(3/2)*a^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x
+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*b)*b^3+(a+b)^(1/2)*(a-b)^(3/2)*ln
(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a+b
)^(1/2)+2*a*cos(d*x+c)+cos(d*x+c)*b+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos(d*x
+c)/(cos(d*x+c)+1)^2)^(1/2)+b)/(cos(d*x+c)-1))*a^3*b-(a+b)^(1/2)*(a-b)^(3/2
)*ln(-2*(2*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*
(a+b)^(1/2)+2*a*cos(d*x+c)+cos(d*x+c)*b+2*(a+b)^(1/2)*((b+a*cos(d*x+c))*cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+b)/(cos(d*x+c)-1))*a^2*b^2-4*(a-b)^(3/2)*((
b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2*cos(d*x+c)-4*(a-
b)^(3/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^3*cos(d*x
+c)+2*(a-b)^(3/2)*ln(4*cos(d*x+c)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+
1)^2)^(1/2)*a^(1/2)+4*a*cos(d*x+c)+4*a^(1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(
```

```
cos(d*x+c)+1)^2)^(1/2)+2*b)*a^(1/2)*b^4-4*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^2*b^2-4*(a-b)^(3/2)*((b+a*cos(d*x+c))*cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a*b^3-ln(1/(a-b)^(1/2)*(2*(a-b)^(1/2)*((b+a
*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+2*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+cos(d*x+c
)*b-b)/(cos(d*x+c)+1))*cos(d*x+c)*a^6-ln(1/(a-b)^(1/2)*(2*(a-b)^(1/2)*((b+a
*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+2*((b+a*cos(d*x+
c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+cos(d*x+c
)*b-b)/(cos(d*x+c)+1))*a^5*b*cos(d*x+c)+ln(1/(a-b)^(1/2)*(2*(a-b)^(1/2)*((b
+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+2*((b+a*cos(d*
x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+cos(d*x
+c)*b-b)/(cos(d*x+c)+1))*cos(d*x+c)*a^4*b^2+ln(1/(a-b)^(1/2)*(2*(a-b)^(1/2)
*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+2*((b+a*co
s(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+cos
(d*x+c)*b-b)/(cos(d*x+c)+1))*a^3*b^3*cos(d*x+c)-ln(1/(a-b)^(1/2)*(2*(a-b)^(
1/2)*((b+a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+2*((b+
a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)
+cos(d*x+c)*b-b)/(cos(d*x+c)+1))*a^5*b-ln(1/(a-b)^(1/2)*(2*(a-b)^(1/2)*((b+
a*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+2*((b+a*cos(d*x
+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+cos(d*x+
c)*b-b)/(cos(d*x+c)+1))*a^4*b^2+ln(1/(a-b)^(1/2)*(2*(a-b)^(1/2)*((b+a*cos(d
*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+2*((b+a*cos(d*x+c))*co
s(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+cos(d*x+c)*b-b)
/(cos(d*x+c)+1))*a^3*b^3+ln(1/(a-b)^(1/2)*(2*(a-b)^(1/2)*((b+a*cos(d*x+c))*
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+2*((b+a*cos(d*x+c))*cos(d*x+c
)/(cos(d*x+c)+1)^2)^(1/2)*(a-b)^(1/2)-2*a*cos(d*x+c)+cos(d*x+c)*b-b)/(cos(d
*x+c)+1))*a^2*b^4)*cos(d*x+c)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/((b+a
*cos(d*x+c))*cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)/(cos(d*x+c)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(122) = 244$.

Time = 24.54 (sec) , antiderivative size = 3924, normalized size of antiderivative = 27.63

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(8*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x +
 c) + 2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*
sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d
*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))
) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*
sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)
```

*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4*(4*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - (a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4*(2*(a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) - 2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))) - (a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4*(4*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)) + 2*(a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) - (a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^2 + b^2 - 4*((2*a + b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b + 3*b^2)*cos(d*x + c))/(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), 1/4*(2*(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*

```
x + c)/((2*a + b)*cos(d*x + c) + b)) + 8*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x
+ c) + b)/cos(d*x + c))*cos(d*x + c) + 2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 -
2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(a)*log(-8*a^2*cos(d*x + c)^2 - 8*a*b*
cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + b)/cos(d*x + c))) - (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 +
2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*co
s(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c))*sqrt(a -
 b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*cos(d*x + c
))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*c
os(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/4*(4*(a^4*b - 2*a^2*b^3
+ b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(2*a*cos(d*x + c) + b)
) - 2*(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c)
)*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)
)*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - 8*(a^3*b^2 - a*b^4)*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) + (a^4*b + 2*a^3*b^2 + a^2*b^3
 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b
 + b^2)*cos(d*x + c)^2 + b^2 + 4*((2*a - b)*cos(d*x + c)^2 + b*cos(d*x + c)
)*sqrt(a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)) + 2*(4*a*b - 3*b^2)*c
os(d*x + c))/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/((a^7 - 2*a^5*b^2 + a^
3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/2*((a^4*b + 2*
a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(-a + b)*ar
ctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c)/(
(2*a - b)*cos(d*x + c) + b)) - (a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*
b + a^3*b^2)*cos(d*x + c))*sqrt(-a - b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d
*x + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a + b)*cos(d*x + c) + b)) - 4*(
a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c) - (a^
4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqrt(a)*log
(-8*a^2*cos(d*x + c)^2 - 8*a*b*cos(d*x + c) - b^2 - 4*(2*a*cos(d*x + c)^2 +
 b*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))))/((a^7 -
2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), -1/
2*(2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sqr
t(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x + c
)/(2*a*cos(d*x + c) + b)) + (a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b +
 a^3*b^2)*cos(d*x + c))*sqrt(-a + b)*arctan(-2*sqrt(-a + b)*sqrt((a*cos(d*x
 + c) + b)/cos(d*x + c))*cos(d*x + c)/((2*a - b)*cos(d*x + c) + b)) - (a^4*
b - 2*a^3*b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cos(d*x + c))*sqrt(-a -
 b)*arctan(2*sqrt(-a - b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*cos(d*x +
 c)/((2*a + b)*cos(d*x + c) + b)) - 4*(a^3*b^2 - a*b^4)*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))*cos(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x +
 c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]
```

Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(cot(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

```
[In] int(cot(c + d*x)/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)/(a + b/cos(c + d*x))^(3/2), x)
```

3.339 $\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 236

$$\begin{aligned}\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}\\ &+\frac{(4a-7b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{5/2}d}\\ &+\frac{(4a+7b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{5/2}d}+\frac{2b^4}{a\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}}\\ &+\frac{\sqrt{a+b\sec(c+dx)}}{4(a+b)^2d(1-\sec(c+dx))}+\frac{\sqrt{a+b\sec(c+dx)}}{4(a-b)^2d(1+\sec(c+dx))}\end{aligned}$$

```
[Out] -2*arctanh((a+b*sec(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+1/4*(4*a-7*b)*arctanh(
(a+b*sec(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d+1/4*(4*a+7*b)*arctanh((a+
b*sec(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/d+2*b^4/a/(a^2-b^2)^2/d/(a+b*s
ec(d*x+c))^(1/2)+1/4*(a+b*sec(d*x+c))^(1/2)/(a+b)^2/d/(1-sec(d*x+c))+1/4*(a
+b*sec(d*x+c))^(1/2)/(a-b)^2/d/(1+sec(d*x+c))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.34, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3970, 912, 1349, 212, 205}

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2b^4}{ad\,(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}} - \frac{b\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{5/2}} + \frac{b\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4d(a+b)^{5/2}} + \frac{(2a-3b)\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{2d(a-b)^{5/2}} + \frac{(2a+3b)\text{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}} + \frac{\sqrt{a+b\sec(c+dx)}}{4d(a+b)^2(1-\sec(c+dx))} + \frac{\sqrt{a+b\sec(c+dx)}}{4d(a-b)^2(\sec(c+dx)+1)}$$

```
[In] Int[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + ((2*a - 3*b)*A
rcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/(2*(a - b)^(5/2)*d) - (b*ArcT
anh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/2)*d) + (b*ArcTanh
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(5/2)*d) + ((2*a + 3*b)*
ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/(2*(a + b)^(5/2)*d) + (2*b^4
)/(a*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + Sqrt[a + b*Sec[c + d*x]]/(
4*(a + b)^2*d*(1 - Sec[c + d*x])) + Sqrt[a + b*Sec[c + d*x]]/(4*(a - b)^2*d
*(1 + Sec[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{b^4\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d}$$

$$= \frac{(2b^4)\,\text{Subst}\left(\int \frac{1}{x^2(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{d}$$

$$= \frac{(2b^4)\,\text{Subst}\left(\int \left(-\frac{1}{a(a-b)^2(a+b)^2x^2} - \frac{1}{ab^4(a-x^2)} - \frac{1}{4(a-b)b^3(a-b-x^2)^2} + \frac{2a-3b}{4(a-b)^2b^4(a-b-x^2)} + \frac{1}{4b^3(a+b)(a+b-x^2)^2}\right.\right.}{d}$$

$$= \frac{2b^4}{a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{ad}$$
$$+ \frac{(2a-3b)\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{2(a-b)^2d}$$
$$- \frac{b\text{Subst}\left(\int \frac{1}{(a-b-x^2)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{2(a-b)d}$$
$$+ \frac{b\text{Subst}\left(\int \frac{1}{(a+b-x^2)^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{2(a+b)d}$$
$$+ \frac{(2a+3b)\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{2(a+b)^2d}$$

$$= -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(2a-3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d} + \frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} + \frac{2b^4}{a(a^2-b^2)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+dx)}}{4(a+b)^2d(1-\sec(c+dx))} + \frac{\sqrt{a+b\sec(c+dx)}}{4(a-b)^2d(1+\sec(c+dx))} - \frac{b\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{4(a-b)^2d} + \frac{b\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sec(c+dx)}\right)}{4(a+b)^2d}$$

$$= -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(2a-3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{5/2}d} + \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{5/2}d} + \frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} + \frac{2b^4}{a(a^2-b^2)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+dx)}}{4(a+b)^2d(1-\sec(c+dx))} + \frac{\sqrt{a+b\sec(c+dx)}}{4(a-b)^2d(1+\sec(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.09 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.33

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{2a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2}\right.}{(a-b)\sqrt{a+b\sec(c+}}}{}$$

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-2*ArcTanh[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a - b]])/Sqrt[a - b] + (2*ArcTan
h[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]])/Sqrt[a + b] - (2*a*Hypergeometric2
F1[-1/2, 1, 1/2, (a + b*Sec[c + d*x])/(a - b)])/((a - b)*Sqrt[a + b*Sec[c +
 d*x]]) + (2*a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sec[c + d*x])/(a + b)
])/((a + b)*Sqrt[a + b*Sec[c + d*x]]) + (4*b*Hypergeometric2F1[-1/2, 1, 1/2
, 1 + (b*Sec[c + d*x])/a])/(a*Sqrt[a + b*Sec[c + d*x]]) + (b^2*Hypergeometr
ic2F1[-1/2, 2, 1/2, (a + b*Sec[c + d*x])/(a - b)])/((a - b)^2*Sqrt[a + b*Se
c[c + d*x]]) - (b^2*Hypergeometric2F1[-1/2, 2, 1/2, (a + b*Sec[c + d*x])/(a
 + b)])/((a + b)^2*Sqrt[a + b*Sec[c + d*x]]))/(2*b*d)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 6366 vs. $2(204) = 408$.

Time = 2.34 (sec) , antiderivative size = 6367, normalized size of antiderivative = 26.98

method	result	size
default	Expression too large to display	6367

```
[In] int(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(202) = 404$.

Time = 48.26 (sec) , antiderivative size = 8098, normalized size of antiderivative = 34.31

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to rounding error%%%{%%%{32,[2,6]%%%}+%%%{-32,[1,7]%%%},[6,1]%%%}+%%%{%%{[%%%{64,[2,6]%%

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^3}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

[In] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(3/2),x)

[Out] int(cot(c + d*x)^3/(a + b/cos(c + d*x))^(3/2), x)

3.340 $\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 530

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d}$$

$$+\frac{2(8a^4-11a^2b^2+3b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{-\frac{b(-1+\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3ab^4\sqrt{a+b}d}$$

$$+\frac{2(2a+b)\left(4a^2+ab-3b^2\right)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{-\frac{b(-1+\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3ab^3\sqrt{a+b}d}$$

$$-\frac{4a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

$$-\frac{2a^2\sec(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}+\frac{2(4a^2-b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2\,(a^2-b^2)\,d}$$

```
[Out] -2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a^2/d+2/3*(8*a^4-11*a^2*b^2+3*b^4)*cot(d*x+c)*EllipticE((a+b*
sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(-b*(-1+sec(d*x+c))/(a+b
))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^4/d/(a+b)^(1/2)+2/3*(2*a+b)*(4
*a^2+a*b-3*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a
+b)/(a-b))^(1/2))*(-b*(-1+sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/a/b^3/d/(a+b)^(1/2)-4*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/
2)+2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2*a^2*sec(d*x+c)*t
an(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*(4*a^2-b^2)*(a+b*sec(d*x
+c))^(1/2)*tan(d*x+c)/b^2/(a^2-b^2)/d
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 907, normalized size of antiderivative = 1.71, number of steps used = 17, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3980, 3870, 4143, 4006, 3869, 3917, 4089, 3921, 4090, 3930, 4167}

$$
\begin{aligned}
\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= -\frac{2\sec(c+dx)\tan(c+dx)a^2}{b(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} \\
&- \frac{4\tan(c+dx)a}{(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} \\
&+ \frac{2(8a^2-5b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}a}{3b^4\sqrt{a+b}d} \\
&- \frac{4\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}a}{b^2\sqrt{a+b}d} \\
&+ \frac{2(4a^2-b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2(a^2-b^2)d} \\
&+ \frac{2(2a+b)(4a+b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{3b^3\sqrt{a+b}d} \\
&- \frac{4\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{b\sqrt{a+b}d} \\
&+ \frac{2b^2\tan(c+dx)}{(a^2-b^2)d\sqrt{a+b\sec(c+dx)}a} \\
&+ \frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{\sqrt{a+b}da} \\
&- \frac{2\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{\sqrt{a+b}da} \\
&- \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{da^2}
\end{aligned}
$$

```
[In] Int[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (4*a*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d)
 + (2*a*(8*a^2 - 5*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
```

```
t[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(3*b^4*Sqrt[a + b]*d) - (2*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(a*Sqrt[a + b]*d) - (4*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*Sqrt[a + b]*d) + (2*(2*a + b)*(
4*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) - (2*Sqrt[a + b]*Cot[c + d*x]*
EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(a^2*d) - (4*a*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c +
 d*x]]) + (2*b^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) -
 (2*a^2*Sec[c + d*x]*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]
]) + (2*(4*a^2 - b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*(a^2 -
b^2)*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
```

```
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3980

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{(a+b\sec(c+dx))^{3/2}}-\frac{2\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}+\frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\right)dx\\
&= -\left(2\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx\right)\\
&\quad+\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx+\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx\\
&= -\frac{4a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}\\
&\quad-\frac{2a^2\sec(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}-\frac{4\int \frac{\sec(c+dx)\left(-\frac{b}{2}-\frac{1}{2}a\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}\\
&\quad-\frac{2\int \frac{\frac{1}{2}(-a^2+b^2)+\frac{1}{2}ab\sec(c+dx)+\frac{1}{2}b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\,(a^2-b^2)}\\
&\quad-\frac{2\int \frac{\sec(c+dx)\left(a^2-\frac{1}{2}ab\sec(c+dx)-\frac{1}{2}(4a^2-b^2)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{b\,(a^2-b^2)}
\end{aligned}$$

$$= -\frac{4a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(4a^2-b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2\,(a^2-b^2)\,d} - \frac{2\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a+b} - \frac{2\int\frac{\frac{1}{2}(-a^2+b^2)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\,(a^2-b^2)} + \frac{(2a)\int\frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2} - \frac{4\int\frac{\sec(c+dx)\left(\frac{1}{4}b(2a^2+b^2)+\frac{1}{4}a(8a^2-5b^2)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{3b^2\,(a^2-b^2)} - \frac{b^2\int\frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx}{a\,(a^2-b^2)}$$

$$= \frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d} - \frac{4a\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2\sqrt{a+b}d} - \frac{4\cot(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}d} - \frac{4a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(4a^2-b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2\,(a^2-b^2)\,d} + \frac{\int\frac{1}{\sqrt{a+b\sec(c+dx)}}dx}{a} - \frac{b\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a+b)} + \frac{((2a+b)(4a+b))\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{3b^2(a+b)} - \frac{(a(8a^2-5b^2))\int\frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx}{3b^2\,(a^2-b^2)}$$

$$= \frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$- \frac{4a\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2\sqrt{a+b}d}$$
$$+ \frac{2a(8a^2-5b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^4\sqrt{a+b}d}$$
$$- \frac{2\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$- \frac{4\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}d}$$
$$+ \frac{2(2a+b)(4a+b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c-}{a-b}}}{3b^3\sqrt{a+b}d}$$
$$- \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d}$$
$$- \frac{4a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$
$$- \frac{2a^2\sec(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(4a^2-b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2\,(a^2-b^2)\,d}$$

Mathematica [A] (warning: unable to verify)

Time = 17.41 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.00

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\Big(2(8a^3+8a^2b}{}$$
$$+ \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(\frac{2(-8a^2+3b^2)\sin(c+dx)}{3ab^3} - \frac{2(-a^2\sin(c+dx)+b^2\sin(c+dx))}{ab^2(b+a\cos(c+dx))} + \frac{2\tan(c+dx)}{3b^2}\right)}{d(a+b\sec(c+dx))^{3/2}}$$

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*(2*(8*a^3 + 8*a^2*b - 3*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 4*a*b*(4*a + b)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 12*b^3*Sqrt[Cos[
```

```
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + ((8*a
^2 - 3*b^2)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(-Sin[(c + d*x)/2] + Si
n[(3*(c + d*x))/2]))/2))/(3*a*b^3*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c +
 d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-8*a^2 + 3*b^2)
*Sin[c + d*x])/(3*a*b^3) - (2*(-(a^2*Sin[c + d*x]) + b^2*Sin[c + d*x]))/(a*
b^2*(b + a*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*b^2)))/(d*(a + b*Sec[c + d*
x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1964 vs. $2(489) = 978$.

Time = 14.84 (sec) , antiderivative size = 1965, normalized size of antiderivative = 3.71

method	result	size
default	Expression too large to display	1965

```
[In] int(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d/a/b^3*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(a*b^2*s
in(d*x+c)-4*a^2*b*sin(d*x+c)-3*b^3*cos(d*x+c)*sin(d*x+c)-8*EllipticE(cot(d*
x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b+3*EllipticE(cot(d*x+c)-cs
c(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2-8*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-cs
c(d*x+c),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)^2+3*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c
)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)^2+8*EllipticF(cot(d*x+c)-c
sc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b+2*EllipticF(cot(d*x+c)-csc(d*x+
c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2-16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x
+c),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)+6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)-6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-cs
c(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)^2-12*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(
d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)+a*b^2*tan(d*x+c)-8
*a^3*cos(d*x+c)*sin(d*x+c)+16*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*a^2*b*cos(d*x+c)+4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a
+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
```

```
s(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)+8*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b
)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)^2+2*EllipticF(cot(d*x+c)-csc(d*x+c)
,((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^2-8*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+
c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*cos(d*x+c)^2+3*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(co
t(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*cos(d*x+c)^2-16*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellip
ticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*cos(d*x+c)+6*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*cos(d*x+c)-6*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3+3*a*b^2*cos
(d*x+c)*sin(d*x+c)-8*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*a^3+3*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^3
+4*a^2*b*cos(d*x+c)*sin(d*x+c))
```

Fricas [F]

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan\left(dx+c\right)^4}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^4\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)**4/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(tan(c + d*x)**4/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^4/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^4}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^4/(a + b/cos(c + d*x))^(3/2), x)
```

3.341 $\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 344

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{ab^2d}$$

$$+\frac{2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{abd}$$

$$+\frac{2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d}$$

$$+\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b
))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/a/b^2/d+2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2)
,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/a/b/d+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b
))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*tan(d*x+c)/a/d/(a+b*sec(d*
x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3979, 4146, 4144, 4006, 3869, 3917, 4089}

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{}}{}\right.\right.}{a^2d}$$
$$+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ab^2d}$$
$$+\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{abd}$$
$$+\frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}$$

```
[In] Int[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b^2*d) + (2*Sqrt[a + b]*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(a*b*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*Tan[c +
d*x])/(a*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4144

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*
x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4146

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \frac{-1+\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(a^2-b^2)+\frac{1}{2}(a^2-b^2)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a(a^2-b^2)}$$

$$= \frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx}{a} - \frac{2\int \frac{\frac{1}{2}(a^2-b^2)-\frac{1}{2}(a^2-b^2)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}$$

$$= \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ab^2d} + \frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx}{a} + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a}$$

$$= \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ab^2d} + \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{abd} + \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d} + \frac{2\tan(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] (warning: unable to verify)

Time = 11.18 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.96

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx = \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(-\frac{2\sin(c+dx)}{ab}+\frac{2\sin(c+dx)}{a(b+a\cos(c+dx))}\right)}{d(a+b\sec(c+dx))^{3/2}} + \frac{4\sqrt{\frac{\cos(c+dx)}{(1+\cos(c+dx))^2}}(b+a\cos(c+dx))\sqrt{\sec(c+dx)}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left((a+b)E\left(\arcsin\left(\tan\left(\right.\right.\right.\right.}{}$$

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*Sin[c + d*x])/(a*b) + (2*Sin[c
 + d*x])/(a*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(3/2)) + (4*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]
*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((a + b)*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)] - 2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (b + a*Cos[c + d*x]
)*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))/(a*b*d*(a + b*Se
c[c + d*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(315) = 630$.

Time = 9.79 (sec) , antiderivative size = 828, normalized size of antiderivative = 2.41

method	result	size
default	Expression too large to display	828

```
[In] int(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/b/a*(EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*cos
(d*x+c)^2+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*cos
(d*x+c)^2-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))
*b*cos(d*x+c)^2+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*a*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*b*cos(d*x+c)-4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^
(1/2))*b*cos(d*x+c)+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*a+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b-2*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c
),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b+a*sin(d*x+c)*
cos(d*x+c)-cos(d*x+c)*sin(d*x+c)*b)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))
/(cos(d*x+c)+1)
```

Fricas [F]

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*tan(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan^2{(c+dx)}}{(a+b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(tan(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan{(dx+c)}^2}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\tan{(dx+c)}^2}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tan(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{{\tan(c+d\,x)}^2}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

```
[In] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(tan(c + d*x)^2/(a + b/cos(c + d*x))^(3/2), x)
```

3.342 $\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 347

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$-\frac{2\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$-\frac{2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d}$$
$$+\frac{2b^2\tan(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b
)^(1/2)-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/
a/d/(a+b)^(1/2)-2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(
d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3870, 4143, 4006, 3869, 3917, 4089}

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{a^2d}$$
$$+\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$
$$-\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}$$
$$+\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}$$

```
[In] Int[(a + b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (
2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tan[c + d*x])/(a*(
a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
```

```
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2\int\frac{\frac{1}{2}(-a^2+b^2)+\frac{1}{2}ab\sec(c+dx)+\frac{1}{2}b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\,(a^2-b^2)} \\
&= \frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} \\
&\quad - \frac{2\int\frac{\frac{1}{2}(-a^2+b^2)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\,(a^2-b^2)} - \frac{b^2\int\frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\,(a^2-b^2)}
\end{aligned}
$$

$$= \frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$+ \frac{2b^2\tan(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx}{a} - \frac{b\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a+b)}$$
$$= \frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$- \frac{2\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$- \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx}{a-b}}}{a^2d}$$
$$+ \frac{2b^2\tan(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 972 vs. $2(347) = 694$.

Time = 6.08 (sec) , antiderivative size = 972, normalized size of antiderivative = 2.80

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}dx = \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(\frac{2b\sin(c+dx)}{a(-a^2+b^2)}+\frac{2b^2\sin(c+dx)}{a(a^2-b^2)(b+a\cos(c+dx))}\right)}{d(a+b\sec(c+dx))^{3/2}}$$
$$+ \frac{2(b+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sqrt{\frac{a+b-a\tan^2\left(\frac{1}{2}(c+dx)\right)+b\tan^2\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(ab\tan\left(\frac{1}{2}(c+dx)\right)+b^2\tan\left(\frac{1}{2}(c+\right.\right.}{}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b*Sin[c + d*x])/(a*(-a^2 + b^2))
 + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Se
c[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)]*(a*b*Tan[(c + d*x)/2] + b^2*Tan[(c + d*x)/2] - 2*a*b*Tan[(c + d*x)/2]^
3 + a*b*Tan[(c + d*x)/2]^5 - b^2*Tan[(c + d*x)/2]^5 + 2*a^2*EllipticPi[-1,
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b^2*El
lipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
)] + 2*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c
 + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
```

```
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + b*(a +
 b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] - a*(a + b)*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Ta
n[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) -
b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(318) = 636$.

Time = 7.08 (sec) , antiderivative size = 1799, normalized size of antiderivative = 5.18

method	result	size
default	Expression too large to display	1799

```
[In] int(1/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a/(a+b)/(a-b)*(-((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*cs
c(d*x+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^2
*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^
2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+
b))^(1/2))*a^2-((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x
+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^2*csc(
d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc
(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(
1/2))*a*b+((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2
-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^2*csc(d*x+c
)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+
c)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))
*a*b+((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^2*csc(d*x+c)^2+1
)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-
a-b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^2+
2*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)*((
1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(
1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b
)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2
-2*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)*(
(1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^
(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-
```

```
b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^
2-((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-c
os(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)*a*b*(1-cos(d*x+c))^3*csc(d*x+c)^3+((1-
cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x
+c))^2*csc(d*x+c)^2+a+b)^(1/2)*b^2*(1-cos(d*x+c))^3*csc(d*x+c)^3+((1-cos(d*
x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2
*csc(d*x+c)^2+a+b)^(1/2)*a*b*(-cot(d*x+c)+csc(d*x+c))-((1-cos(d*x+c))^4*a*c
sc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)
^2+a+b)^(1/2)*b^2*(-cot(d*x+c)+csc(d*x+c)))*((a*(1-cos(d*x+c))^2*csc(d*x+c)
^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^
(1/2)/((1-cos(d*x+c))^4*a*csc(d*x+c)^4-(1-cos(d*x+c))^4*b*csc(d*x+c)^4-2*a*
(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(1/2)/(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b
*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
```

Fricas [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c)
+ a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(1/(a + b/cos(c + d*x))^(3/2), x)
```

3.343 $\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 449

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d} \\
&+ \frac{2(a^2+b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{-\frac{b(-1+\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a(a-b)(a+b)^{3/2}d} \\
&- \frac{(a^2-ab+2b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{-\frac{b(-1+\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a(a-b)(a+b)^{3/2}d} \\
&- \frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}} + \frac{b^2\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} \\
&+ \frac{2b^2(a^2+b^2)\tan(c+dx)}{a\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}}
\end{aligned}
$$

```
[Out] -cot(d*x+c)/d/(a+b*sec(d*x+c))^(3/2)+2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x
+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*(a^2+b^2)*cot(d*x
+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(-b*(
-1+sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(
3/2)/d-(a^2-a*b+2*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(-b*(-1+sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d+b^2*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*
x+c))^(3/2)+2*b^2*(a^2+b^2)*tan(d*x+c)/a/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/
2)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3981, 3870, 4143, 4006, 3869, 3917, 4089, 3960, 3918, 4088, 4090}

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a}}{}\right.\right.}{a^2d}$$
$$-\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{4ab^2\tan(c+dx)}{d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}$$
$$+\frac{b^2\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$
$$+\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}$$
$$-\frac{(3a-b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d(a-b)(a+b)^{3/2}}$$
$$-\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}$$
$$+\frac{4a\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d(a-b)(a+b)^{3/2}}$$
$$-\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}}$$

```
[In] Int[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*a*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) - (2*Cot[c + d*x]*EllipticE[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a +
 b]*d) - ((3*a - b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/
Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-(
(b*(1 + Sec[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) + (2*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(a*Sqrt[a + b]*d) + (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) -
Cot[c + d*x]/(d*(a + b*Sec[c + d*x])^(3/2)) + (b^2*Tan[c + d*x])/((a^2 - b^
2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*a*b^2*Tan[c + d*x])/((a^2 - b^2)^2*d*
```

```
Sqrt[a + b*Sec[c + d*x]]) - (2*b^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3981

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
```

```
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{(a+b\sec(c+dx))^{3/2}}+\frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\right)\,dx\\
&= -\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx+\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx\\
&= -\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}}-\frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}\\
&\quad -\frac{1}{2}(3b)\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx+\frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\frac{1}{2}ab\sec(c+dx)+\frac{1}{2}b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}\\
&= -\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}}+\frac{b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}\\
&\quad -\frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+\frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}\\
&\quad +\frac{b\int \frac{\sec(c+dx)\left(-\frac{3a}{2}+\frac{1}{2}b\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx}{a^2-b^2}+\frac{b^2\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}\\
&= -\frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}\\
&\quad -\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}}+\frac{b^2\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}\\
&\quad +\frac{4ab^2\tan(c+dx)}{\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}}-\frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}\\
&\quad -\frac{\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx}{a}+\frac{b\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a(a+b)}-\frac{(2b)\int \frac{\sec(c+dx)\left(\frac{1}{4}\left(3a^2+b^2\right)+ab\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{\left(a^2-b^2\right)^2}
\end{aligned}
$$

$$= -\frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$+\frac{2\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$+\frac{2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d}$$
$$-\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}}+\frac{b^2\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}}$$
$$+\frac{4ab^2\tan(c+dx)}{(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}-\frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$
$$-\frac{((3a-b)b)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{2(a-b)(a+b)^2}-\frac{(2ab^2)\int\frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{(a^2-b^2)^2}$$

$$=\frac{4a\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{(a-b)(a+b)^{3/2}d}$$
$$-\frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$-\frac{(3a-b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{(a-b)(a+b)^{3/2}d}$$
$$+\frac{2\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$+\frac{2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d}$$
$$-\frac{\cot(c+dx)}{d(a+b\sec(c+dx))^{3/2}}+\frac{b^2\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}}$$
$$+\frac{4ab^2\tan(c+dx)}{(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}-\frac{2b^2\tan(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] (verified)

Time = 10.81 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.04

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(-a(b+a\cos(c+dx))\left(-2ab+(a^2+b^2)\cos\right.\right.}{}$$

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(-(a*(b + a*Cos[c + d*x])*(-2*a*b + (a
^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]) + 2*b^4*Sin[c + d*x] - 2*b*(a^2 + b^2
)*(b + a*Cos[c + d*x])*Sin[c + d*x] + 2*Cos[(c + d*x)/2]^2*(2*b*(a^3 + a^2*
b + a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)] + a*(2*a^3 - a^2*b - 6*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 4*(a^2 - b^2)^2*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(a
^2 + b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2])))/(a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2061 vs. 2(416) = 832.

Time = 8.20 (sec) , antiderivative size = 2062, normalized size of antiderivative = 4.59

method	result	size
default	Expression too large to display	2062

```
[In] int(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a/(a-b)^2/(a+b)^2*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))*(-8*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2*cos(d*x+c)-a
^3*b*cot(d*x+c)+2*a^2*b^2*cot(d*x+c)-8*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,
((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+
c),-1,((a-b)/(a+b))^(1/2))*b^4*cos(d*x+c)-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc
(d*x+c),((a-b)/(a+b))^(1/2))*b^4*cos(d*x+c)-2*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-c
sc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*cos(d*x+c)+4*(cos(d*x+c)/(cos(d*x+c)+1))
```

```
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c
)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*cos(d*x+c)-2*EllipticF(cot(d*x+c)-
csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+4*EllipticPi(cot(d*x+c)-csc(d*x+
c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+4*EllipticPi(cot(d*x+c)-csc(d*x+c),-1
,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*b^4-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*a^3*b-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*a^2*b^2-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot
(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
*b^3+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-c
sc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b+6*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+
c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2+3*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3-3*a*b^3*cot(d*x+
c)+2*a^3*b*cos(d*x+c)*cot(d*x+c)-a^2*b^2*cos(d*x+c)*cot(d*x+c)+2*a*b^3*cos(
d*x+c)*cot(d*x+c)+EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*a^3*b*cos(d*x+c)+6*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*a^2*b^2*cos(d*x+c)+3*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*a*b^3*cos(d*x+c)-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-
b)/(a+b))^(1/2))*a^3*b*cos(d*x+c)-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*b^2*cos(d*x+c)-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))*a*b^3*cos(d*x+c)-a^4*cos(d*x+c)*cot(d*x+c)-2*b^4
*cos(d*x+c)*cot(d*x+c)+2*b^4*cot(d*x+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cot(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(cot(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^2}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^2/(a + b/cos(c + d*x))^(3/2), x)
```

3.344 $\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 23, antiderivative size = 245

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx = \frac{3ab^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{a^3\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{3a^2b\cos^2(e+fx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{b^3\cos^2(e+fx)^{\frac{4+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{4+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec^3(e+fx)(d\tan(e+fx))^{1+n}}{df(1+n)}$$

```
[Out] 3*a*b^2*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+a^3*hypergeom([1, 1/2+1/2*n],[3/2+1/
2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+3*a^2*b*(cos(f*x+e)^2)^(
1+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*sec(f*x+e
)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+b^3*(cos(f*x+e)^2)^(2+1/2*n)*hypergeom([2+
1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*sec(f*x+e)^3*(d*tan(f*x+e))^(1+
n)/d/f/(1+n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

$= \{3971, 3557, 371, 2697, 2687, 32\}$

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx$$
$$= \frac{a^3(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(e+fx)\right)}{df(n+1)}$$
$$+\frac{3a^2b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{df(n+1)}$$
$$+\frac{3ab^2(d\tan(e+fx))^{n+1}}{df(n+1)}$$
$$+\frac{b^3\sec^3(e+fx)\cos^2(e+fx)^{\frac{n+4}{2}}(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+4}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{df(n+1)}$$

```
[In] Int[(a + b*Sec[e + f*x])^3*(d*Tan[e + f*x])^n,x]
```

```
[Out] (3*a*b^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (a^3*Hypergeometric2F1[1
, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1
+ n)) + (3*a^2*b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2,
(2 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e + f*x])^(1 + n)
)/(d*f*(1 + n)) + (b^3*(Cos[e + f*x]^2)^((4 + n)/2)*Hypergeometric2F1[(1 +
n)/2, (4 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(d*Tan[e + f*x])
^(1 + n))/(d*f*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
```

```
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3971

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3(d\tan(e+fx))^n+3a^2b\sec(e+fx)(d\tan(e+fx))^n\right.\\
&\qquad \left.+3ab^2\sec^2(e+fx)(d\tan(e+fx))^n+b^3\sec^3(e+fx)(d\tan(e+fx))^n\right)\,dx\\
&= a^3\int(d\tan(e+fx))^n\,dx+\left(3a^2b\right)\int\sec(e+fx)(d\tan(e+fx))^n\,dx\\
&\quad+\left(3ab^2\right)\int\sec^2(e+fx)(d\tan(e+fx))^n\,dx+b^3\int\sec^3(e+fx)(d\tan(e+fx))^n\,dx\\
&= \frac{3a^2b\cos^2(e+fx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+fx)}{df(1+n)}\\
&\quad+\frac{b^3\cos^2(e+fx)^{\frac{4+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{4+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec^3(e+fx)(d\tan(e+fx}{df(1+n)}\\
&\quad+\frac{\left(3ab^2\right)\text{Subst}\left(\int(dx)^n\,dx,x,\tan(e+fx)\right)}{f}\\
&\quad+\frac{\left(a^3d\right)\text{Subst}\left(\int\frac{x^n}{d^2+x^2}\,dx,x,d\tan(e+fx)\right)}{f}\\
&= \frac{3ab^2(d\tan(e+fx))^{1+n}}{df(1+n)}\\
&\quad+\frac{a^3\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)}\\
&\quad+\frac{3a^2b\cos^2(e+fx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+f}{df(1+n)}\\
&\quad+\frac{b^3\cos^2(e+fx)^{\frac{4+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{4+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec^3(e+fx)(d\tan(e+fx}{df(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.97

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx$$
$$= \frac{d(d\tan(e+fx))^{-1+n}\left(-\tan^2(e+fx)\right)^{-n/2}\left(9a^2b(1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3}{2},\sec^2(e+fx)\right)\sec(\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^3*(d*Tan[e + f*x])^n,x]
```

```
[Out] (d*(d*Tan[e + f*x])^(-1 + n)*(9*a^2*b*(1 + n)*Hypergeometric2F1[1/2, (1 - n
)/2, 3/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2] + b^3*(1 + n)*
Hypergeometric2F1[3/2, (1 - n)/2, 5/2, Sec[e + f*x]^2]*Sec[e + f*x]^3*Sqrt[
-Tan[e + f*x]^2] - 3*a^3*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e
+ f*x]^2]*(-Tan[e + f*x]^2)^((2 + n)/2) + 9*a*b^2*(Sqrt[-Tan[e + f*x]^2] -
(-Tan[e + f*x]^2)^((2 + n)/2))))/(3*f*(1 + n)*(-Tan[e + f*x]^2)^(n/2))
```

Maple [F]

$$\int (a+b\sec(fx+e))^3\,(d\tan(fx+e))^n\,dx$$

```
[In] int((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x)
```

```
[Out] int((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx = \int (b\sec(fx+e)+a)^3(d\tan(fx+e))^n\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e
) + a^3)*(d*tan(f*x + e))^n, x)
```

Sympy [F]

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx = \int (d\tan{(e+fx)})^n\,(a+b\sec{(e+fx)})^3\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**3*(d*tan(f*x+e))**n,x)
[Out] Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

Maxima [F]

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx = \int (b\sec{(fx+e)}+a)^3(d\tan{(fx+e)})^n\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

Giac [F]

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx = \int (b\sec{(fx+e)}+a)^3(d\tan{(fx+e)})^n\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^3*(d*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^3*(d*tan(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(e+fx))^3(d\tan(e+fx))^n\,dx = \int (d\tan(e+f\,x))^n\left(a+\frac{b}{\cos{(e+f\,x)}}\right)^3\,dx$$

```
[In] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^3,x)
[Out] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^3, x)
```

3.345 $\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx=\frac{b^2(d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{a^2\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)}+\frac{2ab\cos^2(e+fx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+fx))^{1+n}}{df(1+n)}$$

```
[Out] b^2*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+a^2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n]
,-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)+2*a*b*(cos(f*x+e)^2)^(1+1/2*
n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*sec(f*x+e)*(d*t
an(f*x+e))^(1+n)/d/f/(1+n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3971, 3557, 371, 2697, 2687, 32}

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx$$

$$=\frac{a^2(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(e+fx)\right)}{df(n+1)}+\frac{2ab\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{df(n+1)}+\frac{b^2(d\tan(e+fx))^{n+1}}{df(n+1)}$$

[In] Int[(a + b*Sec[e + f*x])^2*(d*Tan[e + f*x])^n,x]

[Out] (b^2*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (a^2*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n)) + (2*a*b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e + f*x])^(1 + n))/(d*f*(1 + n))

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 371

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 2687

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 2697

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e + f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]

Rule 3557

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3971

Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]

Rubi steps

$$\text{integral} = \int \left(a^2(d\tan(e+fx))^n + 2ab\sec(e+fx)(d\tan(e+fx))^n + b^2\sec^2(e+fx)(d\tan(e+fx))^n\right)\,dx$$

$$= a^2\int(d\tan(e+fx))^n\,dx + (2ab)\int\sec(e+fx)(d\tan(e+fx))^n\,dx + b^2\int\sec^2(e+fx)(d\tan(e+fx))^n\,dx$$

$$= \frac{2ab\cos^2(e+fx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{b^2\text{Subst}\left(\int(dx)^n\,dx,x,\tan(e+fx)\right)}{f} + \frac{(a^2d)\,\text{Subst}\left(\int\frac{x^n}{d^2+x^2}\,dx,x,d\tan(e+fx)\right)}{f}$$

$$= \frac{b^2(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{a^2\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{2ab\cos^2(e+fx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+fx))^{1+n}}{df(1+n)}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.11

$$\int(a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx = \frac{d(d\tan(e+fx))^{-1+n}\left(-\tan^2(e+fx)\right)^{-n/2}\left(2ab(1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3}{2},\sec^2(e+fx)\right)\sec(e\right.}{}$$

```
[In] Integrate[(a + b*Sec[e + f*x])^2*(d*Tan[e + f*x])^n,x]
```

```
[Out] (d*(d*Tan[e + f*x])^(-1 + n)*(2*a*b*(1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, 3/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2] - a^2*Hypergeome
tric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(-Tan[e + f*x]^2)^((2 + n
)/2) + b^2*(Sqrt[-Tan[e + f*x]^2] - (-Tan[e + f*x]^2)^((2 + n)/2))))/(f*(1
+ n)*(-Tan[e + f*x]^2)^(n/2))
```

Maple [F]

$$\int (a+b\sec(fx+e))^2\,(d\tan(fx+e))^n\,dx$$

```
[In] int((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x)
[Out] int((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx = \int (b\sec(fx+e)+a)^2(d\tan(fx+e))^n\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*tan(f*x + e))^n
, x)
```

Sympy [F]

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx = \int (d\tan(e+fx))^n\,(a+b\sec(e+fx))^2\,dx$$

```
[In] integrate((a+b*sec(f*x+e))**2*(d*tan(f*x+e))**n,x)
[Out] Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx = \int (b\sec(fx+e)+a)^2(d\tan(fx+e))^n\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

Giac [F]

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx = \int (b\sec(fx+e)+a)^2(d\tan(fx+e))^n\,dx$$

```
[In] integrate((a+b*sec(f*x+e))^2*(d*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^2*(d*tan(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(e+fx))^2(d\tan(e+fx))^n\,dx = \int (d\tan(e+f\,x))^n\left(a+\frac{b}{\cos(e+f\,x)}\right)^2\,dx$$

```
[In] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^2,x)
[Out] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x))^2, x)
```

3.346 $\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 129

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx = \frac{a\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)} + \frac{b\cos^2(e+fx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+fx))^{1+n}}{df(1+n)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(1+n)/
d/f/(1+n)+b*(cos(f*x+e)^2)^(1+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/
2*n],sin(f*x+e)^2)*sec(f*x+e)*(d*tan(f*x+e))^(1+n)/d/f/(1+n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3969, 3557, 371, 2697}

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx = \frac{a(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(e+fx)\right)}{df(n+1)} + \frac{b\sec(e+fx)\cos^2(e+fx)^{\frac{n+2}{2}}(d\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{df(n+1)}$$

```
[In] Int[(a + b*Sec[e + f*x])*(d*Tan[e + f*x])^n,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e + f*x]^2]*(d*Tan[e + f
*x])^(1 + n))/(d*f*(1 + n)) + (b*(Cos[e + f*x]^2)^((2 + n)/2)*Hypergeometri
c2F1[(1 + n)/2, (2 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(d*Tan[e
 + f*x])^(1 + n))/(d*f*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3969

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(e*Cot[c + d*x])^m, x], x] + Dist[b, Int[(e
*Cot[c + d*x])^m*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= a\int (d\tan(e+fx))^n\,dx + b\int \sec(e+fx)(d\tan(e+fx))^n\,dx\\
&= \frac{b\cos^2(e+fx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+fx))^{1+n}}{df(1+n)}\\
&\quad+\frac{(ad)\text{Subst}\left(\int\frac{x^n}{d^2+x^2}\,dx,x,d\tan(e+fx)\right)}{f}\\
&= \frac{a\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)(d\tan(e+fx))^{1+n}}{df(1+n)}\\
&\quad+\frac{b\cos^2(e+fx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)\sec(e+fx)(d\tan(e+fx))^{1+n}}{df(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx$$
$$=\frac{(d\tan(e+fx))^n\left(\frac{a\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)\tan(e+fx)}{1+n}+b\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{}\right.\right.}{f}$$

```
[In] Integrate[(a + b*Sec[e + f*x])*(d*Tan[e + f*x])^n,x]
```

```
[Out] ((d*Tan[e + f*x])^n*((a*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[e +
 f*x]^2]*Tan[e + f*x])/(1 + n) + b*Csc[e + f*x]*Hypergeometric2F1[1/2, (1 -
 n)/2, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^((1 - n)/2)))/f
```

Maple [F]

$$\int (a+b\sec(fx+e))\,(d\tan(fx+e))^n\,dx$$

```
[In] int((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x)
```

```
[Out] int((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx=\int (b\sec(fx+e)+a)(d\tan(fx+e))^n\,dx$$

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Sympy [F]

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx=\int (d\tan(e+fx))^n\,(a+b\sec(e+fx))\,dx$$

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))**n,x)
```

```
[Out] Integral((d*tan(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx = \int (b\sec(fx+e)+a)(d\tan(fx+e))^n\,dx$$

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Giac [F]

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx = \int (b\sec(fx+e)+a)(d\tan(fx+e))^n\,dx$$

```
[In] integrate((a+b*sec(f*x+e))*(d*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)*(d*tan(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(e+fx))(d\tan(e+fx))^n\,dx = \int (d\tan(e+f\,x))^n\left(a+\frac{b}{\cos(e+f\,x)}\right)\,dx$$

```
[In] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x)),x)
[Out] int((d*tan(e + f*x))^n*(a + b/cos(e + f*x)), x)
```

3.347 $\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 266

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$
$$= \frac{d\,\text{AppellF1}\left(1-n,\frac{1-n}{2},\frac{1-n}{2},2-n,\frac{a+b}{a+b\sec(e+fx)},\frac{a-b}{a+b\sec(e+fx)}\right)\left(-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}\right)^{\frac{1-n}{2}}\left(\frac{b(1+\sec(e+fx))}{a+b\sec(e+fx)}\right)^{\frac{1-n}{2}}(d\,\text{ta}}{af(1-n)}$$
$$-\frac{d\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)(d\tan(e+fx))^{-1+n}\left(-\tan^2(e+fx)\right)^{\frac{1-n}{2}+\frac{1+n}{2}}}{af(1+n)}$$

```
[Out] d*AppellF1(1-n,1/2-1/2*n,1/2-1/2*n,2-n,(a-b)/(a+b*sec(f*x+e)),(a+b)/(a+b*se
c(f*x+e)))*(-b*(1-sec(f*x+e))/(a+b*sec(f*x+e)))^(1/2-1/2*n)*(b*(1+sec(f*x+e
))/(a+b*sec(f*x+e)))^(1/2-1/2*n)*(d*tan(f*x+e))^(-1+n)/a/f/(1-n)+d*hypergeo
m([1, 1/2+1/2*n],[3/2+1/2*n],-tan(f*x+e)^2)*(d*tan(f*x+e))^(-1+n)*tan(f*x+e
)^2/a/f/(1+n)
```

Rubi [F]

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

```
[In] Int[(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]),x]
[Out] Defer[Int][(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 786 vs. $2(266) = 532$.

Time = 4.22 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.95

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

$$= \frac{}{f(a+b\sec(e+fx))\left(\left((a+b)\operatorname{AppellF1}\left(\frac{1+n}{2},n,1,\frac{3+n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-b\operatorname{Appel}\right.\right.}$$

```
[In] Integrate[(d*Tan[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] (2*((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2,
 ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]*(d*Tan[e + f*x])^n
)/(f*(a + b*Sec[e + f*x])*(((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n
)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Sec[(e + f*
x)/2]^2 - 16*n*((a + b)*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(e
+ f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Cos[(e + f*x)/2]*Csc[e
+ f*x]^3*Sec[e + f*x]*Sin[(e + f*x)/2]^5 + 2*n*((a + b)*AppellF1[(1 + n)/2,
 n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*AppellF1[(1
+ n)/2, n, 1, (3 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(
a + b)])*Csc[e + f*x]*Sec[e + f*x]*Tan[(e + f*x)/2] - (2*(1 + n)*((a - b)*b
*AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e +
 f*x)/2]^2)/(a + b)] + (a + b)^2*(AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 + n, 1, (5 +
 n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]) + b*(a + b)*n*AppellF1[(3
+ n)/2, 1 + n, 1, (5 + n)/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^
2)/(a + b)])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^2)/((a + b)*(3 + n))))
```

Maple [F]

$$\int \frac{(d\tan(fx+e))^n}{a+b\sec(fx+e)}dx$$

```
[In] int((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

```
[Out] int((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\tan(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

```
[In] integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
[Out] integral((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

```
[In] integrate((d*tan(f*x+e))**n/(a+b*sec(f*x+e)),x)
[Out] Integral((d*tan(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\tan(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

```
[In] integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\tan(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

```
[In] integrate((d*tan(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
[Out] integrate((d*tan(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^n}{a+b\sec(e+fx)}\,dx=\int \frac{\cos\left(e+f\,x\right)\,\left(d\tan(e+f\,x)\right)^n}{b+a\,\cos\left(e+f\,x\right)}\,dx$$

```
[In] int((d*tan(e + f*x))^n/(a + b/cos(e + f*x)),x)
[Out] int((cos(e + f*x)*(d*tan(e + f*x))^n)/(b + a*cos(e + f*x)), x)
```

3.348 $\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx=\mathrm{Int}\big((a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m,x\big)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx=\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

Rubi steps

$$\text{integral}=\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

Mathematica [N/A]

Not integrable

Time = 29.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx=\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m,x]
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Tan[c + d*x])^m, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\left(e\tan\left(dx+c\right)\right)^{m}dx$$

```
[In] int((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx=\int \left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\left(e\tan\left(dx+c\right)\right)^{m}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Sympy [N/A]

Not integrable

Time = 69.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx=\int \left(e\tan\left(c+dx\right)\right)^{m}\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(e*tan(d*x+c))**m,x)
[Out] Integral((e*tan(c + d*x))**m*(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Giac [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\tan(dx+c))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*tan(d*x + c))^m, x)
```

Mupad [N/A]

Not integrable

Time = 19.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b\sec(c+dx))^{3/2}(e\tan(c+dx))^m\,dx = \int (e\tan(c+d\,x))^m\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

```
[In] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(3/2),x)
[Out] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(3/2), x)
```

3.349 $\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx = \operatorname{Int}\left(\sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx$$

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
```

```
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

Rubi steps

$$\text{integral} = \int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx=\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx$$

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m,x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Tan[c + d*x])^m, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt{a+b\sec\left(dx+c\right)}\left(e\tan\left(dx+c\right)\right)^{m}dx$$

```
[In] int((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx=\int \sqrt{b\sec\left(dx+c\right)+a}\left(e\tan\left(dx+c\right)\right)^{m}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Sympy [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx=\int \left(e\tan\left(c+dx\right)\right)^{m}\sqrt{a+b\sec\left(c+dx\right)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**(1/2)*(e*tan(d*x+c))**m,x)
[Out] Integral((e*tan(c + d*x))**m*sqrt(a + b*sec(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}(e\tan\left(dx+c\right))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}(e\tan\left(dx+c\right))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^(1/2)*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m, x)
```

Mupad [N/A]

Not integrable

Time = 14.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{a+b\sec(c+dx)}(e\tan(c+dx))^m\,dx = \int (e\tan(c+d\,x))^m\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(1/2),x)
[Out] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^(1/2), x)
```

3.350 $\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \operatorname{Int}\left(\frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

```
[In] Int[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\text{integral} = \int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 15.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

```
[In] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[(e*Tan[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(e\tan(dx+c))^m}{\sqrt{a+b\sec(dx+c)}}dx$$

```
[In] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
[Out] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{(e\tan(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

```
[In] integrate((e*tan(d*x+c))**m/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral((e*tan(c + d*x))**m/sqrt(a + b*sec(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\tan(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\tan(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 20.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\tan(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\tan(c+d\,x))^m}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(1/2), x)
```

3.351 $\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx=\operatorname{Int}\left(\frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

```
[In] Int[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\text{integral}=\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 19.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

```
[In] Integrate[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[(e*Tan[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(e\tan(dx+c))^m}{(a+b\sec(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
[Out] int((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(dx+c))^m}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*tan(d*x + c))^m/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 4.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan{(c+dx)})^m}{(a+b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*tan(d*x+c))**m/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((e*tan(c + d*x))**m/(a + b*sec(c + d*x))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(dx+c))^m}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((e*tan(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\tan(dx+c))^m}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*tan(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((e*tan(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 28.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(e\tan(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{(e\tan(c+d\,x))^m}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((e*tan(c + d*x))^m/(a + b/cos(c + d*x))^(3/2), x)
```

3.352 $\int (a+b\sec(c+dx))^n (e\tan(c+dx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sec(c+dx))^n (e\tan(c+dx))^m\,dx = \text{Int}((a+b\sec(c+dx))^n (e\tan(c+dx))^m, x)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n (e\tan(c+dx))^m\,dx = \int (a+b\sec(c+dx))^n (e\tan(c+dx))^m\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

Rubi steps

$$\text{integral} = \int (a+b\sec(c+dx))^n (e\tan(c+dx))^m\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*(e*Tan[c + d*x])^m, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{n}\left(e\tan\left(dx+c\right)\right)^{m}dx$$

```
[In] int((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \int \left(b\sec\left(dx+c\right)+a\right)^{n}\left(e\tan\left(dx+c\right)\right)^{m}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Sympy [N/A]

Not integrable

Time = 33.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \int \left(e\tan\left(c+dx\right)\right)^{m}\left(a+b\sec\left(c+dx\right)\right)^{n}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n*(e*tan(d*x+c))**m,x)
[Out] Integral((e*tan(c + d*x))**m*(a + b*sec(c + d*x))**n, x)
```

Maxima [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \int (b\sec\left(dx+c\right)+a)^n(e\tan\left(dx+c\right))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Giac [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \int (b\sec\left(dx+c\right)+a)^n(e\tan\left(dx+c\right))^m\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*(e*tan(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*(e*tan(d*x + c))^m, x)
```

Mupad [N/A]

Not integrable

Time = 15.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int (a+b\sec(c+dx))^n(e\tan(c+dx))^m\,dx = \int (e\tan(c+d\,x))^m\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

```
[In] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^n,x)
[Out] int((e*tan(c + d*x))^m*(a + b/cos(c + d*x))^n, x)
```

3.353 $\int (a+b\sec(c+dx))^n \tan^5(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 177

$$\begin{aligned}
&\int (a+b\sec(c+dx))^n \tan^5(c+dx)\,dx \\
&= -\frac{a(a^2-2b^2)\,(a+b\sec(c+dx))^{1+n}}{b^4d(1+n)} \\
&\quad - \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} \\
&\quad + \frac{(3a^2-2b^2)\,(a+b\sec(c+dx))^{2+n}}{b^4d(2+n)} - \frac{3a(a+b\sec(c+dx))^{3+n}}{b^4d(3+n)} + \frac{(a+b\sec(c+dx))^{4+n}}{b^4d(4+n)}
\end{aligned}$$

```
[Out] -a*(a^2-2*b^2)*(a+b*sec(d*x+c))^(1+n)/b^4/d/(1+n)-hypergeom([1, 1+n],[2+n],
1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a/d/(1+n)+(3*a^2-2*b^2)*(a+b*sec(d
*x+c))^(2+n)/b^4/d/(2+n)-3*a*(a+b*sec(d*x+c))^(3+n)/b^4/d/(3+n)+(a+b*sec(d*
x+c))^(4+n)/b^4/d/(4+n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

$= \{3970, 966, 1634, 67\}$

$$\int (a+b\sec(c+dx))^n \tan^5(c+dx)\,dx$$
$$= -\frac{a(a^2-2b^2)\,(a+b\sec(c+dx))^{n+1}}{b^4d(n+1)} + \frac{(3a^2-2b^2)\,(a+b\sec(c+dx))^{n+2}}{b^4d(n+2)}$$
$$- \frac{3a(a+b\sec(c+dx))^{n+3}}{b^4d(n+3)} + \frac{(a+b\sec(c+dx))^{n+4}}{b^4d(n+4)}$$
$$- \frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] -((a*(a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(1 + n))/(b^4*d*(1 + n))) - (Hyperg
eometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(
1 + n))/(a*d*(1 + n)) + ((3*a^2 - 2*b^2)*(a + b*Sec[c + d*x])^(2 + n))/(b^4
*d*(2 + n)) - (3*a*(a + b*Sec[c + d*x])^(3 + n))/(b^4*d*(3 + n)) + (a + b*S
ec[c + d*x])^(4 + n)/(b^4*d*(4 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+x)^n (b^2-x^2)^2}{x}\,dx, x, b\sec(c+dx)\right)}{b^4 d}\\
&= \frac{(a+b\sec(c+dx))^{4+n}}{b^4 d(4+n)}\\
&\quad+\frac{\text{Subst}\left(\int \frac{(a+x)^n (b^4(4+n)-a^3(4+n)x-(3a^2+2b^2)(4+n)x^2-3a(4+n)x^3)}{x}\,dx, x, b\sec(c+dx)\right)}{b^4 d(4+n)}\\
&= \frac{(a+b\sec(c+dx))^{4+n}}{b^4 d(4+n)}\\
&\quad+\frac{\text{Subst}\left(\int \left(-a(a^2-2b^2)(4+n)(a+x)^n+\frac{(4b^4+b^4n)(a+x)^n}{x}+(3a^2-2b^2)(4+n)(a+x)^{1+n}-3a\right.\right.}{b^4 d(4+n)}\\
&= -\frac{a(a^2-2b^2)(a+b\sec(c+dx))^{1+n}}{b^4 d(1+n)}+\frac{(3a^2-2b^2)(a+b\sec(c+dx))^{2+n}}{b^4 d(2+n)}\\
&\quad-\frac{3a(a+b\sec(c+dx))^{3+n}}{b^4 d(3+n)}+\frac{(a+b\sec(c+dx))^{4+n}}{b^4 d(4+n)}\\
&\quad+\frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= -\frac{a(a^2-2b^2)(a+b\sec(c+dx))^{1+n}}{b^4 d(1+n)}\\
&\quad-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}\\
&\quad+\frac{(3a^2-2b^2)(a+b\sec(c+dx))^{2+n}}{b^4 d(2+n)}\\
&\quad-\frac{3a(a+b\sec(c+dx))^{3+n}}{b^4 d(3+n)}+\frac{(a+b\sec(c+dx))^{4+n}}{b^4 d(4+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.77

$$\int (a+b\sec(c+dx))^n\tan^5(c+dx)\,dx = \frac{(a+b\sec(c+dx))^{1+n}\left(-\frac{a^3-2ab^2}{1+n}-\frac{b^4\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)}{a+an}+\frac{(3a^2-2b^2)(a+b\sec(c+dx))}{2+n}-\frac{3a(a+b\sec(c+dx))^2}{3+n}+\frac{(a+b\sec(c+dx))^3}{4+n}\right)}{b^4d}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^5,x]
```

```
[Out] ((a + b*Sec[c + d*x])^(1 + n)*(-((a^3 - 2*a*b^2)/(1 + n)) - (b^4*Hypergeome
tric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a])/(a + a*n) + ((3*a^2 - 2*b
^2)*(a + b*Sec[c + d*x]))/(2 + n) - (3*a*(a + b*Sec[c + d*x])^2)/(3 + n) +
(a + b*Sec[c + d*x])^3/(4 + n)))/(b^4*d)
```

Maple [F]

$$\int (a+b\sec(dx+c))^n\tan(dx+c)^5\,dx$$

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x)
```

Fricas [F]

$$\int (a+b\sec(c+dx))^n\tan^5(c+dx)\,dx = \int (b\sec(dx+c)+a)^n\tan(dx+c)^5\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^n\tan^5(c+dx)\,dx = \int (a+b\sec(c+dx))^n\tan^5(c+dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**5,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**5, x)
```

Maxima [F]

$$\int (a+b\sec(c+dx))^n\tan^5(c+dx)\,dx=\int (b\sec(dx+c)+a)^n\tan(dx+c)^5\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Giac [F]

$$\int (a+b\sec(c+dx))^n\tan^5(c+dx)\,dx=\int (b\sec(dx+c)+a)^n\tan(dx+c)^5\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^5,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^n\tan^5(c+dx)\,dx=\int \tan(c+d\,x)^5\left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^5*(a + b/cos(c + d*x))^n, x)
```

3.354 $\int (a+b\sec(c+dx))^n \tan^3(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int (a+b\sec(c+dx))^n \tan^3(c+dx)\,dx = -\frac{a(a+b\sec(c+dx))^{1+n}}{b^2d(1+n)} + \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} + \frac{(a+b\sec(c+dx))^{2+n}}{b^2d(2+n)}$$

```
[Out] -a*(a+b*sec(d*x+c))^(1+n)/b^2/d/(1+n)+hypergeom([1, 1+n],[2+n],1+b*sec(d*x+
c)/a)*(a+b*sec(d*x+c))^(1+n)/a/d/(1+n)+(a+b*sec(d*x+c))^(2+n)/b^2/d/(2+n)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3970, 966, 81, 67}

$$\int (a+b\sec(c+dx))^n \tan^3(c+dx)\,dx = -\frac{a(a+b\sec(c+dx))^{n+1}}{b^2d(n+1)} + \frac{(a+b\sec(c+dx))^{n+2}}{b^2d(n+2)} + \frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] -((a*(a + b*Sec[c + d*x])^(1 + n))/(b^2*d*(1 + n))) + (Hypergeometric2F1[1,
 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a*d*(
1 + n)) + (a + b*Sec[c + d*x])^(2 + n)/(b^2*d*(2 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(a+x)^n(b^2-x^2)}{x}\,dx, x, b\sec(c+dx)\right)}{b^2d} \\
&= \frac{(a+b\sec(c+dx))^{2+n}}{b^2d(2+n)} - \frac{\text{Subst}\left(\int \frac{(a+x)^n(b^2(2+n)+a(2+n)x)}{x}\,dx, x, b\sec(c+dx)\right)}{b^2d(2+n)} \\
&= -\frac{a(a+b\sec(c+dx))^{1+n}}{b^2d(1+n)} + \frac{(a+b\sec(c+dx))^{2+n}}{b^2d(2+n)} - \frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx, x, b\sec(c+dx)\right)}{d}
\end{aligned}
$$

$$= -\frac{a(a+b\sec(c+dx))^{1+n}}{b^2d(1+n)} + \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} + \frac{(a+b\sec(c+dx))^{2+n}}{b^2d(2+n)}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int (a+b\sec(c+dx))^n\tan^3(c+dx)\,dx = \frac{(a+b\sec(c+dx))^{1+n}\left(b^2(2+n)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)+a(-a+b(1+n)\,\mathrm{se}\right.}{ab^2d(1+n)(2+n)}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^3,x]
```

```
[Out] ((a + b*Sec[c + d*x])^(1 + n)*(b^2*(2 + n)*Hypergeometric2F1[1, 1 + n, 2 +
n, 1 + (b*Sec[c + d*x])/a] + a*(-a + b*(1 + n)*Sec[c + d*x])))/(a*b^2*d*(1
+ n)*(2 + n))
```

Maple [F]

$$\int (a+b\sec(dx+c))^n\tan(dx+c)^3\,dx$$

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x)
```

Fricas [F]

$$\int (a+b\sec(c+dx))^n\tan^3(c+dx)\,dx = \int (b\sec(dx+c)+a)^n\tan(dx+c)^3\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^n\tan^3(c+dx)\,dx = \int (a+b\sec{(c+dx)})^n\tan^3{(c+dx)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**3,x)
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**3, x)
```

Maxima [F]

$$\int (a+b\sec(c+dx))^n\tan^3(c+dx)\,dx = \int (b\sec{(dx+c)}+a)^n\tan{(dx+c)}^3\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Giac [F]

$$\int (a+b\sec(c+dx))^n\tan^3(c+dx)\,dx = \int (b\sec{(dx+c)}+a)^n\tan{(dx+c)}^3\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^3,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^n\tan^3(c+dx)\,dx = \int \tan(c+d\,x)^3\left(a+\frac{b}{\cos{(c+d\,x)}}\right)^n\,dx$$

```
[In] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^3*(a + b/cos(c + d*x))^n, x)
```

3.355 $\int (a + b \sec(c + dx))^n \tan(c + dx)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int (a+b\sec(c+dx))^n \tan(c+dx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}$$

```
[Out] -hypergeom([1, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a/d/(1+n
)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3970, 67}

$$\int (a+b\sec(c+dx))^n \tan(c+dx)\,dx = -\frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a*d*(1 + n)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
|| GtQ[-d/(b*c), 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx, x, b\sec(c+dx)\right)}{d}$$
$$= -\frac{\text{Hypergeometric2F1}\left(1, 1+n, 2+n, 1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n \tan(c+dx)\,dx$$
$$= -\frac{\text{Hypergeometric2F1}\left(1, 1+n, 2+n, 1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x],x]
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a*d*(1 + n)))
```

Maple [F]

$$\int (a+b\sec(dx+c))^n \tan(dx+c)\,dx$$

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c),x)
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c),x)
```

Fricas [F]

$$\int (a+b\sec(c+dx))^n\tan(c+dx)\,dx=\int (b\sec(dx+c)+a)^n\tan(dx+c)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Sympy [F]

$$\int (a+b\sec(c+dx))^n\tan(c+dx)\,dx=\int (a+b\sec(c+dx))^n\tan(c+dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c),x)
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x), x)
```

Maxima [F]

$$\int (a+b\sec(c+dx))^n\tan(c+dx)\,dx=\int (b\sec(dx+c)+a)^n\tan(dx+c)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Giac [F]

$$\int (a+b\sec(c+dx))^n\tan(c+dx)\,dx=\int (b\sec(dx+c)+a)^n\tan(dx+c)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^n\tan(c+dx)\,dx=\int \tan(c+d\,x)\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

```
[In] int(tan(c + d*x)*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)*(a + b/cos(c + d*x))^n, x)
```

3.356 $\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 162

$$\begin{aligned}\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx &= -\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)}\\ &\quad-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{2(a+b)d(1+n)}\\ &\quad+\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}\end{aligned}$$

```
[Out] -1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n
)/(a-b)/d/(1+n)-1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*s
ec(d*x+c))^(1+n)/(a+b)/d/(1+n)+hypergeom([1, 1+n],[2+n],1+b*sec(d*x+c)/a)*(
a+b*sec(d*x+c))^(1+n)/a/d/(1+n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used

$= \{3970, 975, 67, 845, 70\}$

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx$$
$$= -\frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)} - \frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)} + \frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$

```
[In] Int[Cot[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a +
 b*Sec[c + d*x])^(1 + n))/((a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n
, 2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a
+ b)*d*(1 + n)) + (Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/
a]*(a + b*Sec[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b^2\text{Subst}\left(\int \frac{(a+x)^n}{x(b^2-x^2)}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= -\frac{b^2\text{Subst}\left(\int \left(\frac{(a+x)^n}{b^2x}-\frac{x(a+x)^n}{b^2(-b^2+x^2)}\right)dx, x, b\sec(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx, x, b\sec(c+dx)\right)}{d}+\frac{\text{Subst}\left(\int \frac{x(a+x)^n}{-b^2+x^2}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}\\
&\quad+\frac{\text{Subst}\left(\int \left(-\frac{(a+x)^n}{2(b-x)}+\frac{(a+x)^n}{2(b+x)}\right)dx, x, b\sec(c+dx)\right)}{d}\\
&= \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}\\
&\quad-\frac{\text{Subst}\left(\int \frac{(a+x)^n}{b-x}\,dx, x, b\sec(c+dx)\right)}{2d}+\frac{\text{Subst}\left(\int \frac{(a+x)^n}{b+x}\,dx, x, b\sec(c+dx)\right)}{2d}\\
&= -\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)}\\
&\quad-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{2(a+b)d(1+n)}\\
&\quad+\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{\left(a(a+b)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)+(a-b)\left(a\,\text{Hypergeometric2F1}\left(1,1+n\right.\right.\right.}{2a(a-b)}$$

```
[In] Integrate[Cot[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] -1/2*((a*(a + b)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a
 - b)] + (a - b)*(a*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])
/(a + b)] - 2*(a + b)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x
])/a]))*(a + b*Sec[c + d*x])^(1 + n))/(a*(a - b)*(a + b)*d*(1 + n))
```

Maple [F]

$$\int \cot(dx+c)\left(a+b\sec(dx+c)\right)^n dx$$

```
[In] int(cot(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx = \int \left(b\sec(dx+c)+a\right)^n \cot(dx+c)\,dx$$

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Sympy [F]

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx = \int \left(a+b\sec(c+dx)\right)^n \cot(c+dx)\,dx$$

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*cot(c + d*x), x)
```

Maxima [F]

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Giac [F]

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n\cot(dx+c)\,dx$$

```
[In] integrate(cot(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot(c+d\,x)\left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

```
[In] int(cot(c + d*x)*(a + b/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)*(a + b/cos(c + d*x))^n, x)
```

3.357 $\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 279

$$
\begin{aligned}
\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx &= \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)} \\
&+ \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{2(a+b)d(1+n)} \\
&- \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} \\
&- \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} \\
&+ \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{4(a+b)^2d(1+n)}
\end{aligned}
$$

```
[Out] 1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n)
/(a-b)/d/(1+n)+1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*se
c(d*x+c))^(1+n)/(a+b)/d/(1+n)-hypergeom([1, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a
+b*sec(d*x+c))^(1+n)/a/d/(1+n)-1/4*b*hypergeom([2, 1+n],[2+n],(a+b*sec(d*x+
c))/(a-b))*(a+b*sec(d*x+c))^(1+n)/(a-b)^2/d/(1+n)+1/4*b*hypergeom([2, 1+n],
[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*sec(d*x+c))^(1+n)/(a+b)^2/d/(1+n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3970, 975, 70, 67, 845}

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx$$
$$= \frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}$$
$$+\frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)}$$
$$-\frac{(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{ad(n+1)}$$
$$-\frac{b(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{4d(n+1)(a-b)^2}$$
$$+\frac{b(a+b\sec(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{a+b\sec(c+dx)}{a+b}\right)}{4d(n+1)(a+b)^2}$$

```
[In] Int[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(2*(a - b)*d*(1 + n)) + (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a + b
)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*
(a + b*Sec[c + d*x])^(1 + n))/(a*d*(1 + n)) - (b*Hypergeometric2F1[2, 1 + n
, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a
- b)^2*d*(1 + n)) + (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*
x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a + b)^2*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b^4\text{Subst}\left(\int \frac{(a+x)^n}{x(b^2-x^2)^2}\,dx, x, b\sec(c+dx)\right)}{d}\\
&= \frac{b^4\text{Subst}\left(\int \left(\frac{(a+x)^n}{4b^3(b-x)^2}+\frac{(a+x)^n}{b^4x}-\frac{(a+x)^n}{4b^3(b+x)^2}-\frac{x(a+x)^n}{b^4(-b^2+x^2)}\right)dx, x, b\sec(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+x)^n}{x}\,dx, x, b\sec(c+dx)\right)}{d}-\frac{\text{Subst}\left(\int \frac{x(a+x)^n}{-b^2+x^2}\,dx, x, b\sec(c+dx)\right)}{d}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{(a+x)^n}{(b-x)^2}\,dx, x, b\sec(c+dx)\right)}{4d}-\frac{b\text{Subst}\left(\int \frac{(a+x)^n}{(b+x)^2}\,dx, x, b\sec(c+dx)\right)}{4d}
\end{aligned}
$$

$$= -\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} - \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} + \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{4(a+b)^2d(1+n)} - \frac{\text{Subst}\left(\int\left(-\frac{(a+x)^n}{2(b-x)}+\frac{(a+x)^n}{2(b+x)}\right)dx, x, b\sec(c+dx)\right)}{d}$$

$$= -\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} - \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} + \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{4(a+b)^2d(1+n)} + \frac{\text{Subst}\left(\int\frac{(a+x)^n}{b-x}dx, x, b\sec(c+dx)\right)}{2d} - \frac{\text{Subst}\left(\int\frac{(a+x)^n}{b+x}dx, x, b\sec(c+dx)\right)}{2d}$$

$$= \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)} + \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{2(a+b)d(1+n)} - \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{ad(1+n)} - \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} + \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx))^{1+n}}{4(a+b)^2d(1+n)}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.67

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx$$
$$=\frac{\left(\frac{2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a-b}\right)}{a-b}+\frac{2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\sec(c+dx)}{a+b}\right)}{a+b}-\frac{4\,\text{Hypergeometric2F1}\left(1,1+n,2+\right.}{a}\right.}{}$$

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (((2*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)])/(a -
 b) + (2*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a + b)])/
(a + b) - (4*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a])/a
- (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)])/(a -
 b)^2 + (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a + b)]
)/(a + b)^2)*(a + b*Sec[c + d*x])^(1 + n))/(4*d*(1 + n))
```

Maple [F]

$$\int \cot\left(dx+c\right)^{3}\left(a+b\sec\left(dx+c\right)\right)^{n}dx$$

```
[In] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx=\int \left(b\sec\left(dx+c\right)+a\right)^{n}\cot\left(dx+c\right)^{3}\,dx$$

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Sympy [F]

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx = \int (a+b\sec(c+dx))^n \cot^3(c+dx)\,dx$$

```
[In] integrate(cot(d*x+c)**3*(a+b*sec(d*x+c))**n,x)
[Out] Integral((a + b*sec(c + d*x))**n*cot(c + d*x)**3, x)
```

Maxima [F]

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \cot(dx+c)^3\,dx$$

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Giac [F]

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n \cot(dx+c)^3\,dx$$

```
[In] integrate(cot(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot(c+d\,x)^3 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

```
[In] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^3*(a + b/cos(c + d*x))^n, x)
```

3.358 $\int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx = \text{Int}\big((a+b\sec(c+dx))^n \tan^4(c+dx),x\big)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx = \int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4, x]
```

Rubi steps

$$\text{integral} = \int (a+b\sec(c+dx))^n \tan^4(c+dx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 13.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\tan^4(c+dx)\,dx=\int (a+b\sec(c+dx))^n\tan^4(c+dx)\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^4, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (a+b\sec(dx+c))^n\tan(dx+c)^4\,dx$$

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\tan^4(c+dx)\,dx=\int (b\sec(dx+c)+a)^n\tan(dx+c)^4\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Sympy [N/A]

Not integrable

Time = 29.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int (a+b\sec(c+dx))^n\tan^4(c+dx)\,dx=\int (a+b\sec(c+dx))^n\tan^4(c+dx)\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**4,x)
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**4, x)
```

Maxima [N/A]

Not integrable

Time = 19.76 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\tan^4(c+dx)\,dx=\int (b\sec(dx+c)+a)^n\tan(dx+c)^4\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Giac [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\tan^4(c+dx)\,dx=\int (b\sec(dx+c)+a)^n\tan(dx+c)^4\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^4,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Mupad [N/A]

Not integrable

Time = 17.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int (a+b\sec(c+dx))^n\tan^4(c+dx)\,dx=\int \tan(c+d\,x)^4\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int(tan(c + d*x)^4*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^4*(a + b/cos(c + d*x))^n, x)
```

3.359 $\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$$
$$= \frac{\sqrt{2}(a+b)\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-}}{bd\sqrt{1+\sec(c+dx)}}$$
$$-\frac{\sqrt{2}a\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}\tan(c}{bd\sqrt{1+\sec(c+dx)}}$$
$$-\mathrm{Int}((a+b\sec(c+dx))^n,x)$$

```
[Out] (a+b)*AppellF1(1/2,-1-n,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*
(a+b*sec(d*x+c))^n*2^(1/2)*tan(d*x+c)/b/d/(((a+b*sec(d*x+c))/(a+b))^n)/(1+s
ec(d*x+c))^(1/2)-a*AppellF1(1/2,-n,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*s
ec(d*x+c))*(a+b*sec(d*x+c))^n*2^(1/2)*tan(d*x+c)/b/d/(((a+b*sec(d*x+c))/(a+
b))^n)/(1+sec(d*x+c))^(1/2)-Unintegrable((a+b*sec(d*x+c))^n,x)
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx = \int (a+b\sec(c+dx))^n \tan^2(c+dx)\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sec[c + d*x])/2, (b*(
1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^n*Tan[c + d*x])/(b*d*Sqrt[
1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^n) - (Sqrt[2]*a*AppellF1[1
/2, 1/2, -n, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a
+ b*Sec[c + d*x])^n*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c
 + d*x])/(a + b))^n) - Defer[Int][(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$\text{integral} = \int (a+b\sec(c+dx))^n \left(-1+\sec^2(c+dx)\right) dx$$

$$= \frac{\int (-b-a\sec(c+dx))(a+b\sec(c+dx))^n\,dx}{b} + \frac{\int \sec(c+dx)(a+b\sec(c+dx))^{1+n}\,dx}{b}$$

$$= -\frac{a\int \sec(c+dx)(a+b\sec(c+dx))^n\,dx}{b} - \frac{\tan(c+dx)\text{Subst}\left(\int \frac{(a+bx)^{1+n}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \int (a+b\sec(c+dx))^n\,dx$$

$$= \frac{(a\tan(c+dx))\text{Subst}\left(\int \frac{(a+bx)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}} + \frac{\left((-a-b)(a+b\sec(c+dx))^n\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{-n}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{1+n}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \int (a+b\sec(c+dx))^n\,dx$$

$$= \frac{\sqrt{2}(a+b)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\,\text{s}}{}\right.}{bd\sqrt{1+\sec(c+dx)}} + \frac{\left(a(a+b\sec(c+dx))^n\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{-n}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^n}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \int (a+b\sec(c+dx))^n\,dx$$

$$= \frac{\sqrt{2}(a+b)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\,\text{s}}{}\right.}{bd\sqrt{1+\sec(c+dx)}} - \frac{\sqrt{2}a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right.}{bd\sqrt{1+\sec(c+dx)}} - \int (a+b\sec(c+dx))^n\,dx$$

Mathematica [N/A]

Not integrable

Time = 12.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\tan^2(c+dx)\,dx=\int (a+b\sec(c+dx))^n\tan^2(c+dx)\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.98 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{n}\tan\left(dx+c\right)^{2}dx$$

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x)
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\tan^2(c+dx)\,dx=\int \left(b\sec\left(dx+c\right)+a\right)^{n}\tan\left(dx+c\right)^{2}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 3.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int (a+b\sec(c+dx))^n\tan^2(c+dx)\,dx=\int \left(a+b\sec\left(c+dx\right)\right)^{n}\tan^{2}\left(c+dx\right)dx$$

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**2,x)
[Out] Integral((a + b*sec(c + d*x))**n*tan(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 5.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\tan^2(c+dx)\,dx=\int (b\sec{(dx+c)}+a)^n\tan{(dx+c)}^2\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sec(c+dx))^n\tan^2(c+dx)\,dx=\int (b\sec{(dx+c)}+a)^n\tan{(dx+c)}^2\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 15.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int (a+b\sec(c+dx))^n\tan^2(c+dx)\,dx=\int \tan(c+d\,x)^2\,\left(a+\frac{b}{\cos{(c+d\,x)}}\right)^n\,dx$$

```
[In] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^2*(a + b/cos(c + d*x))^n, x)
```

3.360 $\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \operatorname{Int}\left(\cot^2(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

```
[In] Int[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
[Out] Defer[Int][Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$\text{integral} = \int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

Mathematica [N/A]

Not integrable

Time = 8.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx$$

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
[Out] Integrate[Cot[c + d*x]^2*(a + b*Sec[c + d*x])^n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cot\left(dx+c\right)^{2}\left(a+b\sec\left(dx+c\right)\right)^{n}dx$$

```
[In] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
[Out] int(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int \left(b\sec\left(dx+c\right)+a\right)^{n}\cot\left(dx+c\right)^{2}\,dx$$

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 19.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^{n}\cot^{2}\left(c+dx\right)dx$$

```
[In] integrate(cot(d*x+c)**2*(a+b*sec(d*x+c))**n,x)
[Out] Integral((a + b*sec(c + d*x))**n*cot(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 5.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n\cot(dx+c)^2\,dx$$

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int (b\sec(dx+c)+a)^n\cot(dx+c)^2\,dx$$

```
[In] integrate(cot(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cot^2(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot(c+d\,x)^2\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^2*(a + b/cos(c + d*x))^n, x)
```

3.361 $\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx = \mathrm{Int}\left(\cot^4(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

```
[In] Int[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] Defer[Int][Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$\text{integral} = \int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Mathematica [N/A]

Not integrable

Time = 13.64 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx=\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
[Out] Integrate[Cot[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cot\left(dx+c\right)^{4}\left(a+b\sec\left(dx+c\right)\right)^{n}dx$$

```
[In] int(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
[Out] int(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx=\int \left(b\sec\left(dx+c\right)+a\right)^{n}\cot\left(dx+c\right)^{4}\,dx$$

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx=\text{Timed out}$$

```
[In] integrate(cot(d*x+c)**4*(a+b*sec(d*x+c))**n,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 18.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx=\int (b\sec{(dx+c)}+a)^n\cot{(dx+c)}^4\,dx$$

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx=\int (b\sec{(dx+c)}+a)^n\cot{(dx+c)}^4\,dx$$

```
[In] integrate(cot(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*cot(d*x + c)^4, x)
```

Mupad [N/A]

Not integrable

Time = 28.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cot^4(c+dx)(a+b\sec(c+dx))^n\,dx=\int \cot(c+d\,x)^4\,\left(a+\frac{b}{\cos{(c+d\,x)}}\right)^n\,dx$$

```
[In] int(cot(c + d*x)^4*(a + b/cos(c + d*x))^n,x)
[Out] int(cot(c + d*x)^4*(a + b/cos(c + d*x))^n, x)
```

3.362 $\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx = \mathrm{Int}\Big((a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx),x\Big)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx = \int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2), x]
```

Rubi steps

$$\text{integral} = \int (a+b\sec(c+dx))^n \tan^{\frac{3}{2}}(c+dx)\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)\,dx=\int (a+b\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2),x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Tan[c + d*x]^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(a+b\sec\left(dx+c\right)\right)^{n}\tan\left(dx+c\right)^{\frac{3}{2}}dx$$

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)\,dx=\int \left(b\sec\left(dx+c\right)+a\right)^{n}\tan\left(dx+c\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)\,dx=\int (b\sec(dx+c)+a)^n\tan(dx+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)\,dx=\int (b\sec(dx+c)+a)^n\tan(dx+c)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*tan(d*x + c)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 19.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n\tan^{\frac{3}{2}}(c+dx)\,dx=\int \tan(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int(tan(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n, x)
```

3.363 $\int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\, dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\, dx = \text{Int}\Big((a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}, x\Big)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\, dx = \int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\, dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]], x]
```

Rubi steps

$$\text{integral} = \int (a+b\sec(c+dx))^n \sqrt{\tan(c+dx)}\, dx$$

Mathematica [N/A]

Not integrable

Time = 10.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx=\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]],x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Tan[c + d*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (a+b\sec(dx+c))^n\sqrt{\tan(dx+c)}dx$$

```
[In] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
[Out] int((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx=\int (b\sec(dx+c)+a)^n\sqrt{\tan(dx+c)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 6.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx=\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n*tan(d*x+c)**(1/2),x)
[Out] Integral((a + b*sec(c + d*x))**n*sqrt(tan(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx=\int (b\sec(dx+c)+a)^n\sqrt{\tan(dx+c)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx=\int (b\sec(dx+c)+a)^n\sqrt{\tan(dx+c)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n*tan(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(tan(d*x + c)), x)
```

Mupad [N/A]

Not integrable

Time = 14.74 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sec(c+dx))^n\sqrt{\tan(c+dx)}\,dx=\int \sqrt{\tan(c+d\,x)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

```
[In] int(tan(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n,x)
[Out] int(tan(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n, x)
```

3.364 $\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]], x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 23.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx=\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]],x]
[Out] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Tan[c + d*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sec(dx+c))^n}{\sqrt{\tan(dx+c)}}dx$$

```
[In] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
[Out] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx=\int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Sympy [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx=\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n/tan(d*x+c)**(1/2),x)
[Out] Integral((a + b*sec(c + d*x))**n/sqrt(tan(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx=\int \frac{(b\sec{(dx+c)}+a)^n}{\sqrt{\tan{(dx+c)}}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx=\int \frac{(b\sec{(dx+c)}+a)^n}{\sqrt{\tan{(dx+c)}}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(tan(d*x + c)), x)
```

Mupad [N/A]

Not integrable

Time = 18.95 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sqrt{\tan{(c+d\,x)}}}\,dx$$

```
[In] int((a + b/cos(c + d*x))^n/tan(c + d*x)^(1/2),x)
[Out] int((a + b/cos(c + d*x))^n/tan(c + d*x)^(1/2), x)
```

3.365 $\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

```
[In] Int[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2),x]
[Out] Integrate[(a + b*Sec[c + d*x])^n/Tan[c + d*x]^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sec(dx+c))^n}{\tan(dx+c)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
[Out] int((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Sympy [N/A]

Not integrable

Time = 52.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))**n/tan(d*x+c)**(3/2),x)
[Out] Integral((a + b*sec(c + d*x))**n/tan(c + d*x)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sec(d*x+c))^n/tan(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n/tan(d*x + c)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 20.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\tan(c+d\,x)^{3/2}}\,dx$$

```
[In] int((a + b/cos(c + d*x))^n/tan(c + d*x)^(3/2),x)
[Out] int((a + b/cos(c + d*x))^n/tan(c + d*x)^(3/2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                         convert(ExpnType_result,string)," vs. order ",
                         convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```